Homework Assingment # 2

Physics 230A: QFT I; Spring 2006

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(1) Consider the Lagrangian of a harmonic oscillator,

$$\mathcal{L} = \frac{m}{2}\dot{q}^2 - \frac{m\omega^2}{2}q^2.$$

Using the path integral formulation, evaluate the transition amplitude

$$\langle q_F, t_F | q_I, t_I \rangle,$$

and show that the initial wave packet of the Gaussian form

$$\Psi(q,0) = \left(\frac{m\omega}{\pi}\right)^{1/4} \exp\left(-\frac{m\omega}{2}(q-q_0)^2\right)$$

does not spread with time.

(2) Consider a free scalar field $\tilde{\phi}$, near the infrared fixed point in which the physics is dominated by the nonzero mass of $\tilde{\phi}$. Using the dimension assignment to $\tilde{\phi}$ and its powers that is natural near this fixed point (as discussed in class), evaluate the engineering dimensions of $(\partial\tilde{\phi})^{2m}\tilde{\phi}^{2n}$ for any positive integer m, n. Calculate the dimensions of the corresponding couplings $\lambda_{m,n}$, and decide which ones are irrelevant, relevant or marginal. What is the intuitive physical interpretation of the result?

(3) Consider a free scalar field in D spacetime dimensions, described by the following action, which is of higher order in derivatives:

$$S = \int d^D x \, (\partial_\mu \partial^\mu \phi)^2.$$

What is the natural dimension of ϕ at this RG fixed point? With this dimension assignment, what is the dimension of $(\partial_\mu\phi\partial^\mu\phi)^m\phi^{2n}$ for positive integers m, n? List all values of m, n for which the corresponding coupling would be relevant in $D = 5$ spacetime dimensions.