Physics 250: Introduction to String Theory

Homework Assignment III

This assignment is worth 10 points, and is due in class on Thu, March 7.

1. (A supersymmetric variation on Problem 1.4 of [Polchinski, Vol. I]):
   Consider open superstrings in the Green-Schwarz formalism, in light-cone gauge. (a): Show that the open string states at the first massive level above the massless level (which contained the massless vector supermultiplet discussed in class) form complete representations of $SO(9)$ (or, more precisely, of $\mathrm{Spin}(9)$); list the irreducible representations of $SO(9)$ that appear.
   (b) Verify that the number of fermionic states matches the number of bosonic states, as expected from a (spacetime) supersymmetric theory.
   (c) Noticing that $SO(9)$ would be the little group of massless particles in eleven spacetime dimensions, compare this spectrum of string states at the first massive level to the massless spectrum of a hypothetical supersymmetric theory in eleven spacetime dimensions.
2. Repeat (1.a, b, c) in the Neveu-Schwarz-Ramond formulation of the open superstring, carefully incorporating the GSO projection.
3. Consider a Majorana fermion ψ_α on a two-dimensional world-sheet Σ in the background of a general zweibein field ${e_a}^A$ and its associated spin connection ω_a^{AB}. (Here $a, b, \ldots$ are worldsheet vector indices, $A, B, \ldots$ are the internal indices of a two-dimensional Lorentz group $SO(1,1)$, and the spinor index α on ψ will be kept implicit.) The action for ψ is given by
   $$S = \frac{i}{2\pi}\int_\Sigma d^2\sigma\, e\,\overline{\psi}\rho^a D_a\psi, \tag{1}$$
   where $e \equiv \det({e_b}^A)$ and the covariant derivative D_a is defined on ψ by
   $$D_a\psi \equiv (\partial_a + \frac{1}{4}\omega_a^{AB}\rho_{AB})\psi, \tag{2}$$
   with $\rho_{AB} \equiv \frac{1}{2}[\rho_A, \rho_B]$ the antisymmetrized product of two gamma matrices of the two-dimensional Lorentz group $SO(1,1)$. Verify that the covariant derivative can be replaced in Eqn. (1) by the ordinary partial derivative ∂_a (i.e., the spin-connection-dependent term in (1) vanishes identically). Which properties of the two-dimensional gamma-matrix algebra and of Majorana fermions do you have to use in this proof? [Thanks to Kenji Kadota for asking this question in class! :-)]