

*Searching for flavour
symmetries: old data, new
tricks*

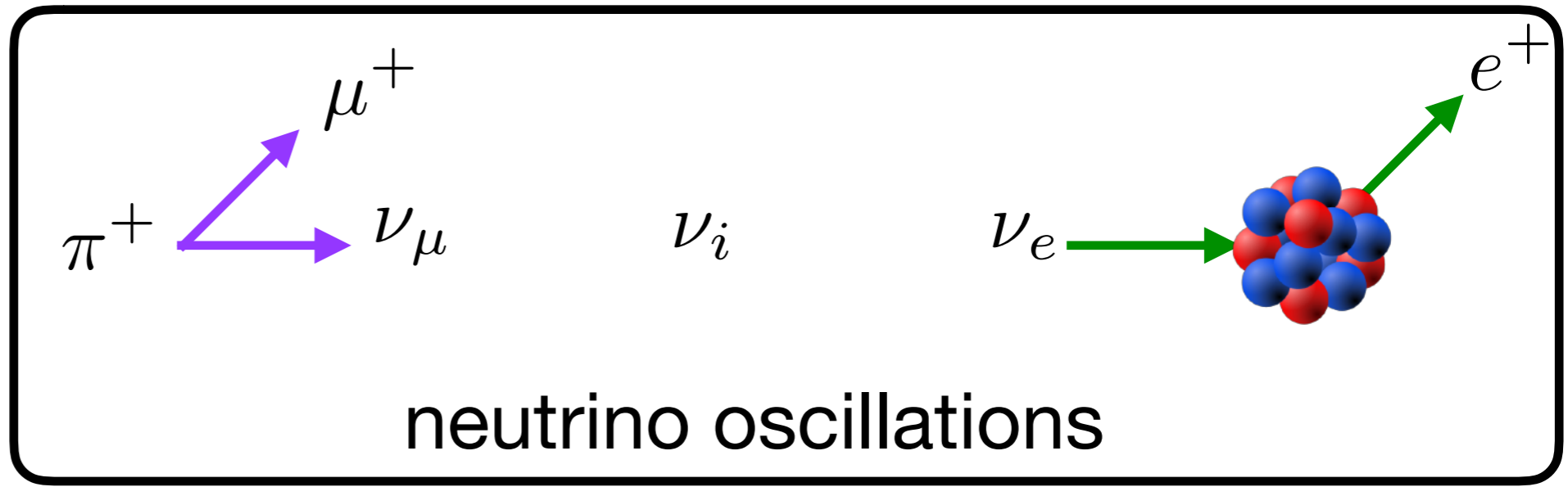
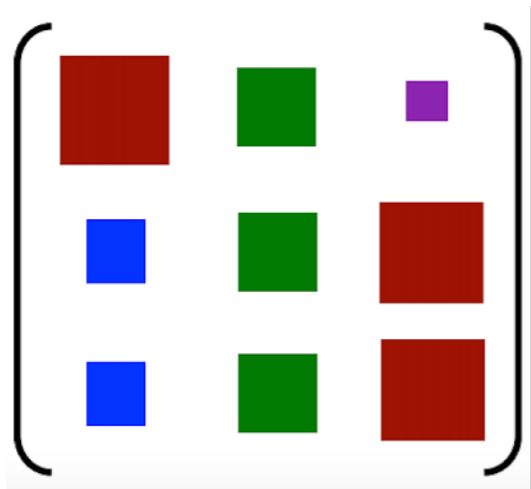
Jessica Turner
Fermilab

LBNL Seminar 20 Mar 2019

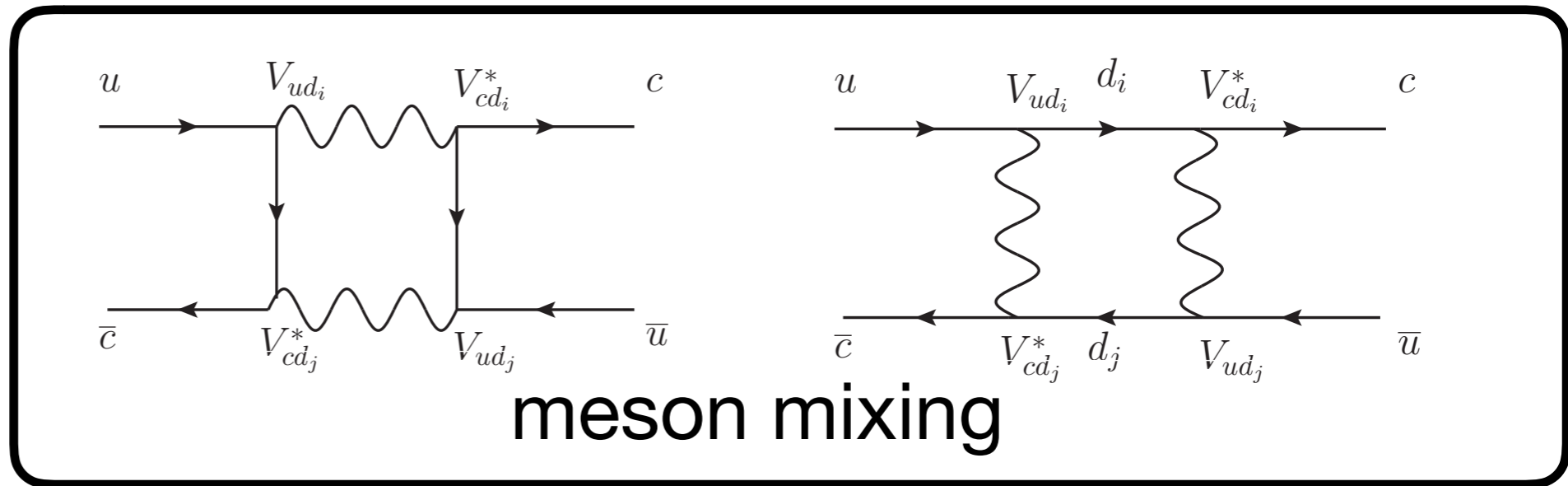
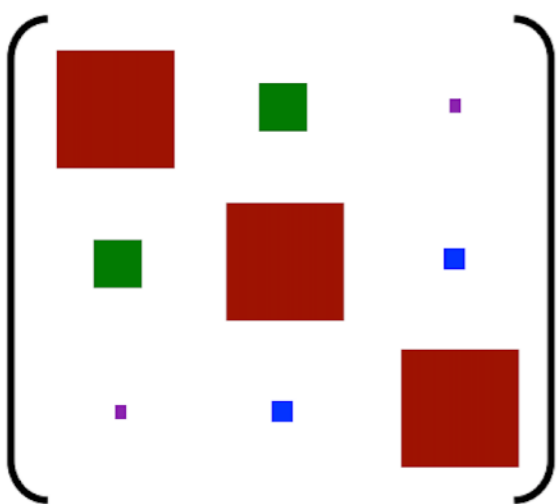
Based on [arXiv:1810.05648](https://arxiv.org/abs/1810.05648) with
L. Heinrich, H. Schulz, Y. L. Zhou,



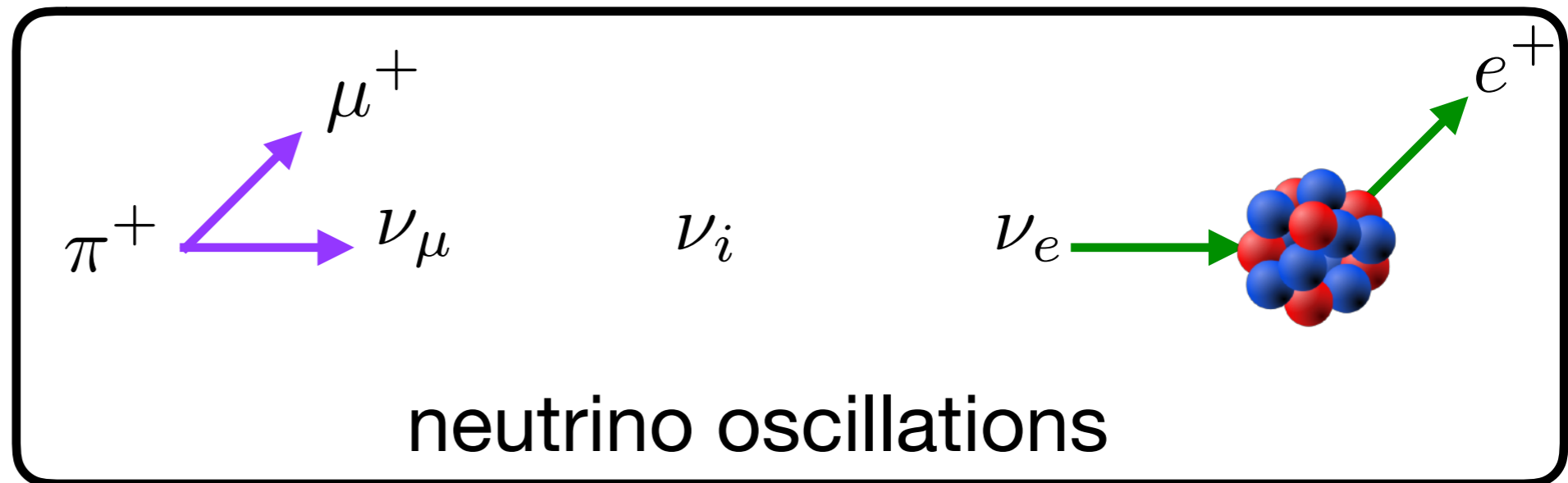
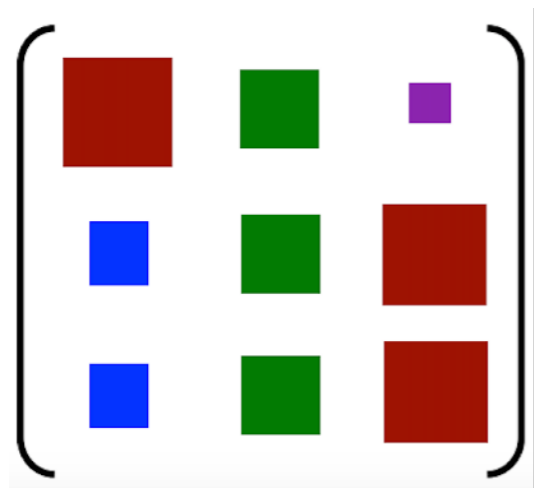
leptonic mixing



quark mixing



leptonic mixing



Why does the leptonic mixing matrix have its peculiar structure?

If a flavour symmetry is present, how can we test it?

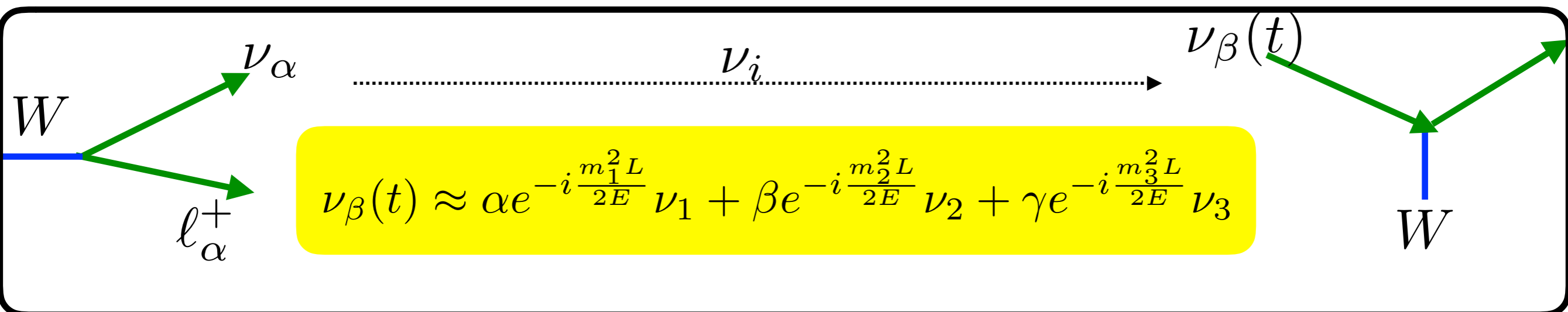
Outline

- Brief Overview of Neutrino Masses and Mixing
- Experimental Status of Leptonic Mixing Matrix
- Basic underlying paradigm and principles of flavour models
- Flavour Model, its parameter space and constraints
- Tool chain and how to calculate exclusion regions
- Results

Current Status of Neutrino Oscillation Parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad U m_\nu U^\dagger = m_\nu \text{diag}$$

flavour states PMNS matrix mass states



$P(\nu_\mu \rightarrow \nu_\mu)$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$ $46.40 \leq \theta_{23}(\circ) \leq 52.40$	$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ $\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}$ $8.22 \leq \theta_{13}(\circ) \leq 8.97$ $133 \leq \delta(\circ) \leq 337$	$P(\nu_\mu \rightarrow \nu_e)$ $\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $31.54 \leq \theta_{12}(\circ) \leq 36.16$
--	---	--

nufit 2018

Current Status of Neutrino Parameters

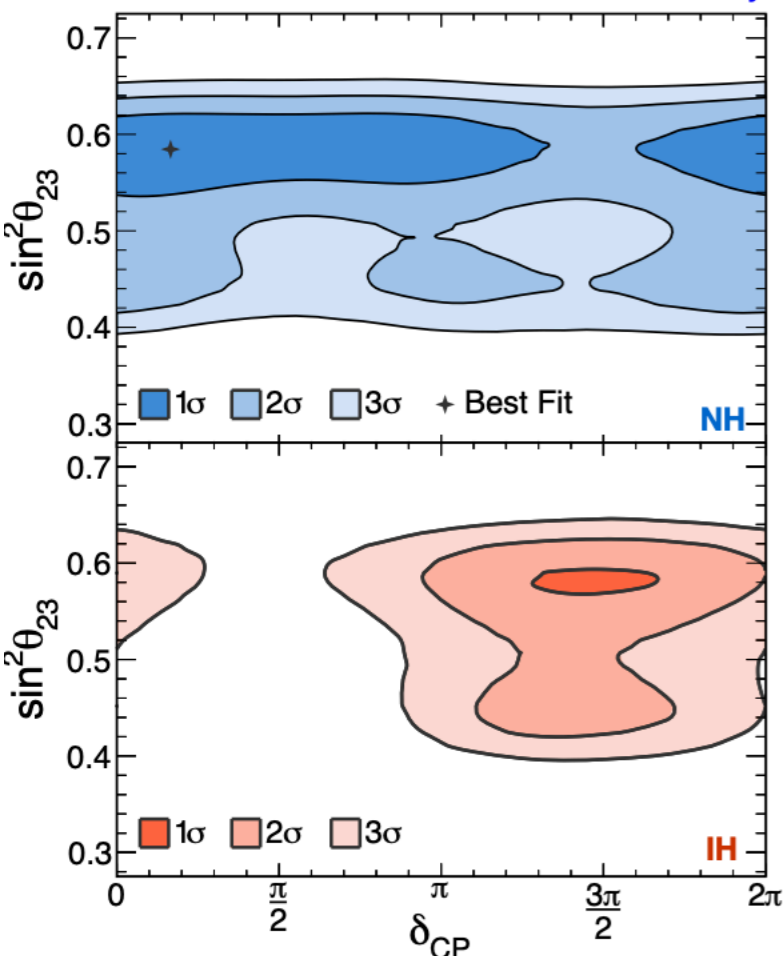
$$6.79 \leq \frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2} \leq 8.02$$

$$2.427 \leq \frac{\Delta m_{3\ell}^2}{10^{-3} \text{eV}^2} \leq 2.625$$

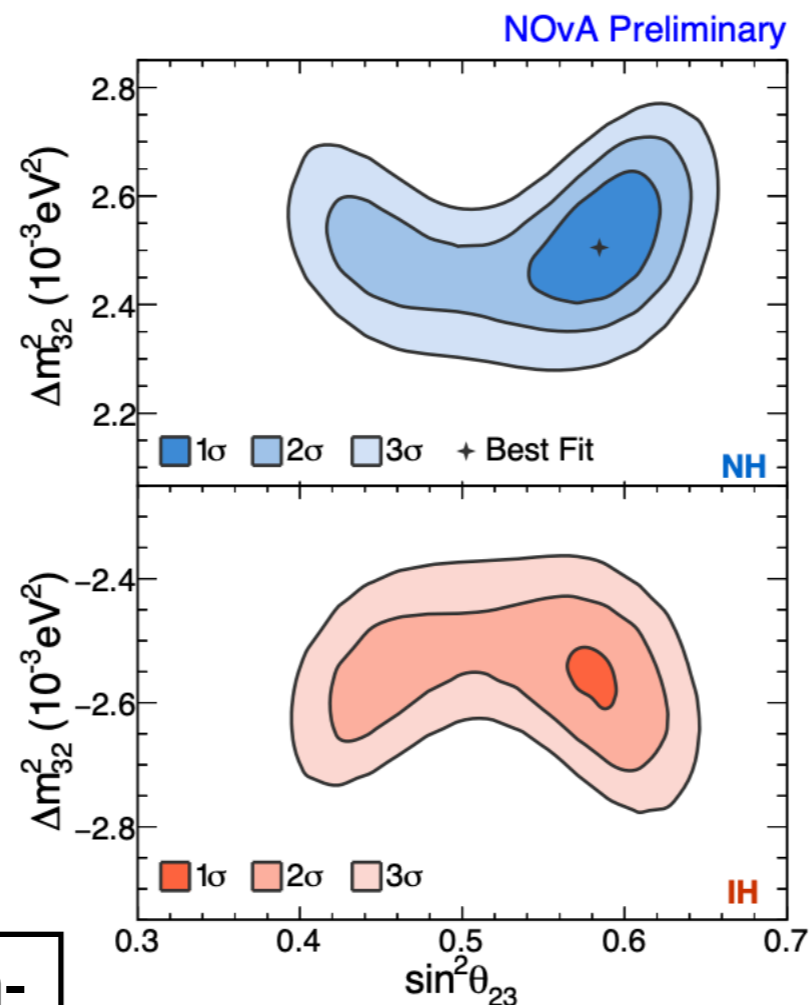
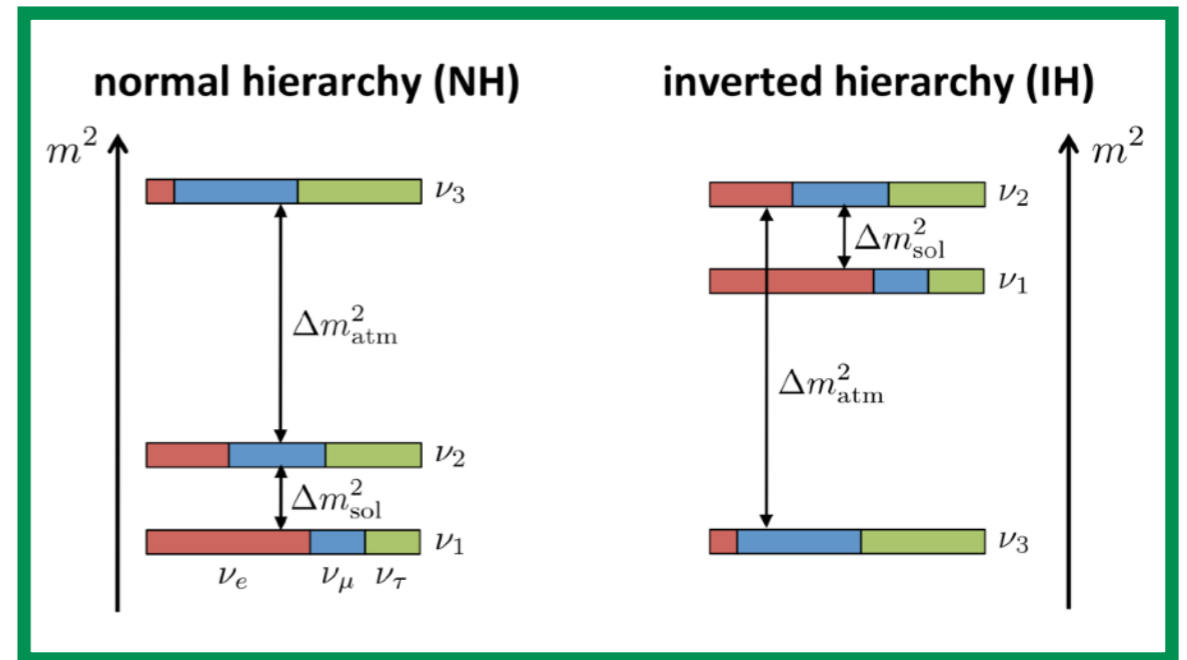
nufit 2018

M. Sanchez,
Neutrino 2018

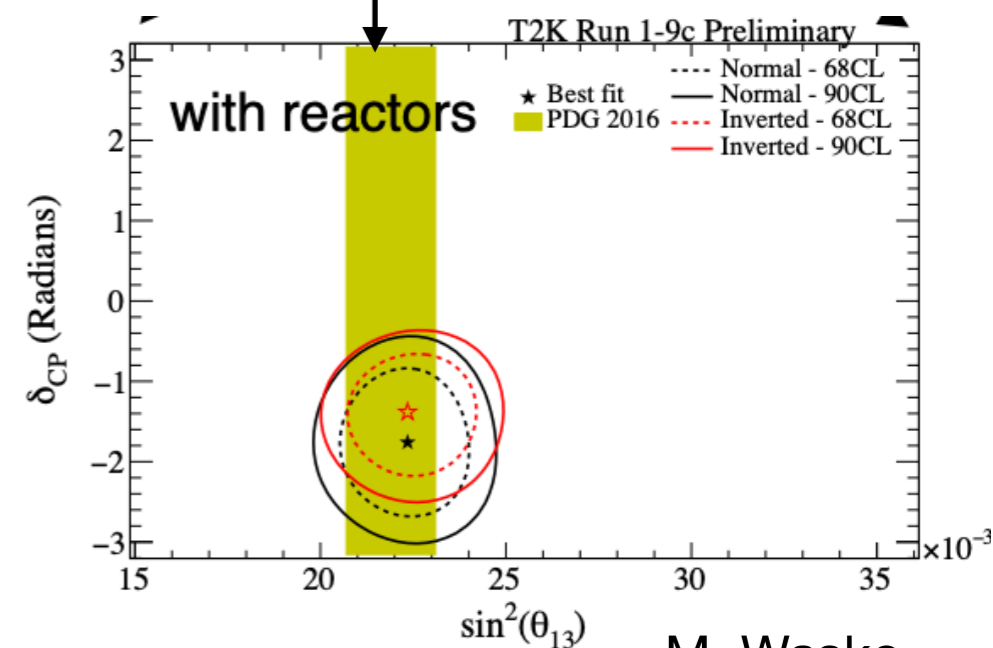
NOvA Preliminary



Slight preference for non-maximality and maximal CPV



Preference for maximal CPV from LBL

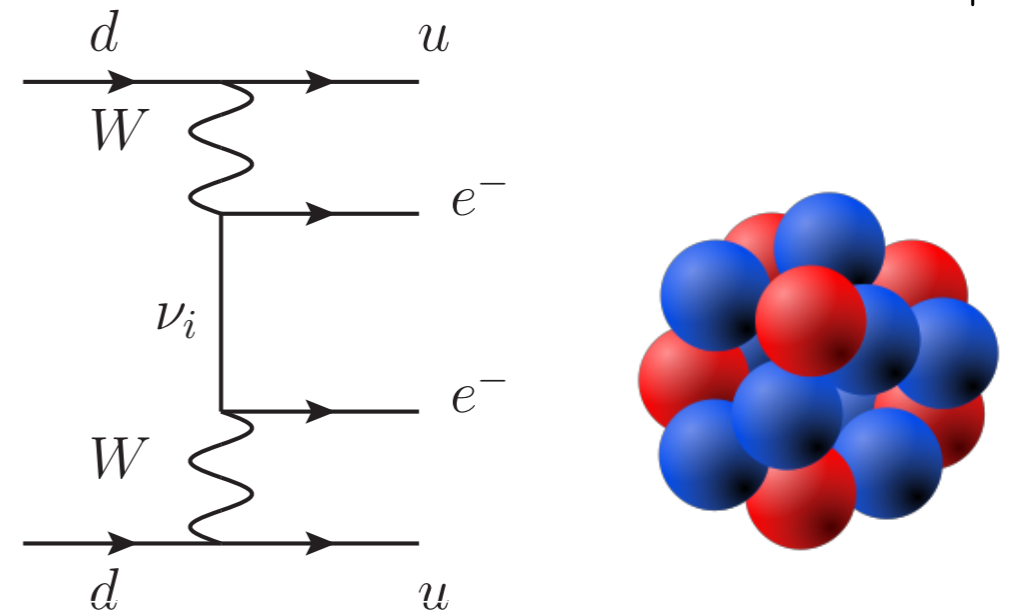
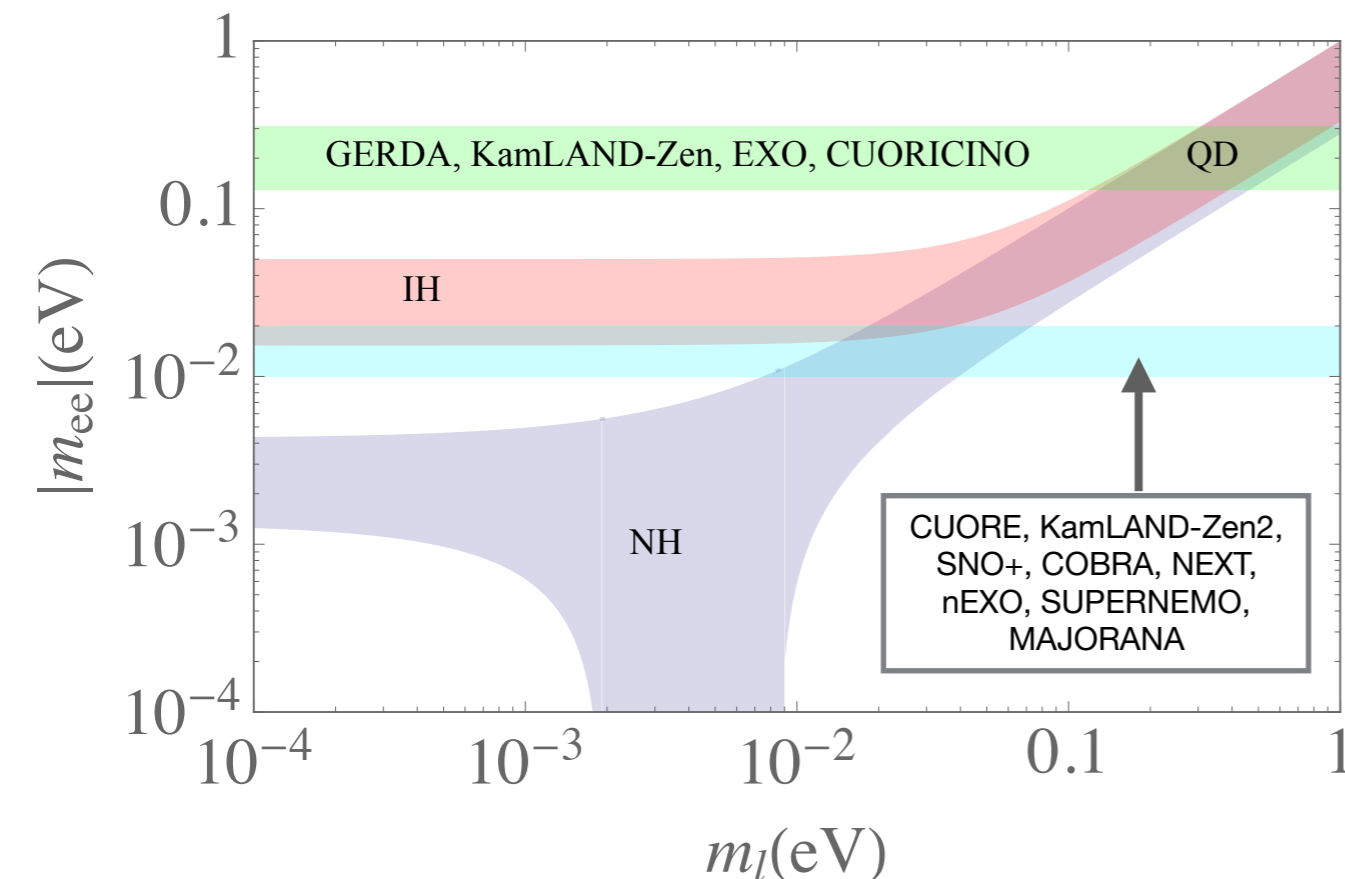


M. Wasko,
Neutrino 2018

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

- Observation of LNV process e.g. neutrinoless double beta decay would indicate neutrinos are Majorana in nature.
- Half life proportionate to effective Majorana mass

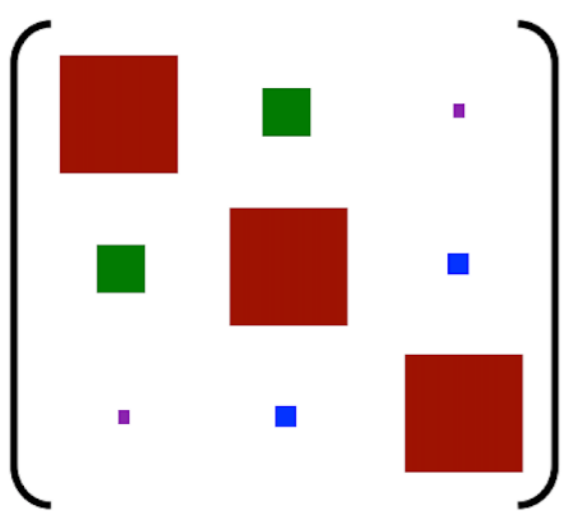
$$|m_{ee}| = |m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{i\alpha_{21}} + m_3 \sin^2 \theta_{13} e^{i(\alpha_{31} - 2\delta)}|$$



NDBD can give information on neutrino mass ordering, absolute mass scale and CPV phases.

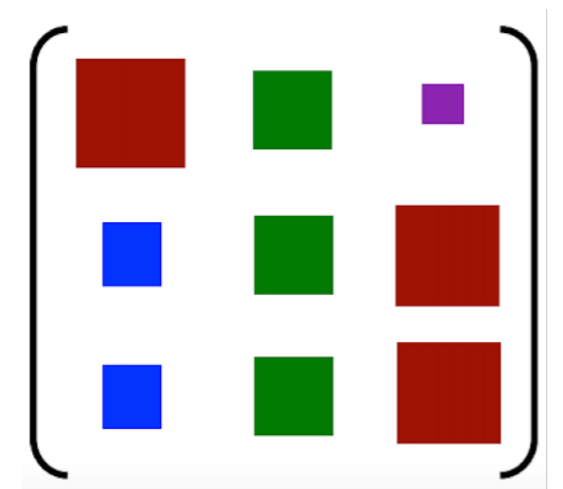
Motivation for Flavour Models

quark mixing



Perturbed
Identity Matrix
Small Mixing
small CPV

leptonic mixing



entries
resemble CG
coefficient of
discrete
groups

Anarchy

PMNS matrix
described as
the result of a
random draw
from unbiased 3
x 3 unitary
matrix

Does not work
for CKM

Symmetry

PMNS matrix
results from the
breaking of a
non-Abelian
symmetry at
high energy
scales

Difficult to apply
to quark sector

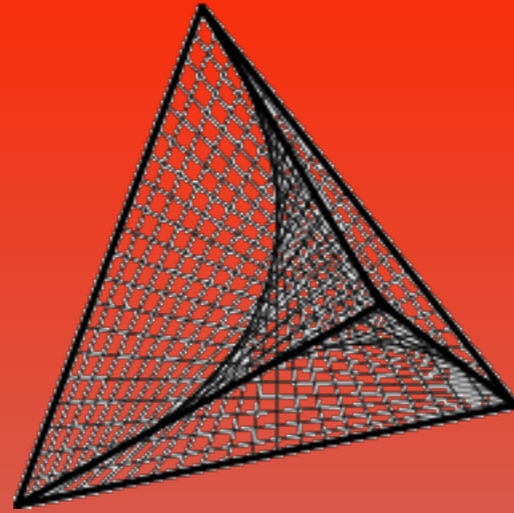
Hall, de Gouvea, Murayama

Altarelli, Everett,
Feruglio, King, Ding,
Hagedorn, Petrov, M. C
Chen, Harrison, Perkins,
Scott, Luhn.....

ENERGY



A_4 unbroken



**Flavour Symmetry
Breaking**



Charged Lepton

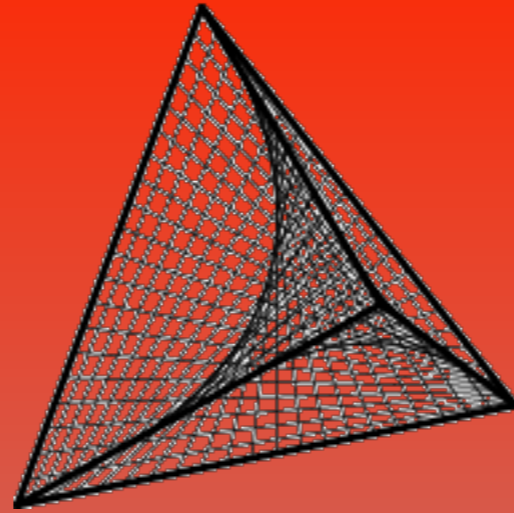
Neutrino Sector

A_4 broken

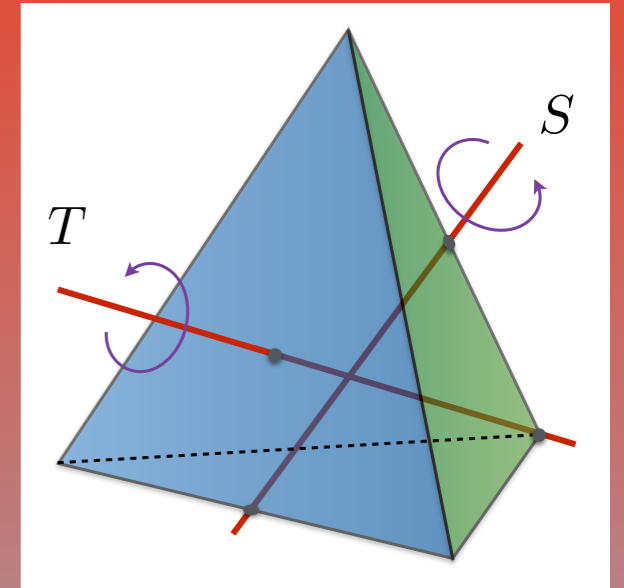
ENERGY



A₄ unbroken



A₄



Flavour Symmetry Breaking

$$\ell_L \rightarrow T\ell_L$$

$$\nu_L \rightarrow S\nu_L$$

$$T^\dagger m_\ell m_\ell^\dagger T = m_\ell m_\ell^\dagger$$

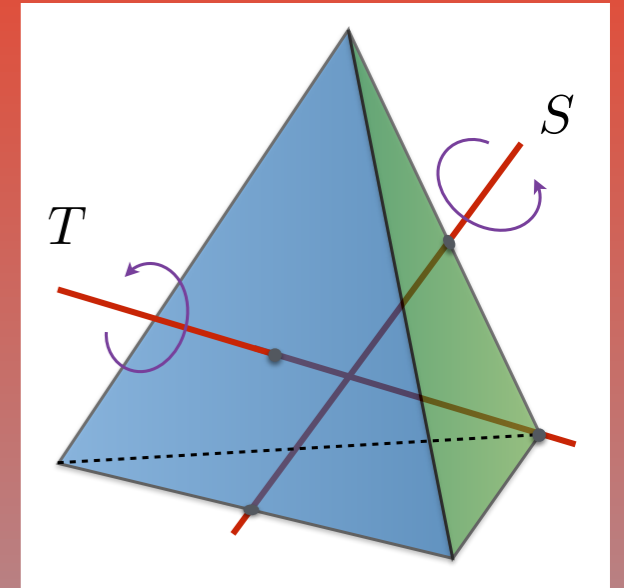
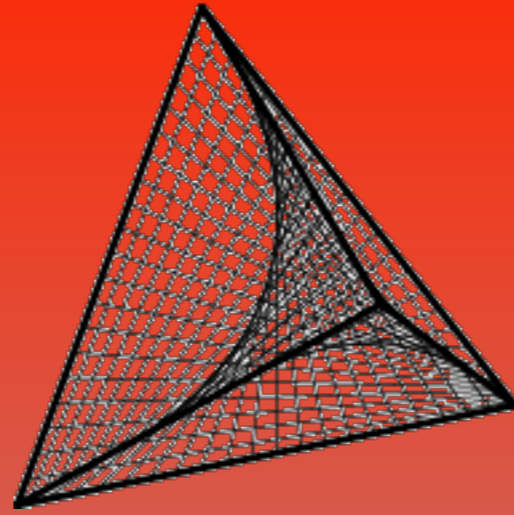
$$S^T m_\nu S = m_\nu$$

A₄ broken

ENERGY

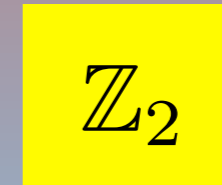
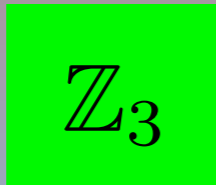


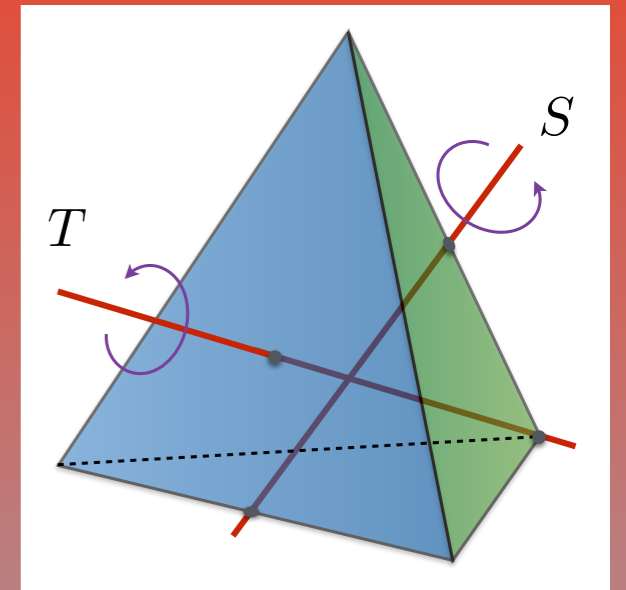
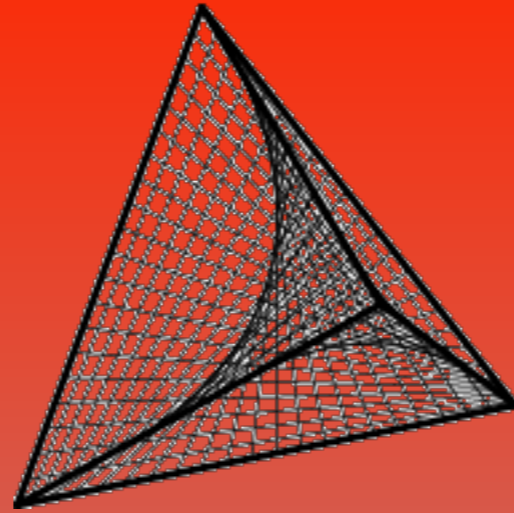
A₄ unbroken



**Flavour Symmetry
Breaking**

A₄ broken

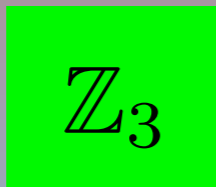




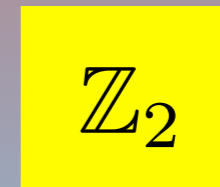
Alteralli-Feruglio
Basis

Flavour Symmetry
Breaking

$$\omega = e^{\frac{2\pi i}{3}}$$



$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

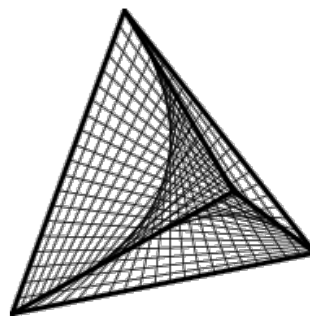


$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Field Content

$$T\langle\varphi\rangle = \langle\varphi\rangle \quad S\langle\chi\rangle = \langle\chi\rangle$$

flavon
pseudo-real
triplets



$$\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \sim \mathbf{3}, \quad \chi = (\chi_1, \chi_2, \chi_3)^T \sim \mathbf{3}$$

$$\ell_L = (\ell_{eL}, \ell_{\mu L}, \ell_{\tau L})^T \sim \mathbf{3}, \quad e_R \sim \mathbf{1}, \quad \mu_R \sim \mathbf{1}', \quad \tau_R \sim \mathbf{1}''$$

SM
fields

Lagrangian terms for charged lepton and neutrinos

$$-\mathcal{L}_l = \frac{y_e}{\Lambda} (\bar{\ell}_L \varphi)_1 e_R H + \frac{y_\mu}{\Lambda} (\bar{\ell}_L \varphi)_{1''} \mu_R H + \frac{y_\tau}{\Lambda} (\bar{\ell}_L \varphi)_{1'} \tau_R H + \text{h.c.},$$

$$-\mathcal{L}_\nu = \frac{y_1}{2\Lambda\Lambda_W} ((\bar{\ell}_L \tilde{H} \tilde{H}^T \ell_L^c)_{\mathbf{3}_S} \chi)_1 + \frac{y_2}{2\Lambda_W} (\bar{\ell}_L \tilde{H} \tilde{H}^T \ell_L^c)_1 + \text{h.c.}$$

Assume neutrino Majorana

$$\langle\varphi\rangle = (1, 0, 0)^T \frac{v_\varphi}{\sqrt{n}}$$

$$\langle\chi\rangle = (1, 1, 1)^T \frac{v_\chi}{\sqrt{3n}}$$

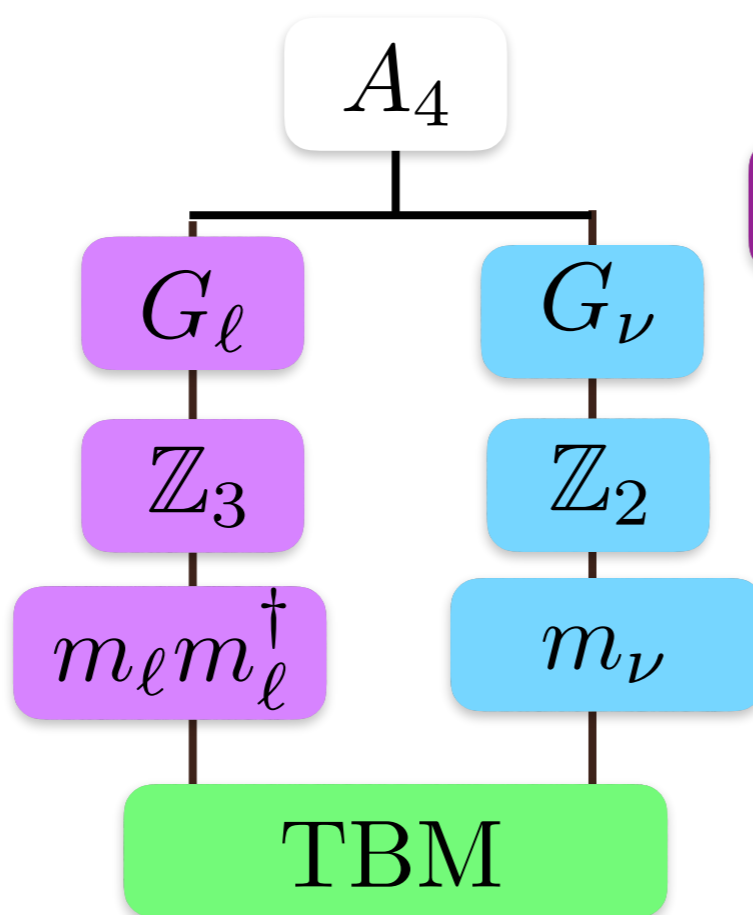
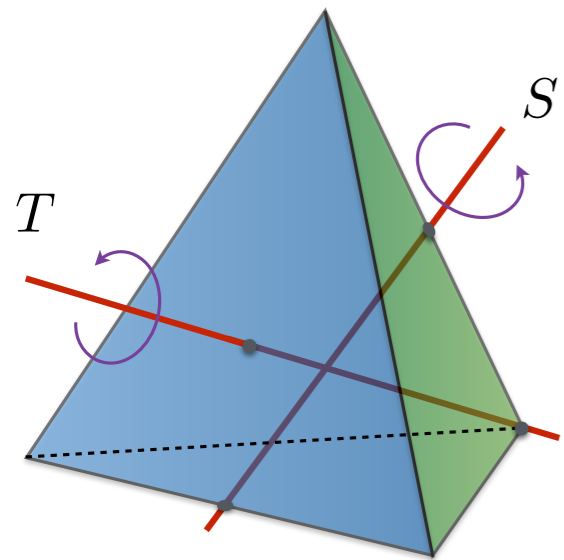
$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{v v_\varphi}{\sqrt{2n\Lambda}}$$

$$M_\nu = \begin{pmatrix} 2a + b & -a & -a \\ -a & 2a & -a + b \\ -a & -a + b & 2a \end{pmatrix}$$

Results in TBM mixing of PMNS matrix

$$a \equiv y_1 v_\chi v^2 / (4\sqrt{3n\Lambda\Lambda_W})$$

$$b \equiv y_2 v^2 / 2\Lambda_W$$



- Need corrections to TBM
- break Z_2 or Z_3
- modify mass matrices
- sizeable θ_{13} and δ

$$U_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$V_{Z_3}(\varphi) = \frac{1}{2} A(\varphi_2^2 + 2\varphi_1\varphi_2^*) + \text{h.c.},$$

Parametrises EXPLICIT breaking of Z_3

How does this flavour sector
communicate with us?

Scalar Sector

- Higgs-Flavon cross-coupling

$$V_{\text{cross}}(H, \varphi) = \frac{1}{2} \epsilon H^\dagger H (\varphi\varphi)_1$$

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2. \quad (\varphi\varphi)_1 = (\varphi_1^2 + 2\varphi_2\varphi_2^*)$$

- Flavon potential

$$V(\varphi) = \frac{1}{2} \mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2, \quad I_{2\varphi} = \frac{1}{3} \varphi_1^4 - \frac{2}{3} \varphi_1 (\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Scalar Sector

- Higgs-Flavon cross-coupling

$$V_{\text{cross}}(H, \varphi) = \frac{1}{2} \epsilon H^\dagger H (\varphi\varphi)_1$$

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2. \quad (\varphi\varphi)_1 = (\varphi_1^2 + 2\varphi_2\varphi_2^*)$$

- Flavon potential

$$V(\varphi) = \frac{1}{2} \mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2, \quad I_{2\varphi} = \frac{1}{3} \varphi_1^4 - \frac{2}{3} \varphi_1 (\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Scalar Sector

- Higgs-Flavon cross-coupling

$$V_{\text{cross}}(H, \varphi) = \frac{1}{2} \epsilon H^\dagger H (\varphi\varphi)_1$$

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2. \quad (\varphi\varphi)_1 = (\varphi_1^2 + 2\varphi_2\varphi_2^*)$$

6 real parameters

- Flavon potential

$$V(\varphi) = \frac{1}{2} \mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2, \quad I_{2\varphi} = \frac{1}{3} \varphi_1^4 - \frac{2}{3} \varphi_1 (\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Yukawa Sector

charged lepton flavour conserving

$$-\mathcal{L}_{\text{clfc}}^{\tilde{h}, \tilde{\varphi}_1} = \sum_{l=e, \mu, \tau} \frac{m_l}{v_H} \bar{l} l \tilde{h} + \frac{m_l}{v_\varphi} \bar{l} l \tilde{\varphi}_1 + \frac{m_l}{v_H v_\varphi} \bar{l} l \tilde{\varphi}_1 \tilde{h},$$

$$\begin{aligned} -\mathcal{L}_{\text{clfv}}^{\tilde{\varphi}_2} &= \frac{m_e}{v_\varphi} (\bar{\mu}_L e_R \tilde{\varphi}_2 + \bar{\tau}_L e_R \tilde{\varphi}_2^*) + \frac{m_e}{v_H v_\varphi} (\bar{\mu}_L e_R \tilde{\varphi}_2 + \bar{\tau}_L e_R \tilde{\varphi}_2^*) \tilde{h} \\ &+ \frac{m_\mu}{v_\varphi} (\bar{\tau}_L \mu_R \tilde{\varphi}_2 + \bar{e}_L \mu_R \tilde{\varphi}_2^*) + \frac{m_\mu}{v_H v_\varphi} (\bar{\tau}_L \mu_R \tilde{\varphi}_2 + \bar{e}_L \mu_R \tilde{\varphi}_2^*) \tilde{h} \\ &+ \frac{m_\tau}{v_\varphi} (\bar{e}_L \tau_R \tilde{\varphi}_2 + \bar{\mu}_L \tau_R \tilde{\varphi}_2^*) + \frac{m_\tau}{v_H v_\varphi} (\bar{e}_L \tau_R \tilde{\varphi}_2 + \bar{\mu}_L \tau_R \tilde{\varphi}_2^*) \tilde{h} + \text{h.c.}, \end{aligned}$$

Final state tau dominated

charged lepton flavour violating

Model Parameter Space

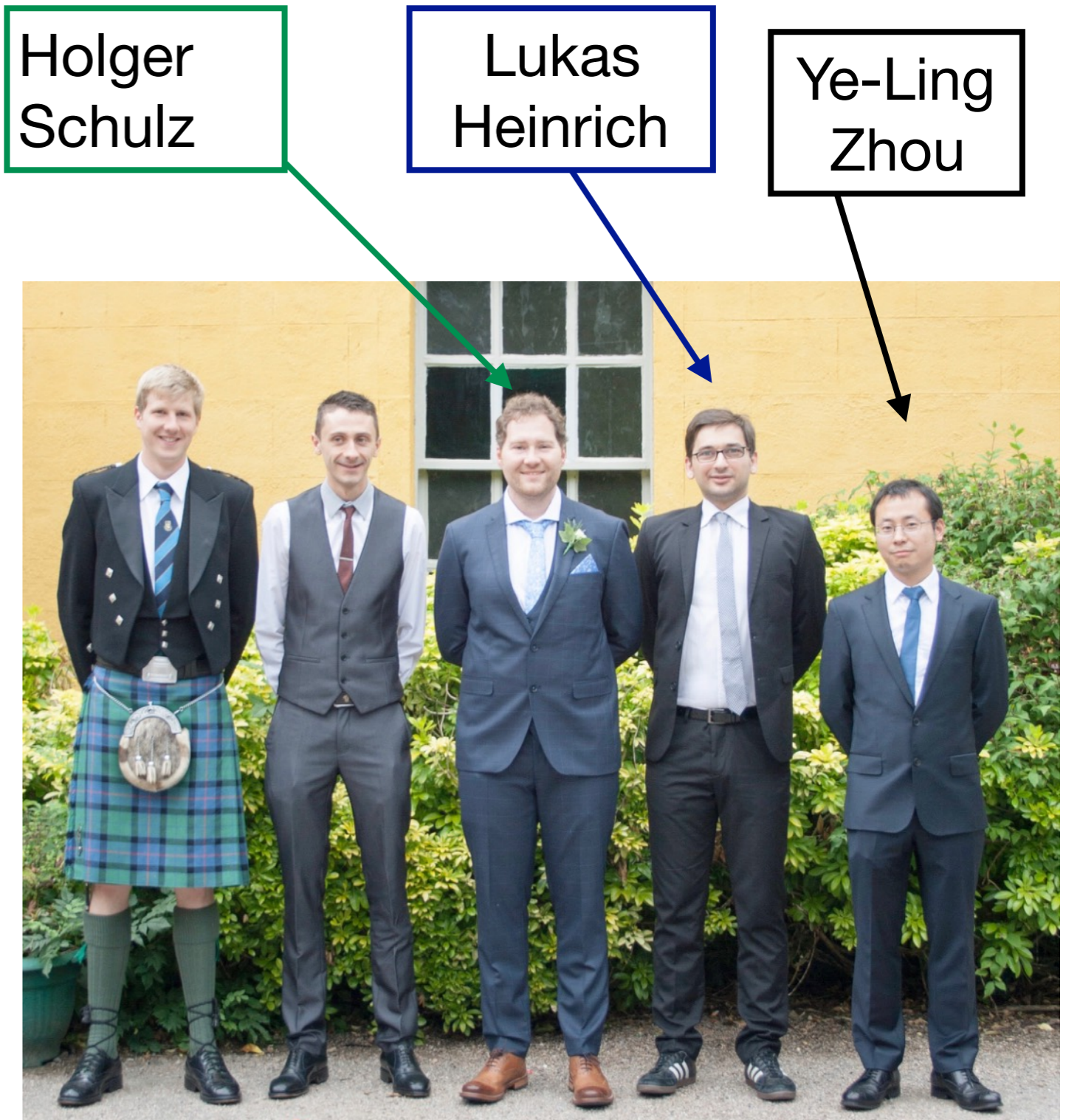
Parameter p	$\min(p)$	$\max(p)$
$\log_{10}(v_\varphi)$	1	3
$\log_{10}(\varepsilon)$	-3	0.5
$\log_{10}(g_1)$	-4	0
$-\log_{10}(g_2)$	-4	0
$\log_{10}(\epsilon_\varphi)$	-3	0.5
θ_φ	0	2π

Table 1: Parameter sampling boundaries.

1. Any flavon mass is too light, i.e. $m(s_i) < 10$ GeV, $i = 1 \dots 3$.
2. All flavon masses are > 1 TeV.
3. Any flavon mass is too close to the Higgs — $|m(s_i) - m_H| < 5$ GeV for $i = 1, 2, 3$.
4. Any flavon mass which is not the Higgs is close to degenerate — $|m(s_i) - m(s_j)| < 100$ MeV for $i, j = 1, 2, 3$.
5. $\lambda g < \frac{\varepsilon}{4}$
6. $g_1 + \frac{g_2}{3} < 0$

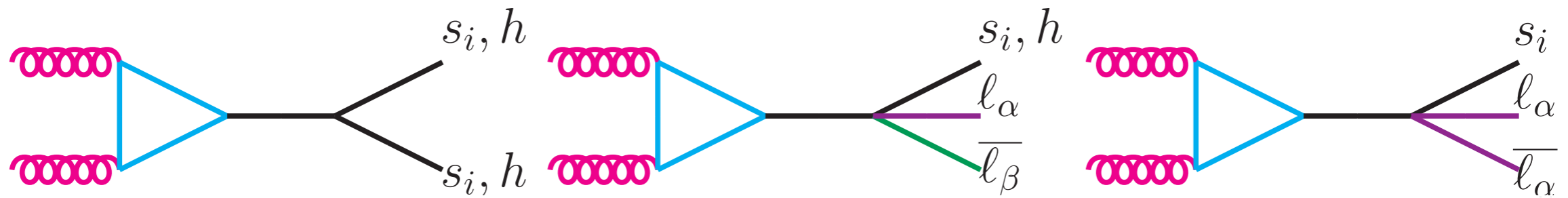
Conditions for Physicality

- **Model:** and Z_3 -breaking (realistic) flavour model
- **Identify:** collider signatures
- **Constraints:** Higgs-width, Higgs-scalar mixing, $g-2$, CLFV BRs.
- **Analysis:** recast 8 TeV ATLAS multi-lepton search
- **Tools:** MC event generation and CL_s method.
- **Results: 1810.05648**



Collider Constraints

- Flavons mix with the Higgs and decay via CLFV and CLFC processes.



- Measured upper limits Higgs width ~ 22 MeV versus 4 MeV SM calculation

1405.3455

- Make sure the Higgs is mostly comprised of the Higgs mass eigenstate.

Robens, Stefaniak, Pruna,
Godunov, Roznanov, Vysotsky, Zhemchugov

1303.1150, 1501.02234,
1503.01618, 1502.01361

G-2 and MEG Constraints



E821 (BNL) measures muon anomalous magnetic moment

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.87 \pm 0.8) \times 10^{-9} \quad (3.6\sigma)$$

0602035, 1311.2198

MEG experiment measures $\mu \rightarrow e\gamma$

1605.05081

$$\text{Br}(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13} \quad \text{at 90\% C.L.}$$

The Collider Analysis in a nutshell

ATLAS Analysis: 8 TeV

1411.2921

Search for new phenomena in events with three or more charged leptons in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector

20.3⁻¹ fb

no-OSSF

OSSF

distributions have not been corrected for detector effect i.e. not unfolded

no-OSSF
 $\geq 3e/\mu$

S1

no-OSSF
 $2e/\mu \geq 1\tau_{had}$

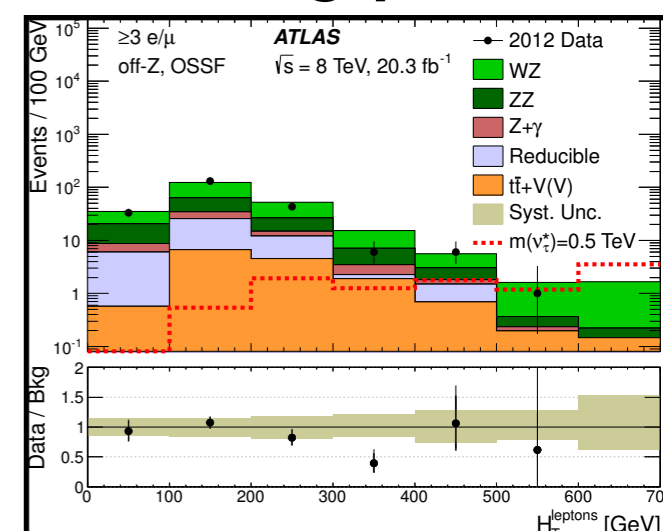
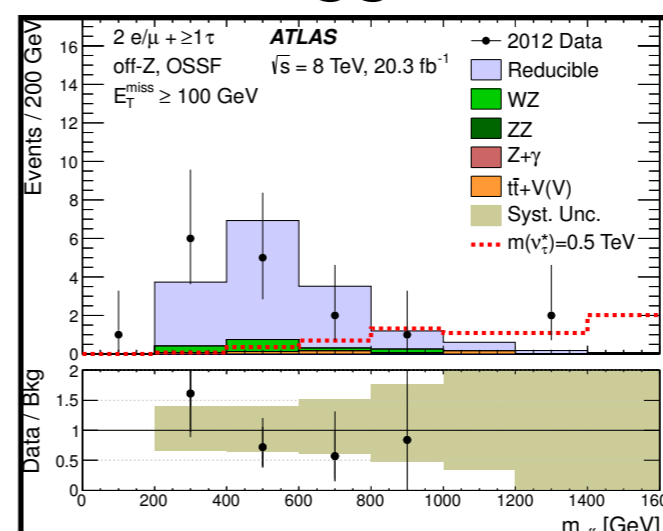
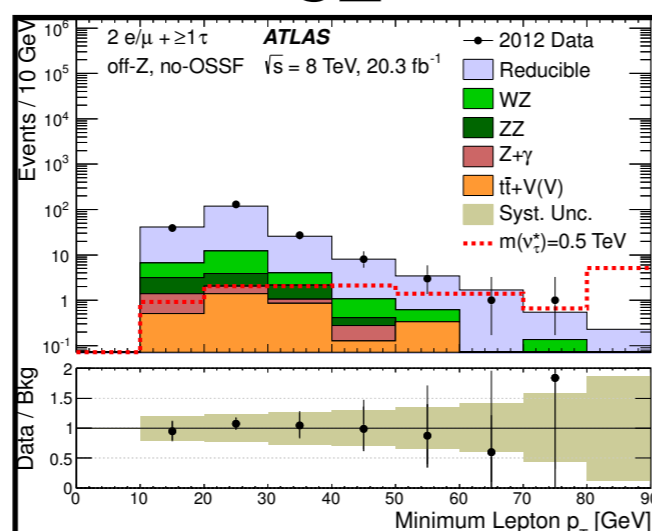
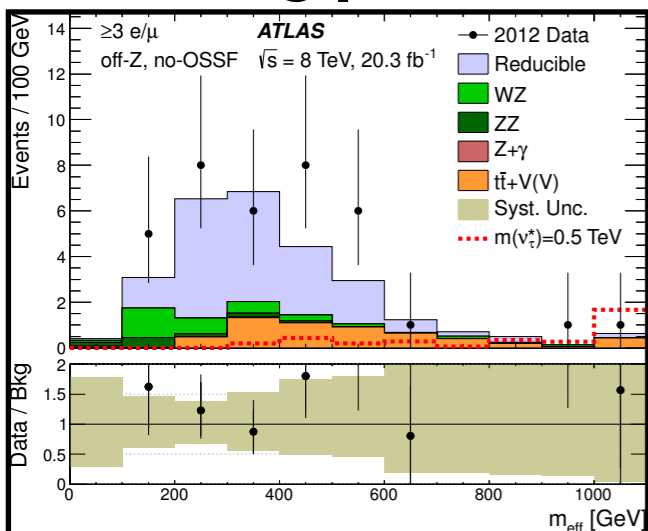
S2

OSSF
 $\geq 3e/\mu$

S3

OSSF
 $2e/\mu \geq 1\tau_{had}$

S4



m_{eff} : effective mass of event combining sum of jets, missing energy and lepton p_T

H_T^{lepton} : scalar sum of lepton p_T used to characterise event

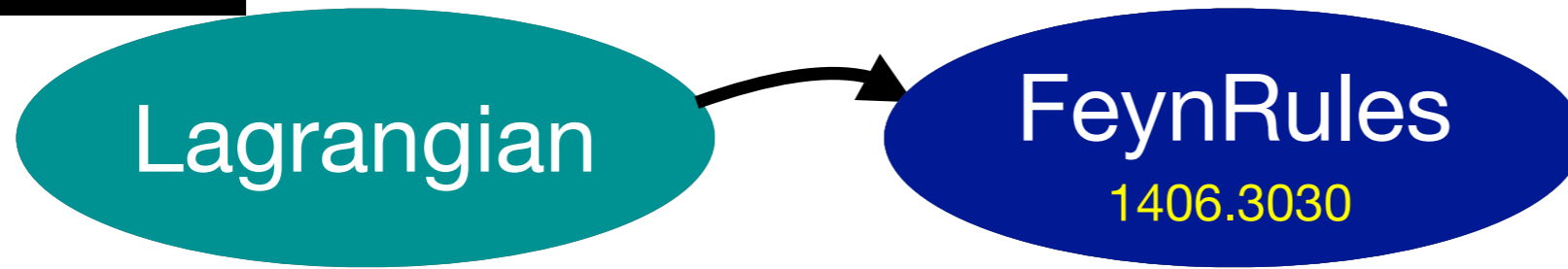
Tool Chain

SM + flavon
interactions

Lagrangian

Tool Chain

SM + flavon interactions



Tool Chain

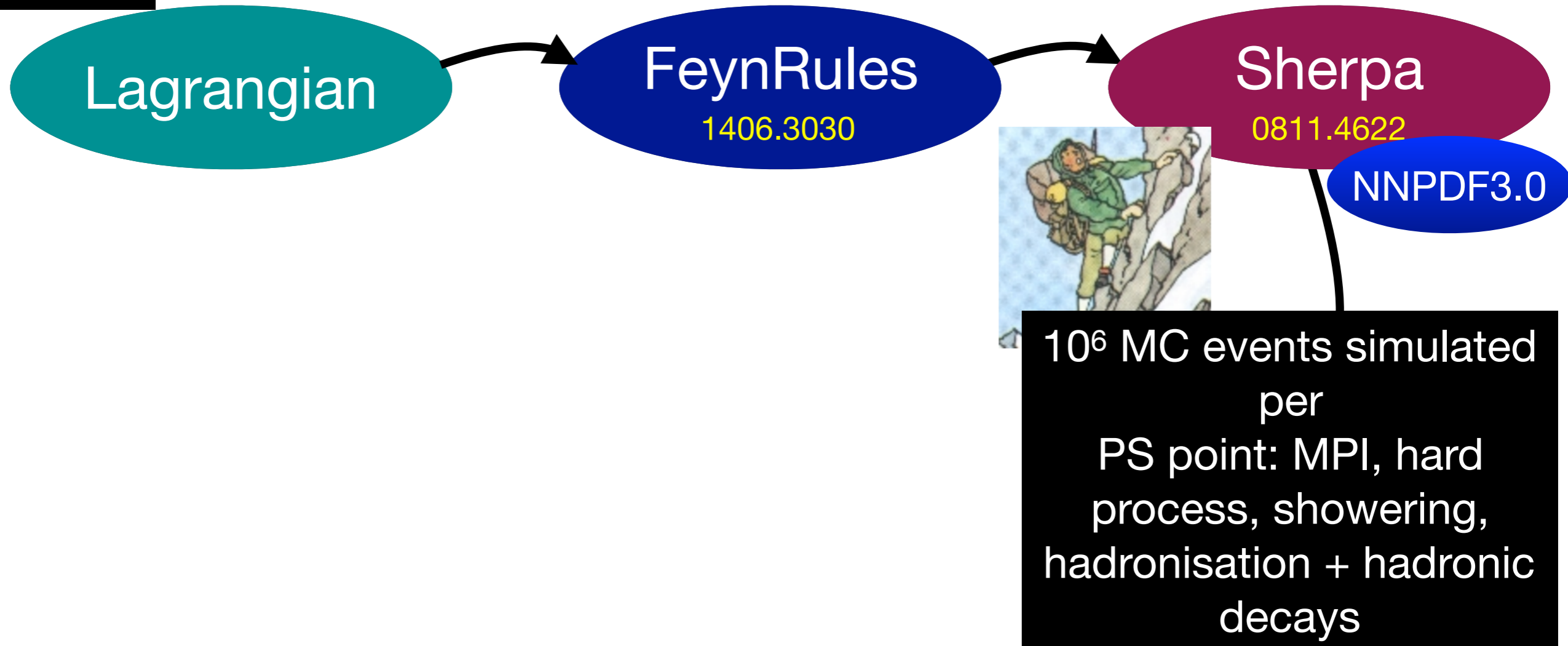
SM + flavon interactions



Thanks to UK HEP Grid Computing for resources
Fermilab

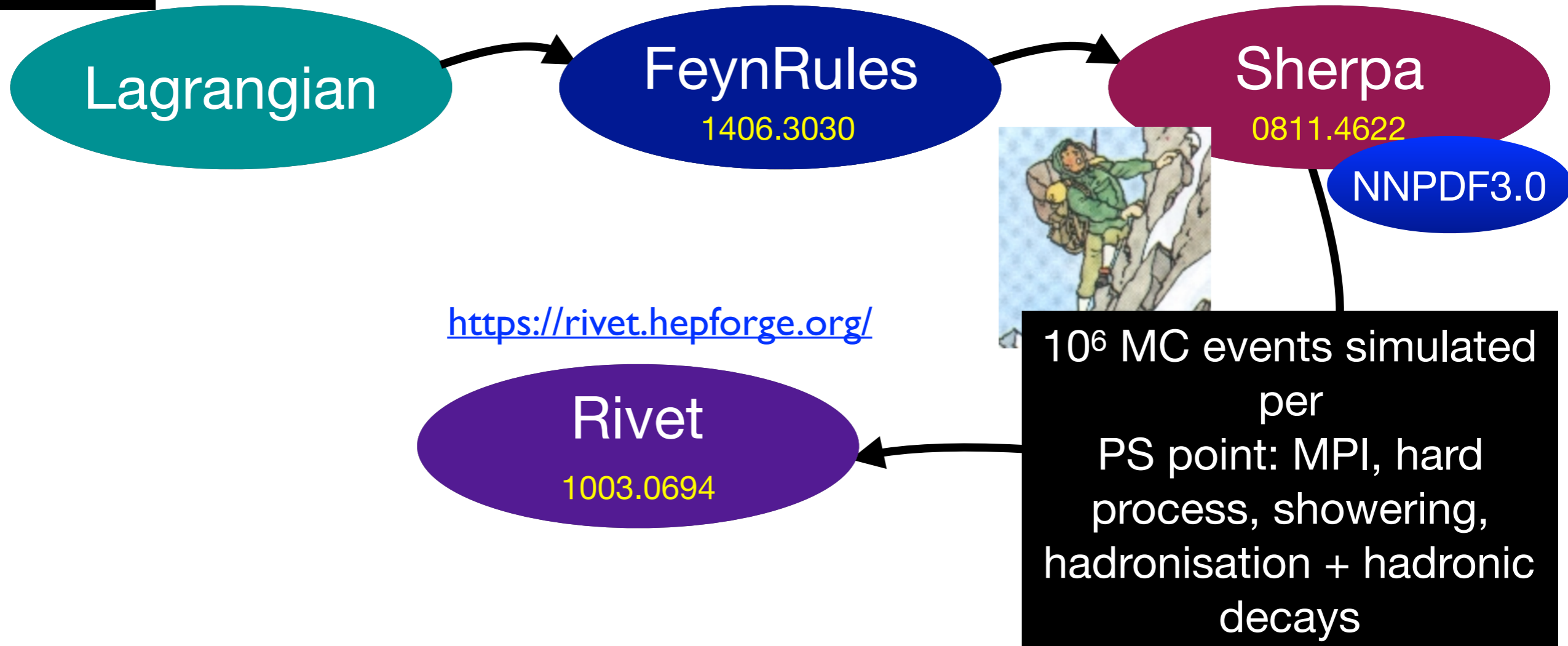
Tool Chain

SM + flavon interactions



Tool Chain

SM + flavon interactions



Tool Chain

SM + flavon interactions

Lagrangian

FeynRules
1406.3030

Sherpa
0811.4622


NNPDF3.0



10⁶ MC events simulated per PS point: MPI, hard process, showering, hadronisation + hadronic decays

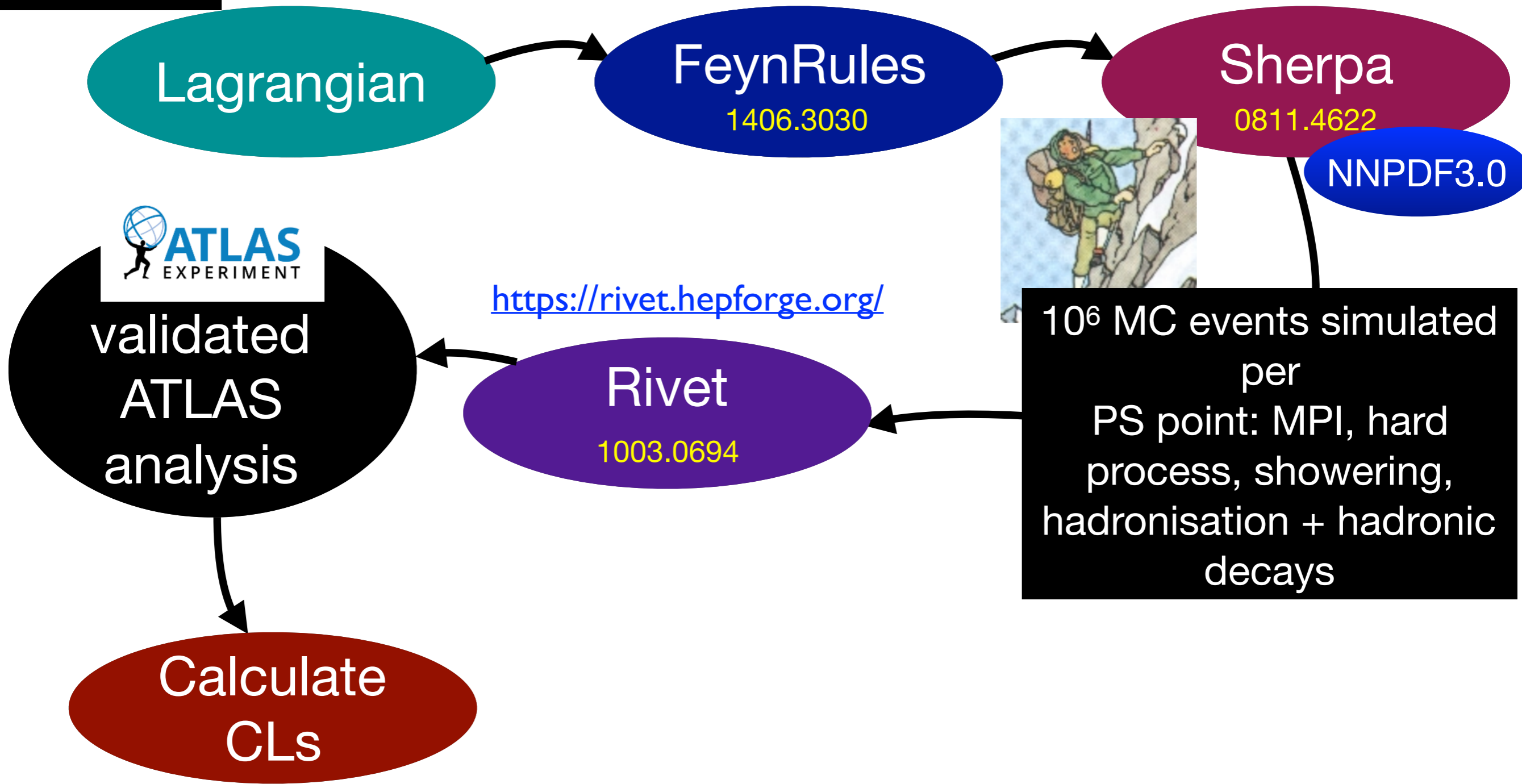
<https://rivet.hepforge.org/>

Rivet
1003.0694


validated ATLAS analysis

Tool Chain

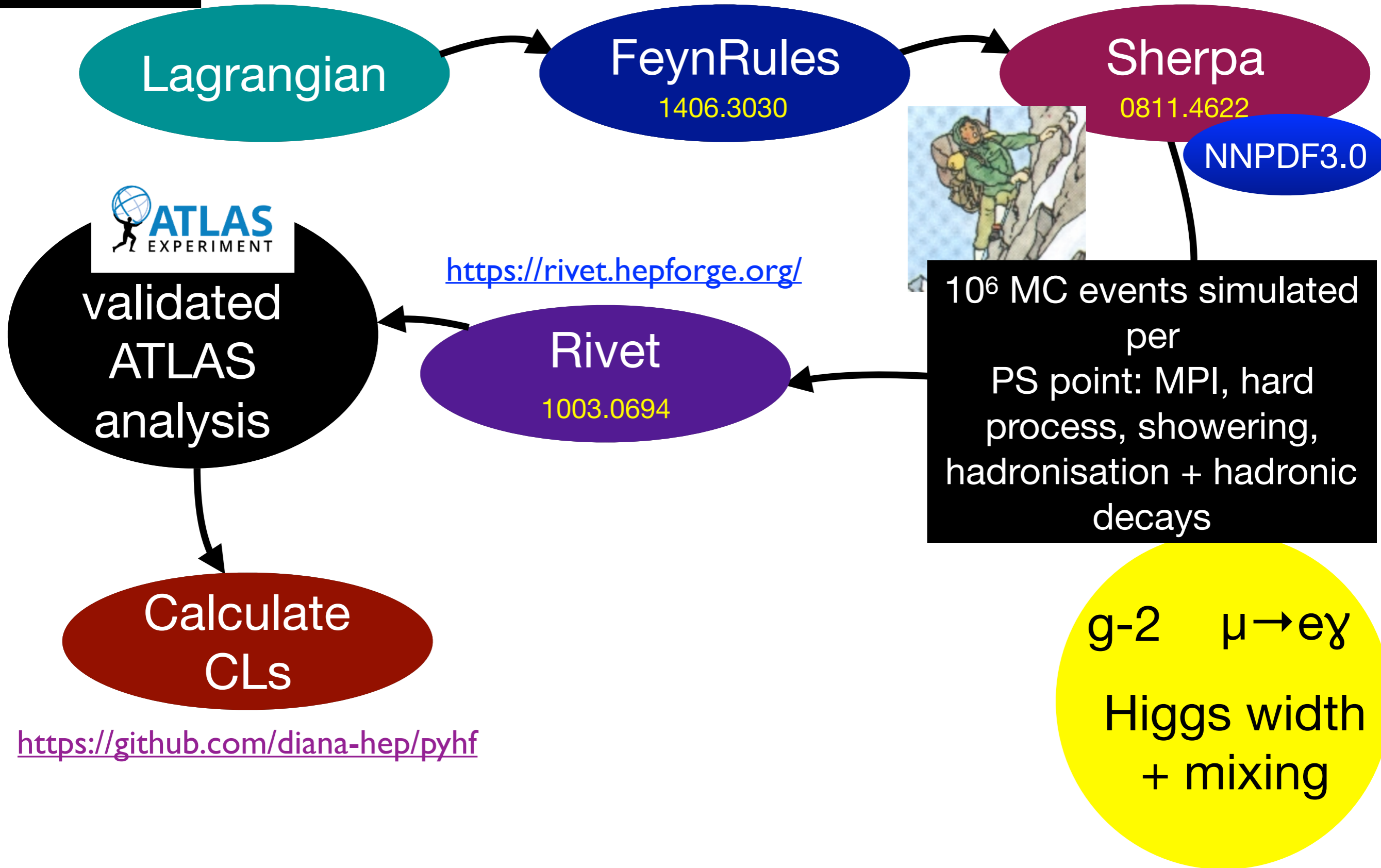
SM + flavon interactions



<https://github.com/diana-hep/pyhf>

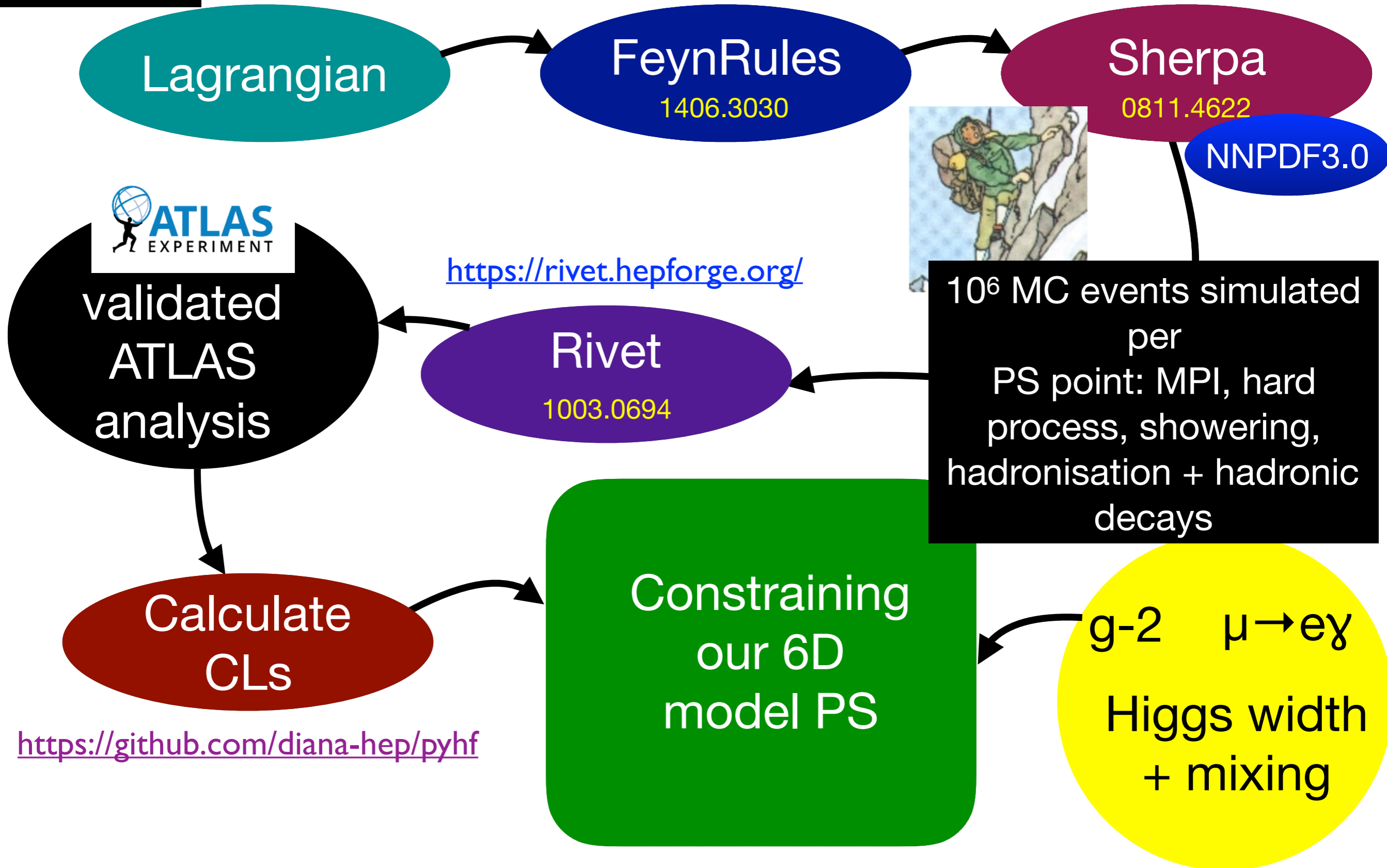
Tool Chain

SM + flavon interactions



SM + flavon interactions

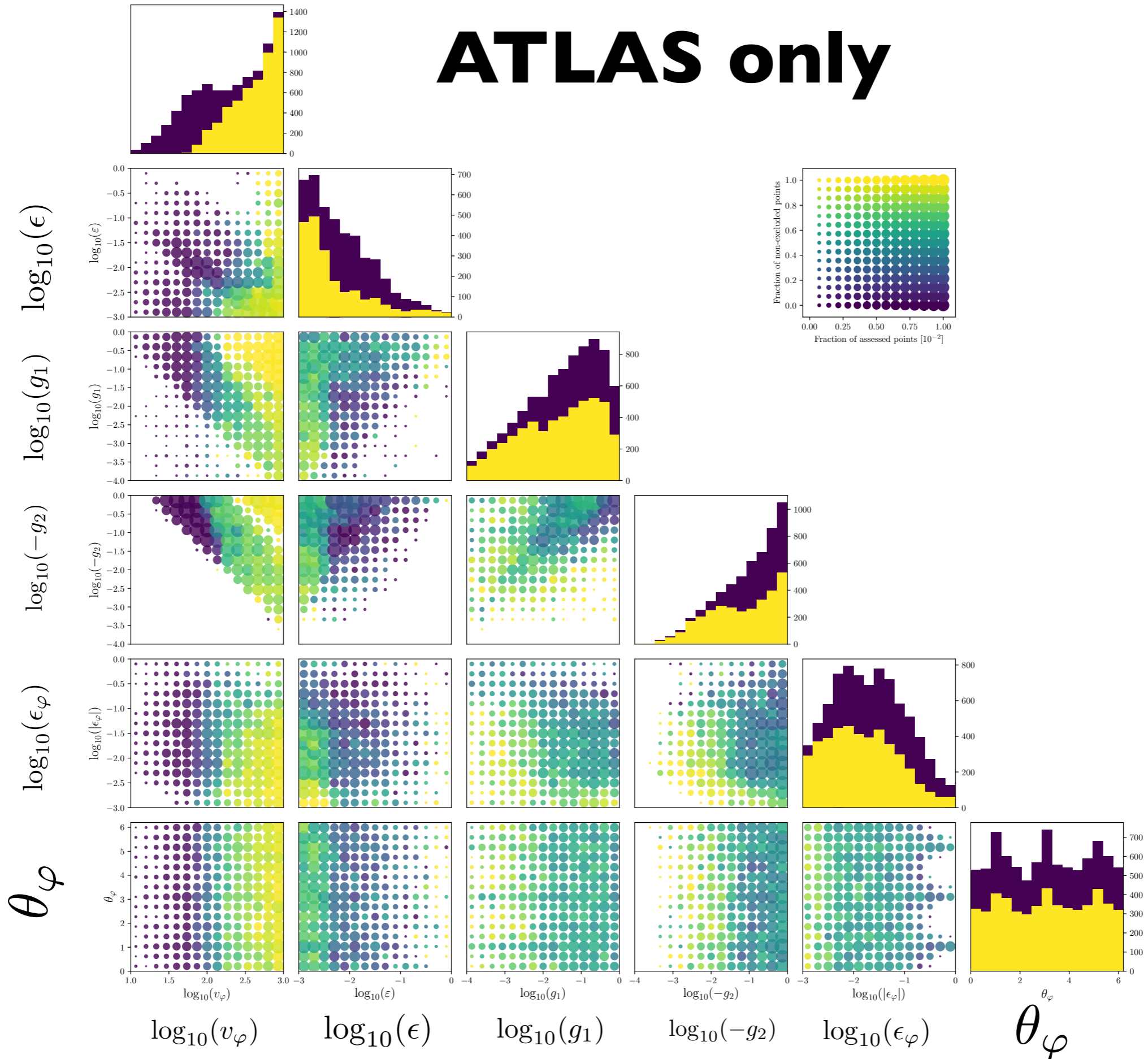
Tool Chain



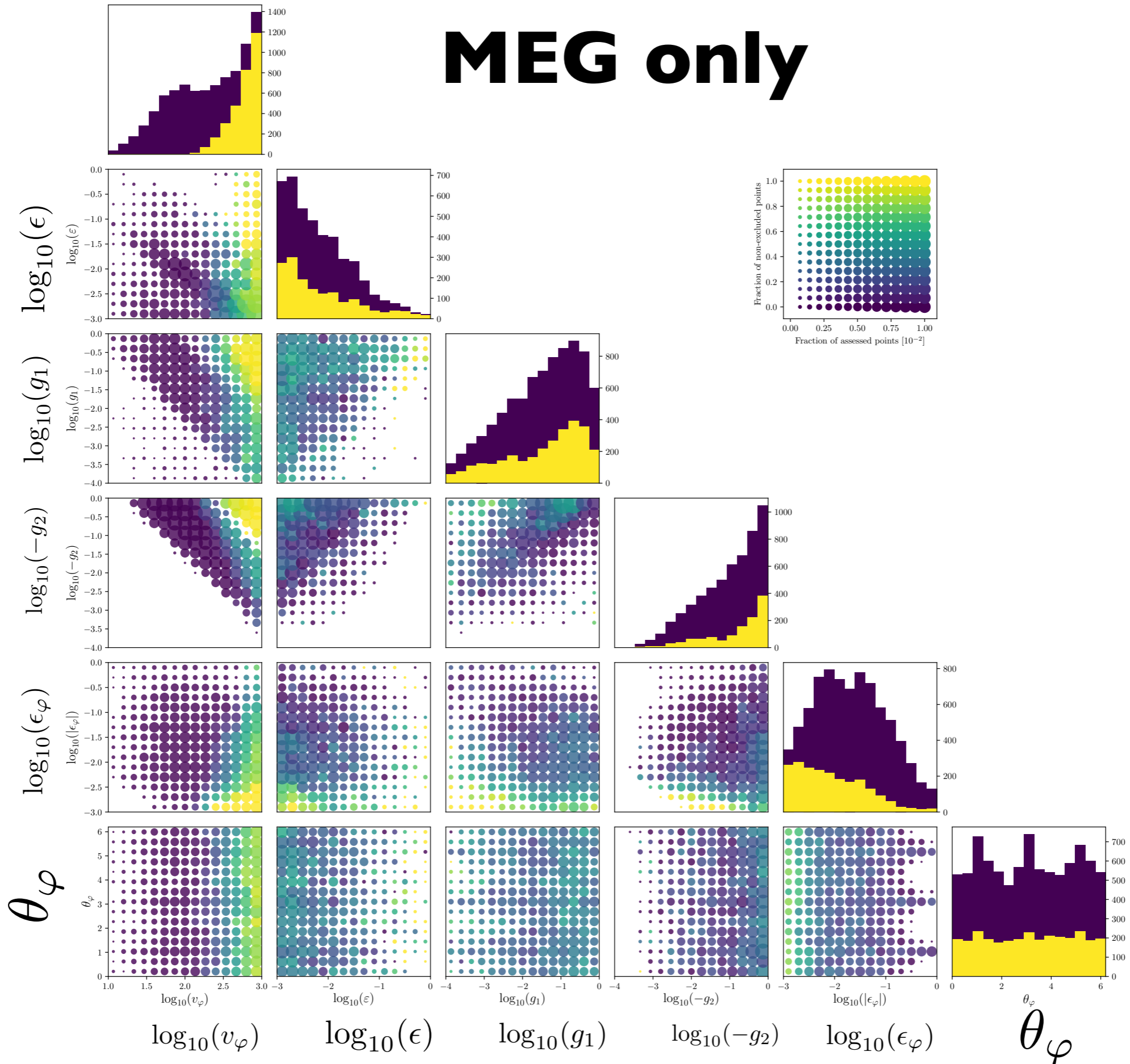
Thanks to UK HEP Grid Computing for resources

Results

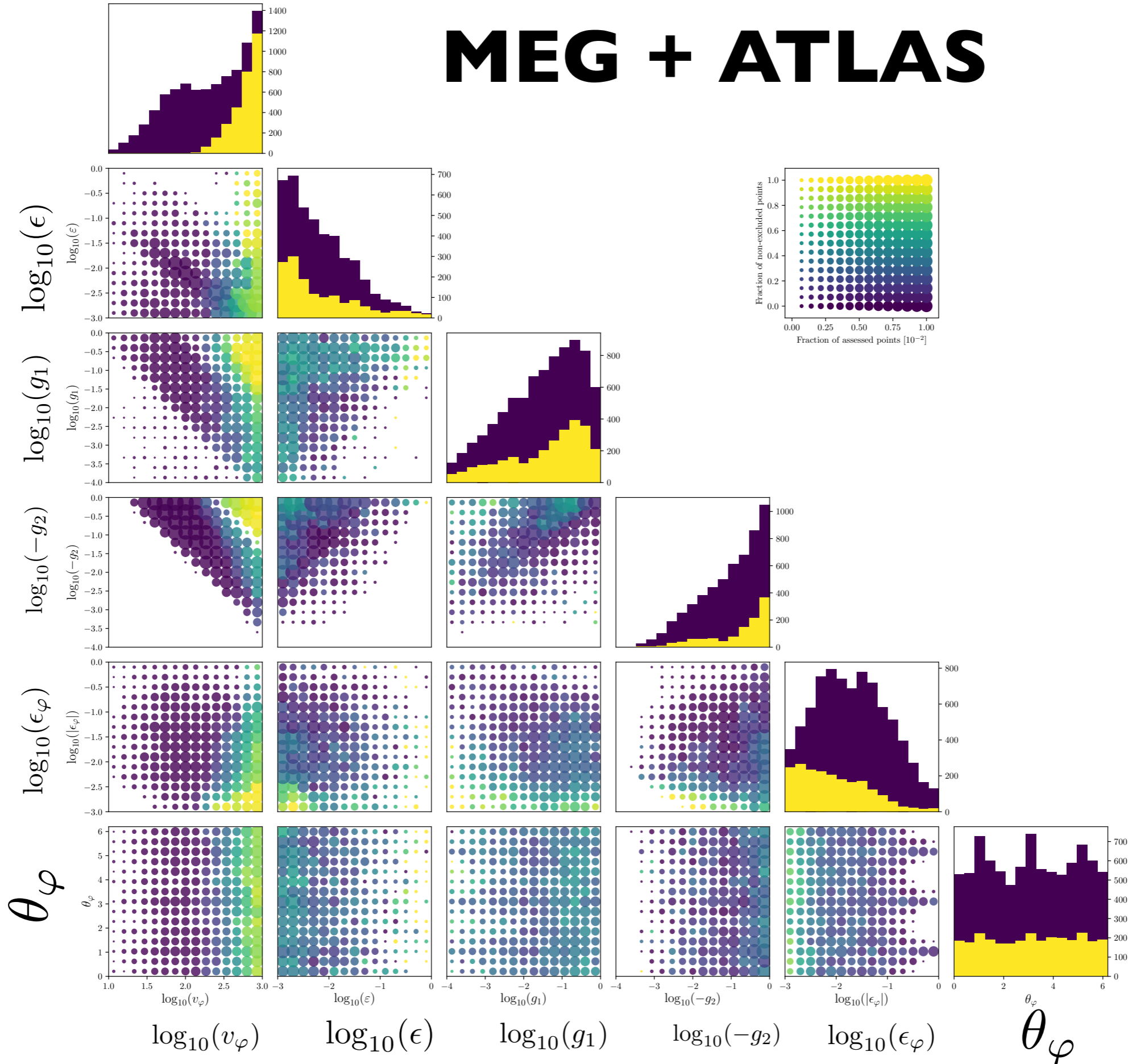
ATLAS only



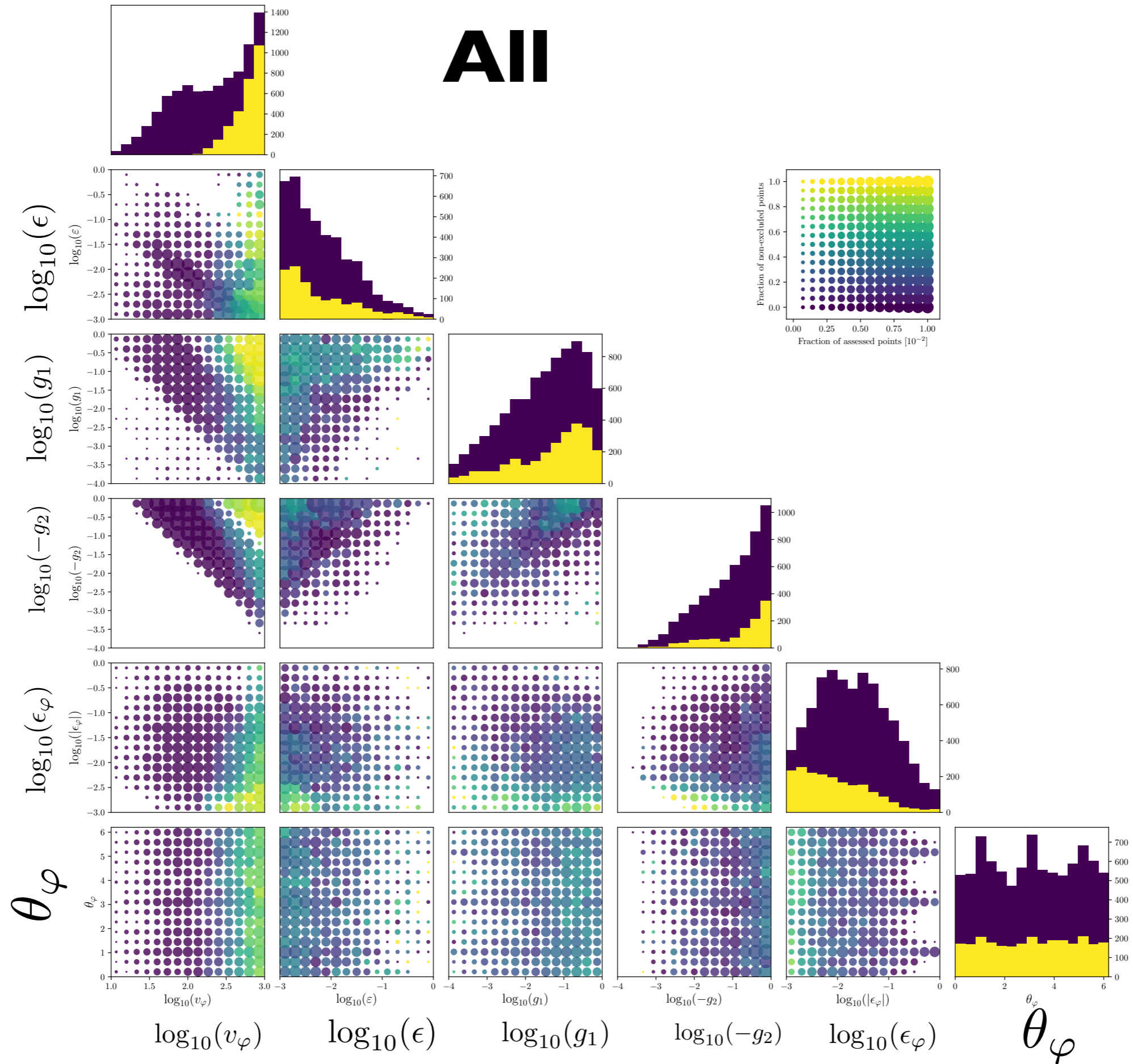
MEG only

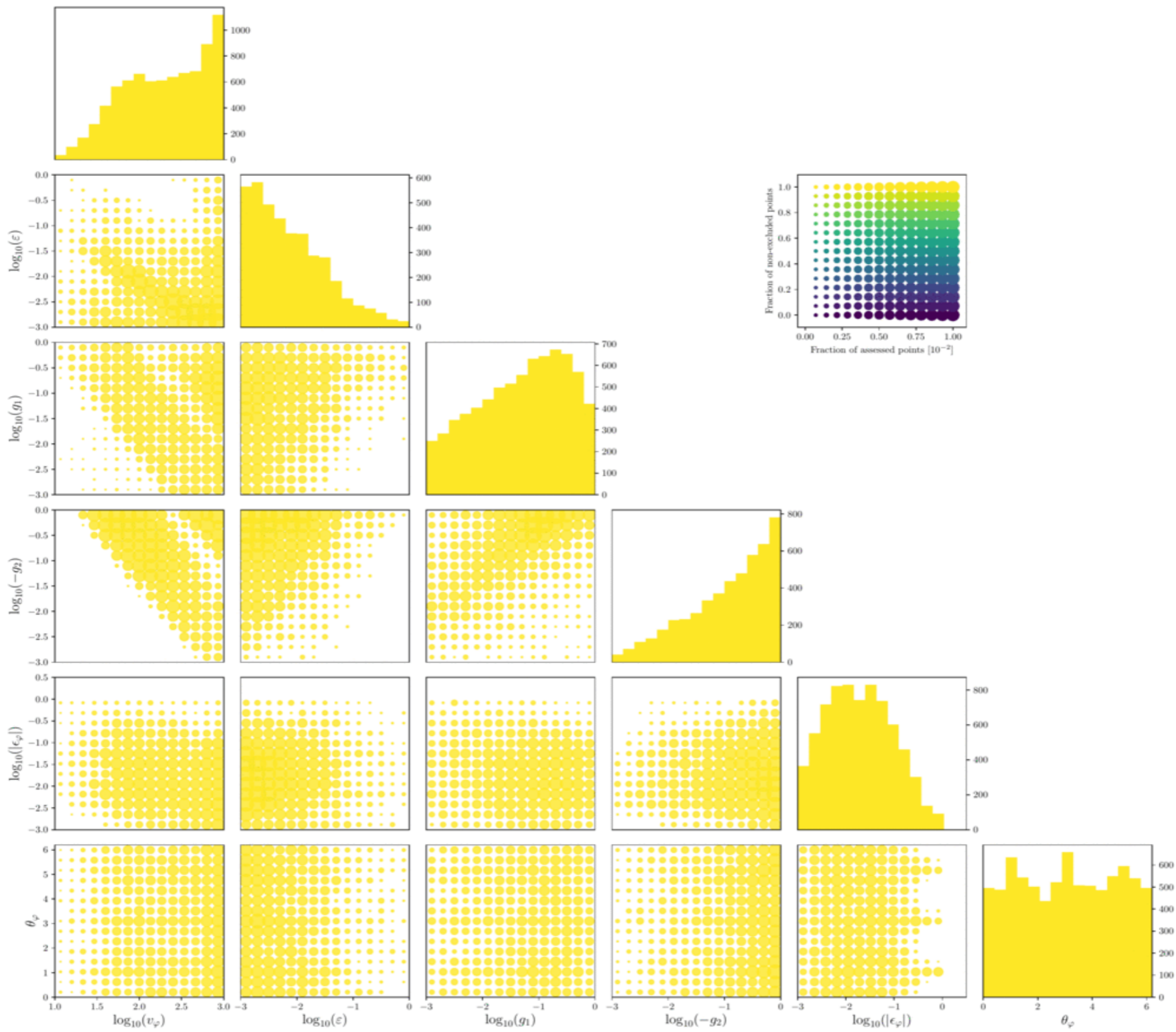


MEG + ATLAS



All

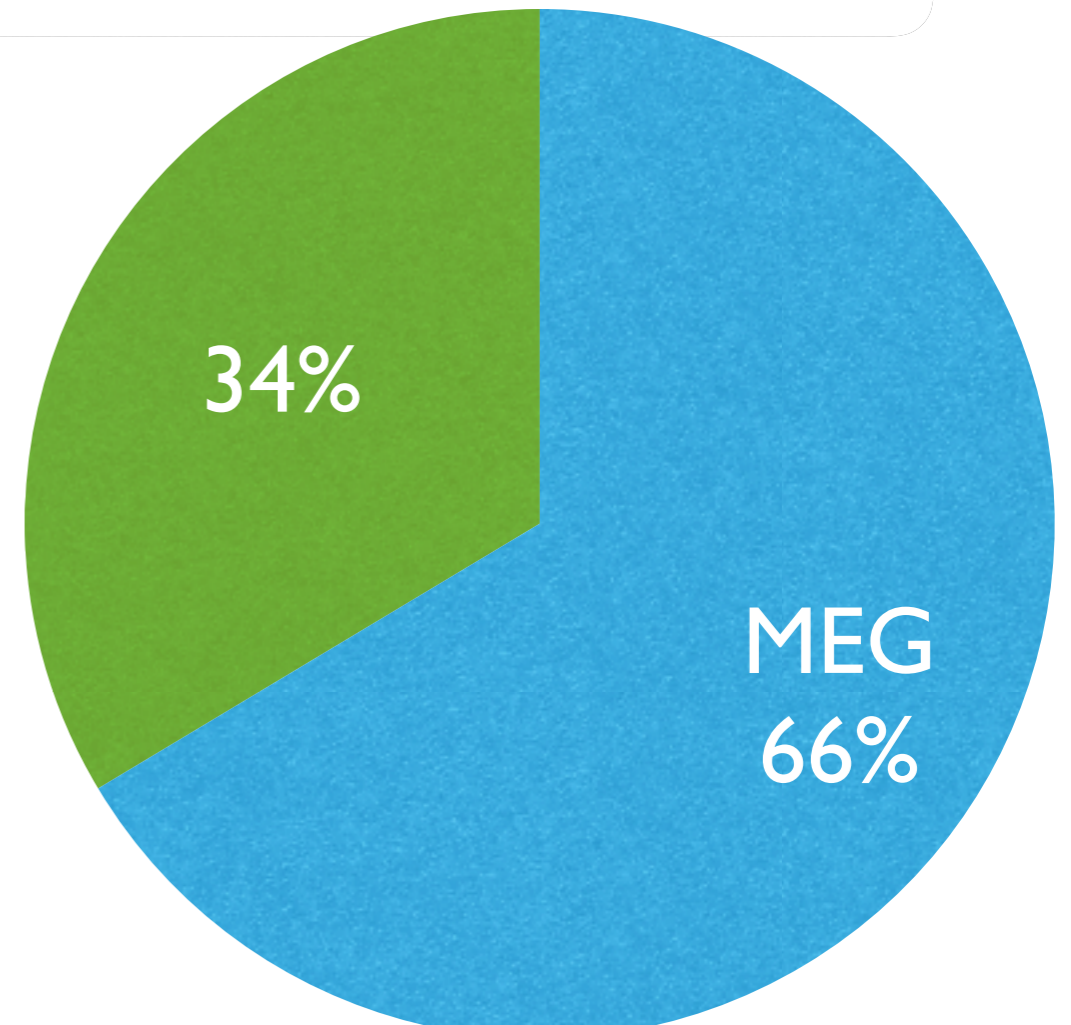
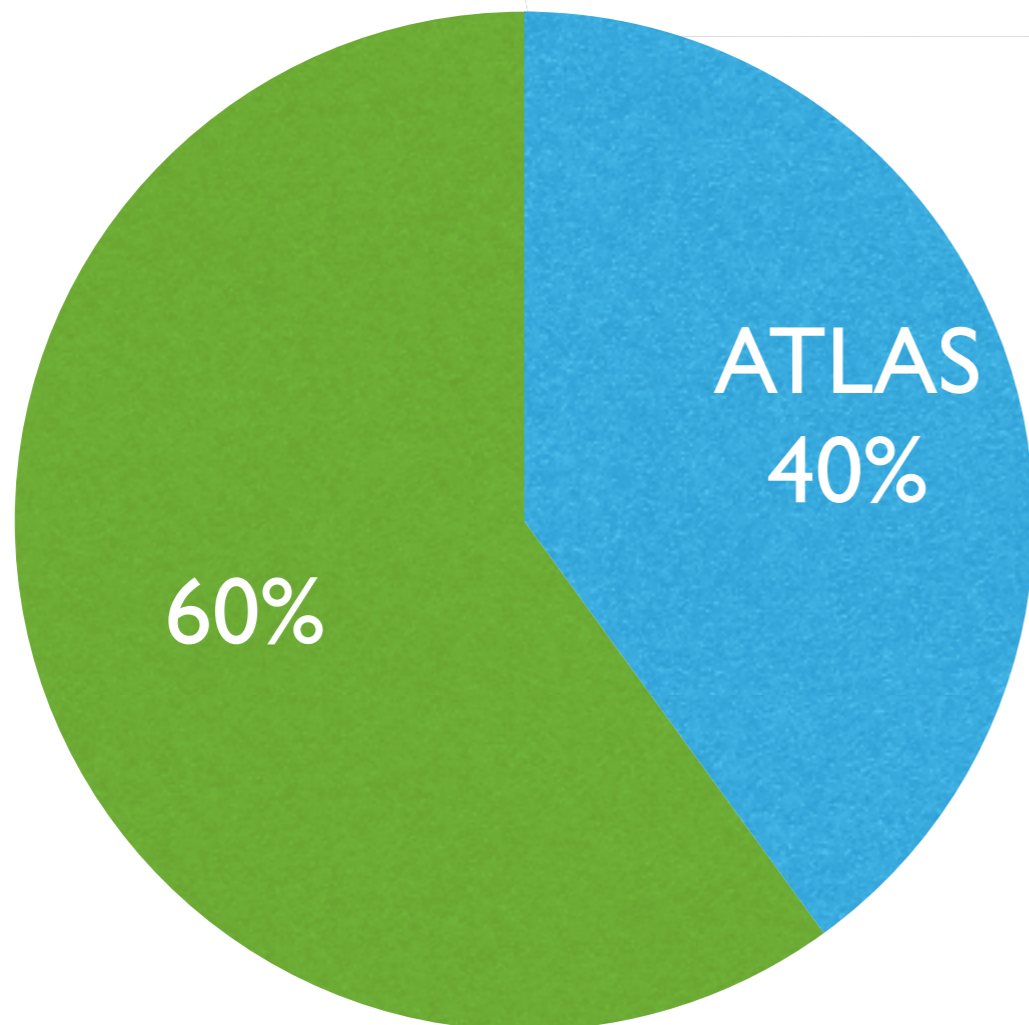




Exclusionary Power

$$\text{exclusion power} = \frac{N_{\text{tot}} - N_{\text{pass}}}{N_{\text{tot}}}$$

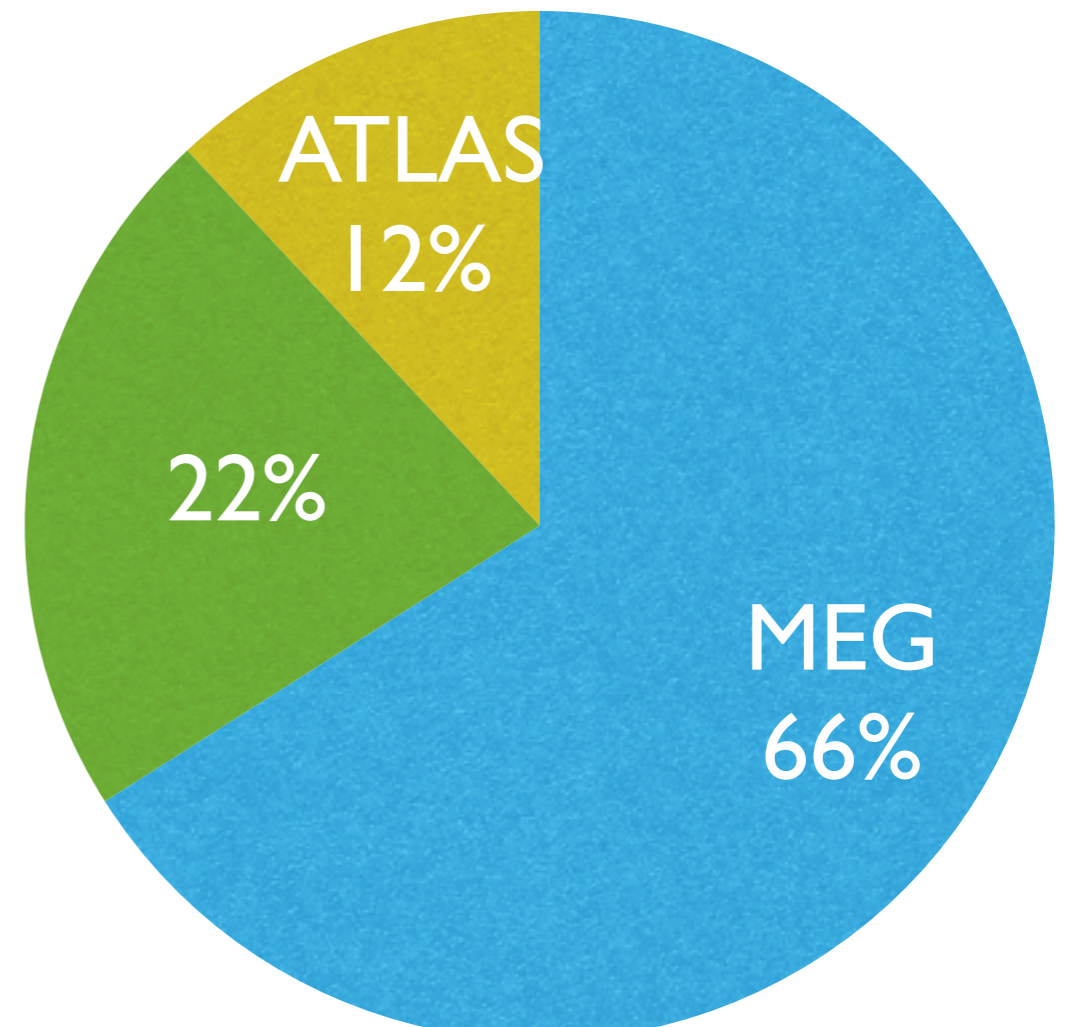
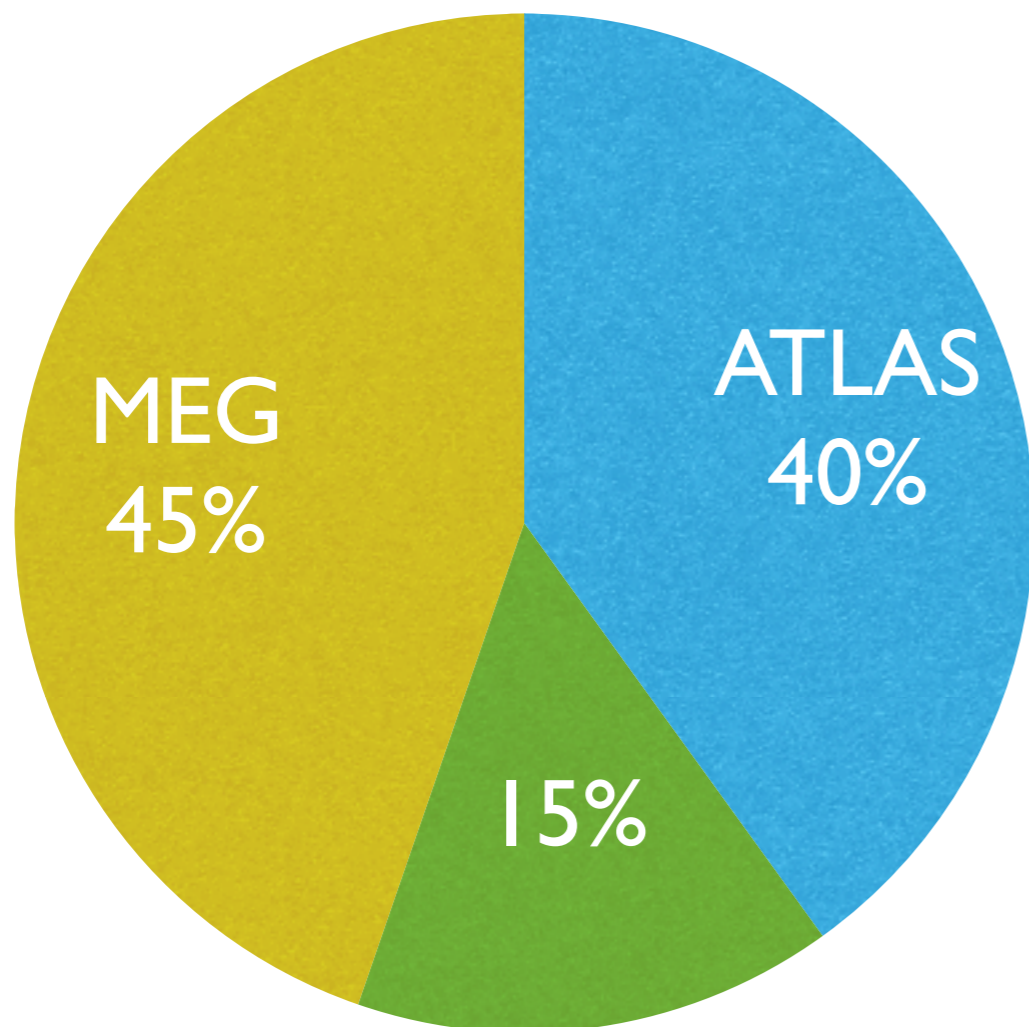
Experimental data	Exclusion power [%]
MEG	65.6
ATLAS	40.0
Higgs-width	6.0
Higgs-mixing	1.7
$g - 2$	0.7



Exclusionary Power

$$\text{exclusion power} = \frac{N_{\text{tot}} - N_{\text{pass}}}{N_{\text{tot}}}$$

Experimental data	Exclusion power [%]
MEG	65.6
ATLAS	40.0
Higgs-width	6.0
Higgs-mixing	1.7
$g - 2$	0.7



Conclusions

- A priori it is not clear the flavour breaking scale should be close to the GUT scale. Can we exclude a lower value of this scale?
- Experiments such as MEG place highly competitive constraints on flavour model P.S (we were skeptical the collider would be able to compete!)
- We demonstrated **collider searches** for high multiplicity leptonic final states **can compete and complement** MEG and $g-2$ experimental constraints.
- Why? The collider has sensitivity to flavon coupling to Higgs, MEG and $g-2$ are not.
- The chosen model P.S is largely excluded through synergy of these experiments.

Thank you!

Back-up Slides

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$(ab)_{\mathbf{1}} = a_1 b_1 + a_2 b_3 + a_3 b_2$$

$$(ab)_{\mathbf{1}'} = a_3 b_3 + a_1 b_2 + a_2 b_1$$

$$(ab)_{\mathbf{1}''} = a_2 b_2 + a_1 b_3 + a_3 b_1$$

$$(ab)_{\mathbf{3}_S} = \frac{1}{2} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \end{pmatrix}, \quad (ab)_{\mathbf{3}_A} = \frac{1}{2} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix}.$$

Back-up Slides

Minimise the flavon and Higgs potential

$$\mu_H^2 + \lambda v_H^2 + \frac{1}{2} \epsilon v_\varphi^2 (1 + 2|\epsilon_\varphi|^2) = 0,$$

$$\mu_\varphi^2 + g_1 v_\varphi^2 (1 + 2|\epsilon_\varphi|^2) + \frac{1}{3} g_2 v_\varphi^2 [1 - \text{Re}(\epsilon_\varphi^3)] + \frac{1}{2} \epsilon v_H^2 + A \epsilon_\varphi^* + A^* \epsilon_\varphi = 0,$$

$$\mu_\varphi^2 \epsilon_\varphi + g_1 v_\varphi^2 (1 + 2|\epsilon_\varphi|^2) \epsilon_\varphi + \frac{1}{2} g_2 v_\varphi^2 [-\epsilon_\varphi^{*2} + |\epsilon_\varphi|^2 \epsilon_\varphi] + \frac{1}{2} \epsilon \epsilon_\varphi v_H^2 + A + A^* \epsilon_\varphi^* = 0.$$

$$A \epsilon_\varphi^* + A^* \epsilon_\varphi^{*2} + 2 \text{Re}(A^* \epsilon_\varphi) |\epsilon_\varphi|^2 = \underbrace{-\frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*3} + \frac{1}{3} g_2 v_\varphi^2 |\epsilon_\varphi|^2 \left[1 - \text{Re}(\epsilon_\varphi^3) - \frac{3}{2} |\epsilon_\varphi|^2\right]}_x$$

$$A = \frac{(\epsilon_\varphi^*)^2 x^* - \epsilon_\varphi (x + 2i |\epsilon_\varphi|^2 \Im[x])}{|\epsilon_\varphi|^2 (-|\epsilon_\varphi|^2 + \epsilon_\varphi^{*3} + \epsilon_\varphi^3 - 1)}.$$

Back-up Slides

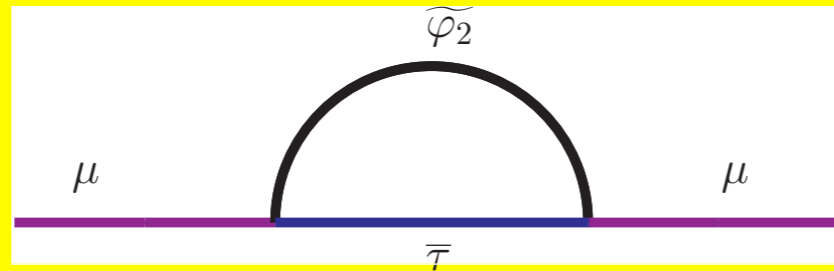
$$\begin{aligned}
 (M_{\tilde{\Phi}}^2)_{11} &= 2\lambda v_H^2, \\
 (M_{\tilde{\Phi}}^2)_{22} &= 2gv_\varphi^2 + \frac{1}{3}g_2v_\varphi^2\text{Re}(\epsilon_\varphi^3) - 2\text{Re}(A\epsilon_\varphi^*), \\
 (M_{\tilde{\Phi}}^2)_{33} &= -\frac{1}{3}g_2v_\varphi^2[1 - \text{Re}(\epsilon_\varphi^3)] + \frac{1}{2}g_2v_\varphi^2|\epsilon_\varphi|^2 - 2\text{Re}(A\epsilon_\varphi^*) + \text{Re}\left(-g_2v_\varphi^2(\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1v_\varphi^2\epsilon_\varphi^2 + A^*\right), \\
 (M_{\tilde{\Phi}}^2)_{44} &= -\frac{1}{3}g_2v_\varphi^2[1 - \text{Re}(\epsilon_\varphi^3)] + \frac{1}{2}g_2v_\varphi^2|\epsilon_\varphi|^2 - 2\text{Re}(A\epsilon_\varphi^*) - \text{Re}\left(-g_2v_\varphi^2(\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1v_\varphi^2\epsilon_\varphi^2 + A^*\right), \\
 (M_{\tilde{\Phi}}^2)_{12} &= v_Hv_\varphi\epsilon, \\
 (M_{\tilde{\Phi}}^2)_{13} &= \sqrt{2}v_Hv_\varphi\epsilon\text{Re}(\epsilon_\varphi), \\
 (M_{\tilde{\Phi}}^2)_{14} &= \sqrt{2}v_Hv_\varphi\epsilon\text{Im}(\epsilon_\varphi), \\
 (M_{\tilde{\Phi}}^2)_{23} &= \sqrt{2}\text{Re}\left(2g_1v_\varphi^2\epsilon_\varphi - \frac{1}{2}g_2v_\varphi^2\epsilon_\varphi^{*2} + A\right), \\
 (M_{\tilde{\Phi}}^2)_{24} &= \sqrt{2}\text{Im}\left(2g_1v_\varphi^2\epsilon_\varphi - \frac{1}{2}g_2v_\varphi^2\epsilon_\varphi^{*2} + A\right), \\
 (M_{\tilde{\Phi}}^2)_{34} &= \text{Im}\left(-g_2v_\varphi^2(\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1v_\varphi^2\epsilon_\varphi^2 + A^*\right), \tag{2.19}
 \end{aligned}$$

Diagonalise mass matrix ensuring (1,1) entry is the Higgs mass

Relating gauge to mass basis

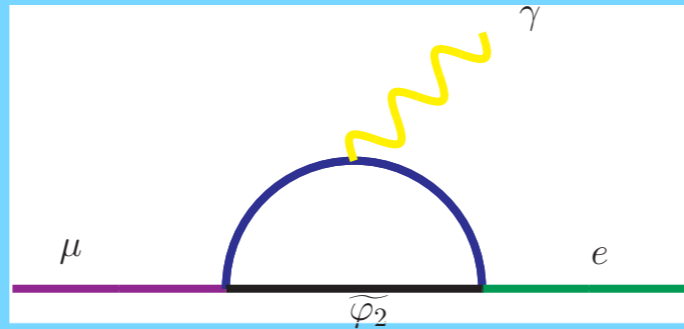
$$\begin{pmatrix} \tilde{h} \\ \tilde{\varphi}_1 \\ \sqrt{2}\text{Re}(\varphi_2) \\ \sqrt{2}\text{Im}(\varphi_2) \end{pmatrix} = \begin{pmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{pmatrix} \begin{pmatrix} h \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

g-2 Constraint



$$\Delta a_\mu = \frac{m_\mu^2 m_\tau^2}{24\pi^2 v_\varphi^2} \left[\frac{(|W_{13}|^2 - |W_{14}|^2)}{m_h^2} + \frac{(|W_{23}|^2 - |W_{24}|^2)}{m_{s_1}^2} + \frac{(|W_{33}|^2 - |W_{34}|^2)}{m_{s_2}^2} + \frac{(|W_{43}|^2 - |W_{44}|^2)}{m_{s_3}^2} \right].$$

$\mu \rightarrow e \gamma$ Constraint



$$A(h) = \frac{1}{128\pi^2} \frac{1}{m_h^2 v_\varphi^2} \left[m_\mu m_\tau^2 G_2 \left(\frac{m_\tau^2}{m_H^2} \right) (W_{13} + iW_{14})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2 \left(\frac{m_\tau^2}{m_H^2} \right) (|W_{13}|^2 + |W_{14}|^2) \right],$$

$$A(s_1) = \frac{1}{128\pi^2} \frac{1}{m_{s_1}^2 v_\varphi^2} \left[m_\mu m_\tau^2 G_2 \left(\frac{m_\tau^2}{m_1^2} \right) (W_{23} + iW_{24})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2 \left(\frac{m_\tau^2}{m_1^2} \right) (|W_{23}|^2 + |W_{24}|^2) \right],$$

$$A(s_2) = \frac{1}{128\pi^2} \frac{1}{m_{s_2}^2 v_\varphi^2} \left[m_\mu m_\tau^2 G_2 \left(\frac{m_\tau^2}{m_2^2} \right) (W_{33} + iW_{34})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2 \left(\frac{m_\tau^2}{m_2^2} \right) (|W_{33}|^2 + |W_{34}|^2) \right],$$

$$A(s_3) = \frac{1}{128\pi^2} \frac{1}{m_{s_3}^2 v_\varphi^2} \left[m_\mu m_\tau^2 G_2 \left(\frac{m_\tau^2}{m_3^2} \right) (W_{43} + iW_{44})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2 \left(\frac{m_\tau^2}{m_3^2} \right) (|W_{43}|^2 + |W_{44}|^2) \right].$$

$$G_2(x) = -\log x - \frac{11}{6}$$

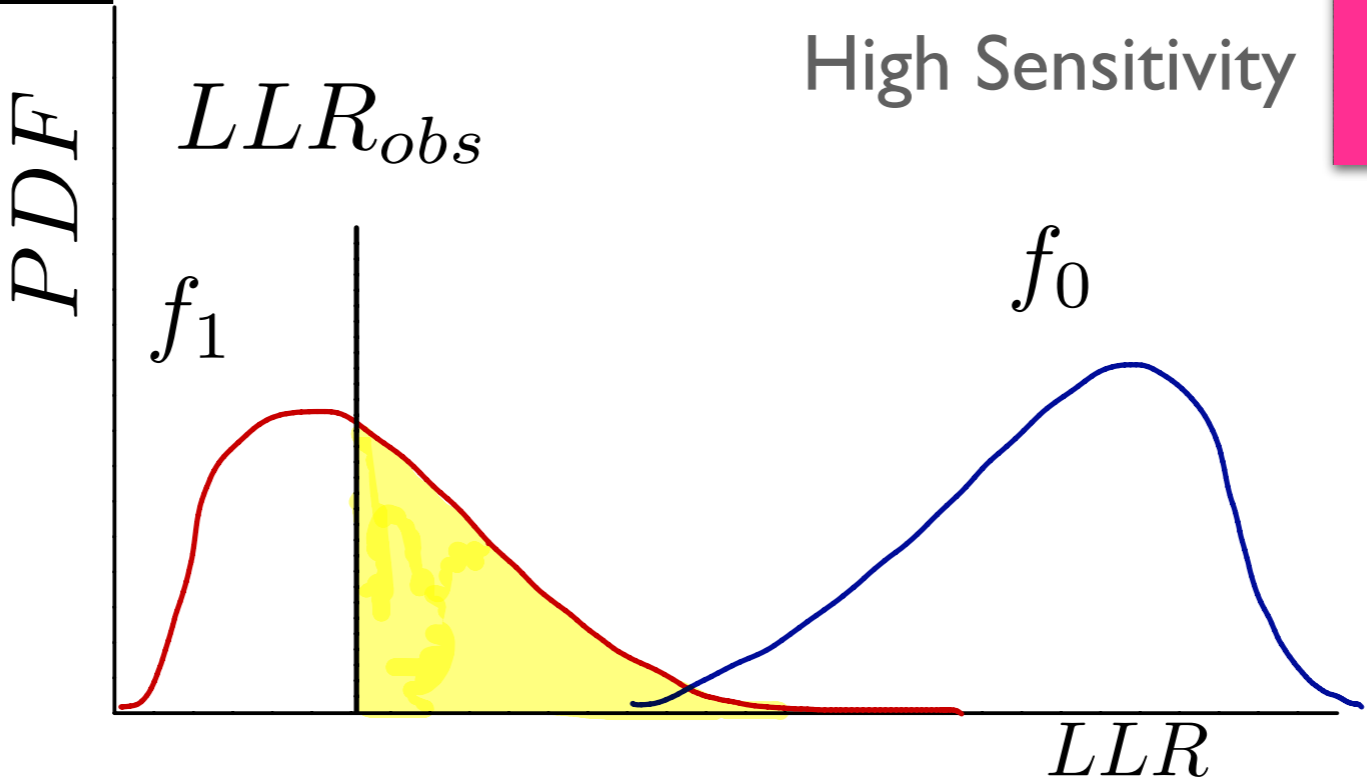
$$\Gamma(\mu \rightarrow e \gamma) = \frac{m_\mu^3 |A|^2}{16\pi}, \quad \Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu \gamma) = \frac{G_F^2 m_\mu^5}{192\pi^3},$$

CLs Method for Recast

PDF generated through possible fluctuations (Asimov data set) 1007.1727

Calculated using PyHF:

<https://github.com/diana-hep/pyhf>



$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR$$

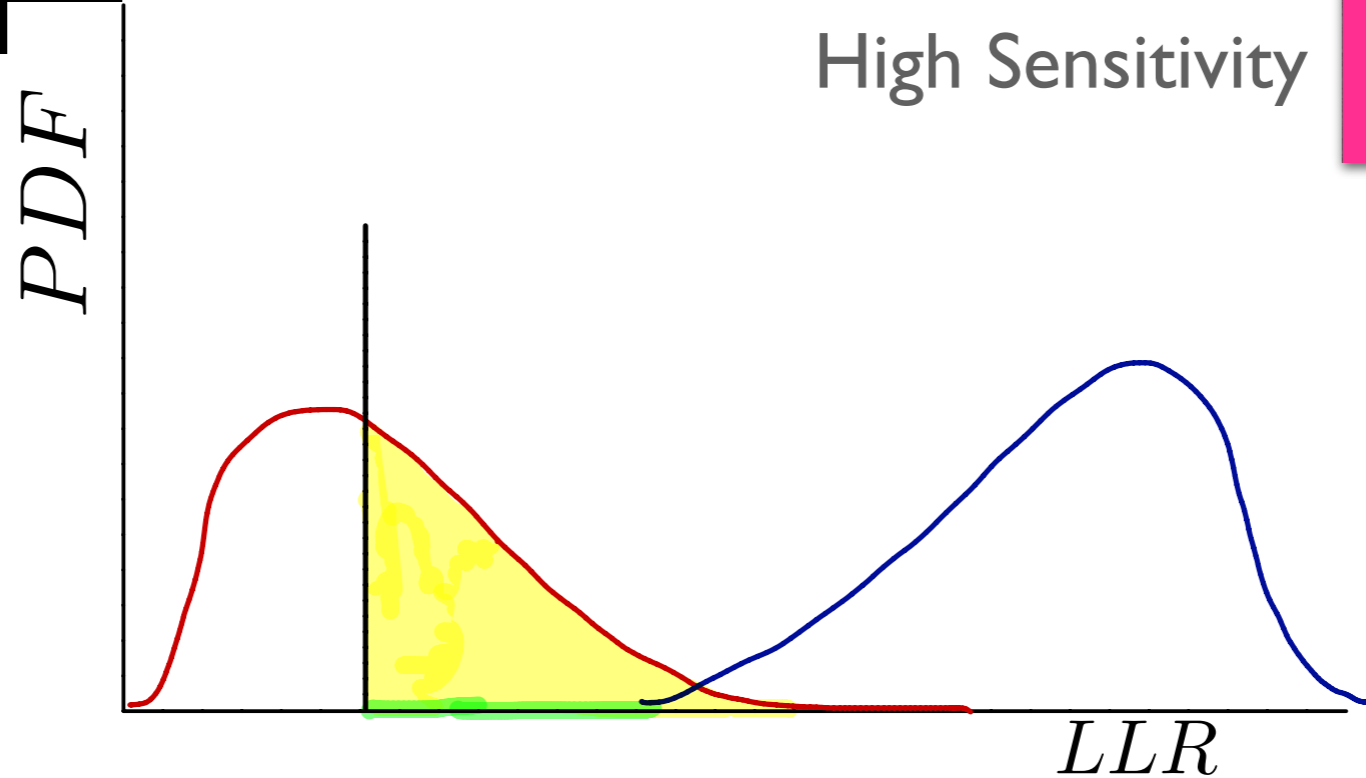
$$CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

CLs Method for Recast

PDF generated through possible fluctuations (Asimov data set) 1007.1727



<https://github.com/diana-hep/pyhf>

$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR$$

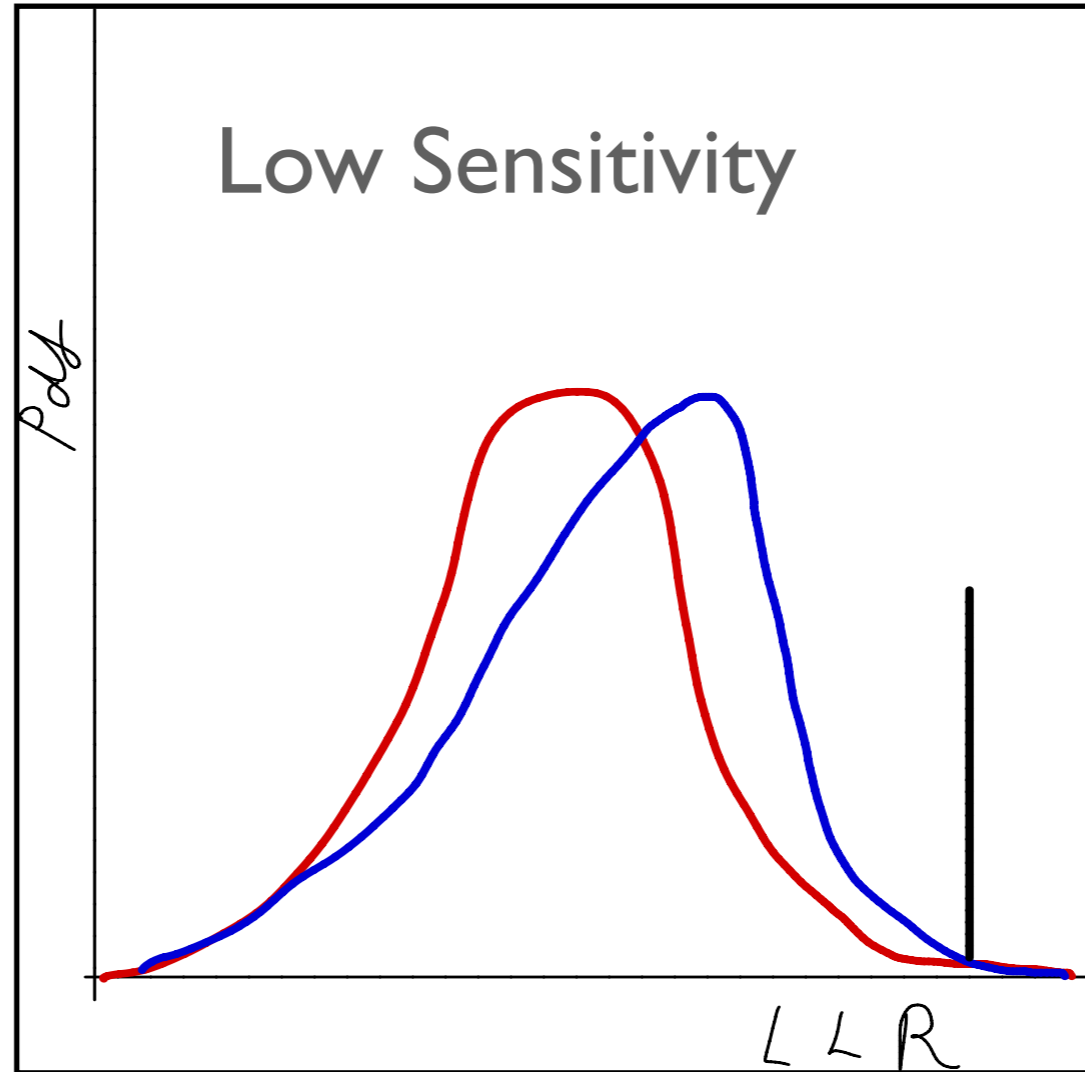
$$CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

$CL_s > 0.05$, H_1 is not excluded 95% C.L.

CLs Method for Recast



CL_s is conservative against overestimating exclusionary power in case of low signal sensitivity

<https://github.com/diana-hep/pyhf>

CL_b becomes small therefore CL_s becomes large and H₁ cannot be excluded

$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR$$

$$CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

CL_s < 0.05, H₁ is excluded 95% C.L.