Physics 250 – Introduction to M-Theory
Final Take-Home Exam

Here is the final take-home exam. The answers can be brief. The completed exam is due on Thursday, December 5, you should turn it in during our last class that day. The questions are based closely on topics discussed explicitly in class, although they may require some slightly creative thinking. If you feel that you do need some more background information on any of these topics I have listed the relevant references below. Enjoy!

1. **D-brane Charges from M-Theory Superalgebra.**
   Write down the anticommutation relations of the M-theory superalgrebra (the maximally centrally extended super Poincaré algebra in eleven dimensions). Imagine compactifying to Type IIA superstring theory, i.e., from $\mathbf{R}^{11}$ to $\mathbf{R}^{10} \times \mathbf{S}^1$. Rewrite the anticommutation relations of the M-theory algebra using the ten-dimensional notation of Type IIA string theory, and identify the M-theory origin of all the D-brane charges.
   [reference for Problem 1: hep-th/9712004.]
2. **Bianchi Identity in the Presence of Gluino Condensate.**
   Consider heterotic M-theory. In Calabi-Yau compactifications, the hidden E_8 gauge sector (residing at the other, "hidden" boundary of spacetime) can develop a gaugino condensate, i.e., a non-zero vacuum expectation value of the fermion bilinear $\overline{\chi}^a \Gamma^{IJK} \chi^a$ (here χ^a is the gaugino of the E_8 super Yang-Mills theory at the hidden boundary, a is the adjoint representation of E_8, and Γ^{IJK} is the completely antisymmetrized product of three gamma matrices with all three indices parallel to the boundary). It turns out that in thre presence of such a non-zero gaugino condensate, the four-form field strength G_{MNPQ} of heterotic M-theory (with $M, N, \ldots$ going over all eleven indices) is modified such that the new field strength is given by

$$\begin{aligned} \widetilde{G}_{IJKy} &= G_{IJKy} - \frac{\sqrt{2}}{16\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3} \delta(y) \overline{\chi}^a \Gamma_{IJK} \chi^a, \\ \widetilde{G}_{IJKL} &= G_{IJKL}. \end{aligned} \tag{1}$$

   Here $I, J, \ldots$ are the ten-dimensional indices parallel to the boundary, y is the coordinate along the extra, $\mathbf{S^1/Z_2}$ dimension normal to the boundary, and G_{MNPQ} is the four-form field strength as usually defined in heterotic M-theory (i.e., as defined in class).
   (a) Using the Bianchi identity for G_{MNPQ} as discussed in class, write down the Bianchi identity for $\widetilde{G}_{MNPQ}$.
   (b) On a Calabi-Yau manifold the gaugino condensate is typically given by a constant multiple of the holomorphic three-form Ω^{IJK} of the Calabi-Yau manifold,

$$\overline{\chi}^a \Gamma_{IJK} \chi^a = c\Omega_{IJK}, \tag{2}$$

   with c a real constant independent of anything. How does the Bianchi identity for $\widetilde{G}_{MNPQ}$ simplify for this particular form of the gaugino condensate?
   [reference for Problem 2: hep-th/9608019]

3. **Scales in Matrix Theory.** Consider Matrix theory, describing DLCQ of M-theory in eleven dimensions. Using the holographic UV-IR correspondence between energy scales E and radial distances r (as discussed in class), identify the energy scale E that corresponds in the Matrix quantum mechanics to the radial distance of the order the eleven-dimensional Planck length, $r \sim M^{-1}$. Is either the supergravity description or the Matrix quantum mechanics description good for r smaller than this distance? [reference for Problem 3: hep-th/9903165.]