Week 9 Homework

(due Thu, March 31)

In this week's homework assignment, consider a non-Abelian Yang-Mills gauge theory in $3+1$ spacetime dimensions, described by gauge potential A^a_μ, with a the adjoint representation index of a compact group G.

Problem 1.

The "axial gauge" condition is given by

$$n^\mu A^a_\mu = 0,$$

with n^μ a fixed, constant spacelike vector, i.e., $n^2 < 0$. Show that in this gauge, the gauge boson propagator in momentum space is of the form

$$\Delta^{\mu\nu}_{ab}(k) = \frac{\delta_{ab}}{k^2}\left\{-g^{\mu\nu} + \frac{1}{k\cdot n}\left(n^\mu k^\nu + n^\nu k^\mu\right) + \frac{(k^2/\xi - n^2)}{(n\cdot k)^2} k^\mu k^\nu\right\},$$

with ξ an arbitrary constant.

Problem 2.

The "Coulomb gauge" condition is given by

$$\partial_i A^a_i = 0,$$

where $i = 1, 2, 3$ goes over the space components of the potential. Calculate the propagator of the gauge bosons in this gauge.

Problem 3.

Consider the Yang-Mills theory coupled to a Dirac fermion ψ in some representation R of the gauge group. The Lagrangian of this theory will be given by

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar\psi\gamma^\mu D_\mu\psi - m\bar\psi\psi,$$

with $D_\mu\psi = (\partial_\mu - igt^a A^a_\mu)\psi$, g the gauge coupling, and t^a the matrices representing the generators of the gauge symmetry, in the corresponding representation R.

Show that, to the lowest order in the gauge coupling g, the fermion two-on-two scattering amplitude

$$\psi, \psi \to \psi, \psi$$

is the same in the covariant, the axial, and the Coulomb gauges. (Here "covariant gauge" refers to the gauge fixing condition $\partial^\mu A^a_\mu = 0$.)

[Hint: Write the amplitude in the schematic form $J_\mu \Delta^{\mu\nu} J_\nu$, and use the fact that the fermion currents J_μ are conserved.]