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b Production at the Tevatron

B Physics at Tevatron Workshop
FNAL, September 23-25, 1999

Results obtained in collaboration with Paolo Nason

Results obtained using the FMNR code (available upon request): heavy quark production at next-to-leading order in QCD with the subtraction method $\rightarrow$ fully exclusive distributions.

For a recent review on Heavy Flavour production, see: S. Frixione, M. Mangano, P.Nason, G.R. hep-ph/9702287 in “Heavy Flavour II” (Buras and Lindner eds.) and references therein.

The subtraction method

Neglect collinear divergences; $x = m^2_{QQ}/s$, η soft cut-off.

$$\frac{d\sigma}{dx} = \delta(1-x) + c\alpha\left[\delta(1-x)\log\eta + \frac{1}{1-x+\eta}\right]$$

Let $f(x)$ be any quantity of which we want to compute the weighted average (e.g. a product of θ functions defining a hystogram bin).

$$\log\eta = \log(1+\eta) - \int_0^1 dx \frac{1}{1-x+\eta}$$

$$\begin{aligned}\int_0^1 dx\, f(x)\frac{d\sigma}{dx} &= f(1)\\ &+ c\alpha\Big[f(1)\log(1+\eta) + \int_0^1 dx\frac{f(x)-f(1)}{1-x+\eta}\Big]\end{aligned}$$

The limit $\eta \to 0$ can be taken safely!

Alternatively (slicing method): keep η small but nonzero, and check η independence of numerical results.

To perform the integration by Monte Carlo techniques:

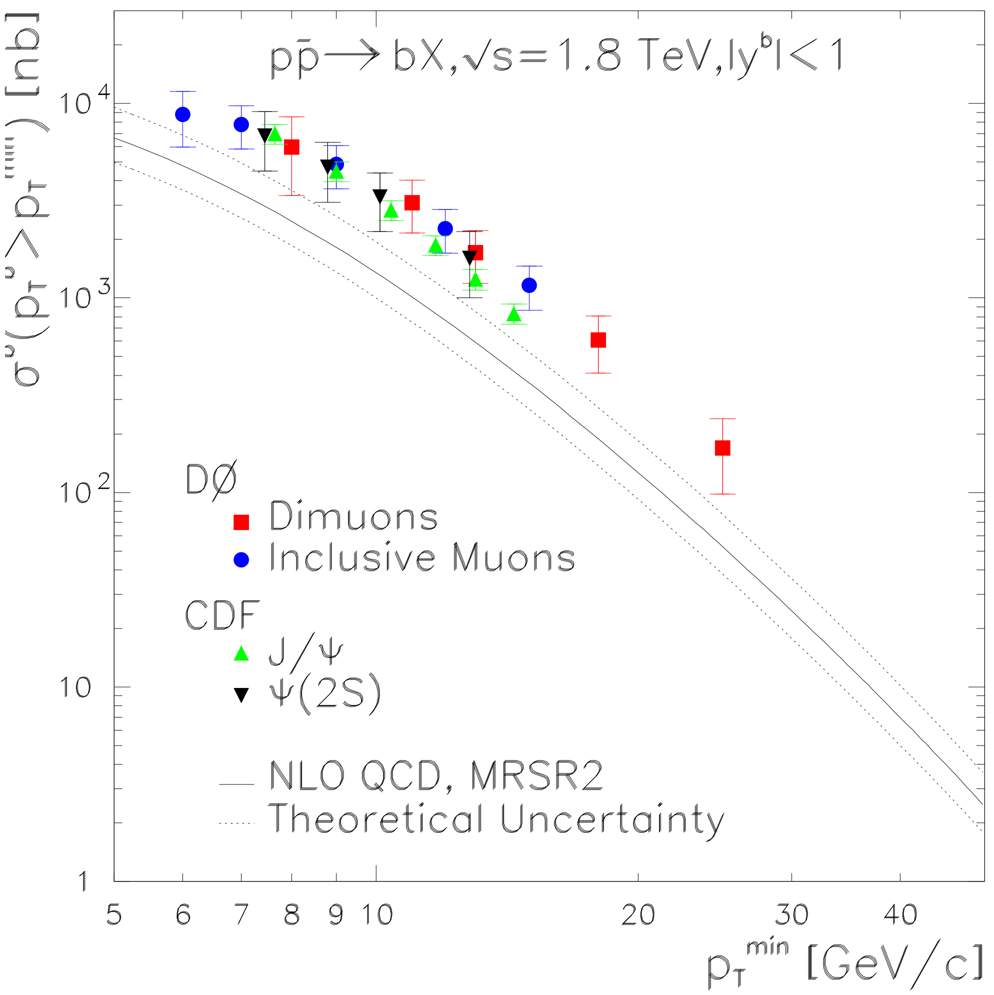
- Pick a random number $\tilde{x}$ between 0 and 1
- Generate one event with weight $c/(1-\tilde{x})$ and $x=\tilde{x}$
- Generate one event with weight $-c/(1-\tilde{x})$ and $x=1$
- Iterate the above steps N times
- Normalize the result by a factor $1/N$

Summing the weights multiplied by $f(x)$ for the events generated in this way, a Monte Carlo estimate of the integral is obtained. Any cut can be applied.

Precision only limited by statistics.

A well-known problem in understanding Tevatron data: the integrated p_T distribution at the Tevatron is larger than the NLO QCD prediction.

Inclusive b-quark production cross section at $\sqrt{s} = 1.8$ TeV for $|y^b| < 1$ (CDF, D0)



- Good agreement between different measurements
- Data/Theory $\sim$ 2.5

What are the possible explanations of the eccess of collider data over theoretical predictions?

- Higher order terms may well do it; after all, radiative corrections are huge.
- Resummation of terms enhanced by powers of $\log(p_T/m)$: there are indications that these terms lead to an enhancement of the cross section in the moderate p_T region.
- Fragmentation function: theoretical predictions without the inclusion of fragmentation effects describe the data reasonably well. Also, recent data favours harder fragmentation.
- Non perturbative effects, like an intrinsic k_T of incoming partons, are sometimes advocated (for example, in direct photon production). How much k_T would we need to explain Tevatron data? Is it consistent with correlation measurements?

Resummation of large logs of p_T/m

Two approaches to the computation of the p_T spectrum:

1. Compute the cross section for the production of a massive quark at fixed order in QCD;
2. Treat all flavors as massless, and convolute the cross section with the perturbative fragmentation functions of partons into a heavy quark, evolved with Altarelli-Parisi equations from the typical scale of the production process, $\mu = \mu_0 \sim m$, up to $\mu \sim p_T$.

Fragmentation functions at order α_{S}:

$$
\begin{aligned}
D_h(x,\mu_0) &= \delta(1-x) \\
&\quad + \frac{\alpha_s(\mu_0) C_{\mathrm{f}}}{2\pi} \left[\frac{1+x^2}{1-x} \left(\log \frac{\mu_0^2}{m^2} - 2\log(1-x) - 1 \right) \right]_+ \\
D_g(x,\mu_0) &= \frac{\alpha_s(\mu_0) T_{\mathrm{f}}}{2\pi} (x^2 + (1-x)^2) \log \frac{\mu_0^2}{m^2} \\
D_i(x,\mu_0) &= 0\,, \quad \text{for } i \neq g, h
\end{aligned}
$$

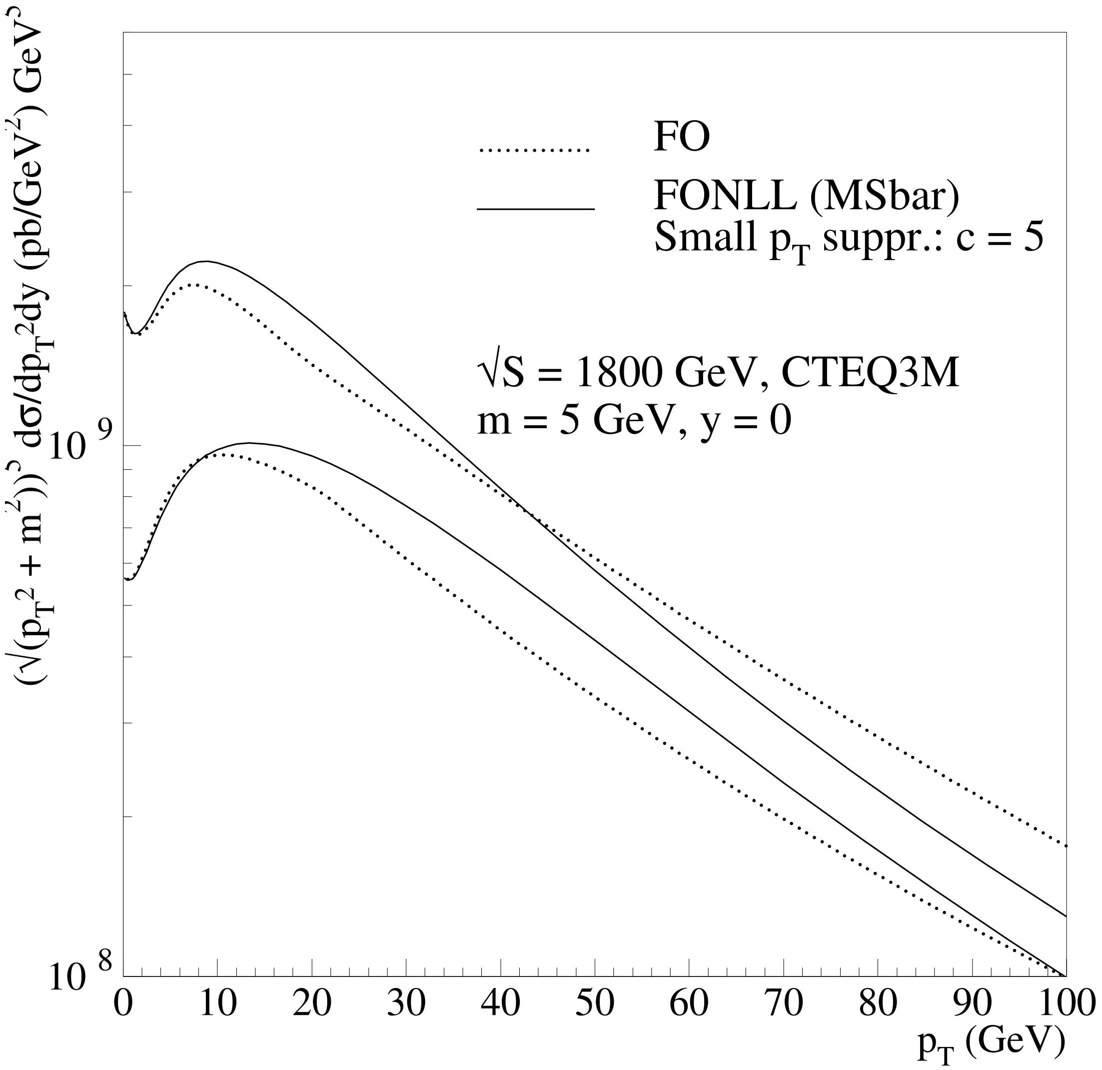
In approach 1., dependence on m fully taken into account; unreliable when $p_T \gg m$ because of large logarithms in the coefficients. Coefficients known up to order α_S^3 (Nason, Dawson, Ellis; Mangano, Nason, R.).

In approach 2., terms suppressed by powers of m/p_T systematically neglected; more accurate than 1. at large p_T, since all terms of order $\alpha_S^2(\alpha_S \log(p_T/m))^k$ (LL) and $\alpha_S^3(\alpha_S \log(p_T/m))^k$ (NLL) are taken into account.

It is possible (Cacciari, Greco, Nason) to use 1 in the region $p_T \sim m$ and 2 in the region $p_T \gg m$, with a suitable matching procedure:

$$
\begin{aligned}
\frac{d\sigma}{dp_T^2} &= A(m)\alpha_S^2 + B(m)\alpha_S^3 \\
&+ \left[\alpha_S^2 \sum_{k=2}^{\infty} a_k (\alpha_S \log\frac{\mu}{m})^k + \alpha_S^3 \sum_{k=1}^{\infty} b_k (\alpha_S \log\frac{\mu}{m})^k\right] \\
&\qquad \times G(m/p_T) \\
&+ \mathcal{O}(\alpha_S^4 (\alpha_S \log\frac{\mu}{m})^k) + \mathcal{O}(\alpha_S^4 \times \text{PST})
\end{aligned}
$$

Resummation of $\log(p_T/m)$ at the NLL level, suitably merged with the NLO fixed order result, favours an enhancement at moderate p_T. This is difficult to quantify; at small p_T it makes no sense to sum transverse momentum logarithms. At high p_T uncertainties become smaller.



M. Cacciari, M. Greco, P. Nason

hep-ph/9803400

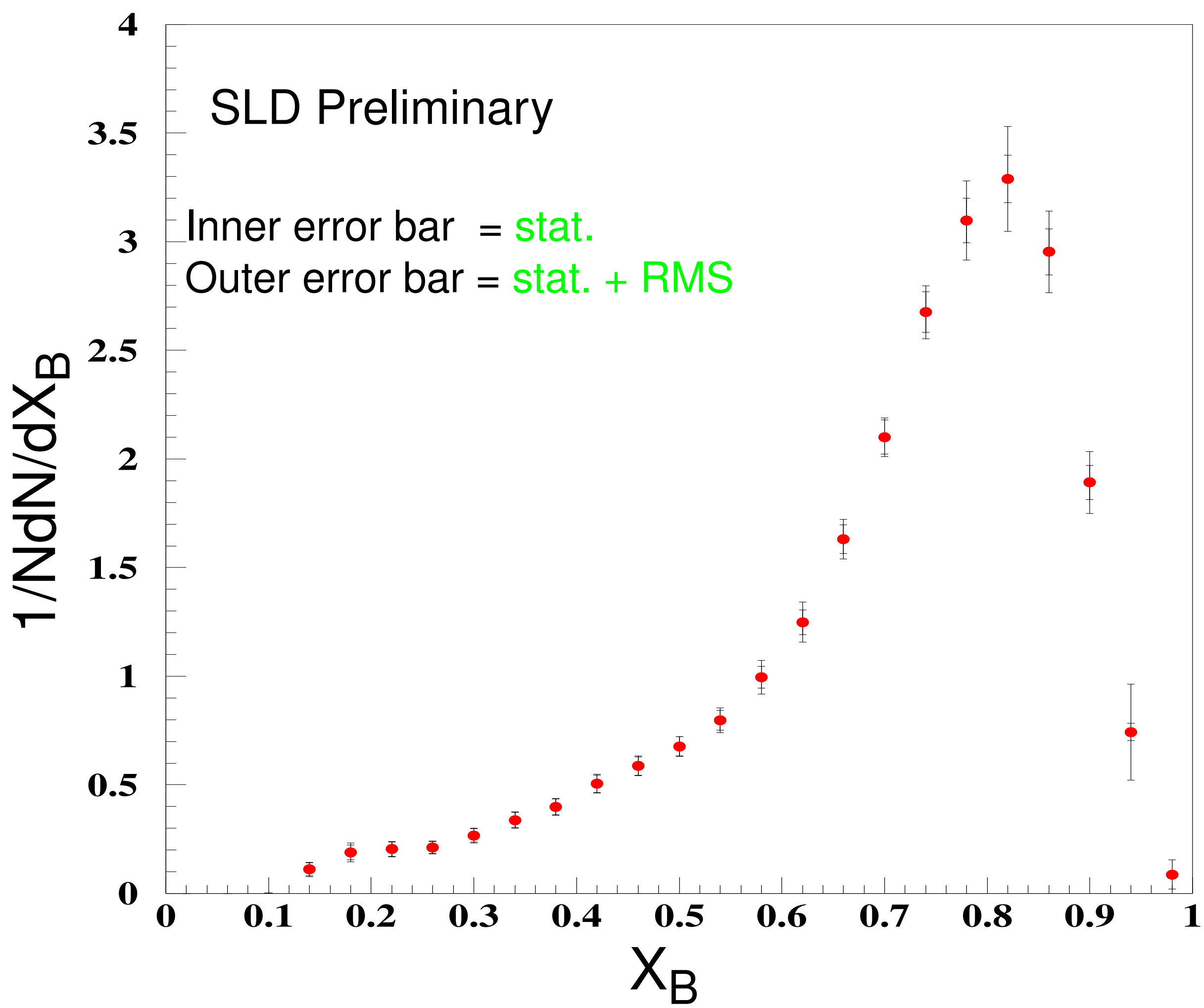
Effects of non-perturbative fragmentation

Fragmentation functions are better understood after LEP and SLD; LEP data indicate that the b-fragmentation function is harder than previously thought.

Recent, very precise SLD measurements (talk by D. Dong at Moriond '99).

How sensitive are cross section values to the shape of the non-perturbative fragmentation function?

Unfolded B Energy Distribution



$$U_i = \left(\sum_{j=1}^{n} M_{ij} \cdot (D_j - B_j) \right) / \varepsilon_i$$

- This method smooths the dist. and is explicitly model-dependent
- M_{ij} is found using MC weighted by each best fit function
- Stat. errors are “local”. Bin-to-bin correlation to be expected
- Total errors provide an “envelope” for acceptable shapes

ALEPH data seems more peaked (B^{**} correction?).

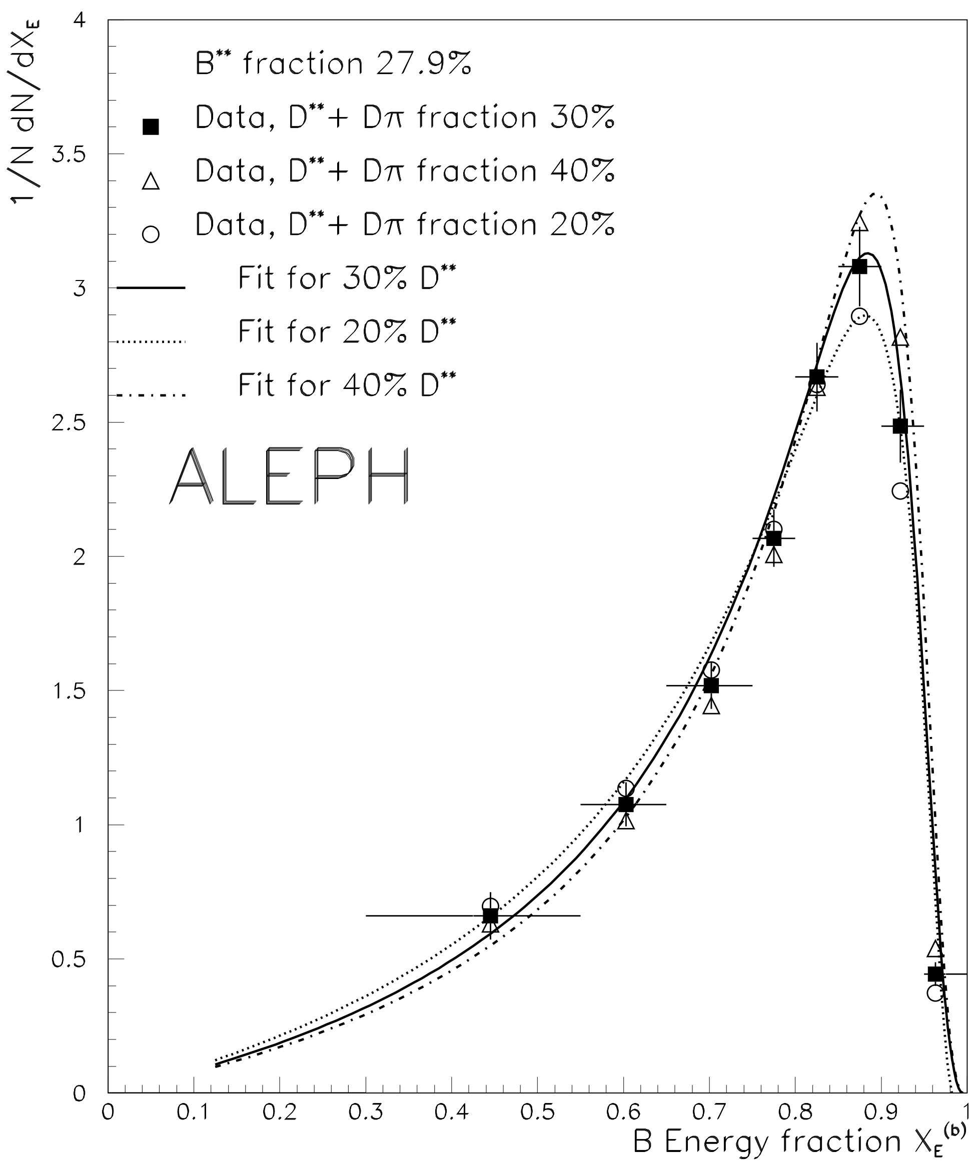


Figure 6: The acceptance corrected $x_E^{(b)}$ spectrum of the leading b-mesons combining all channels, for $f_{D^{**}}$=20, 30 and 40%. The error shown is statistical only and does not account for the point-to-point correlations induced by the deconvolution process. It is only shown for $f_{D^{**}}$=30%. Also shown are the fit results of Eq. 16 for $f_{D^{**}}$=20, 30 and 40%.

The effect of a non-perturbative fragmentation function on the p_T spectrum is easily quantified if one assumes a steeply-falling transverse momentum distribution for the produced b quark:

$$\frac{d\sigma}{dp_T} = A p_T^{-M} \,,$$

The corresponding distribution for the hadron is

$$\begin{aligned} \frac{d\sigma_{\rm had}}{dp_T} &= A \int \hat{p}_T^{-M} \, \delta(p_T - z\hat{p}_T) D(z) \, dz \, d\hat{p}_T \\ &= A p_T^{-M} \int_0^1 dz z^{M-1} D(z) \,. \end{aligned}$$

The hadron spectrum is proportional to the quark spectrum times the $M^{\rm th}$ moment of the fragmentation function $D(z)$. The larger the moment, the larger the enhancement of the spectrum.

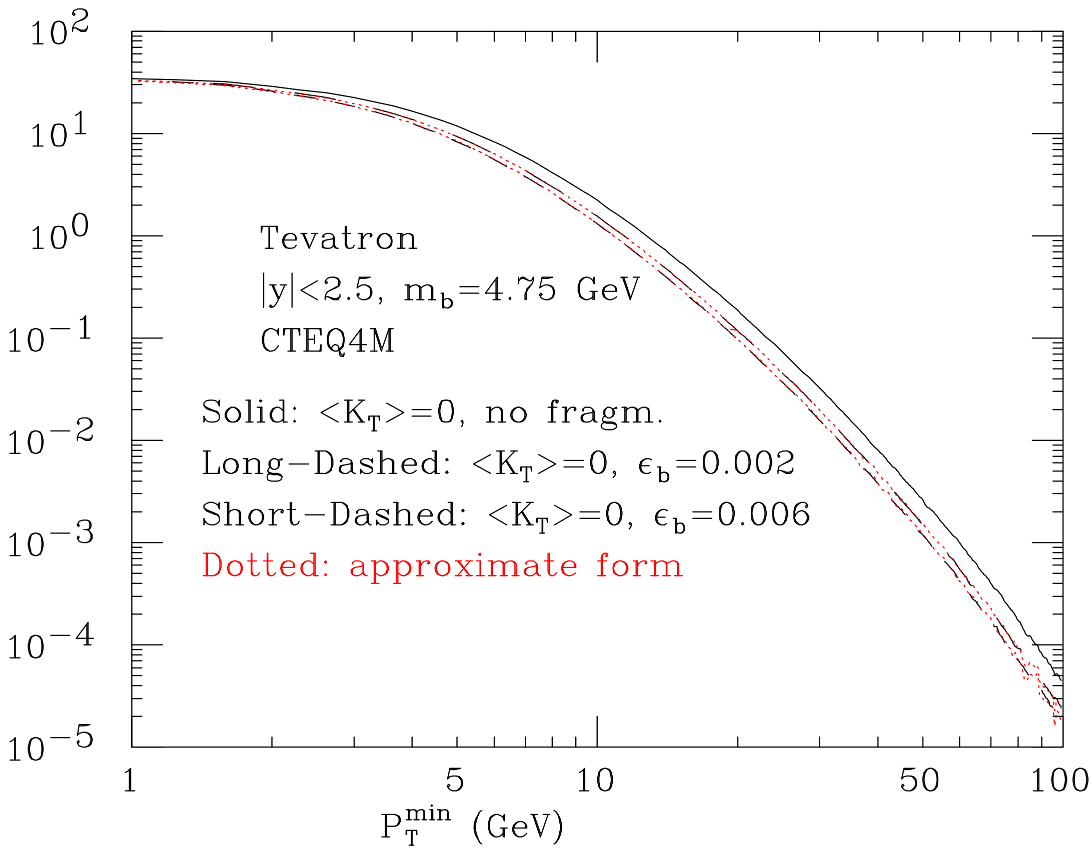
In practice, M is slightly dependent on p_T. Define a p_T dependent M value:

$$\frac{d\log\sigma(p_T > p_T^{\rm cut})}{d\log p_T^{\rm cut}} = -M(p_T^{\rm cut}) + 1$$

and

$$\sigma_{\rm had}(p_T > p_T^{\rm cut}) = \sigma(p_T > p_T^{\rm cut}) \times \int_0^1 dz z^{M(p_T^{\rm cut})-1} D(z)\,.$$

Excellent approximation to the effect of the fragmentation function:



P. Nason, G.R.

- At Tevatron energies and p_T, M ranges between 4 and 6
- The second moment (average momentum fraction) of the fragmentation function is well constrained by e^+e^- data.

How large can we make the M^{th} moment of $D(z)$, $\langle z^{M-1} \rangle$, keeping $\langle z \rangle$ fixed?

Imposing the further constraint that $D(z)$ should be monotonically increasing, the maximum $\langle z^{M-1} \rangle$ is achieved with

$$D(z) = A + B\delta(1-z)\,,$$

which gives

$$\langle z \rangle = \frac{A/2 + B}{A+B}\,; \quad \langle z^{M-1} \rangle = \frac{A/M + B}{A+B}\,.$$

We computed numerically the M^{th} moments of different forms:

1. Peterson:

$$D(z) \propto \frac{1}{z\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^2}$$

2. Kartvelishvili:

$$D(z) \propto z^{\alpha}(1-z)$$

for which

$$\langle z^{M-1} \rangle = \frac{\Gamma(\alpha+M)\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\alpha+\beta+M+1)}\,,$$

3. Collins-Spiller:

$$D(z) \propto \frac{\left(\frac{1-z}{z}+\frac{(2-z)\epsilon}{1-z}\right)(1+z^2)}{\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^2}$$

at fixed values of $\langle z \rangle$ corresponding to the choices $\epsilon = 0.002$ and 0.006 in the Peterson form.

$\langle z \rangle = 0.879$	$M = 4$	$M = 5$	$M = 6$
Peterson	0.711	0.649	0.595
Kartvelishvili	0.694	0.622	0.562
Collins-Spiller	0.729	0.677	0.633
Maximal	0.818	0.806	0.798

Table 1: Values of the $4^{\rm th}$, $5^{\rm th}$ and $6^{\rm th}$ moment, at fixed $\langle z \rangle$ (corresponding to $\epsilon = 0.002$ in the Peterson form), for different forms of the fragmentation function.

$\langle z \rangle = 0.828$	$M = 4$	$M = 5$	$M = 6$
Peterson	0.611	0.535	0.474
Kartvelishvili	0.594	0.513	0.447
Collins-Spiller	0.626	0.559	0.505
Maximal	0.742	0.724	0.713

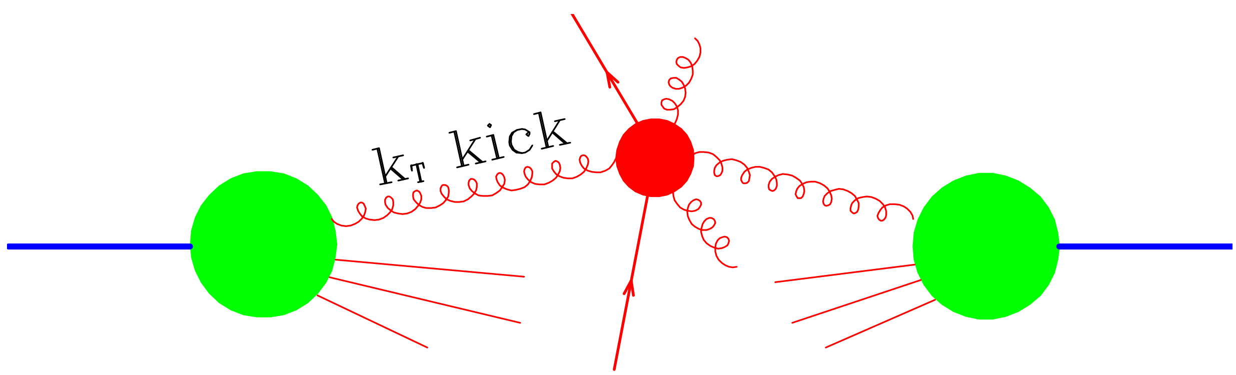
Table 2: Values of the $4^{\rm th}$, $5^{\rm th}$ and $6^{\rm th}$ moment, at fixed $\langle z \rangle$ (corresponding to $\epsilon = 0.006$ in the Peterson form), for different forms of the fragmentation function.

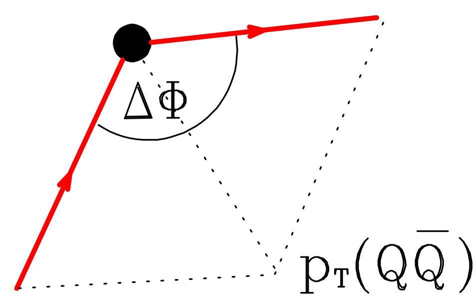
Keeping $\langle z \rangle$ fixed, the enhancement of the hadronic p_T distribution obtained by varying the shape of the fragmentation function among commonly used models is between 5% and 13%.

With the extreme choice $D(z) = A + B\delta(1-z)$ one gets at most a 50% enhancement, for the largest value of ϵ and M (compatible with e^+e^- data?).

Other non-perturbative effects:

A primordial transverse momentum of incoming partons:



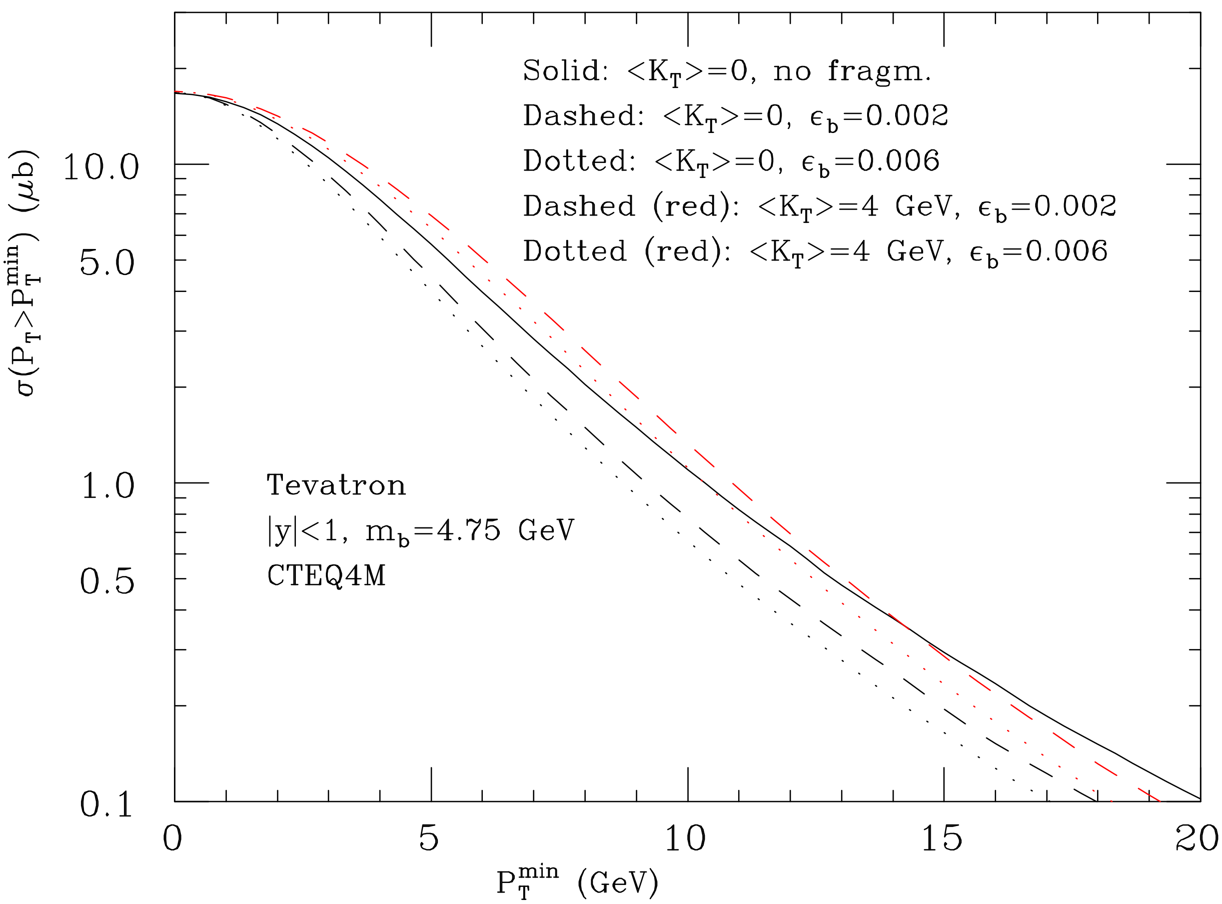


Assuming that partons inside hadrons have a transverse momentum distributed as

$$\frac{1}{N}\frac{dN}{dk_{\mathrm{T}}^2} = \frac{1}{\langle k_{\mathrm{T}}^2\rangle}\exp(-k_{\mathrm{T}}^2/\langle k_{\mathrm{T}}^2\rangle)$$

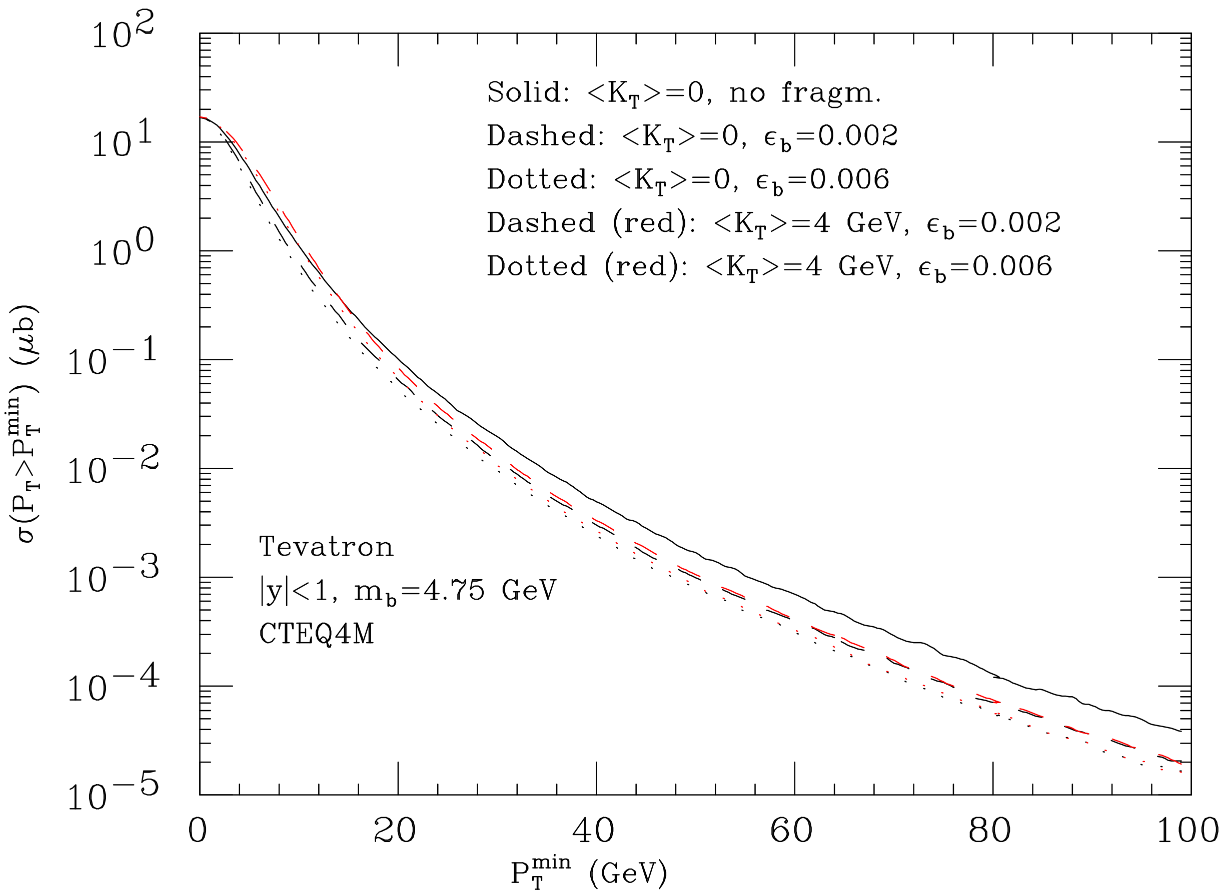
one can attempt fitting both the $P_{\mathrm{T}}(Q\bar{Q})$ and the $\Delta\Phi$ distributions.

What is the effect of intrinsic k_T and fragmentation?



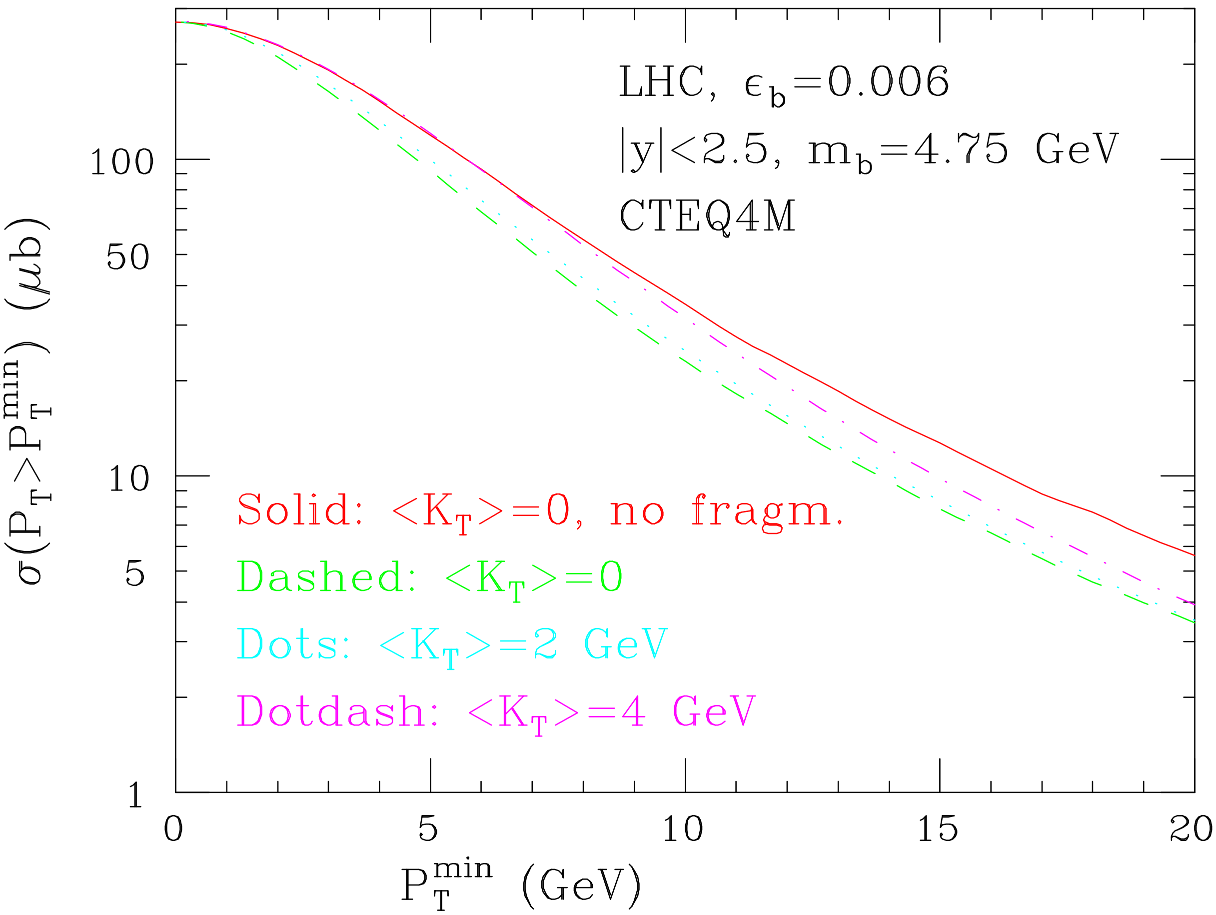
$\langle k_T \rangle$ up to 4 GeV suggested by prompt photon production studies.

N.B.: $\langle k_T \rangle$=4 GeV corresponds to $\sqrt{\langle k_T^2 \rangle} \simeq$5 GeV



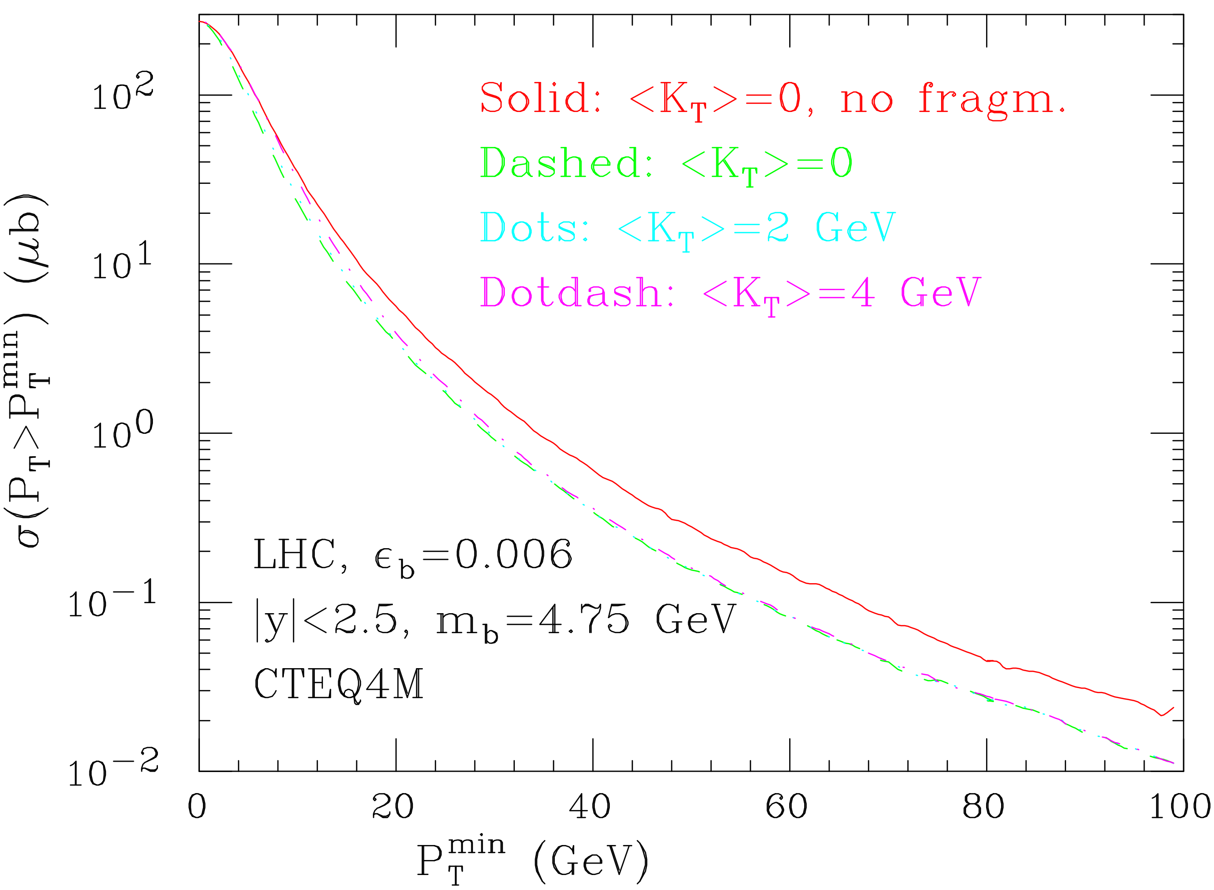
Comments:

- For $p_T^{min} < 20$ GeV, the k_T effect is sizeable, even in presence of fragmentation;
- Uncertainty on the fragmentation function (ϵ parameter) relatively small.



Comments:

- $\langle k_T \rangle = 2$ GeV has almost no effect;
- Even with $\langle k_T \rangle = 4$ GeV the effect is small (never above the pure perturbative result).



At the LHC, the effect of an intrinsic transverse momentum of the colliding partons is much less important.

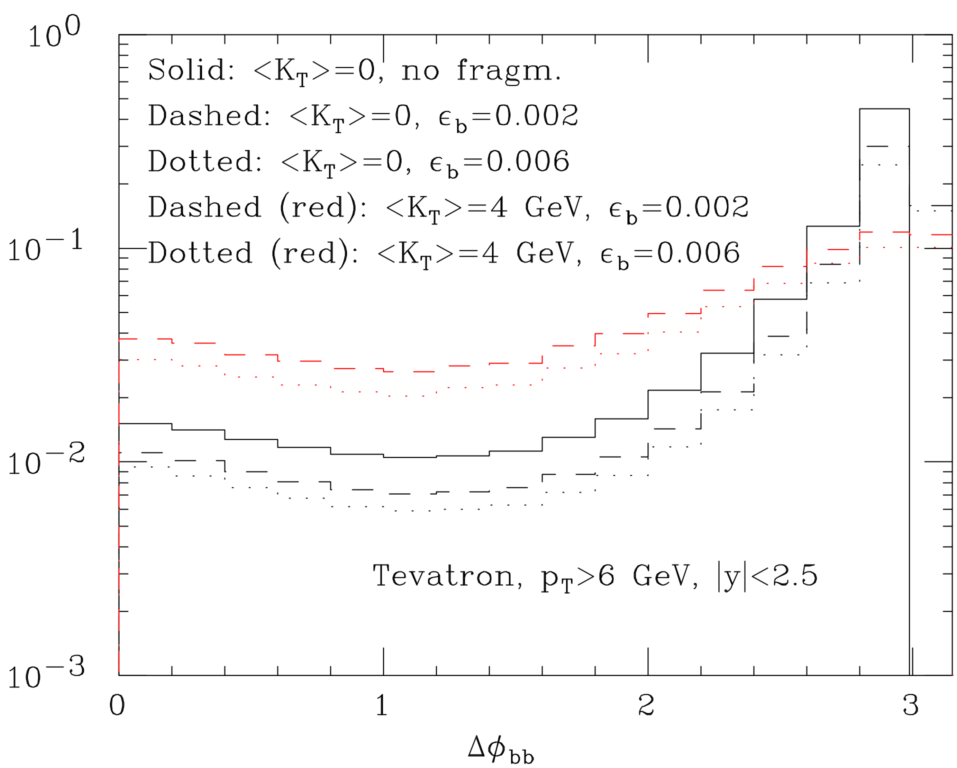
Angular correlation in the transverse plane

Are such large values of $\langle k_T \rangle$ compatible with other observables?

The $\Delta\phi$ distribution is trivial at leading order: b and $\bar{b}$ are emitted back-to-back, so

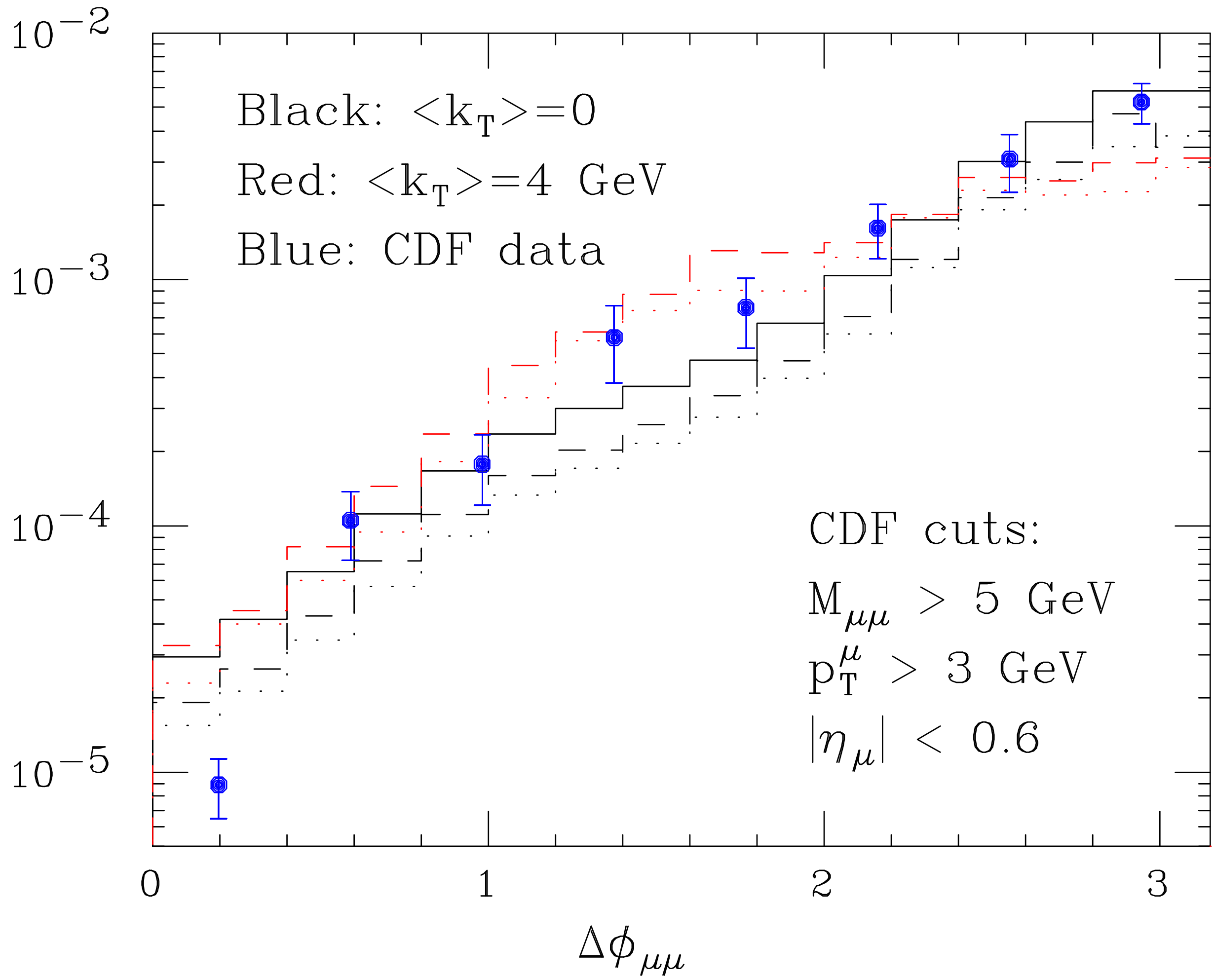
$$\frac{d\sigma}{d\Delta\phi} \propto \delta(\phi - \pi).$$

This correlation is particularly sensitive to the effect of an intrinsic k_T.

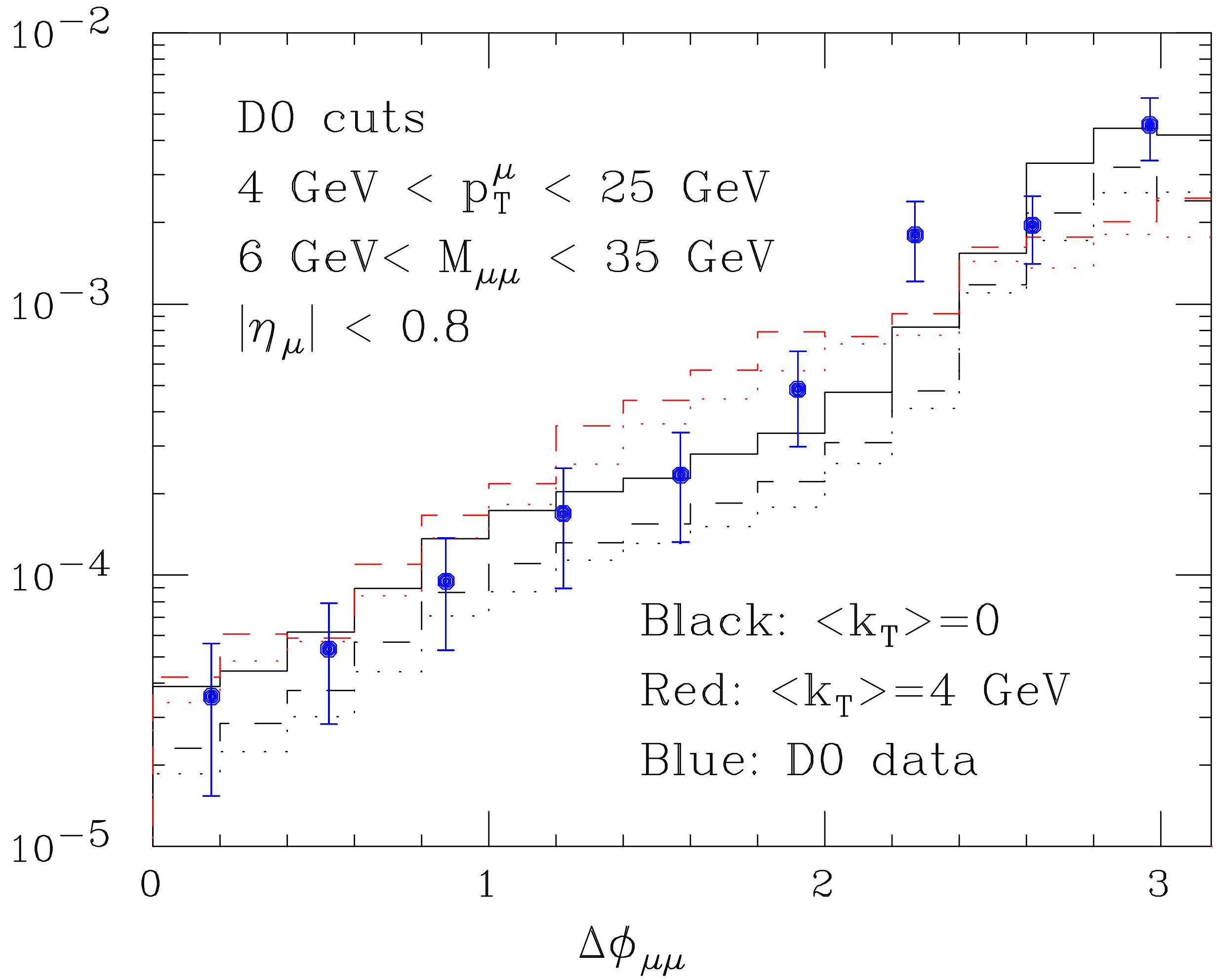


A dramatic effect

CDF and D0 analyses based on cuts on the momenta of muons produced in b decays. We have implemented a rough model of b decays in the FMNR code, in order to reproduce the set of cuts adopted by the Tevatron collaborations.



The data seems to disfavour a large intrinsic k_T effect. Deeper investigation might be useful.



D0 cuts
4 GeV < p_T^μ < 25 GeV
6 GeV< $M_{\mu\mu}$ < 35 GeV
$|\eta_\mu|$ < 0.8
Black: <k_T>=0
Red: <k_T>=4 GeV
Blue: D0 data
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
1
2
3
$\Delta\phi_{\mu\mu}$

Summary and conclusions

A combination of effects may explain the observed excess in the p_t spectrum:

- higher order corrections
- large logarithms of p_T/m
- intrinsic parton k_T

Different fragmentation shapes have small impact.

Accurate study of correlations might prove very useful.