# B Physics from the Lattice

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Source material:
Review of "$B$ Decays on the Lattice" at Lattice '99
Shoji Hashimoto, hep-lat 9909136

See also T. Draper review at Lattice '98, hep-lat 9810065
T. Onogi review at Lattice '97, hep-lat 9802027

# Plan of Talk

General questions:

- How much can you trust them; how good/how necessary is the quenched approximation?
- How do you interpret error analyses in lattice results?
- Which quantities are:
  - easy to do and done decently now?
  - doable with current methods and more work?
  - in need of new methods?

Specific topics (focusing on quantities maximizing ease of calculation, ease of experiment, physics impact):

- $f_B$, $f_{B_s}$
- $m_b$
- $B_B$, $B_{B_s}$
- $B \to Dl\nu$
- $B \to \pi l\nu$

# Which quantities are easiest/most believable on the lattice?

Most important dividing line: one hadron present at a time.

Naive treatment of $t \to i t_E$ gets final state interactions wrong for multihadron states.

Moreover, mesons easier than baryons; Lightest hadrons with given quarks have best statistics. $B$ easiers than $B^{**}$.

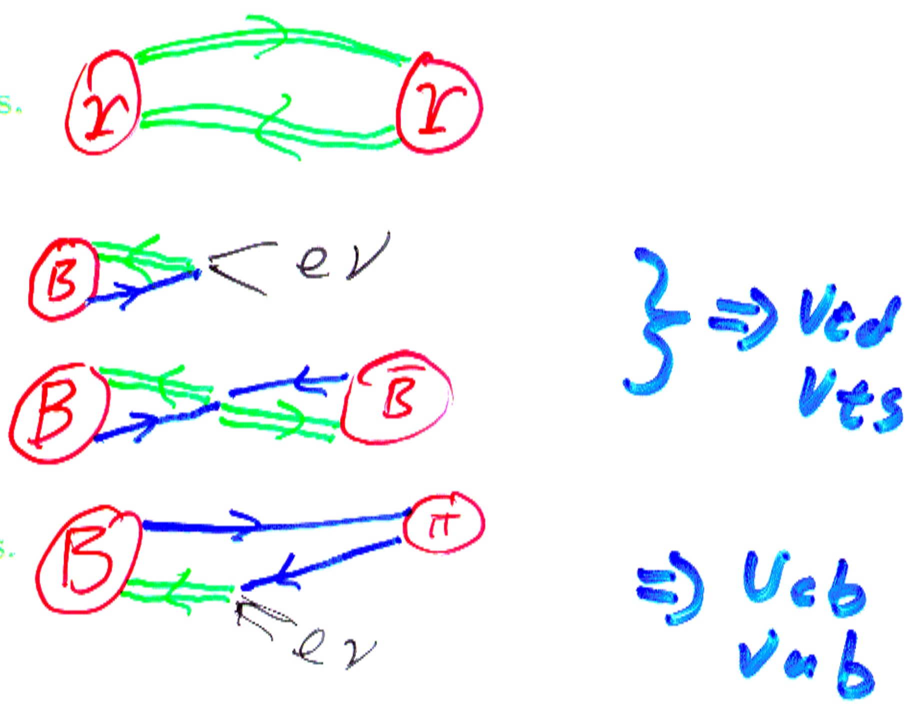
"Easy" quantities, done decently now.

Heavy quark masses.

Decay constants.

B parameters.

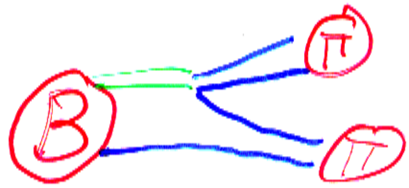
Semileptonic decays.




ALSO SPECTRUM
MASS OF $B_0$ $B_+$ $B_s$ $B_c$
MAYBE $\Lambda_b$ ...

"Hard" quantities, need better methods.

Nonleptonic decays.




"Medium" quantities, need more work

Other bag parameters.

HQET expectation values.

# How fast will unquenched errors approach those of the best current quenched errors?

(Quenched approximation: leave out light quark loops, the effects of sea quarks. Speeds up calculations by orders of magnitude.)
Reasonable guesses:

- No unforeseen problems, and no new ideas: 5 years?
- Unquenching harder than it looks now: 5-10 years?
- New ideas, better methods: ?

# How believable is any quenched result?

The quenched approximation is the leading term in a loop expansion, BUT higher order corrections are as hard to calculate as the full unquenched theory.

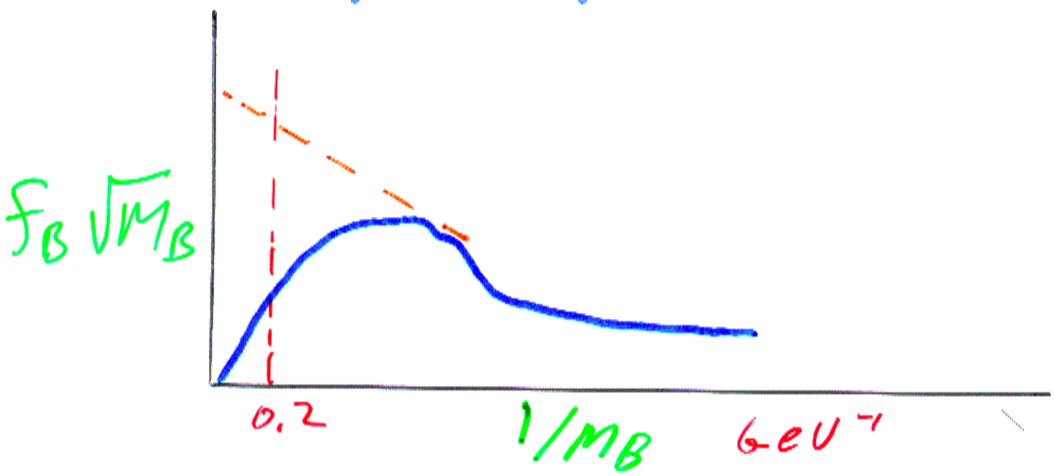
Compare:

- Quark models. Not first step in any systematic expansion.
- Sum rules. Formally correct expansion, but no small parameter.
- Quenched approximation. Expansion exists, but second order corrections as hard the complete theory.
- Unquenched lattice calculations. The complete theory.

# Putting the $b$ quark on the lattice.

Standard light quark actions give singular results at large $m$.

Before NRQCD and HQET:




extrapolate light quark results to $\infty$ $m$

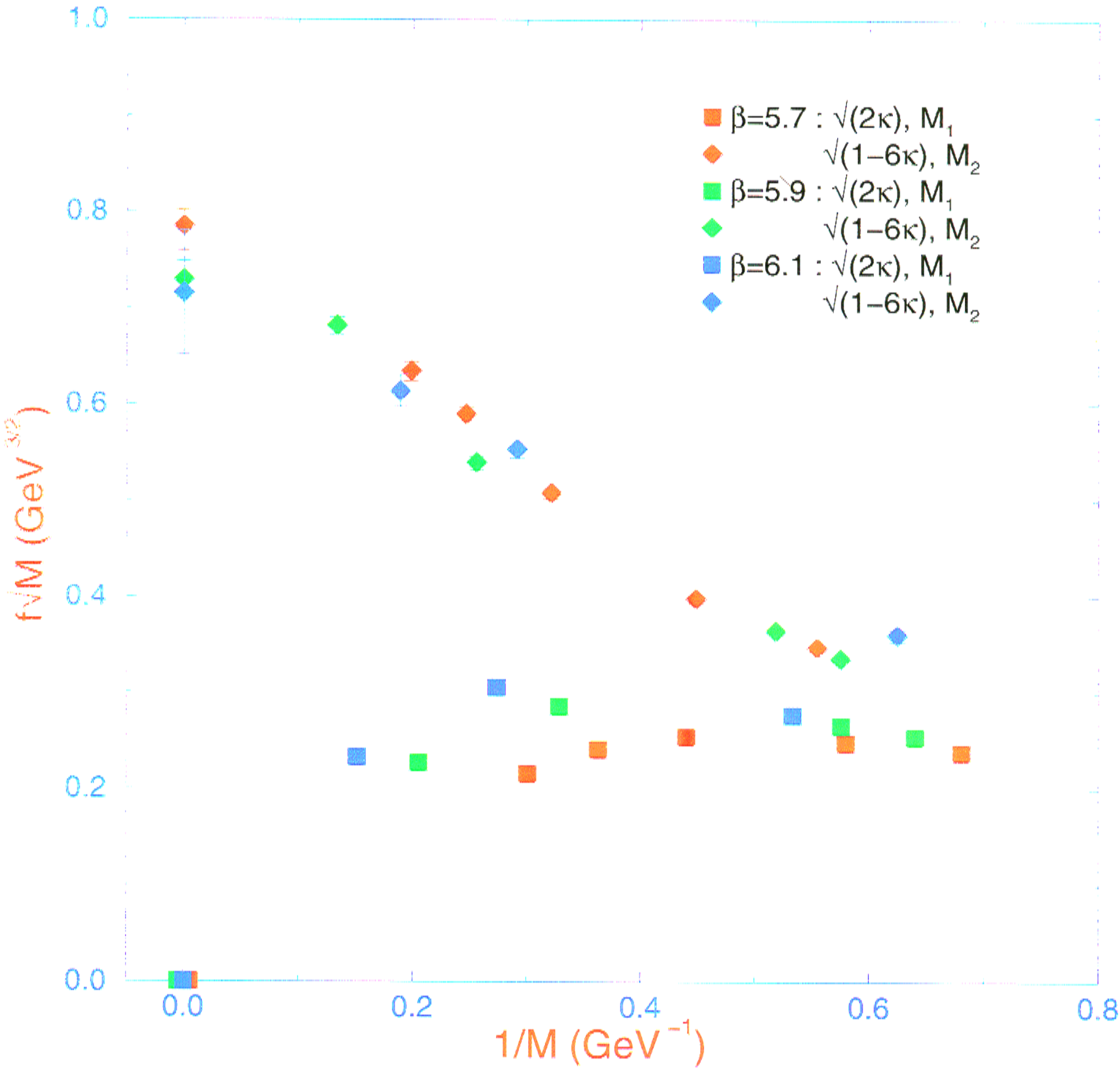
Better way:

Discretize nonrelativistic action:

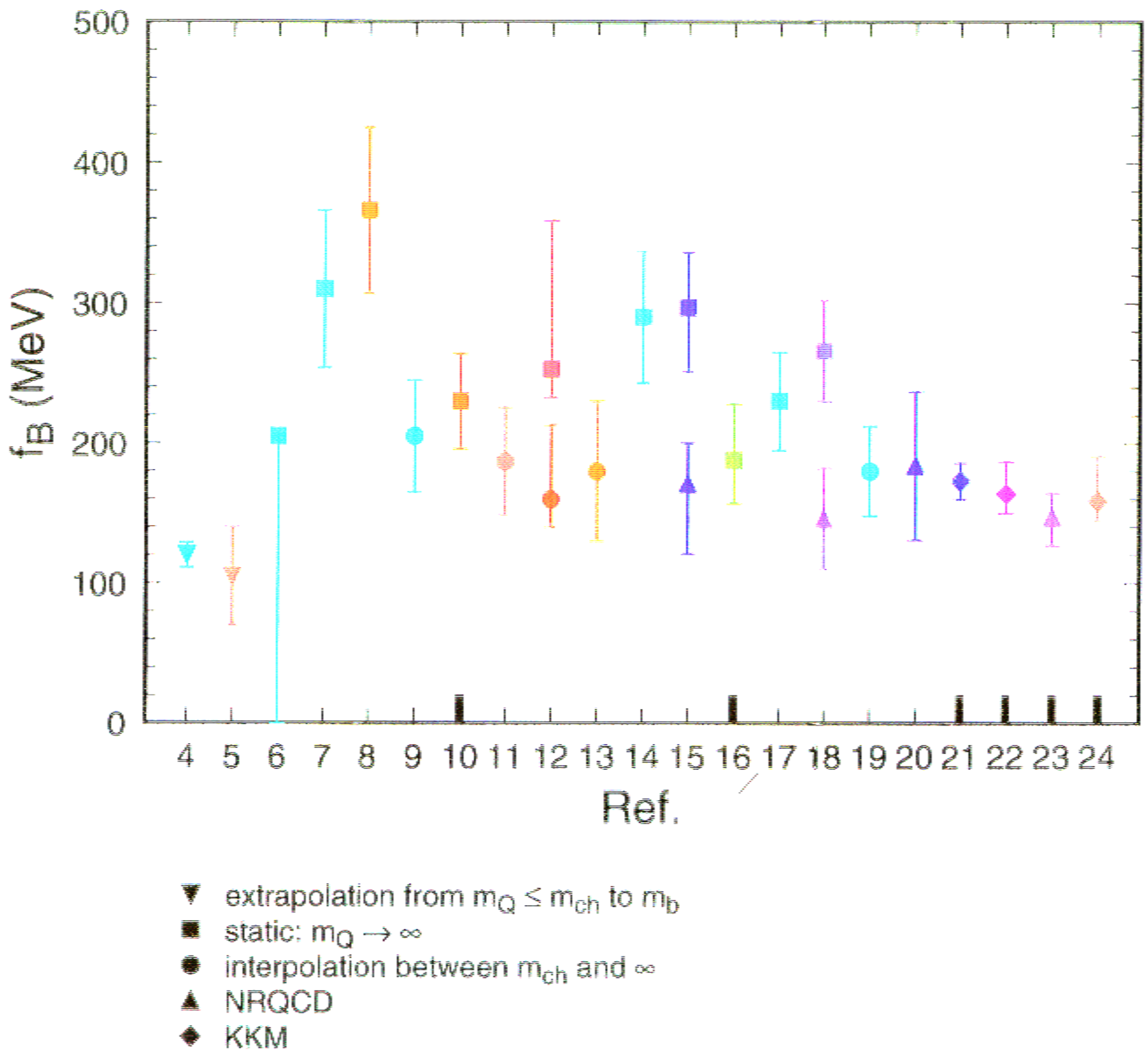
$$\mathcal{L} = \psi^\dagger \left( iD_t + \frac{D^2}{2m} + \frac{\sigma \cdot B}{2m} + \dots \right) \tag{1}$$

Main methods:

- NRQCD (Lepage)
- Static approximation (Eichten and Hill)
- "Relativistic" heavy fermions (El-Khadra, Kronfeld, Mackenzie)



β=5.7 : √(2κ), M1
√(1−6κ), M2
β=5.9 : √(2κ), M1
√(1−6κ), M2
β=6.1 : √(2κ), M1
√(1−6κ), M2
f√M (GeV 3/2)
1/M (GeV −1)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8



500
400
300
200
100
0
f_B (MeV)
4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Ref.
extrapolation from $m_Q \leq m_{ch}$ to $m_b$
static: $m_Q \to \infty$
interpolation between $m_{ch}$ and $\infty$
NRQCD
KKM

# LATTICE MONTE CARLO

CARTOON VERSION:

**0.** $A_{x\mu} \to U_{x\mu} = \exp\left[\int_x^{a+a\hat{\mu}} d\ell \cdot A\right]$

$\mathcal{L}: F_{\mu\nu}^2 \to 1 - \mathrm{Tr}\, U\, U U U + O(a^2)$

$\square$ PLAQUETTES

$(\not{D} + m)\psi_x \to m\psi_x + \frac{1}{2}[\psi_{x+\hat{\mu}a} - \psi_{x-\hat{\mu}a}]\gamma_\mu + \ldots$

$Z \to \int [d\psi][d\bar{\psi}][dU]\, e^{-\sum_x S(x)}$

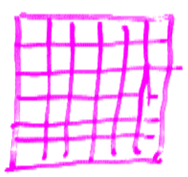
↑ LARGE BUT FINITE DIM. INT.

## I. PATH INTEGRAL: MONTE CARLO

METROPOLIS METHOD

OVERRELAXED

⋮



BIG, FINITE SET OF CONFIGURATIONS

## II QUARK PROPAGATORS: $\frac{1}{\not{D} + m}$ ← SPARCE MATRIX

SPARSE MATRIX METHODS

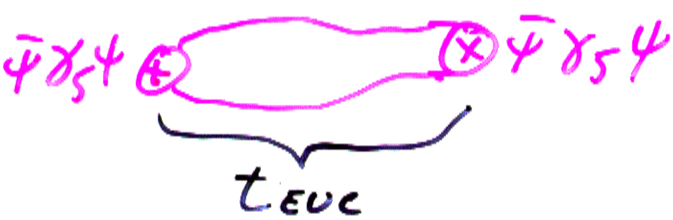
CONJUGATE GRADIENT

MINIMUM RESIDUAL

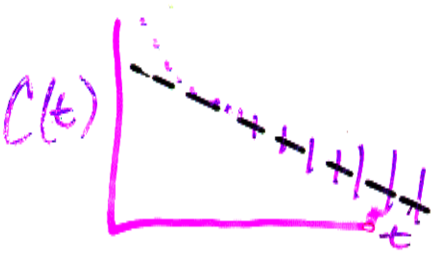
⋮

## III HADRON CORRELATION FUNCTIONS

PUT QUARK PROPS TOGETHER




## IV FIT TO EXPONENTIALS




$C(t) = A e^{-Et} + A' e^{-E't} + \ldots$

↑ EXCITED STATES

# How do you interpret reported uncertainties?

Lattice uncertainties consist of several pieces:

- Statistics. Usually can be done reliably.
- Quenched approximation. Usually a guess, perhaps 10%. Checked to some extent by using several quantities to set the physical scale. However, $\Delta_{\mathrm{QUENCHING}}(\alpha_s) \sim 25\%$, $\Delta_{\mathrm{QUENCHING}}(m_b) \sim 0\%$.
- Other systematic uncertainties.
  - Perturbation theory. Can be estimated objectively, e.g., $\mathcal{O}\alpha^2$.
  - Finite lattice spacing. Do you get the same answer at different lattice spacings?
  - Excited state contamination.
  - Chiral extrapolation.
  - Finite volume.

# $f_B$ unquenched.

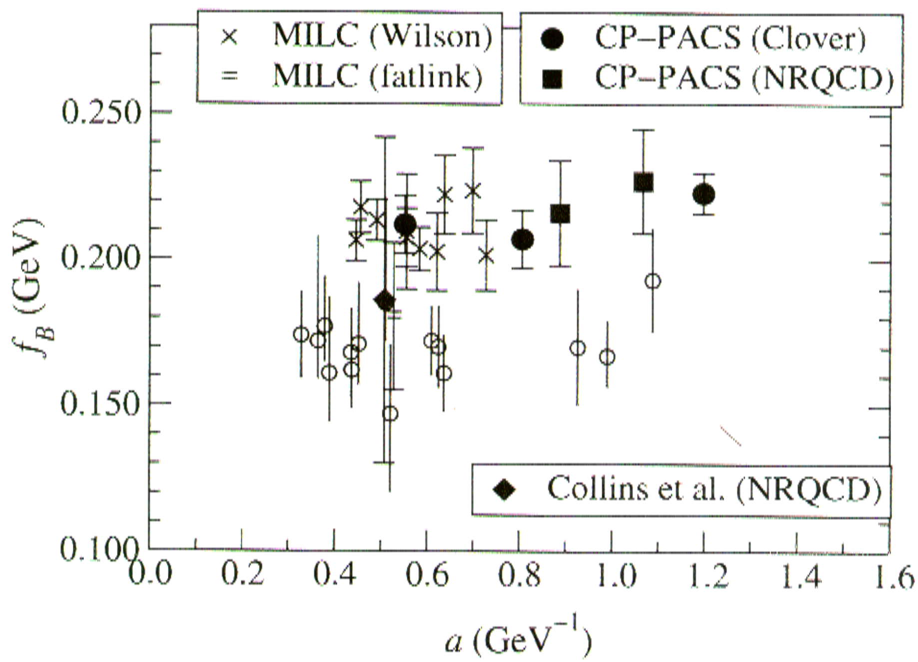



Figure 1: Dynamical lattice calculations of $f_B$. Results are from MILC, Collins *et al.*, and CP-PACS. Quenched results are also plotted with small open symbols.

## Hashimoto, Lattice '99:

| | $N_F$=2 | $N_F$=0 |
|---|---|---|
| $f_B$ (MeV) | 210 ± 30 | 170 ± 20 |
| $f_{B_s}$ (MeV) | 245 ± 30 | 195 ± 20 |
| $f_{B_s}/f_B$ | 1.16 ± 4 | 1.15 ± 4 |

# $b$ quark mass.

Earlier work by NRQCD ($*$, quenched, $+$, $n_f$=2) and APE with HQET (squares).

New two-loop calculation by Martinelli and Sachrajda agrees.

**Hashimoto, Lattice '99:** $\overline{m_b}(m_b) = 4.26 \pm 0.11$ GeV.

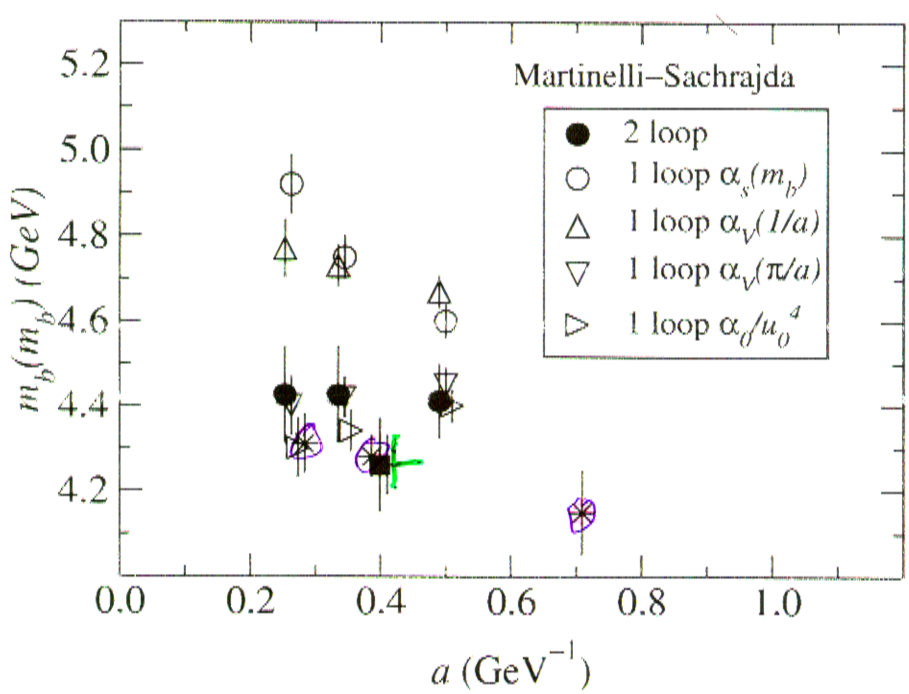



Figure 2: Lattice calculation of the $b$ quark mass. Martinelli-Sachrajda's two-loop results (filled circles) are plotted together with the corresponding one-loop matching results with various definitions of coupling constant (open symbols). APE's HQET result with $N_F = 2$ (square), and NRQCD result with $N_F = 0$ (star) and with $N_F = 2$ (plus) are also shown. From Hashimoto.

0 +

# $B_B$.

Not yet as close agreement between different methods as for quenched $f_B$.

$B_B(m_b) = 0.80(0.15)$

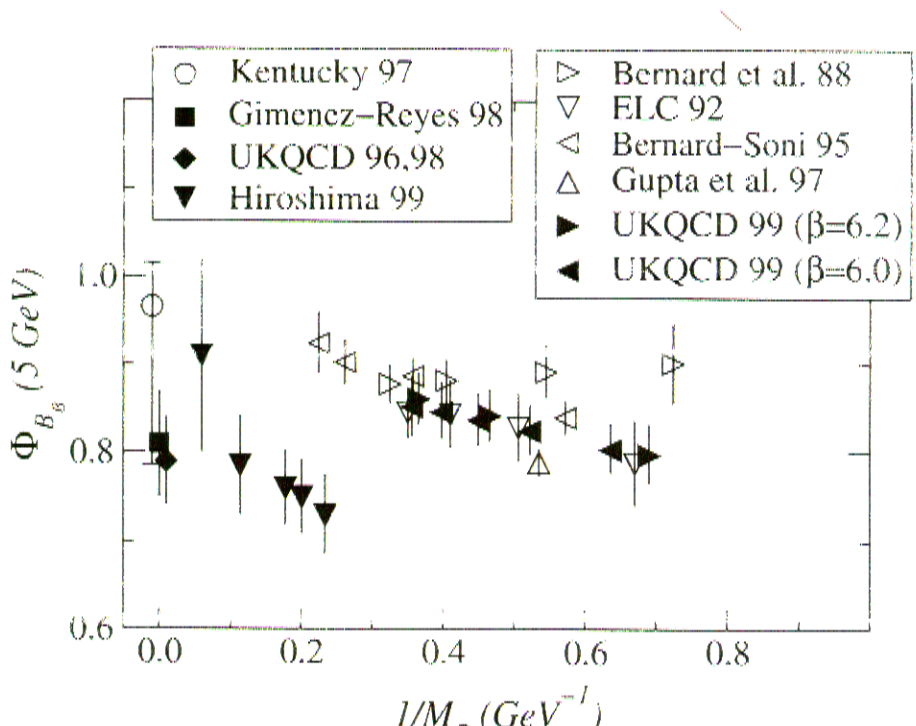



Figure 3: $1/M$ dependence of $\Phi_{B_B}(5GeV)$. The static and NRQCD data are from Kentucky, Giménez-Reyes, UKQCD, and Hiroshima. The relativistic calculations are Bernard *et al.*, ELC, Bernard-Soni, Gupta *et al.*, and UKQCD. Open symbols are obtained with Wilson quark for heavy and/or light quarks, and filled ones are $O(a)$-improved. Most points have statistical errors only.

# Semileptonic decays.

- More amplitudes, many momenta.
- Errors more qualitative, worse statistics, finite lattice spacing errors.

Arbitrarily choose two topics where progress could be made.

$B \to D^{(*)} l\nu$

Shape of amplitude can be measured experimentally; need normalization at zero recoil to get $V_{cb}$. Deviation from 1 known only from sum rules.

Need $< 5\%$ errors.
Ratios of amplitudes produce errors vanishing in the $M_B = M_D$ symmetry limit.

$$|h_+^{B\to D}(1)|^2 = \frac{\langle D|V_0^{cb}|B\rangle\langle B|V_0^{bc}|D\rangle}{\langle D|V_0^{cc}|D\rangle\langle B|V_0^{bb}|B\rangle}, \tag{2}$$

**Prototype calculation (Hashimoto et. al.):**

$$\mathcal{F}(1) = 1.060 \pm 0.016 \pm 0.002 \pm 0.003^{+0.001}_{-0.010}, \tag{3}$$

where errors are from statistics, heavy quark masses, higher-order matching for $h_+(1)$, and the omitted rotation for $h_-(1)$, but not quenching or finite lattice spacing.

## $B \to \pi l \nu, \quad \rho l \nu$

- Much work.
- Different quantities reported, harder to compare.
- Often extrapolates to $q^2$ regions ($q^2 = 0$) and sometimes $1/M_B$ regions not covered by calculations.

A better way: calculate the decay rate directly (not the form factors). Compare theory and experiment in a $q^2$ region where both are good.

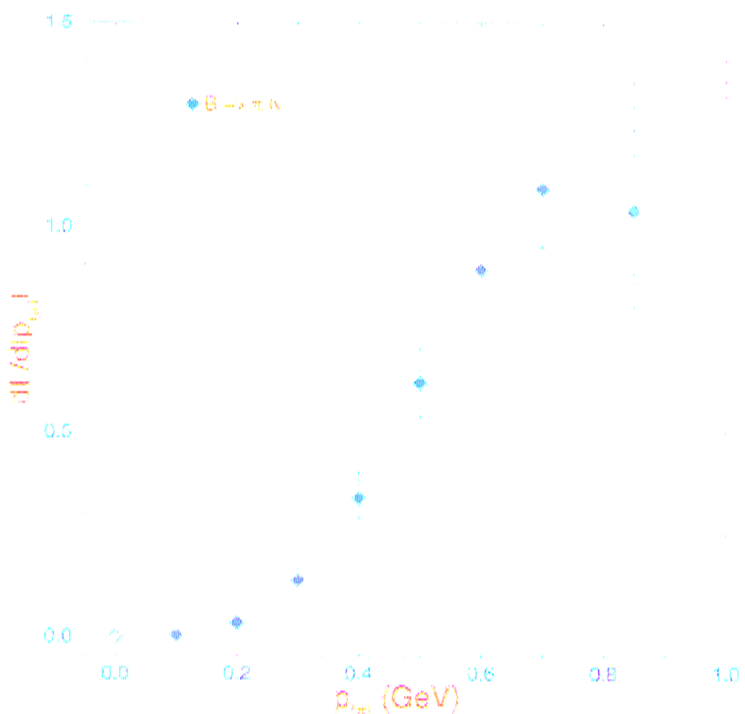


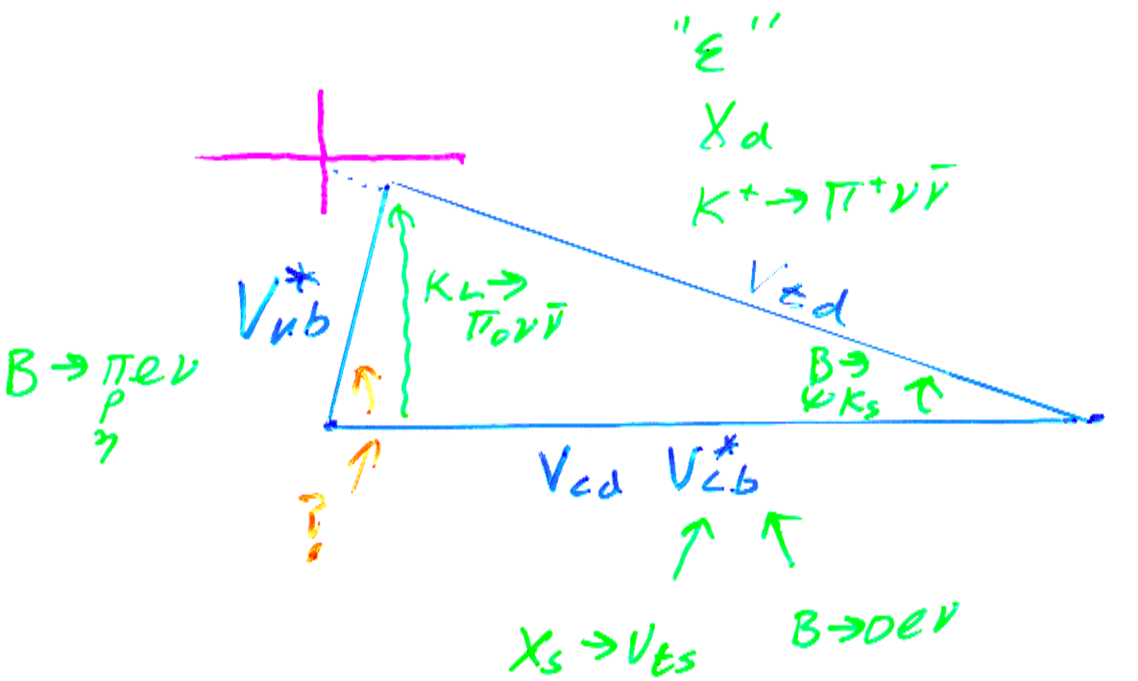

Figure 4. $B \to \pi l \nu$ differential decay rate at $\beta = 5.7$ S. Ryan, Lattice '99.

# MY GOLD PLATED LIST

FOR CKM 3rd ROW + COLUMN

(FOCUS + KTEV + LATTICE FOR 1st 2×2)

| NEEDED | | | BEST AT FNAL? |
|---|---|---|---|
| $f_{B_s}\sqrt{B_{B_s}}$ | $X_s$ | $V_{ts}$ } $V_{ts}/V_{cb}$ | ✓ |
| | $B \to D e \nu$ | $\lvert V_{cb} \rvert$ | ? |
| | $B \to \pi, \rho, \eta \; e\nu$ | $\lvert V_{ub} \rvert$ | ? |
| $B_K$ | $\varepsilon$ | $V_{ts}^* V_{td}$ | ✓ |
| $f_B\sqrt{B_B}$ | $X_d$ | $V_{td}$ | |
| DON'T NEED LATTICE | $B \to \psi K_s$ | $\beta$ (arg $V_{td}$) | |
| | $K_L \to \pi_0 \nu\bar{\nu}$ | $\mathrm{Im}\, V_{ts}^* V_{td}$ | ✓ |
| | $K^+ \to \pi^+ \nu\bar{\nu}$ | $V_{td}$ | ✓ |



# Summary.

Some of the most important quantities in $B$ phenomenology are also some of the easiest things for lattice QCD to calculate. These include:

- $f_B$, $f_{B_s}$
- $B_B$, $B_{B_s}$, $\frac{f_B^2 B_B}{f_{B_s}^2 B_{B_s}}$, etc.
- $m_b$
- $B \to \pi l\nu$, $\rho l\nu$
- $B \to Dl\nu$, $D^* l\nu$

Now: **10-20% accuracy.**

Goal in 10 years: 2% accuracy?

Sooner if new ideas (coarse lattice methods?), later if roadblocks (chiral extrapolation harder at higher accuracy?)

Should add many new quantities to Buras's "gold-plated" list in time for his retirement.