

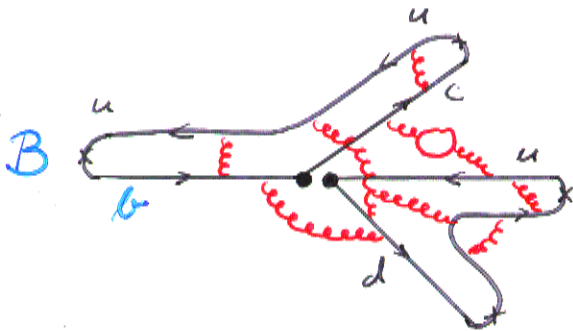
# Nonleptonic Decays, Lifetimes and Mixing

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2. Inclusive Decays  
HQE ; lifetimes , other applications
3.  $B-\bar{B}$  Mixing
4. Exclusive Decays
5. Summary

# Introduction

## nonleptonic B decays

- $\sim 75\%$  BR
- crucial for  $\tau_B$
- large and important field for CPV studies  
exclusive  $B \rightarrow \psi K_S, \pi\pi, \pi K, \dots$  ; inclusive channels
- theoretical challenge: all-hadronic final state



very difficult in general  
boundstate dynamics, nonpert. QCD

- opportunity: large scale  $m_b \gg \Lambda_{\text{QCD}}$   
→ systematic extraction of short distance physics

short distance  $\leftrightarrow$  long distance factorization  
↑ calculable ↑ parametrize;  $\leftrightarrow$  nonpert. input

tools: OPE, HQE, HQET, ...

→ important simplifications

# Inclusive Decays

• optical theorem  $\Gamma_H = \frac{1}{2M_H} \langle H | \mathcal{T} | H \rangle \equiv \langle \mathcal{T} \rangle$

$\mathcal{T} = \text{Im} i \int d^4x T \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0)$

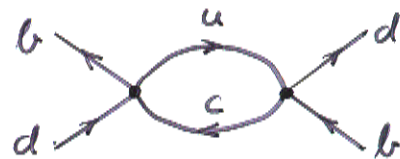
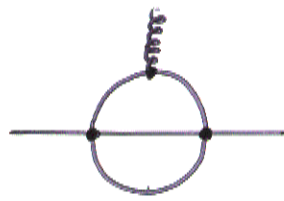
$\Gamma_H \sim \langle H | \mathcal{H}_{\text{eff}} | X \rangle \langle X | \mathcal{H}_{\text{eff}} | H \rangle$

•  $\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{CKM} C_i(\frac{M_W}{\mu}, \alpha_s) Q_i$

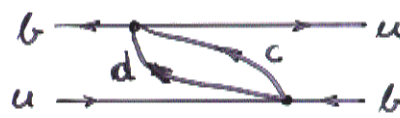


NLO Buras et al., Ciuchini et al.

•  $\mathcal{T} = \Gamma_b \bar{b}b + \frac{z_G}{m_b^2} \bar{b} \sigma G b + \sum \frac{z_i}{m_b^3} \bar{b} \Gamma_i q \bar{q} \Gamma_i b + \dots$



WA  
(B<sub>d</sub>)



PI  
(B<sub>u</sub>)

OPE  $1/m_b$

Bigi, Blok, Shifman, Uraltsev, Vainshtein, Voloshin  
Chay, Georgi, Grinstein

-  $\langle \bar{b}b \rangle = 1 + \frac{1}{2m_b^2} \langle \bar{h} (iD)^2 h \rangle + \frac{1}{4m_b^2} \langle \bar{h} \sigma G h \rangle$

HQET

$\rightarrow \frac{3}{2}(M_{B^*}^2 - M_B^2) ; B$   
 $0 \quad \Lambda_b$

- asymptotically  $\Gamma_H = \Gamma_b \leftarrow \text{NLO } (\mathcal{O}(\alpha_s)) \text{ Bagan et al.}$

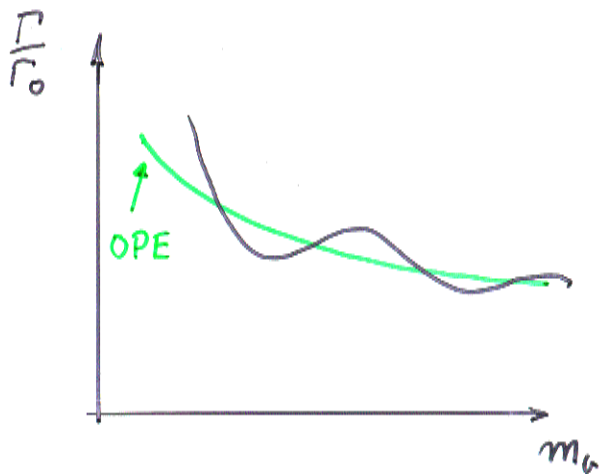
- no  $1/m_b$  correction

-  $1/m_b^3$  spectator effects:  $\times 16\pi^2$  (phase space)

# Local duality

$$\Gamma/\Gamma_0 \stackrel{\text{OPE}}{=} 1 + \sum_{n=2}^{\infty} Z_n \left(\frac{\Lambda}{m_b}\right)^n$$

missed by OPE:  $\exp\left(-\left(\frac{m_b}{\Lambda}\right)^k\right) \sin\left(\frac{m_b}{\Lambda}\right)^k$



•  $\Gamma_B$  in 't Hooft model [QCD<sub>2</sub> (N<sub>c</sub> → ∞)]

duality violation strongly suppressed

• resonance model, instanton model

Grinstein, Lebed

Bigi, Shifman, Uraltsev, Vainshtein

Bigi, Uraltsev

Shifman et al.

## Lifetimes

	exp.	th.
$\tau(B^+)/\tau(B_s^0)$	$1.07 \pm 0.02$	$\sim 1 - 1.1$
$\tau(B_s)/\tau(B_d)$	$0.94 \pm 0.04$	$1 \pm (<10\%)$
$\tau(\Lambda_b)/\tau(B_d)$	$0.79 \pm 0.05$	$0.9 - 1.0$

Bigi et al.

Neubert, Sachrajda

Beneke, G.B., Dunietz

• uncertainties from  $\langle \bar{b} \Gamma q \bar{q} \Gamma b \rangle$

$\Lambda_b$ : Neubert, Sachrajda; Colangelo, DeFazio; Rosner; Voloshin

Lattice (exploratory):  $\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} 0.91(1) & a m_\pi = 0.52(3) \\ 0.93(1) & 0.74(3) \end{cases} \quad a = (1.1 \text{ GeV})^{-1}$

Di Pierro, Sachrajda, Michael

$B_c$  HQE + NRQCD

$$\Gamma_{B_c} = \Gamma_b \left(1 - \frac{v_b^2}{2}\right) + \Gamma_c \left(1 - \frac{v_c^2}{2}\right) + \{PI, WA\} + \mathcal{O}(v^4) = (0.4 - 0.7 \text{ ps})^{-1}$$

Beneke, G.B.

CDF  $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03 \text{ ps}$

Bigi

$$\underline{(\Delta\Gamma/\Gamma)_{B_s}}$$

$$\sim 16\pi^2 (\Lambda_{\text{QCD}}/m_b)^3$$

• Lowest order estimates  $\Delta\Gamma/\Gamma \approx 10-20\%$

*Bigi et al.*

• Large  $\Delta\Gamma \rightarrow$  novel CPV studies w. untagged  $B_s$

*Dunietz  
Fleischer, Dunietz*

• with New Physics  $\Delta\Gamma \leq \Delta\Gamma_{\text{SM}}$

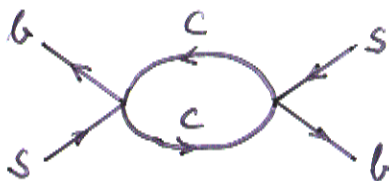
*Grossman*

• HQE test case

$\Delta\Gamma_{B_s}$ : local duality exact in large  $N_c$  + SV limit  $\Lambda_{\text{QCD}} \ll m_b - 2m_c \ll m_b$

*Shifman, Voloshin; Aleksan et al.*

HQE

$$T = \text{Im } i$$


$$= C \left[ \begin{array}{c} b \quad s \\ \swarrow \quad \searrow \\ V-A \quad V-A \\ \nearrow \quad \nwarrow \\ s \quad b \\ Q \end{array} \right] + C_S \left[ \begin{array}{c} b \quad s \\ \swarrow \quad \searrow \\ S-P \quad S-P \\ \nearrow \quad \nwarrow \\ s \quad b \\ Q_S \end{array} \right] + (\text{dim 7 operators})$$

$$\begin{aligned} \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} &= 16\pi^2 \frac{f_{B_s}^2 M_{B_s}}{m_b^3} \left[ c(\alpha_s, \frac{m_c}{m_b}) B + c_S(\alpha_s, \frac{m_c}{m_b}) B_S \right] + R_{1/m_b} \\ &= \left(\frac{f_{B_s}}{210 \text{ MeV}}\right)^2 \left[ 0.006 B(m_b) + 0.150 B_S(m_b) - 0.063 \right] \quad (\text{NDR}) \end{aligned}$$

•  $R_{1/m_b}$ : rel.  $1/m_b$  correction

*Beneke, G.B., Dunietz*

•  $C, C_S$  known at NLO

*Beneke, G.B., Greub, Lenz, Nierste*

• exploratory lattice study  $B_S(m_b) = 0.75 \pm ??$

*Gupta, Bhattacharya, Sharpe*

# HQE - other applications

- $B_{SL} = \frac{\Gamma(B \rightarrow X e \nu)}{\Gamma_{tot}}$

$$r_c = 1 + B(b \rightarrow c \bar{c} s) - B(\text{no charm})$$

Bigi et al.  
G.B., Dunieta, Yamamoto  
Neubert, Sachrajda  
Lenz, Nierste, Ostermaier  
Dunieta et al.  
Kagan

- $V_{ub}$  from  $\frac{\Gamma(b \rightarrow u \bar{c} s)}{\Gamma(b \rightarrow c l \nu)} = K \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( 1 + \mathcal{O}(\alpha_s, \frac{1}{m_c^3}) \right)$

Beneke, G.B., Dunieta  
Falk, Petrov  
Chay et al.

Lead.  $m_b, m_c$  dep. drops out

- CP violation in ...

... mixing  $A_e(t) = \frac{\Gamma(\bar{B}(t) \rightarrow l^+) - \Gamma(B(t) \rightarrow l^-)}{+} = a = \text{Im} \frac{\Gamma_{12}}{M_{12}}$

SM:  $a \lesssim 10^{-3}$

New Physics  $a \sim 10^{-2}$

Cahn, Worah

$$A_{all}(t) = \frac{\Gamma(B(t) \rightarrow all) - \Gamma(\bar{B}(t) \rightarrow all)}{+} = a \left[ \frac{x}{2} \sin \Delta M t - \sin^2 \frac{\Delta M t}{2} \right]; \quad x = \frac{\Delta \Gamma}{\Gamma}$$

Beneke, G.B., Dunieta

... mixing-decay interference

$$A(t) = \frac{\Gamma(B(t) \rightarrow f) - \Gamma(\bar{B}(t) \rightarrow f)}{+} \approx -0.1 \sin 2\alpha \sin \Delta M t$$

$$f = u \bar{u} d \bar{d}$$

Beneke, G.B., Dunieta

... decay (direct)

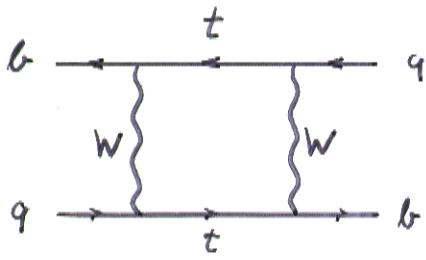
$$\frac{\Gamma(B^+ \rightarrow X) - \Gamma(B^- \rightarrow X)}{+} = \begin{cases} (2.0^{+1.2}_{-1.0}) \% & \Delta S = 0 \\ (-1.0^{+0.5}_{-0.5}) \% & \Delta S = 1 \end{cases}$$

charmless

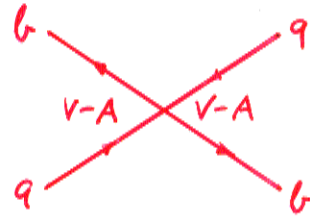
Lenz, Nierste, Ostermaier

# $B_q - \bar{B}_q$ Mixing, $\Delta M_q$

$q = d, s$



$$= C\left(\frac{m_t}{M_W}, \mu\right) \cdot$$



$$+ \mathcal{O}\left(\frac{m_b^2}{M_W^2}\right)$$

coefficient

matrix element

$$\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} m_{B_q} |V_{tq}|^2 S_0\left(\frac{m_t}{M_W}\right) \eta_B \cdot B_{B_q} f_{B_q}^2$$

- clean situation for OPE: top dominance,  $M_W, m_t \gg \Lambda_{QCD}$

$C$  at NLO

Buras, Jamin, Weisz

- theoretical uncertainty  $\leftrightarrow B_{B_q} f_{B_q}^2$

$\hookrightarrow$  reduced in

$$\frac{\Delta M_{Bd}}{\Delta M_{Bs}} = \frac{m_{Bd}}{m_{Bs}} \underbrace{\frac{B_{Bd} f_{Bd}^2}{B_{Bs} f_{Bs}^2}}_{1 + SU(3) \text{ breaking}} \left| \frac{V_{td}}{V_{ts}} \right|^2$$

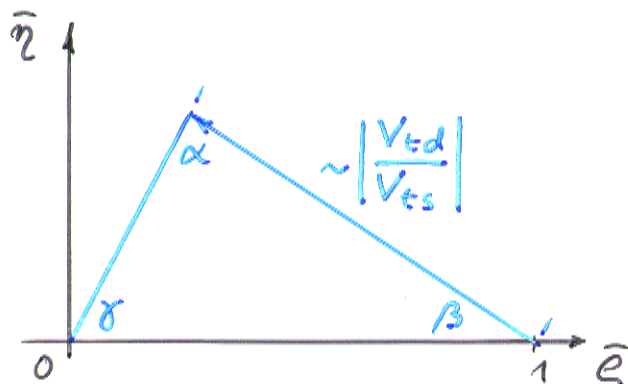
$1 + SU(3) \text{ breaking}$

$$B_{Bs}/B_{Bd} = 1.01 (1) (3)$$

$$f_{Bs}/f_{Bd} = 1.16 (4)$$

Hashimoto  
Lattice 99

- $\Delta M_d/\Delta M_s$  clean measure of  $|V_{td}/V_{ts}|$



exp. avg.:

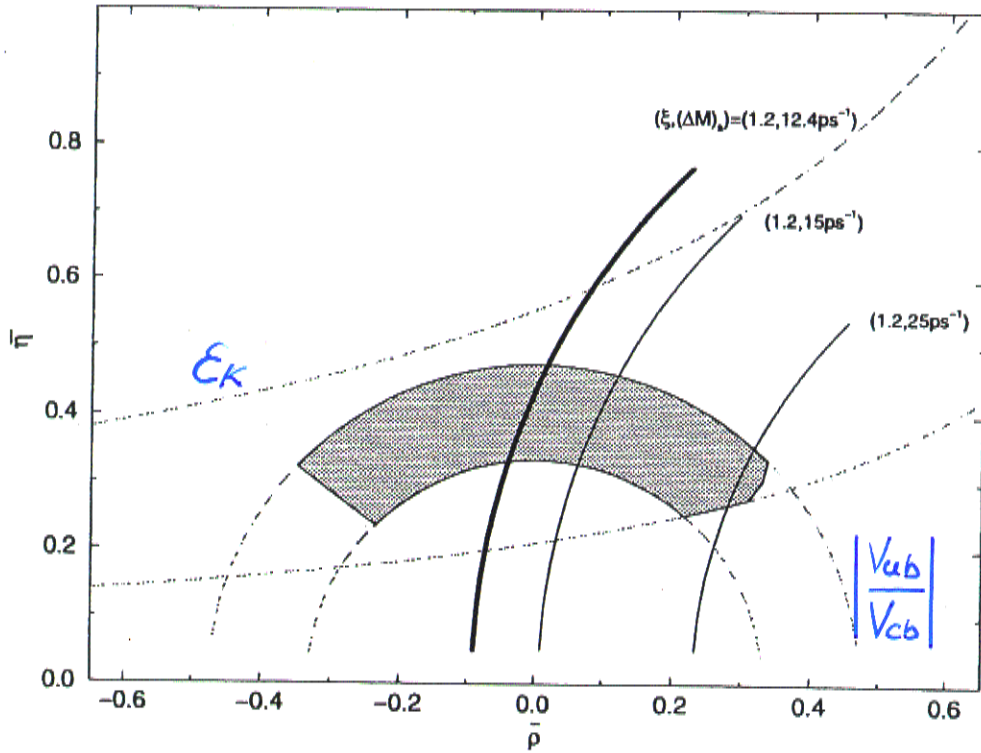
$$\Delta M_d = 0.473 \pm 0.016 \text{ ps}^{-1}$$

$$\Delta M_s > 12.4 \text{ ps}^{-1}$$

UT

Wolfenstein parameters  $\bar{e}, \bar{\eta}$

Buras



Quantity	Central	Error	Reference
$ V_{cb} $	0.040	$\pm 0.002$	[8]
$ V_{ub} $	$3.56 \cdot 10^{-3}$	$\pm 0.56 \cdot 10^{-3}$	[15]
$\hat{B}_K$	0.80	$\pm 0.15$	See Text
$\sqrt{B_d} F_{B_d}$	200 MeV	$\pm 40$ MeV	[48]
$m_t$	165 GeV	$\pm 5$ GeV	[50]
$\Delta M_d$	$0.471 \text{ ps}^{-1}$	$\pm 0.016 \text{ ps}^{-1}$	[51]
$\Delta M_s$	$> 12.4 \text{ ps}^{-1}$	95% C.L.	[51]
$\xi$	1.14	$\pm 0.08$	[48, 52]



# Exclusive Nonleptonic Decays

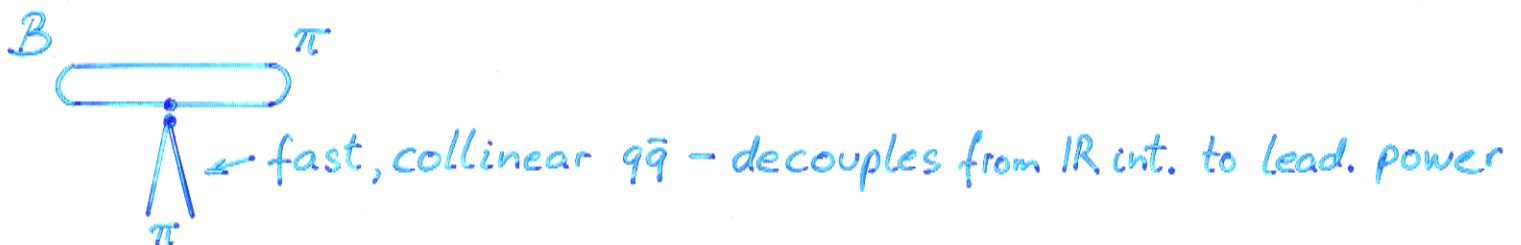
Problem:  $\langle \pi\pi | Q_i | B \rangle$ ;  $\neq$  incl.:  $\sum_f |\langle f | Q | B \rangle|^2 \rightarrow \langle B | Q^{(x)} Q^{(y)} | B \rangle \rightarrow$  OPE

- eliminate matrix elements: use flavor symm. + combine modes  
 $B \rightarrow \pi\pi + SU(2)$  Gronau, London;  $B \rightarrow \pi^+\pi^-, B_s \rightarrow K^+K^- + SU(3)$  Dunietz; Fleischer
- general parametrizations of m.e. Ciuchini et al.; Buras, Silvestrini
- naive factorization
- hard scattering approach Szczepaniak, Henley, Brodsky; Kroll et al.
- LEET for  $B \rightarrow D\pi$  Dugan, Grinstein
- QCD sum rules (corrections to fact.) Blok, Shifman; Khodjamirian, Rückl; Donoghue, Petrou

new approach: QCD factorization Beneke, G.B., Neubert, Sachrajda

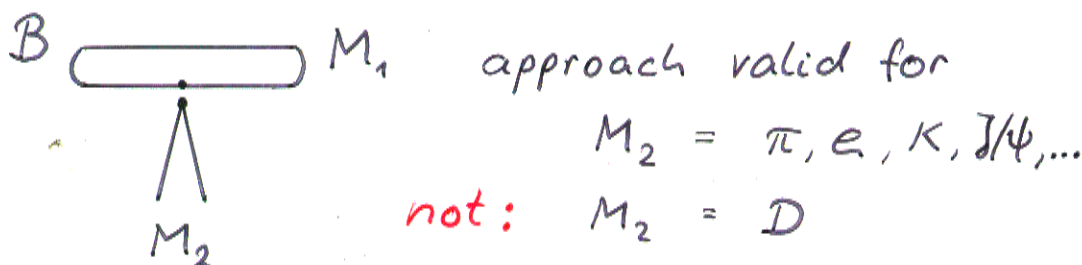
$$m_B \gg \Lambda_{QCD} \Rightarrow$$

$$A(B \rightarrow \pi\pi) \sim \underbrace{\langle \pi | j_1 | B \rangle \langle \pi | j_2 | 0 \rangle}_{\text{calculable in terms of } f_+(0), f_\pi, \phi_\pi, \dots} \left[ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_B}\right) \right]$$



includes: hard spectator int., penguins, rescatt.  $\rightsquigarrow$

strong phases  $\mathcal{O}(\alpha_s; \frac{\Lambda}{m_B})$



$$\frac{\Gamma(B \rightarrow D^* \pi)}{\Gamma(B \rightarrow D \pi)}$$

Politzer, Wise

# Summary

theoretical framework for nonleptonic B decays

$$\langle f | \mathcal{H}_{\text{eff}} | B \rangle \sim C_i \left( \frac{M_W}{m_b}, \alpha_s \right) \cdot \langle f | Q_i | B \rangle$$

short distances  
 $M_W \rightarrow m_b$

short + long distances

$$m_b \gg \Lambda_{\text{QCD}}$$

HQE (inclusive); QCD fact. (exclusive)

□ inclusive decays - HQE

- local quark-hadron duality  
models ... ? ; test predictions  $\leftrightarrow$  experiment
- higher corrections :  $1/m_b, \alpha_s$   $\Delta\Gamma_{B_s}$  ✓
- nonpert. matrix elements ←
- applications - b-hadron lifetimes  
- CPV, CKM  
?  $\tau_{B_b}, B_{SL} \leftrightarrow m_c$

□ exclusive decays (two-body)

- QCD factorization - power corrections ?  
- exp. tests
- phenomenological approaches

→ promising opportunities for  
physics with hadronic B decays