Consider a free nonrelativistic particle in one spatial dimension q. The time-dependent Schrödinger wavefunction is defined by $\Psi(q,t) = \langle q, t | \Psi \rangle$. Assume that the wavefunction is initially given by the Gaussian wave packet

$$\Psi(q,0) = \left(\frac{1}{2\pi\sigma^2} \right)^{1/4} \exp\left(-\frac{(q-q_0)^2}{4\sigma^2} \right).$$

Using the fact that

$$\Psi(q_F, t_F) = \int dq_I \, \langle q_F, t_F | g_I, t_I \rangle \, \Psi(q_I, t_I),$$

show that the wave packet will spread as time evolves, leading to

$$|\Psi(q,t)|^2 = \left(\frac{1}{2\pi\sigma^2(t)} \right)^{1/2} \exp\left(-\frac{(q-q_0)^2}{2\sigma^2(t)} \right).$$

Determine the function $\sigma(t)$.