Continuum Naturalness



Ofri Telem Cornell University LBNL Particle Physics Seminar September 2018



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Motivation

- Higgs naturalness hints at top & gauge partners around the weak scale
 - Superpartners
 - Fermionic top partners and Z'
 - Other
- No new states have been observed so far for 136 fb⁻¹ @ 13TeV. Searches involve:
 - Heavy resonances
 - Same sign dileptons
 - Opposite sign dileptons
 - b-tagging
 - MET
- Do the new states actually have to be particles?

A New Top Partner Road Map



What it is vs. what it isn't

What it is

- A composite Higgs models where the top & gauge partners are continuum states
- Solution to "big" Hierachy problem the same as RS
- A different model for the confining dynamics at Λ : a continuum of composites
- Group theory the same as regular CH: SO(5)/SO(4) or other
- Very different phenomenology

<u>What it isn't</u>

- A new solution to the "big" Hierarchy problem
- Completely hidden from the LHC

Outline

Part I: Continuum composite Higgs

- The 4D action for continuum states: spectral densities
- Modeling a gapped continuum with a 5D Linear Dilaton
- Continuum composite Higgs: setting and results

Part II: The pair-production CS for generic continuum states

- A colorful continuum?
- Dispersion relations
- The generic result

Part I: Continuum Composite Higgs



The 4D action for continuum states



A 4D Weyl fermion

$$\mathcal{L}_{4\mathrm{D}} = -i\bar{\chi}\bar{\sigma}^{\mu}p_{\mu}\chi$$

Two point function:

$$\langle \bar{\chi}\chi
angle = rac{\sigma^{\mu}p_{\mu}}{p^2} \qquad rac{1}{ar{\sigma}^{\mu}p_{\mu}} = rac{\sigma^{\mu}p_{\mu}}{p^2}$$

Has pole at p=0: massless Weyl fermion

A 4D non-local Weyl fermion

$$\mathcal{L}_{4\mathrm{D}} = -i\bar{\chi}\frac{\bar{\sigma}^{\mu}p_{\mu}}{p^{2}G\left(p^{2}\right)}\chi$$

Two point function:

$$\langle \bar{\chi}\chi\rangle = \sigma^{\mu}p_{\mu}G(p^2)$$

Poles of
$$G(p^2)$$
 \longrightarrow particles
Branch cut of $G(p^2)$ \longrightarrow continuum

Can we describe the complex structure of G with a single real-valued function? Yes!

The spectral function

Dispersion relation:

$$G(p^2) = \int_0^\infty \frac{\rho(s)}{s - p^2} \, ds$$

where $\rho(s) = \frac{1}{\pi} \text{Im} \left[G(s) \right]$ is the spectral function

Generically:

$$\rho(s) = -\sum_{i} \delta(s - m_{i}) + \sigma(s)$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$
Tower of Continuum particles

The SM + continuum partners



Goal: SM + continuum as the IR description of some "confining" strong dynamics. More ambitious: composite Higgs

Method: model strong dynamics in 5D Linear Dilaton background

Modeling gapped continua with a 5D Linear Dilaton



Modeling 4D Strong Dynamics in 5D

Green's function for a fermion in RS1 (AdS, with UV brane and IR brane)

A tower of 4D composite fermions

Grossman, Neubert, hep/ph:9912408 Gherghetta, Pomarol, hep-ph/0003129

Green's function for a fermion in AdS₅

(RS2 with UV brane \rightarrow z=0)

Green's function for 4D ungapped continuum fermion

Cacciapaglia, Marandela, Terning, hep/ph:0804.0424

Ungapped continuum fermions (U.C.F)

A U.C.F is just a continuum particle with a specific spectral function:

$$\mathcal{L}_{4D} = -i\bar{\chi}\frac{\bar{\sigma}^{\mu}p_{\mu}}{p^{2}G\left(p^{2}\right)}\chi \qquad G(p^{2}) \propto \frac{\Gamma\left(\frac{5}{2}-d\right)}{4^{d-2}\Gamma\left(d-\frac{3}{2}\right)}\frac{1}{(-p^{2})^{\frac{5}{2}-d}}$$

It can be regarded as either a continuum or a "fractional number of particles"

d is the scaling dimension of the U.C.F. When d \rightarrow 3/2 it approaches a single Weyl fermion.



The AdS/U.C.F. correspondence

A fermion in AdS₅:

Metric:
$$ds^2 = a^2(z) \left(dx^{\mu} dx_{\mu} - dz^2 \right)$$
 Warp factor: $a(z) = R/z$



Bulk fermion Lagrangian:

$$\mathcal{L}_{5\mathrm{D}} = a^4(z) \left[\mathcal{L}_{\mathrm{kin.}} + \frac{\mathbf{C}}{z} \left(\psi \chi + \bar{\chi} \bar{\psi} \right) \right]$$

5D bulk mass ↔ 4D scaling dimension

The AdS/U.C.F. correspondence

5D EOM:

$$\chi'(z) - p\psi(z) + \frac{c-2}{z}\chi(z) = 0$$
$$\psi'(z) + p\chi(z) - \frac{c+2}{z}\psi(z) = 0$$

"IR regular" solution:

$$\chi(z) = A\left(\frac{z}{R}\right)^{\frac{5}{2}} H_{c+\frac{1}{2}}^{(1)}(pz) \qquad \psi(z) = A\left(\frac{z}{R}\right)^{\frac{5}{2}} H_{c-\frac{1}{2}}^{(1)}(pz)$$

Green's function:

$$G(p^2) = \frac{1}{p} \bar{\lim}_{z \to 0} \frac{\psi(z)}{\chi(z)} \propto \frac{\Gamma\left(\frac{1}{2} - c\right)}{4^c \Gamma\left(\frac{1}{2} + c\right)} \frac{1}{(-p^2)^{\frac{1}{2} - c}}$$

What's so special about it? This is the Green's function for a 4D U.CF !

The AdS/U.C.F. correspondence



If we identify @=2+@4D scaling & 5D bulk massdimension

Cacciapaglia, Marandela, Terning, hep/ph:0804.0424

A different viewpoint: the Schrodinger picture

We want to understand why AdS_5 gave rise to a continuum.

Best way: convert to effective Schrodinger eqn.

Bulk EOM:
$$-\chi''(z) + \frac{4}{z}\chi'(z) + \frac{c^2 + c - 6}{z^2}\chi(z) = p^2\chi(z)$$

Define:
$$\chi(z) = \left(\frac{z}{R}\right)^2 \hat{\chi}(z)$$

Schrodinger equation!

$$-\hat{\chi}''(z) + V(z)\hat{\chi}(z) = p^2\hat{\chi}(z)$$
$$V(z) = \frac{c(c+1)}{z^2}$$



Continuum = unbounded solutions to the effective Schrodinger problem

Back to our story: a gapped continuum?

We've seen how to model ungapped Unparticles in AdS₅

For realistic model building, we need to model a gapped continuum

Method: introduce a dilaton in the 5D picture

$$\mathcal{L}_{5D} = e^{\Phi(z)} a^4(z) \left[\mathcal{L}_{kin} + \frac{c + y \Phi(z)}{z} \left(\psi \chi + \bar{\chi} \bar{\psi} \right) \right]$$

This modifies the bulk EOM and the effective Schrodinger potential

Falkowski, Perez-Victoria, hep/ph:0806.1737 Batell, Gherghetta, Sword, hep/ph:0808.3977 Gutsche et al. hep/ph:1108.0346

A gapped continuum!

Effective Schrodinger eqn. with dilaton

$$-\hat{\chi}''(z) + V(z)\hat{\chi}(z) = p^2\hat{\chi}(z)$$
$$V(z) = \frac{c(c+1) + y\Phi(z)(2c + y\Phi(z) + 1) - yz\Phi'(z)}{z^2}$$

For a gapped continuum we need $V(z \rightarrow \infty) = \text{finite gap}$

An IR scale gap appears for a linear dilaton

$$\Phi(z) = \mu (z - R) \qquad \mu \sim \text{TeV}$$

the gap is $y\mu$



Stabilization – not our problem

A natural question is how the scale $\,\mu$ ~ TeV emerges and how the linear dilaton is stabilized.

We don't claim to have a new solution to the "big" Hierarchy problem

Use regular RS – Goldberger – Wise stabilization



In 4D language:

The IR scale is generated as usual by "confinement" / dimensional transmutation. The linear dilaton models the emergence of a gapped continuum below the IR scale 1/R'.

The Linear Dilaton/Gapped continuum Correspondence

"IR regular" solutions to bulk EOM:

See also Cai, Cheng, Medina, Terning

$$\chi(z) = A a^{-2}(z) e^{\frac{4}{3}\Phi(z)} W\left(-\frac{c\mu y}{\Delta}, c + \frac{1}{2}, 2\Delta z\right)$$
$$\psi(z) = A a^{-2}(z) e^{\frac{4}{3}\Phi(z)} W\left(-\frac{c\mu y}{\Delta}, c - \frac{1}{2}, 2\Delta z\right) \frac{\mu y - \Delta}{p}$$

where $\Delta \equiv \sqrt{y^2 \mu^2 - p^2}$ goes imaginary above the gap

Green's function:

$$G(p) = \frac{1}{p} \bar{\lim}_{z \to 0} \frac{\psi(z)}{\chi(z)} = \frac{y\mu - \Delta}{p^2} \frac{\Gamma(1 - 2c)}{\Gamma(1 + 2c)} \frac{\Gamma\left(1 - c \frac{y\mu - \Delta}{\Delta}\right)}{\Gamma\left(1 - c \frac{y\mu + \Delta}{\Delta}\right)} (2\Delta)^{2c}$$

As expected, it has a pole at $p^2=0$ (zero mode) and a branch cut (continuum) for $p^2 \ge y\mu$ when Δ is imaginary

Cai, Cheng, Medina, Terning, hep/ph:0910.3925

The Linear Dilaton/Gapped continuum Correspondence

Fermion spectral functions:



(zero modes not depicted)

Gauge spectral function:





1st Phenomenology Result: No s-channel Resonances

Cross-section for s-channel vector resonance:



In standrad RS, the KK gluons show up as s-channel resonances.

In our setting, the continuum gluon only yields a continuous execess above the SM background \rightarrow no s-channel resonances!

Remember the Road Map?







Composite Higgs models

- A class of solutions to the hierarchy problem
- A strongly interacting sector confines at Λ ~10 TeV
 - → composite vectors and fermions
 - \rightarrow global symmetry G broken to a subgroup H
- SM fermions & gauge bosons: a mixture of composite and elementary states
- The Higgs is the pseudo-Goldstone of G/H
 → no tree level potential
- Higgs potential arises radiatively
 - \rightarrow quadratic divergences are cut off at g_*f
 - \rightarrow composite top & gauge partners at g_{*}f



Georgi, Kaplan, Phys.Lett. 136B (1984) Contino, Nomura, Pomarol 0306259 Agashe, Contino, Pomarol 0412089

Composite Higgs models

- A class of solutions to the hierarchy problem
- A strongly interacting sector confines at Λ ~10 TeV
 - → composite vectors and fermions
 - \rightarrow global symmetry SO(5) broken to a subgroup SO(4)
- SM fermions & gauge bosons: a mixture of composite and elementary states
- The Higgs is the pseudo-Goldstone of SO(5)/SO(4)
 → no tree level potential
- Higgs potential arises radiatively

 \rightarrow quadratic divergences are cut off at g,f

 \rightarrow composite top & gauge partners at g f

This is why it works! (as a solution to the hierarchy problem)



Georgi, Kaplan, Phys.Lett. 136B (1984) Contino, Nomura, Pomarol 0306259 Agashe, Contino, Pomarol 0412089

Composite Higgs from 5D – Gauge Higgs Unification

In RS1:



Where is the Higgs in all this? The 5D gauge fields A_m^a break to (A_{μ}^a, A_{5}^a) . The A_5 of the "Dirichlet" generators has a 4D scalar zero mode \rightarrow the composite Higgs. Bonus: we can calculate the radiative Higgs potential from the 5D Green's functions in the presence of $\langle A_5 \rangle$.

Continuum Composite Higgs

Can model gapped continua in 5D Can construct composite Higgs models in 5D

Let's combine the two!



The gauge fields in SO(4) are continuous across the IR brane The gauge fields in SO(5)/SO(4) get Dirichlet b.c. on the IR brane The Higgs is the A5 of the (-,-) gauge field

Continuum Composite Higgs - Fermions

The bulk fermion reps. are $Q_1(5)$, $T_{R}(5)$, $B_{R}(10)$ of SO(5)



On the IR brane \rightarrow SO(4) invariant mass terms:

$$M_{1}Q_{L}^{1}T_{R}^{1} + M_{4}Q_{L}^{4}T_{R}^{4} + M_{d}Q_{L}^{4}B_{R}^{4}$$

- → SM Yukawa couplings
- → Continuum Yukawa couplings

Feeling Lost? Here's a recap

We want to construct a continuum composite Higgs model

The 4D picture is confinement at Λ ~10 TeV which breaks SO(5) \rightarrow SO(4), with the Higgs a pNGB of SO(5)/SO(4). The low energy spectrum is the SM + continuum states above μ ~2TeV.

The 5D picture is AdS with a bulk symmetry SO(5) and an IR brane where the symmetry is reduced to SO(4). Beyond the IR brane, a linear dilaton turns on and yields continuum spectra. The SM Green's functions have zero modes + branch cuts above μ ~2TeV.

Continuum Compsite Higgs - Results

Parameters

$R/R'=10^{-16}$, $1/R'=2.25$ TeV, $\mu=2.08$ TeV				
y=0.5,	c _Q =0.2,	с _т =-0.2	2,	c _B =-0.01
M ₁ =1.2,		M ₄ =0,	M _d =0.01	

Inverse Green's functions



We get the right mt, mb, mW, mZ

Gauge spectral functions

Fermion spectral functions



Continuum Compsite Higgs – the Higgs potential

The Higgs is a pNGB of SO(5)/SO(4):

- The potential is radiatively generated (Coleman-Weinberg potential)
- The radiative corrections are cut off at the global symmetry breaking scale $g_{*}f$ this is the essence of composite Higgs as a solution to the Hierarchy problem
- We get the potential from the fermion & gauge boson Green's functions

$$V(h) = \frac{3}{16\pi^2} \int dp \, p^3 \left[-4\sum_j \log G_{f_j}(ip) + \sum_k \log G_{g_k}(ip) \right]$$

The Higgs potential - results



We exactly reproduce the Higgs mass and VEV for the compositeness scale f=3v

This is consistent with the bound $v/f \le 3$ for electroweak precision

Continuum Composite Higgs - Summary

- Found a way to model gapped continua in 5D
- Used our 5D setting to construct a fully realistic composite Higgs model
 - All SM masses and couplings are reproduced, consistent with EWP
 - The fermion & gauge continua start at ~1 TeV (can push higher at the cost of tuning)
 - Still need to check tuning (at least as good as regular CH)



Continuum Composite Higgs - Phenomenology

- Very different gauge partner pheno no s-channel resonances!
- LHC pheno no cascade decays. Work in progress for same-sign dilepton bounds (similar or better than CH)
- 100 TeV pheno possible cascade decays between continuum fermions and gluons. Need to calculate total pair-production CS for continuum fermions.
- Flavor constraints the same as standard CH models

Continuum Composite Higgs - Phenomenology

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 - Part II Goal
- Flavor constraints the same as standard CH models

Part II: Continuum Pair-Production



The Pair-production CS for Continuum Fermions

Looking for the tree-level CS

$$\sigma\left(q\bar{q}\to G\to T\bar{T}\right)$$

for a continuum fermion T with a generic 2-pt function G(p)

Looks easy, but is actually hard!

- TTG vertex has momentum dependence
- Final state phase space hard to calculate
 - For Unparticles, LIPS is like a fractional number of particles
 - For generic continuum?

Using the Optical Theorem in Reverse

$$\sigma \left(q\bar{q} \to G \to T\bar{T} \right) = -\frac{4\pi\alpha_S}{3s} \operatorname{Im}\Pi_g(s)$$



We need to:

- Get continuum fermion Feynman rules
- Calculate non-standard loops

Feynman Rules: A Colorful Continuum?



The Continuum Propagator

Take a generic continuum fermion:

$$\mathcal{L}_f = -i\bar{\chi}\bar{\sigma}^{\mu}p_{\mu}\Sigma\left(p^2\right)\chi$$

The propagator is then



The Gluon-Continuum Vertex

Gluon-Continuum interaction uniquely defined by gauge invariance This is not your typical -igT^a, but two momentum dependent vertices

Exact form derived by:

- Switching to position space
- Inserting a Wilson line to preserve gauge invariance
- Switching back to momentum space
 Mandelstam 62', Terning 91'



Vertices satisfy generalized Ward-Takahashi identities

$$q_{\mu} \Gamma^{a;\mu} = i T^{a} \bar{\sigma}^{\mu} \left[(p+q)_{\mu} \Sigma (p+q) - p_{\mu} \Sigma (p) \right]$$
$$q_{\mu} \Gamma^{ab;\mu\nu} = \Gamma^{b;\nu}(p,q) T^{a} - T^{a} \Gamma^{b;\nu}(p,-q)$$

Calculating the Loops: Dispersion Relations

$$G(s) = \frac{1}{\pi} \int_0^\infty \frac{\operatorname{Im} G(s')}{s' - s} \, ds'$$

The Continuum Loop

Focus on:



$$\operatorname{Im} \Pi_{\psi 3}(q) = -\frac{1}{2} \operatorname{Im} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+q)^2 (q^2 + 2k \cdot q)^2} \left[\frac{\Sigma(k+q)}{\Sigma(k)} P_+ + \frac{\Sigma(k)}{\Sigma(k+q)} P_- \right]$$

The Continuum Loop

Focus on:



$$\operatorname{Im} \Pi_{\psi 3}(q) = -\frac{1}{2} \operatorname{Im} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+q)^2 (q^2 + 2k \cdot q)^2} \left[\underbrace{\frac{\Sigma(k+q)}{\Sigma(k)}}_{\Sigma(k)} P_+ + \underbrace{\frac{\Sigma(k)}{\Sigma(k+q)}}_{\Sigma(k+q)} P_- \right]$$

Denominator linear in loop momentum (like HQET) This is from the vertex momentum dependence

Model dependent continuum spectral density. We will mod it out using a dispersion relation

Uniquely defined polynomials in the loop momentum

The Continuum Loop

Defining the following

$$I[P] \equiv \int \frac{d^d k}{(q^2 + 2k \cdot q)^2} P \qquad \sigma_1(p) = \frac{\Sigma(p)}{p^2} , \ \sigma_2(p) = \frac{\Sigma(p)^{-1}}{p^2}$$

We get the "compact" form

Im
$$\Pi_{\psi_3}(q) = -\frac{1}{2(2\pi)^d} \operatorname{Im} I \left[\sigma_1(k+q)\sigma_2(k) P_+ + \sigma_1(k)\sigma_2(k+q) P_- \right]$$

The expressions in red are dependent on the particular spectral density We want to get the out of the loop integral I[P]

A Dispersion Relation for $\sigma_{1,2}$

To mod out the model dependence from our loop integral, we use dispersion relations

$$\sigma_1(p^2) = \frac{c_{-2}}{p^4} + \frac{c_{-1}}{p^2} - \frac{1}{\pi} \int_{\mu^2}^{\infty} \frac{\operatorname{Im} \sigma_1(s) ds}{s - p^2}$$

$$\sigma_2(p^2) = \sigma_2(p_0^2) - \frac{p^2 - p_0^2}{\pi} \int_{\mu^2}^{\infty} \frac{\operatorname{Im} \sigma_2(s) ds}{(s - p_0^2)(s - p^2)}$$

Now all the p² (and loop momentum) dependence is contained in a Feynman numerator/denominator that we can integrate over

$$\operatorname{Im}\Pi_{3}(q) = -\frac{1}{2(2\pi)^{d}} \frac{1}{\pi^{2}} \operatorname{Im} \iint_{\mu^{2}}^{\infty} \frac{\operatorname{Im}\sigma_{1}(s) \operatorname{Im}\sigma_{2}(s') \, ds ds'}{s' - q_{0}^{2}} \underbrace{I_{\operatorname{kernel}}(q, q_{0}, s, s') + \dots}_{\operatorname{Model independent Feynman integral (still non-trivial)}}$$

The Pair-Production Cross Section – Summary

$$\sigma\left(q\bar{q} \to G \to T\bar{T}\right) = -\frac{4\pi\alpha_S}{3s}\operatorname{Im}\Pi_g(s)$$

$$\operatorname{Im}\Pi_g(q) = -\frac{1}{2(2\pi)^d}\frac{1}{\pi^2}\operatorname{Im}\iint_{\mu^2}^{\infty}\frac{\operatorname{Im}\sigma_1(s)\operatorname{Im}\sigma_2(s')\,dsds'}{s'-q_0^2}I_{\text{kernel}}\left(q,q_0,s,s'\right) + \dots$$

- The model independent kernels I(q,q0,s,s') involve a linear denominator and are similar to 2-loop HQET
- We can* calculate them by Schwinger transformations and the Mellin-Barnes technique
- The result has a general significance beyond our model valid for any continuum fermion (can generalize to any particle)

* Wrote our own 1-loop Mellin-Barnes code for linear denominators. Currently struggling with double Harmonic sums using the Sigma package.

Conclusions

- Found a way to model gapped continua in a 5D Linear Dilaton setting
 See also Cai, Cheng, Medina, Terning, hep/ph:0910.3925
- Constructed a fully realistic composite Higgs model with continuum top & gauge partners
- Closing in on a generic formula for the pair production of continuum fermions
- Future study:
 - LHC pheno no resonances, need to finish top partner bounds
 - 100 TeV pheno cascade decays, spherical event shapes (?) and more
 - Cosmology what does a continuum-populated universe look like?

Thank You!



Backup



Stabilizing a Linear Dilaton

- Stabilization by the superpotential method:
 - → Solve Einstein-scalar equations by assuming a "superpotential"

One scalar:
$$W = R^{-1} \left(1 + e^{\phi/R} \right) \longrightarrow$$
 gapped continuum

Carber, von Gersdorff, Quiros, hep/ph: 0907.5361

2 scalars:
$$W(\Phi,T) \sim R^{-1} \left[2e^{\frac{T^2}{\Lambda_T^2}} - \left(1 - \frac{\Phi}{\Lambda_\Phi}\right) e^{\phi/\Lambda_\Phi} \right] \longrightarrow \text{linear dilaton}$$

Batell, Gherghetta, Sword, hep/ph: 0808.3977

As a "little String theory" construction

Aharony, Berkooz, Kutasov, Seiberg, hep/ph:9808149 Antoniadis, Arvanitaki, Dimopoulos, Giveon, hep/ph:1102.4043

• Other:

Guidice, Kats, McCullough, Torre, Urbano, hep/ph:1711.08437