Quantum Information Tools for Simulating Quantum Field Theories

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With: 

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Can quantum computers simulate all physical processes efficiently?

Universality Conjecture:
Quantum circuits can simulate all physical dynamics in $\mathrm{poly}(E, V, t, 1/\epsilon)$ time.

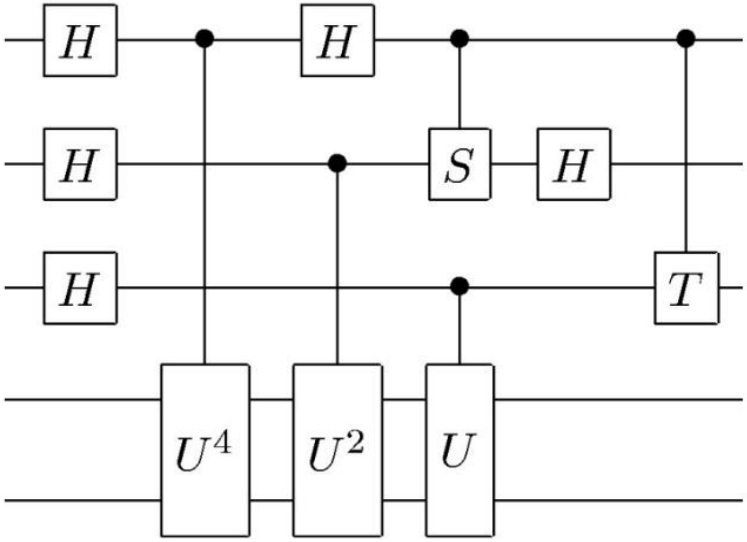
Status:

Non-relativistic QM	Yes: Now being optimized
Quantum Field Theories	Probably: In progress
Quantum Gravity/Strings	Nobody knows

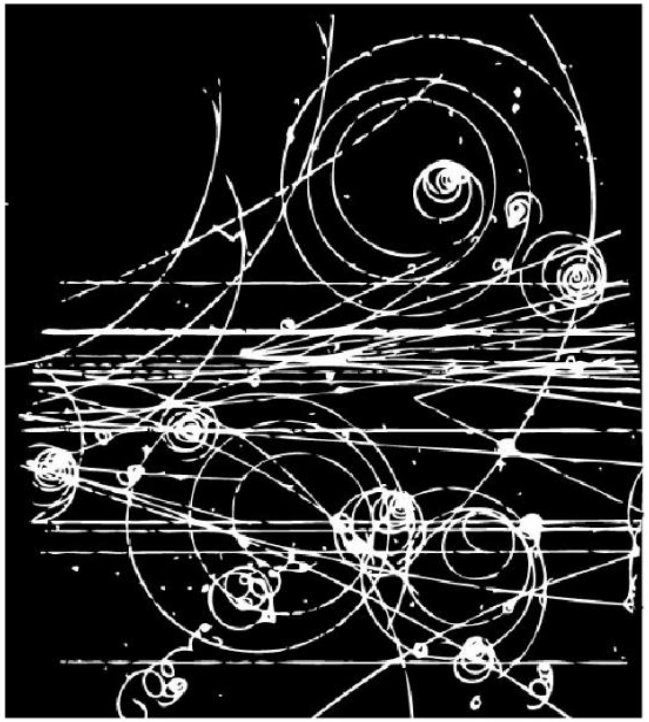
Quantum Field Theory

- Much is known about using quantum computers to simulate quantum systems.
- Why might Quantum Field Theory be different?
  - Field has infinitely many degrees of freedom
  - Relativistic
  - Particle number not conserved
  - Formalism looks different

What is the computational power of our universe?

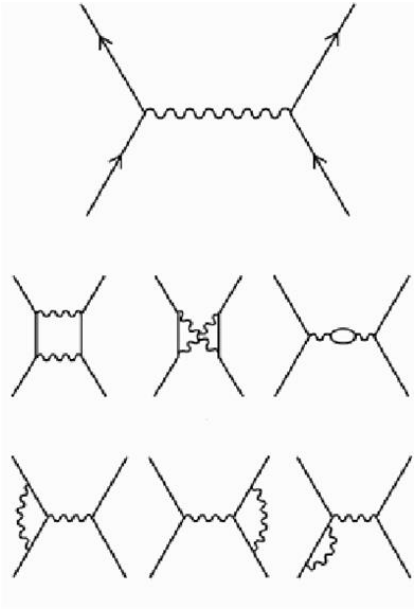


simulate



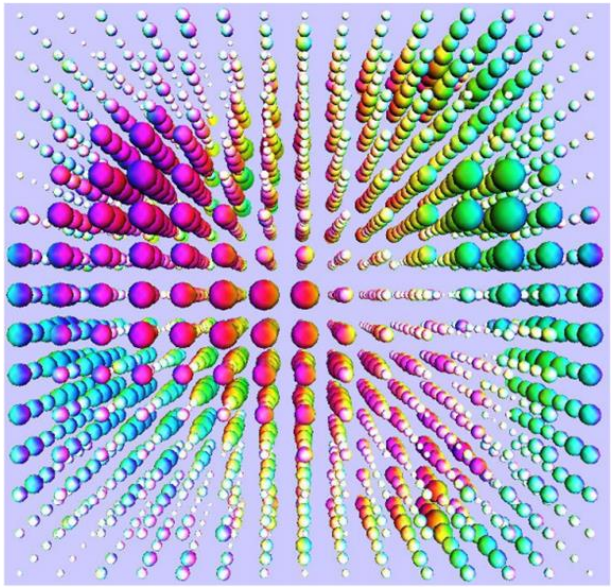
Classical Algorithms

Feynman diagrams



Break down at strong coupling or high precision

Lattice methods



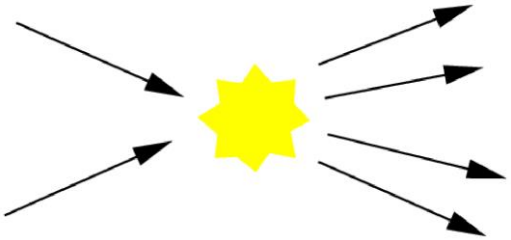
Good for binding energies.
Real-time dynamics difficult.

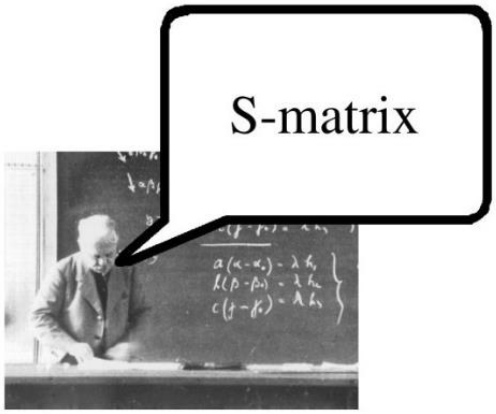
There’s room for exponential speedup by quantum computing.

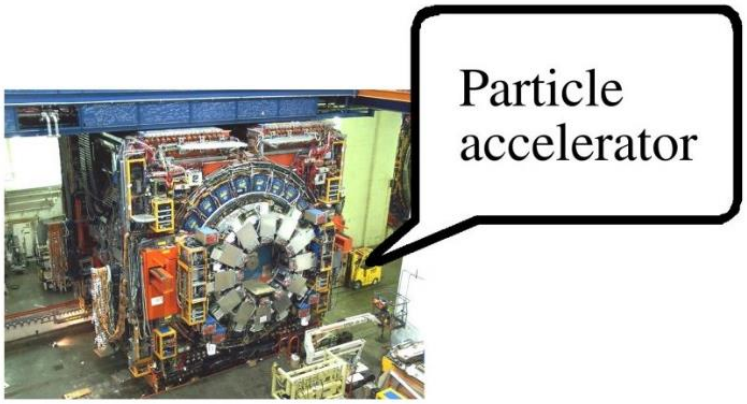
A QFT Computational Problem

Input: a list of momenta of incoming particles.

Output: a list of momenta of outgoing particles.





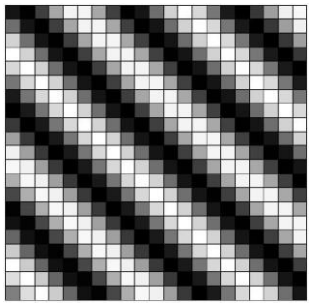


Our Results

- Efficient simulation algorithms for example QFTs:
  - Bosonic: Massive ϕ^4
    [Jordan, Lee, Preskill, *Science* 336:1130, 2012]
  - Fermionic: Massive Gross-Neveu
    [Jordan, Lee, Preskill *ArXiv:1404.7115*, 2014]
- Recent Developments
  - BQP-hardness: classical computers cannot perform certain QFT simulations efficiently
    [Jordan, Krovi, Lee, Preskill, *Quantum* 2, 44, 2018]
  - Better Speed and broken symmetries
    [Moosavian, Jordan, *ArXiv:1711.04006*, 2017]

Representing Quantum Fields

A field is a list of values, one for each location in space.



A quantum field is a superposition over classical fields.

$$\frac{1}{\sqrt{2}}\left|\ \right\rangle - \frac{i}{\sqrt{2}}\left|\ \right\rangle$$

A superposition over bit strings is a state of a quantum computer.

$\phi^4 - \text{theory}$

Lagrangian Density

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

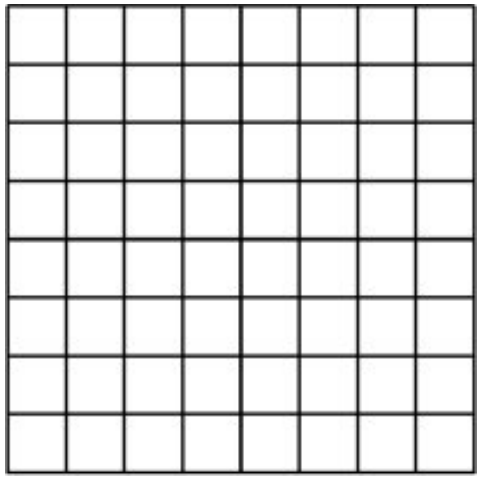
For quantum simulation we prefer Hamiltonian formulation (equivalent)

$$H = \int d^d x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right]$$

$$[\phi(x), \pi(y)] = i\delta^{(d)}(x-y)$$

Our Algorithms

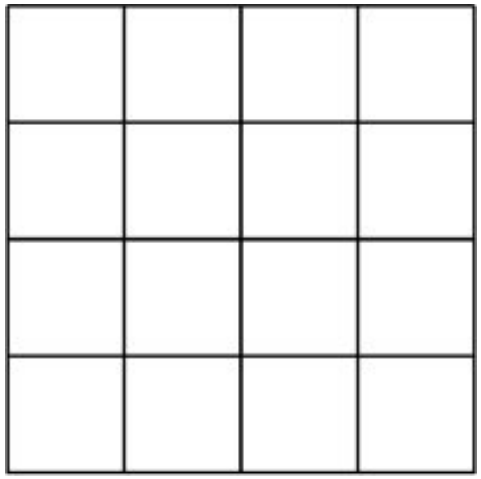
1) Choose a lattice discretization.
   Bound discretization error (renormalization group)
2) Prepare physically realistic initial state.
   Is the most time-consuming step.
   This depends strongly on which QFT is simulated.
3) Implement time-evolution by a quantum circuit.
   Can use Suzuki-Trotter formulae.
4) Perform measurements on final state.
   One must be careful about variance.



Mass: m

Interaction strength: λ

Coarse grain

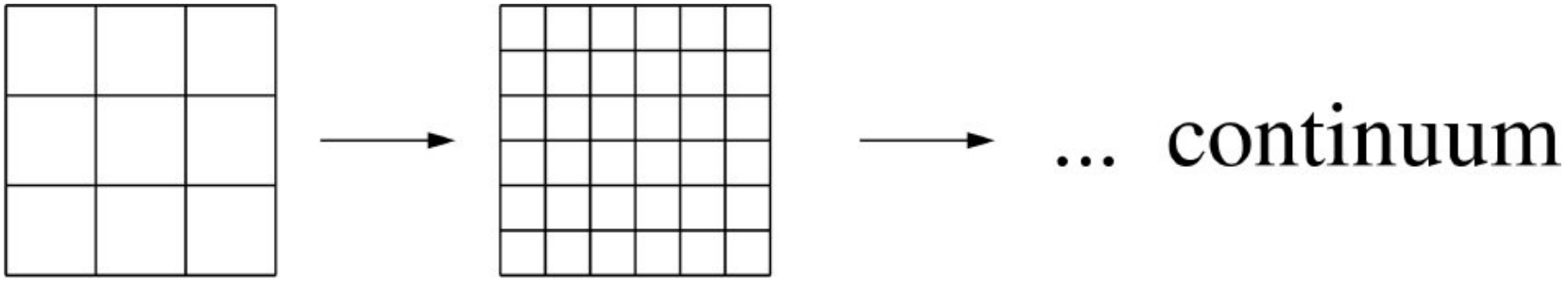


Mass: m'

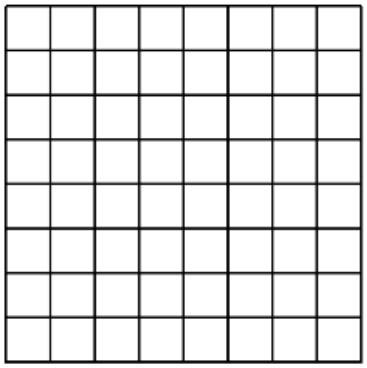
Interaction strength: λ'

Lattice Cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices.

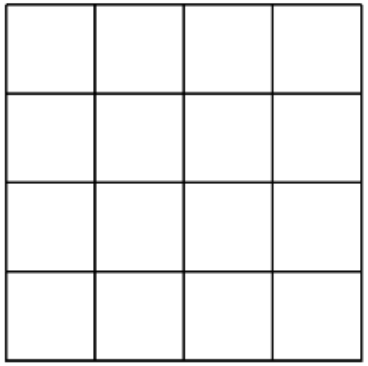


m and λ are functions of lattice spacing!



$$H = \frac{1}{2} \sum_{x \in \Omega} a^d \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]$$

Coarse grain



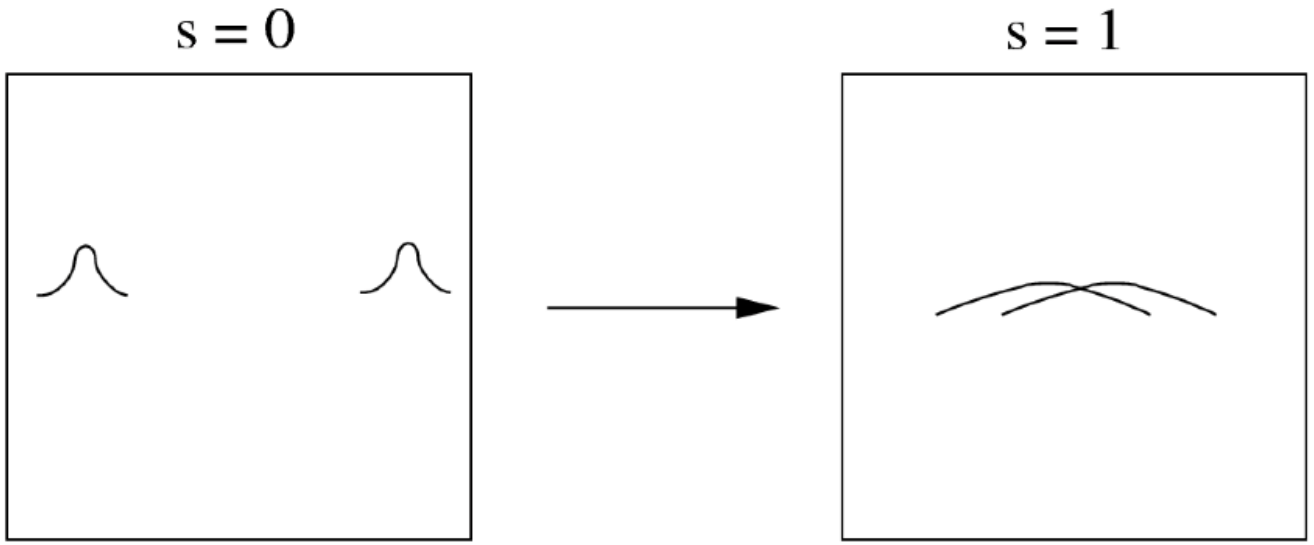
$$H_{\text{eff}} = \frac{1}{2} \sum_{x \in \Omega'} (2a)^d \left[\pi^2 + (\nabla' \phi)^2 + m_{\text{eff}}^2 \phi^2 + \lambda_{\text{eff}} \phi^4 + g\phi^6 + \ldots \right]$$

Simulation converges as a^2

Adiabatic State Preparation

$$H(s) = H_{\text{free}} + sH_{\text{interaction}}$$

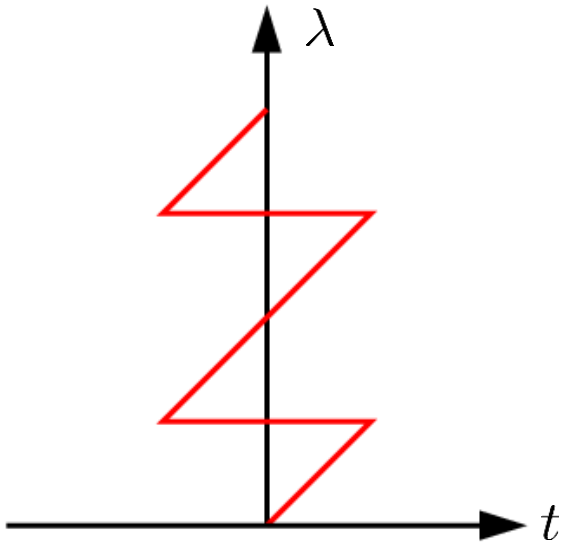
Prepare wavepackets in free theory, then adiabatically turn on interaction. Problem:



Adiabatic State Preparation

Solution: intersperse backward time evolutions with time-independent Hamiltonians.

This winds back dynamical phase on each eigenstate without undoing adiabatic change of basis.



Runtimes

Weak Coupling:

$d = 1$	$(1/\epsilon)^{1.5}$
$d = 2$	$(1/\epsilon)^{2.376}$
$d = 3$	$(1/\epsilon)^{5.5}$

Strong Coupling:

	$\lambda_c - \lambda_0$	p	n_{out}
$d = 1$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^9$	p^4	n_{out}^5
$d = 2$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^{6.3}$	p^6	$n_{\text{out}}^{7.128}$

Fermions

- We consider the massive Gross-Neveu model as an illustrative example:

$$H = \int dx \left[\sum_{j=1}^{N} \bar{\psi}_j \left(m_0 - i\gamma^1 \frac{d}{dx} \right) \psi_j + \frac{g^2}{2} \left(\sum_{j=1}^{N} \bar{\psi}_j \psi_j \right)^2 \right]$$

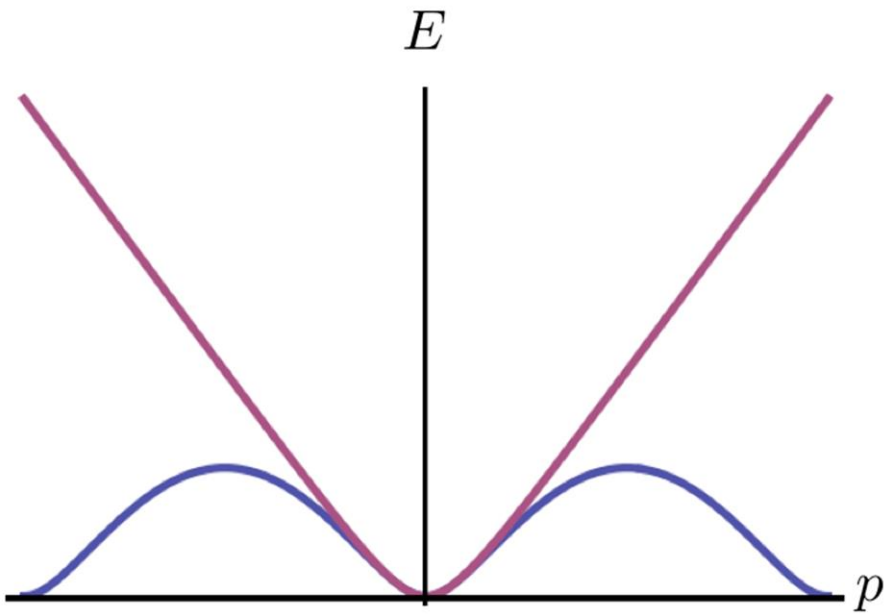
- New challenges encountered:
  - Fermion doubling problem
  - Free vacuum different from Bosonic case

[Jordan, Lee, Preskill, arXiv:1404.7115]

Fermion Doubling Problem

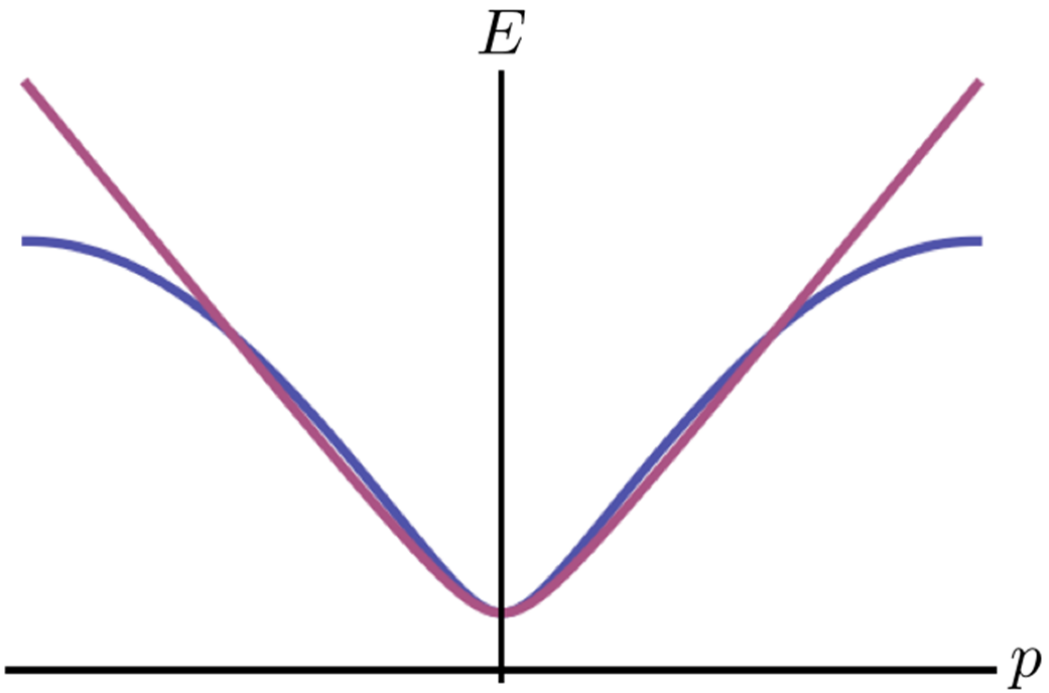
$$\frac{d\psi}{dx} \to \frac{\psi(x+a) - \psi(x-a)}{2a}$$

$$\sqrt{p^2 + m^2} \to \sqrt{\sin^2 p + m^2}$$



Wilson Term

$$H \to H - \frac{r}{2a} \sum_x \bar{\psi}(\psi(x+a) - 2\psi(x) + \psi(x-a))$$

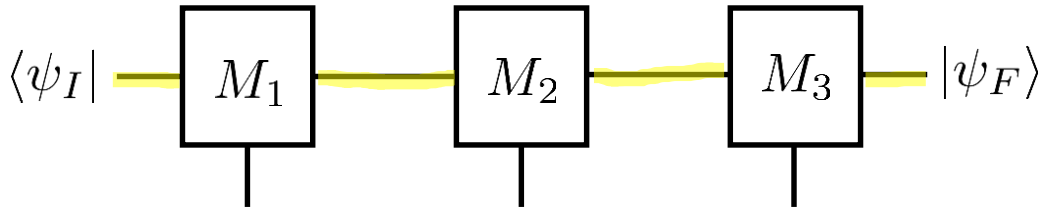


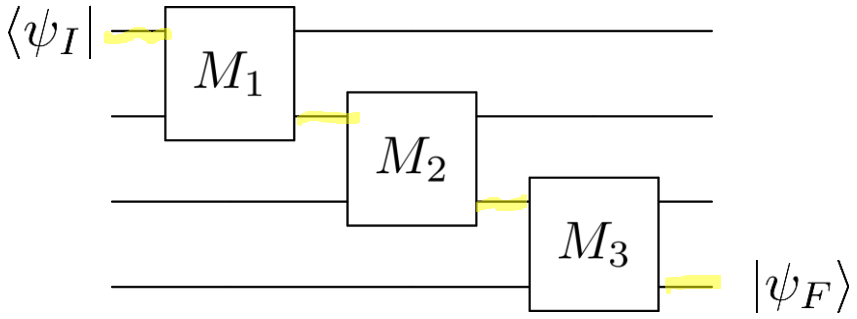
Improved State Preparation

- Two problems with adiabatic state preparation:
  - Cannot reach symmetry-broken phase
  - Runtime bound not practical $O(\epsilon^{-8})$
- A solution for both problems, complexity $O(e^{-3.23})$:
  - Classically compute a Matrix Product State description of the (interacting) vacuum
  - Compile this MPS directly into a quantum circuit that prepares the state
  - Excite single-particle wavepackets by simulating an oscillatory source term

[Moosavian, Jordan, arXiv:1711.04006]

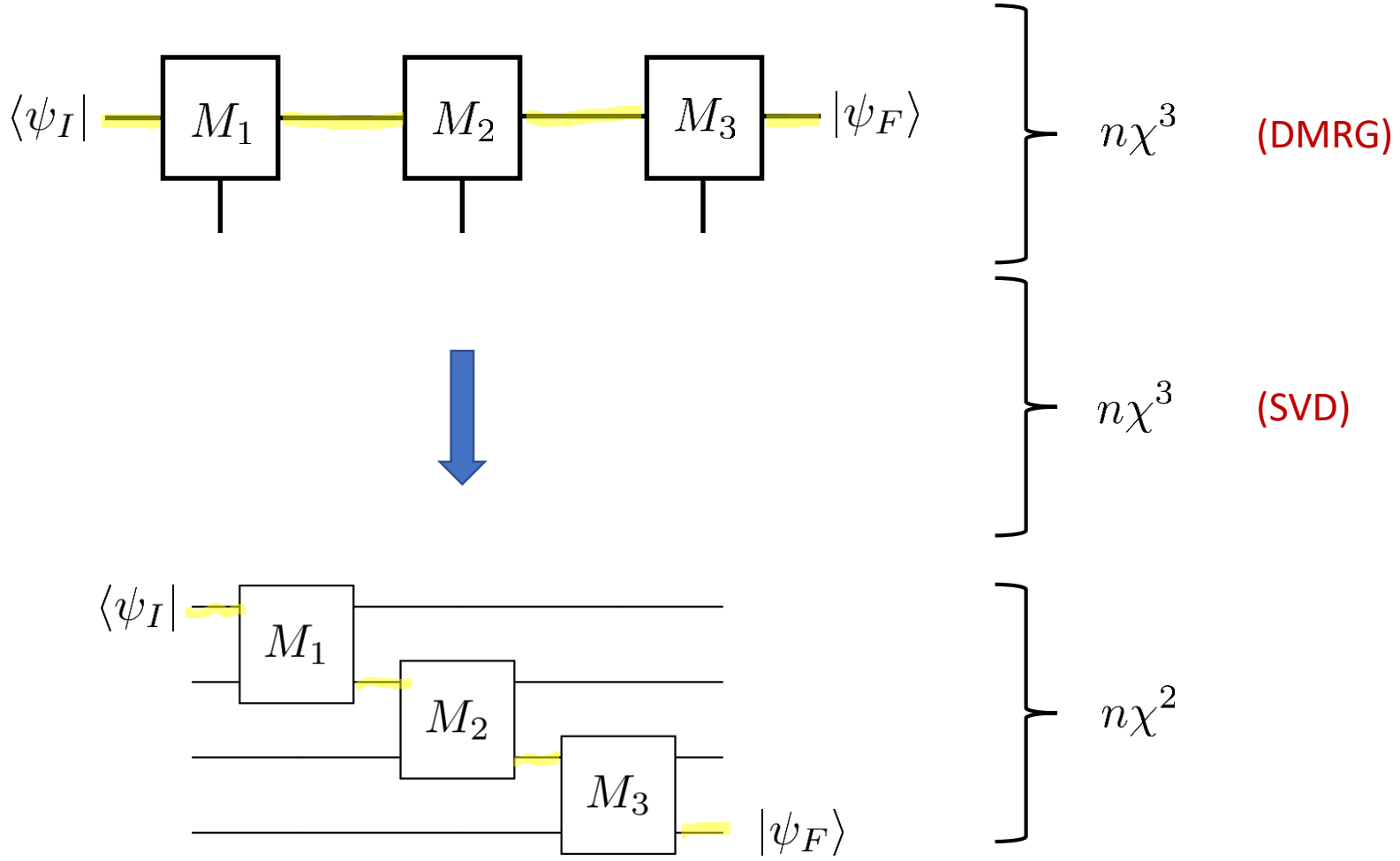
From MPS to Quantum Circuit





[Schon, Hamerer, Wolf, Cirac, Solano, 2006]

From MPS to Quantum Circuit



[Schon, Hamerer, Wolf, Cirac, Solano, 2006]

Bond Dimension

- It suffices to take $\chi = ke^{S_{1/2}}$ where errors shrink superpolynomially with *k*, resulting in $k \sim \epsilon^{-\sqrt{N/3}}$ [Swingle, arXiv:1304.6402]
- For correlation lengths large compared to lattice spacing, estimates of $S_{1/2}$ are available from conformal field theory:

$$S_{1/2} = \frac{N}{6} \log\left(\frac{1}{ma}\right) \qquad ma \ll 1$$

- $\epsilon \sim a$, hence for complexity of preparing interacting vacuum is:

$$\epsilon^{-N/2-1-\sqrt{3N}} \xrightarrow{N=1} \epsilon^{-3.23\ldots}$$

Exciting Particles

- Simulate dynamics with an oscillatory source term:

$$H(t) = H_0 + \lambda \cos(\omega t) W$$ (Rabi Oscillation)

- Ensure resonance with desired state:

$$\omega = \sqrt{p^2 + m^2}$$

- Ensure W selects desired momentum:

$$W = \int dx \left(f(x)\psi(x) + f^* \psi^\dagger(x) \right)$$

$$f(x) \propto e^{ipx - x^2/\sigma^2}$$

Exciting Particles

- How hard to drive the system (choosing λ)?
- How long to drive the system?

- Strategy:
  - Make 2-level approximation. Derive error bound:

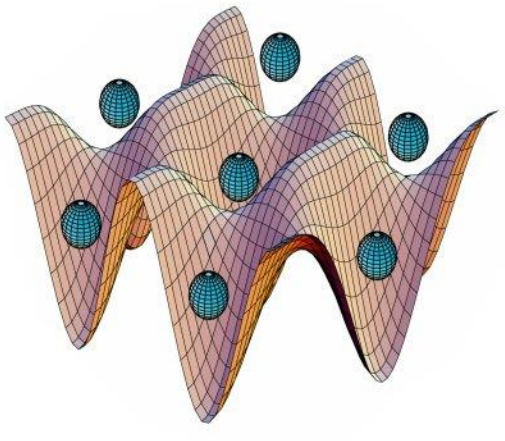
$$|\langle\psi(t)|\psi_2(t)\rangle| \geq 1 - (2\lambda + 3\lambda^2 t)\frac{1}{\delta}$$

  - Analyze 2-level system with Floquet theory:

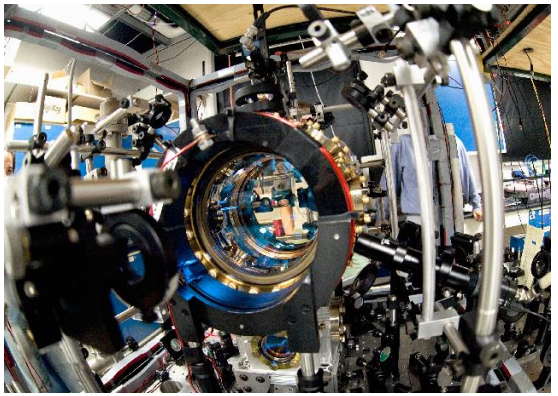
$$|\langle 1|\psi(\pi/\lambda)\rangle| \geq 1 - O\left(\frac{\lambda^2}{\omega^2}\right)$$

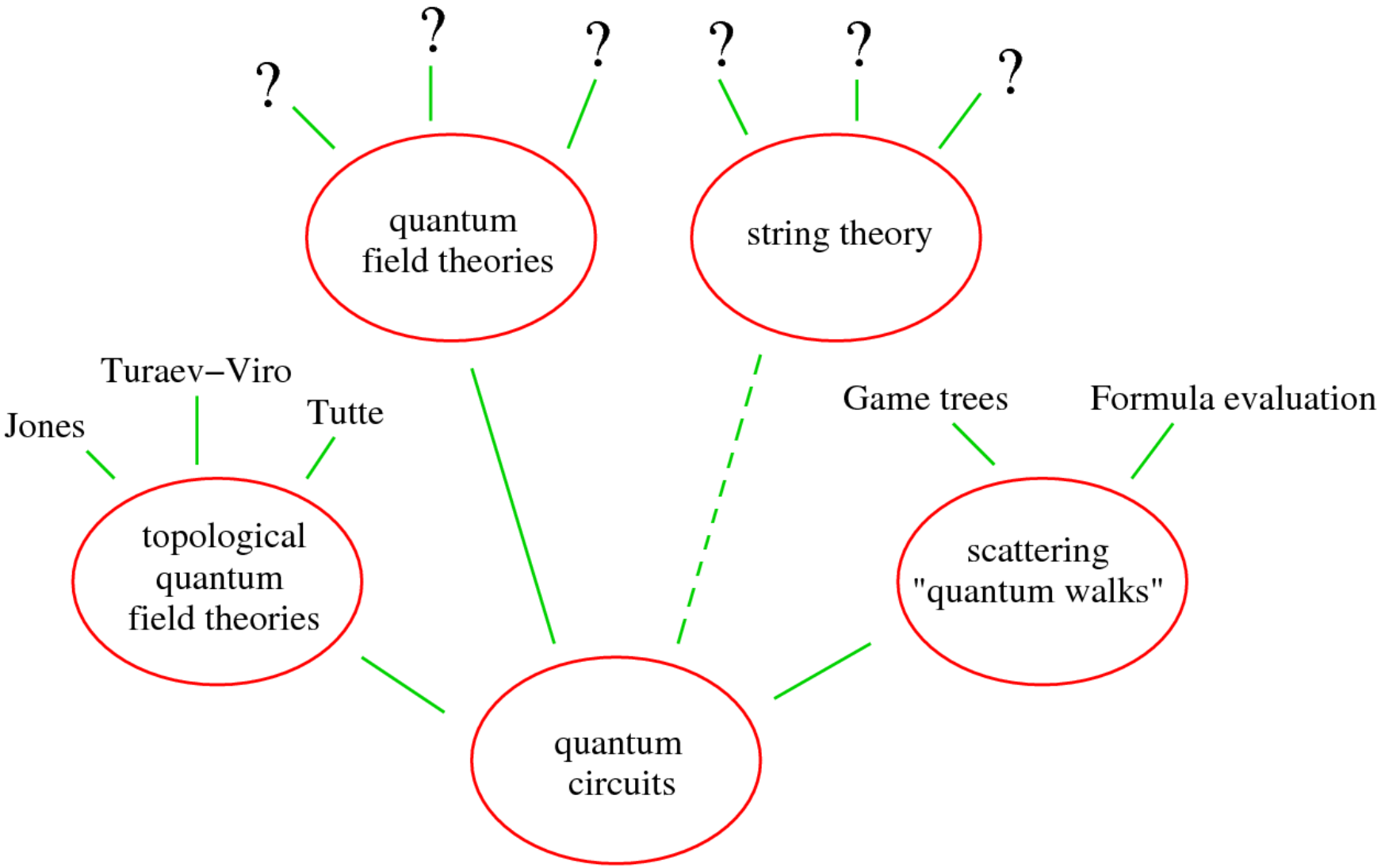
Analog Simulation

- No gates: just tune parameters in Hamiltonian to mimic lattice field theory and let it time-evolve.



- People really do this!




? ? ?
quantum field theories
? ? ?
string theory
Turaev–Viro
Jones
Tutte
topological quantum field theories
Game trees
Formula evaluation
scattering "quantum walks"
quantum circuits

What I'm trying to do is get you people who think about computer simulation to see if you can't invent a different point of view than the physicists have.

-Richard Feynman, 1981

In thinking and trying out ideas about "what is a field theory" I found it very helpful to demand that a correctly formulated field theory should be soluble by computer... It was clear, in the '60s, that no such computing power was available in practice.

-Kenneth Wilson, 1982

Conclusions

- Quantum algorithms with detailed analysis of complexity are being worked out for various QFTs
- State preparation is a key focus
- There are many opportunities for high energy physics and quantum information to contribute to each other's progress.
- Let's simulate the whole Standard Model!

Thanks