A method for extracting

$$\cos(\alpha)$$

trough

$$B^{\pm} \to \pi^{\pm} e^{+} e^{-}$$

work done with

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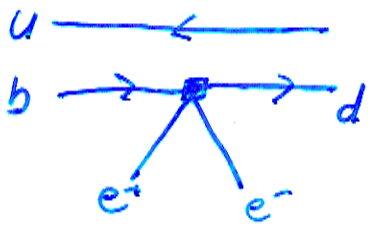
hep-ph/9910245

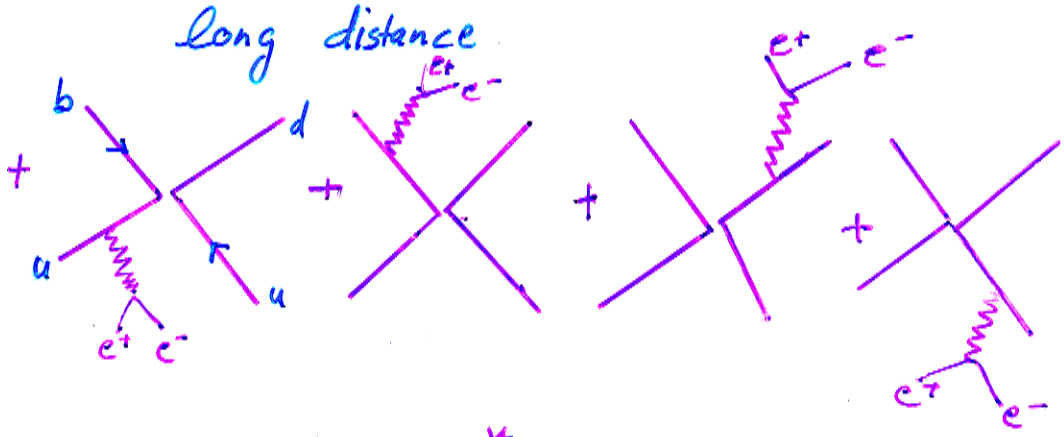
$$B^{\pm} \longrightarrow \pi^{\pm} e^+ e^-$$

short distance



$V_{td} V_{tb}^*$

long distance



$V_{ud} V_{ub}^*$

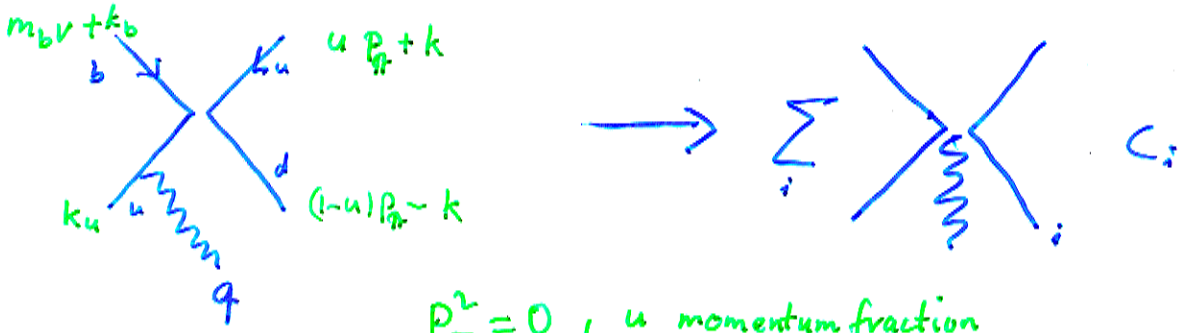
$$\text{Ampl} = V_{td} V_{tb}^* \left(A_{short} + \frac{V_{ud} V_{ub}^*}{V_{td} V_{tb}^*} A_{long} \right)$$

Comments:

$$\frac{|V_{ub}^*| A_{long}}{|V_{td}| A_{short}} = \frac{|V_{ub}| f_B f_\pi}{M_B^2} \cdot \frac{16\pi^2}{|V_{td}|} \propto (7-8)\,\%$$

- Ampl depends on $\cos(\alpha)$ (and not on $\sin 2\alpha$)
- A_{long} interferes with A_{short}

Long distance contribution



$P_\pi^2 = 0$, u momentum fraction

Relevant weak Hamiltonian

$$\mathcal{H}_{eff} = \frac{4 G_F}{\sqrt{2}} (V_{ub} V_{ud}^*) \left[C\left(\frac{\mu}{m_W}\right) \sigma + C'\left(\frac{\mu}{m_W}\right) \sigma' \right]$$

with $\sigma = (\bar{u} \gamma^\nu P_- b)(\bar{d} \gamma_\nu P_- u)$ and $\sigma' = (\bar{u} \gamma^\nu P_- T^A b)(\bar{d} \gamma_\nu P_- T^A u)$

Calculate the hadronic current:

$$h^\mu = \langle \pi | \int d^4x\, e^{iqx}\, T\left(J^\mu_{EM}(x)\, \mathcal{H}_{eff}(0)\right) | B \rangle$$

Factorisation : Dugan/Grinstein Phys Lett B255 (583) 1991
Bjorken Nucl Phys Proc Suppl 11 (325) (1989)

LEET applies, if E_π is large.

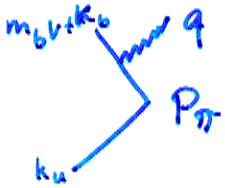
Corrections are of order $\frac{\Lambda_{QCD}}{E_\pi}$

$\Rightarrow$ color octet does not contribute

$$h^\mu = \frac{4 G_F}{\sqrt{2}} (V_{ub} V_{ud}^*)\, C\left(\frac{\mu}{m_W}\right) \int d^4x\, e^{iqx} \Big[\langle \pi | T(J^\mu_{EM}(x)\, j_\lambda(0)) | 0 \rangle \tfrac{1}{2} f_B P_B^\lambda$$
$$+ \langle \pi | j_\lambda(0) | 0 \rangle \langle 0 | T(J^\mu_{EM}(x)\, J^\lambda(0) | B \rangle \Big]$$

Second piece:

(1)



$\longrightarrow \sum_i$

C_i

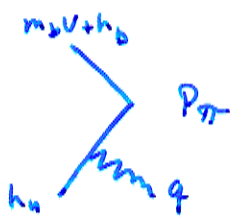
$$\text{Propagator} = \frac{1}{\not{p}_\pi + \not{k}_u - m_b}$$

$$\overset{\text{Taylor}}{\simeq} \frac{\not{p} + m_b}{m_b^2} + \frac{\not{k}_u}{m_b^2} - \frac{2(k\cdot p)(\not{p} + m_b)}{m_b^4} + \text{higher order}$$

– leading corrections are suppressed by

$$\frac{k}{m_b} \sim \frac{\Lambda_{QCD}}{m_b}$$

(2)



$$\text{Propagator} = \frac{-1}{(-m_b \not{v} + \not{p} + \not{k}_u)}$$

$$\overset{\text{Taylor}}{\simeq} \frac{\not{q}}{q} - \frac{\not{k}_u}{q^2} + \frac{2(m_b v\cdot k_u - p\, k_u)\not{q}}{q^4}$$

– leading corrections are suppressed by

$$\frac{k\, m_b}{q^2} \sim \frac{\Lambda_{QCD}\, m_b}{q^2}$$

– q^2 has to scale like m_b^2 !

Isospin - Ward Identity

$$\text{1-term} = \int d^4x \, e^{iqx} \int d^4y \, \frac{1}{2} f_B \, P_B^\lambda e^{-iP_B y} \langle \pi | T(\hat{J}_{Em}(x) j_\lambda(y)) | 0 \rangle$$

with $\quad P_B^\lambda = i \frac{\partial}{\partial y_\lambda}$

and $\quad i \frac{\partial}{\partial y^\lambda} j_\lambda(y) = (m_d - m_u)(\bar{d}u) - (m_d + m_u)(\bar{d}\gamma_5 u) \simeq 0$

$$\text{1. term} = \delta^{(4)}(P_B - q - P_\pi) \int d^3y \, \frac{f_B}{2} e^{-i\vec{P}_B \vec{y}} \langle \pi | [\hat{J}_{Em}(0), j_0(y)] | 0 \rangle$$

$$= \delta^{(4)}(P_B - q - P_\pi) \left(- f_B f_\pi \, P_B^m \right) e$$

Comments:

- no higher twist operators have to be calculated
- the result is independent of the wavefunction of the Pion (Does not depend on momentum fraction u)

Result:

$$h^m = \frac{4 G_F}{\sqrt{2}} (V_{ub} V_{ud}^*) C\left(\frac{m}{M_W}\right) e \left(-\frac{4}{3} f_B f_\pi \right) P_B^m$$

Leading corrections

With $k^\mu \to -iD^\mu$

$$\langle 0|\bar{u}\,(\gamma^\mu\gamma^5\,(-i\overleftarrow{D}^\nu))\,b\,|B\rangle = A\,v^\mu v^\nu + B\,g^{\mu\nu}$$

Apply $g_{\mu\nu}$ and equation of motion for u-quark:

$$0 = A + 4B$$

Apply v_ν and integration by parts:

$$\bar{\Lambda}\,\langle 0|\bar{u}\,(\gamma^\mu\gamma^5)\,b\,|B\rangle = \bar{\Lambda}\,i\,f_B\,m_B\,v^\mu = (A+B)\,v^\mu$$

$$\Rightarrow \langle 0|\bar{u}\,(\gamma^\mu\gamma^5\,\overleftarrow{D}^\nu)\,b\,|B\rangle = i\left(\tfrac{4}{3}\right)\bar{\Lambda}\,f_B\,m_B\left(v_\mu v_\nu - \tfrac{1}{4}\,g^{\mu\nu}\right)$$

Hadronic matrix element:

$$h^\mu = -\tfrac{4}{3}\,e\left(\tfrac{4G_F}{\sqrt{2}}\right)V_{ub}V_{ud}^*\,c(\tfrac{\epsilon}{m_W})\;f_\pi f_B\,p_B^\mu\left(1+\tfrac{2}{3}\bar{\Lambda}\,\frac{m_B}{q^2}\right)$$

Remaining uncertainty:

$$\left(\frac{\Lambda_{QCD}\,m_b}{q^2}\right)^2 \sim 20\% \quad \text{for } q^2 \sim 3.5\,\text{GeV}$$

Short distance contribution + Results

The effective Hamiltonian for the short distance part:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{j=7,9,10} C_j(\mu) \mathcal{O}_j(\mu),$$

where

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{d}\sigma^{\mu\nu} P_+ b) F_{\mu\nu}, \qquad C_7(m_b) = 0.33$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{d}\gamma^\mu P_- b) \bar{e}\gamma_\mu e, \qquad C_9(m_b) = -4.3$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{d}\gamma^\mu P_- b) \bar{e}\gamma_\mu \gamma_5 e. \qquad C_{10}(m_b) = 5.3$$

The long distance contributions from c and u-quark penguins can be put into c_9:

$$\tilde{C}_9 = C_9 + (c + \tfrac{4}{3}c') g(m_c/m_b, \hat{q}^2) + \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} (c + \tfrac{4}{3}c')[g(m_c/m_b, \hat{q}^2) - g(0, \hat{q}^2)]$$

Matrix elements of the short distance piece written in terms of formfactors:

$$\langle \pi(p') | \bar{d}\gamma^\mu b | B(p) \rangle = (p + p')^\mu f_+ + (p - p')^\mu f_-$$
$$\langle \pi(p') | \bar{d}\sigma^{\mu\nu} b | B(p) \rangle = 2ih(p^\nu p'^\mu - p^\mu p'^\nu)$$

Combine short and long distance pieces.
The rate for $B \to \pi e^+ e^-$ is:

$$\frac{d\Gamma}{dq^2} = |V_{tb}V_{td}^*|^2 \frac{G_F^2 \alpha^2 m_B^3}{3 \times 2^9 \pi^5} \frac{(m_B^2 - q^2)^3}{m_B^6}$$
$$\left[|C_{10} f_+|^2 + \left| \tilde{C}_9 f_+ + 2m_b C_7 h - \frac{16\pi^2}{3} \left(\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right) \frac{c(m_b) f_\pi f_B}{q^2} (1 + \frac{2\bar{\Lambda} m_b}{3q^2}) \right|^2 \right]$$

$$\sim \left| \frac{V_{ub}}{V_{td}} \right| \cos(\alpha)$$

- f_+, h should be determined by exp
- V_{ub}, V_{td} should be determined by exp

I. EXAMPLE

CKM elements:

$$V_{ub} = 0.04 \text{ and } V_{td} = 0.08$$

Model for formfactors (Becirevic and Kaidalov: hep-ph/9904490):

$$f_+^{(\mathrm{BK})} = \frac{N(1-\beta)}{(1-\tilde{q}^2)(1-\beta\tilde{q}^2)}, \quad f_0^{(\mathrm{BK})} = \frac{N(1-\beta)}{1-\gamma\tilde{q}^2},$$

where $f_0 \equiv \frac{q^2}{m_B^2 - m_\pi^2} f_- + f_+$,
$\tilde{q}^2 = q^2/M_{B^*}^2$
and the parameters are
$\beta = 0.54$,
$\gamma = 0.8$,
$N = 0.6$,
$M_{B^*} = 5.325$ GeV.
The value of γ is fixed by the Callan-Treiman relation
$f_0(m_B^2) = f_B/f_\pi$.
Using heavy HQ symmetry:

$$h(q^2) = \frac{(f_+(q^2) - f_-(q^2))}{2m_b} + O(1/m_b).$$

— consistent with lattice calculation at high q^2
— correct scaling with heavy mass m_b

Differential Branching Ratio

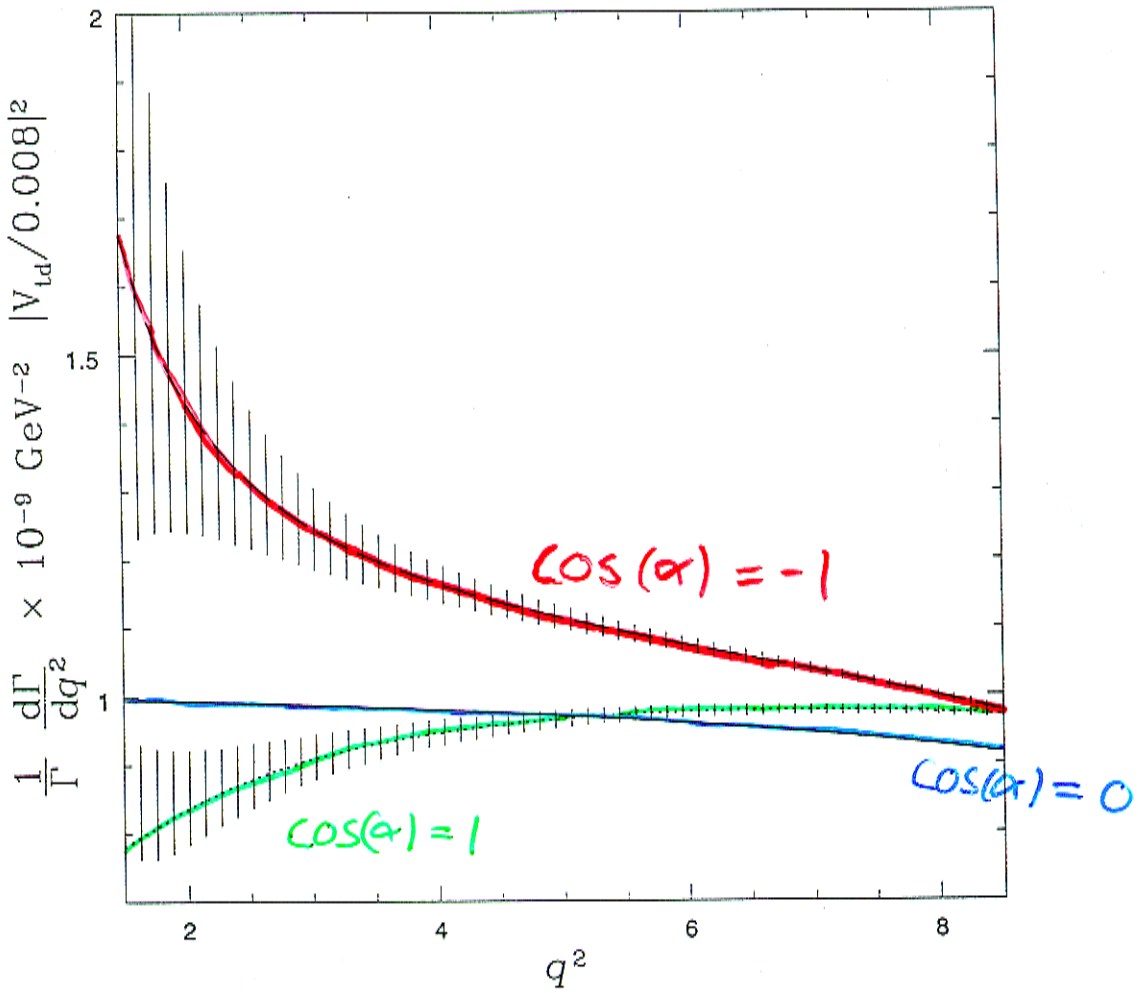


FIG. 1. Differential branching fraction, $\tau_B\, d\Gamma/dq^2$ for $B \to \pi e^+ e^-$. The solid (blue), dashed (red) and dotted (green) lines correspond to $\cos\alpha = 0, -1, 1$, respectively. The shaded region shows the uncertainty in our calculation.

Questions and Conclusions

To make our analysis useful, we need:

- $|V_{ub}|$, $|V_{td}|$

and the formfactors

- $f_+(q^2)$, $h(q^2)$

$\Rightarrow$ lot's of $B^\pm$ $\Rightarrow$ Production Rates at hadron machines big enough ?

$\Rightarrow$ combine results for B^+ and B^- to $\mu^\pm$ and $e^\pm$

Theoretical side:

- clean predictions are possible !

$\Rightarrow$ next higher corrections might be possible to estimate

$\Rightarrow$ quantitative analysis of influence of resonance