Issues in semileptonic B decays

Zoltan Ligeti

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From:  kowalews@uvic.ca
To:    Zoltan Ligeti <ligeti@lbl.gov>
Date:  11/21/04 07:36
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current status of using q2-Mx (i.e., do you still
feel it has benefits relative to other variables?)

global HQE fits to moments
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see: C. Bauer, Z.L., M. Luke, PRD **64** (2001) 113004 [hep-ph/0107074];
C. Bauer, Z.L., M. Luke, PLB **479** (2000) 395 [hep-ph/0002161].

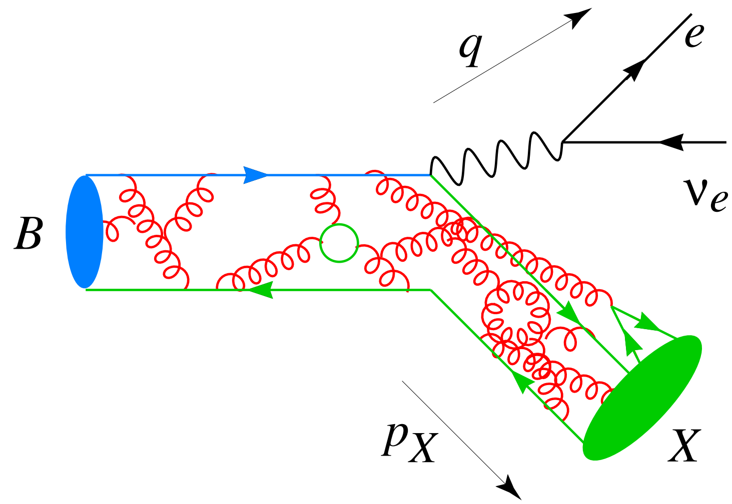
C. Bauer, Z.L., M. Luke, A. Manohar, M. Trott, PRD **70** (2004) 094017 [hep-ph/0408002];
C. Bauer, Z.L., M. Luke, A. Manohar, PRD **67** (2003) 054012 [hep-ph/0210027].

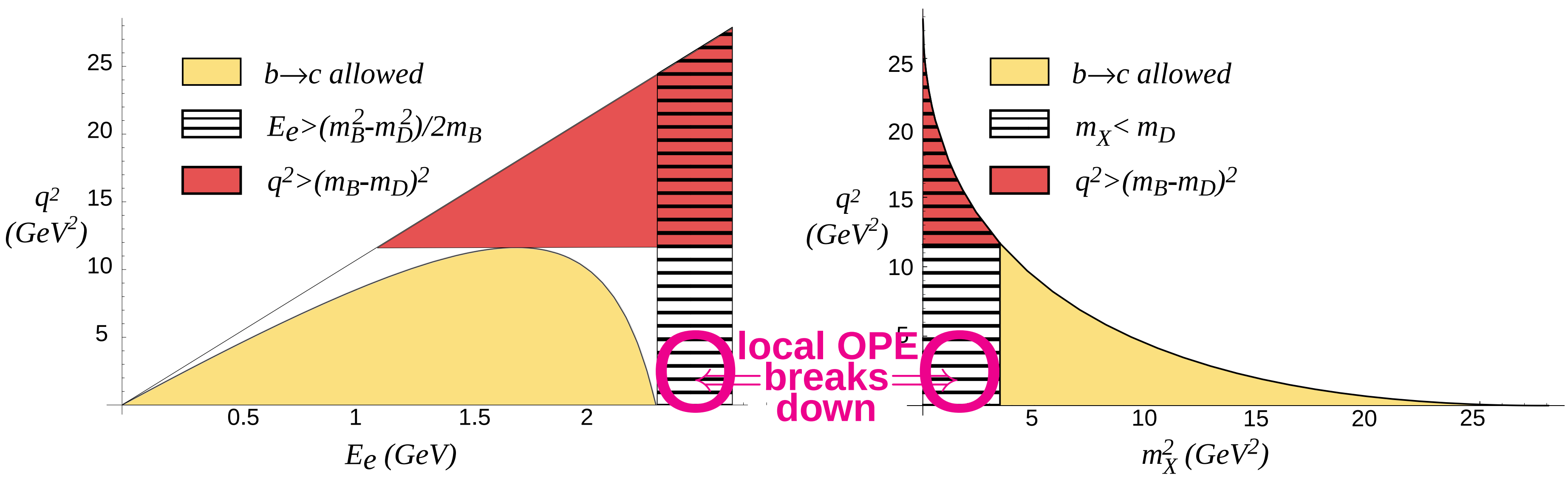
$|V_{ub}|$ — (q^2, m_X) **cuts**

Inclusive $B \to X_u \ell \bar{\nu}$ phase space

Cuts to eliminate $B \to X_c \ell \bar{\nu}$ background:

- Lepton spectrum: $E_\ell > (m_B^2 - m_D^2)/2m_B$
- Hadronic mass spectrum: $m_X < m_D$
- Dilepton mass spectrum: $q^2 > (m_B - m_D)^2$
- Combinations of cuts





V_{ub}: q^2 spectrum

- Large q^2 region: first few terms in local OPE converge (known to $\Lambda_{\rm QCD}^3/m_b^3$, $\alpha_s^2\beta_0$)

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{q}^2} = \left(1+\frac{\lambda_1}{2m_b^2}\right) 2\,(1-\hat{q}^2)^2\,(1+2\hat{q}^2) + \frac{\lambda_2}{m_b^2}\,(3-45\hat{q}^4+30\hat{q}^6) + \mathcal{O}(\alpha_s)$$

No shape fn., because $q^2 > (m_B - m_D)^2$ implies $E_X < m_D$ [$\Rightarrow$ no "hadron jet"]

Price: expansion scale low $\sim \Lambda_{\rm QCD}/m_c$

- Dominant uncertainties: (i) perturbative corrections
  (ii) nonperturbative terms (largest: weak annihilation)

- Combine q^2 cut with m_X cut to increase fraction of included events

Scale of expansion significantly higher:

– reduced uncertainties from perturbation series and nonperturbative corrections

– uncertainty from the b quark light-cone distribution function only turns on slowly

Preliminary comments

- $|V_{ub}|$ is tricky business, so it is important to measure it several independent ways
- Important to have shape function independent measurements
  - Shape fn.'s are numerous and less constrained at $\mathcal{O}(\alpha_s, \Lambda_{\rm QCD}/m_b)$
  - Need $m_B/2 - E_\gamma < m_D/2$ or $m_D/4$? Difference may be power suppressed
- Is summing not-too-large logs, $\ln(\sqrt{\Lambda_{\rm QCD}/m_b}) \sim \ln(m_c/m_b)$, useful in practice?

Can the full reco sample be used to measure $\mathrm{d}\Gamma/\mathrm{d}E_\ell$ (much) below charm end-point (i.e., below 2 GeV)? Has it been tried? Would also eliminate need for SF.

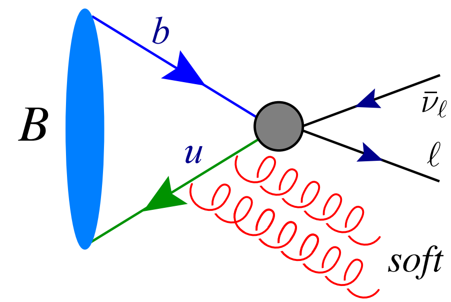
Weak annihilation

- Old story: $16\pi^2 \frac{\Lambda^3_{\rm QCD}}{m_b^3}\,\varepsilon$ in total rate, smeared around $\delta(q^2 - m_B^2)\,\delta(E_\ell - \frac{m_B}{2})$

  Guesstimate: $\sim 3\%$ of $b \to u$ semileptonic rate

  At large E_ℓ: enhanced by $\frac{m_b}{\Lambda_{\rm QCD}}$ $\Rightarrow$ sizable uncertainty

  $\Rightarrow$ Constrain it by comparing D^0 vs. D_s SL widths, or V_{ub} from $B^\pm$ vs. B^0 decay



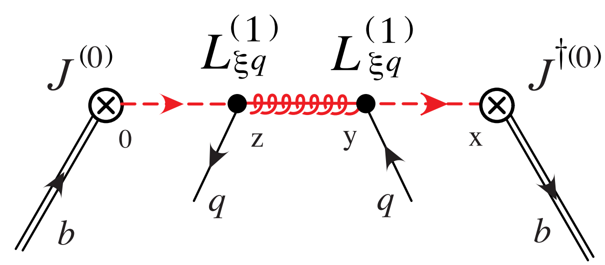
- New story: $4\pi\alpha_s \frac{\Lambda^3_{\rm QCD}}{m_b^3}\,\varepsilon$ in total rate, smeared in usual shape fn. region

  Role / presence of $4\pi\alpha_s$ and ε factors argued

  In shape fn. regions: enhanced by $\frac{m_b^2}{\Lambda^2_{\rm QCD}}$

  Can be absorbed into other subleading shape fn.'s

  $\Rightarrow$ don't yet know how to constrain from data



[Lee & Stewart; Bosch, Neubert, Paz]

Cut dependences & errors

- Fraction of included events when shape fn. gives $< 10\%$ correction to local OPE

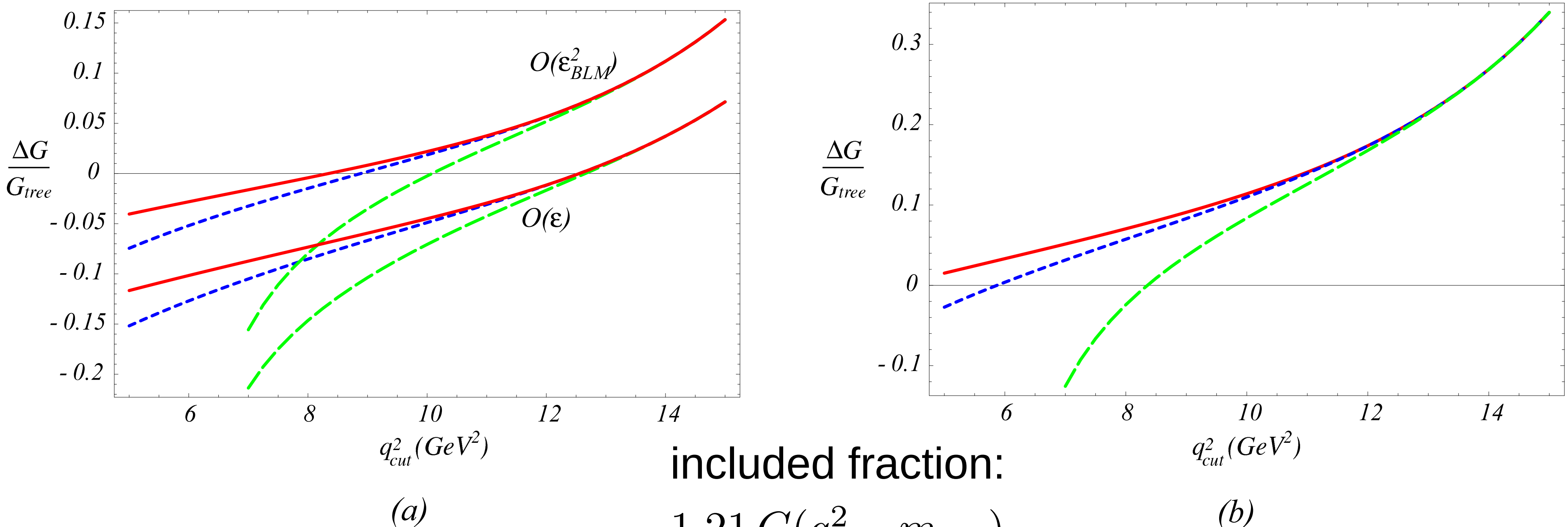
q^2/m_X	1.5	1.6	1.7	1.75	1.8	1.85
12	0.146 $5\pm16\pm11\pm20$	0.153 $2\pm16\pm11\pm20$	0.155 $1\pm16\pm11\pm20$	0.156 $0\pm16\pm11\pm20$	0.156 $0\pm16\pm11\pm20$	0.156 $0\pm16\pm11\pm20$
11	0.177 $7\pm13\pm11\pm16$	0.190 $4\pm13\pm10\pm16$	0.196 $2\pm13\pm10\pm16$	0.198 $1\pm14\pm10\pm16$	0.199 $1\pm14\pm10\pm16$	0.199 $0\pm14\pm10\pm16$
10	0.204 $10\pm9\pm11\pm13$	0.227 $6\pm11\pm10\pm13$	0.239 $3\pm11\pm10\pm13$	0.242 $2\pm11\pm9\pm13$	0.245 $1\pm11\pm9\pm13$	0.247 $1\pm12\pm9\pm13$
9	–	0.262 $8\pm8\pm10\pm10$	0.282 $4\pm9\pm9\pm10$	0.289 $3\pm9\pm9\pm10$	0.294 $2\pm9\pm9\pm10$	0.297 $1\pm9\pm9\pm10$
8	–	0.290 $10\pm4\pm11\pm8$	0.324 $6\pm6\pm9\pm8$	0.335 $4\pm7\pm9\pm8$	0.343 $3\pm7\pm8\pm8$	0.349 $2\pm7\pm8\pm8$
7	–	–	0.362 $8\pm4\pm9\pm7$	0.379 $6\pm4\pm9\pm7$	0.392 $5\pm5\pm8\pm7$	0.402 $3\pm6\pm8\pm7$
6	–	–	0.392 $10\pm5\pm10\pm6$	0.412 $8\pm4\pm9\pm6$	0.438 $6\pm3\pm8\pm6$	0.453 $5\pm4\pm8\pm6$

Errors are in $\%$: structure fn. (neg. correction included); perturbation series; $m_b^{1S} = (4.70 \pm 0.05)$ GeV; order $\Lambda_{\rm QCD}^3/m^3$

[Bauer, ZL, Luke]

- Recall: error of $|V_{ub}|$ is half the above

Uncertainties (1): perturbation series



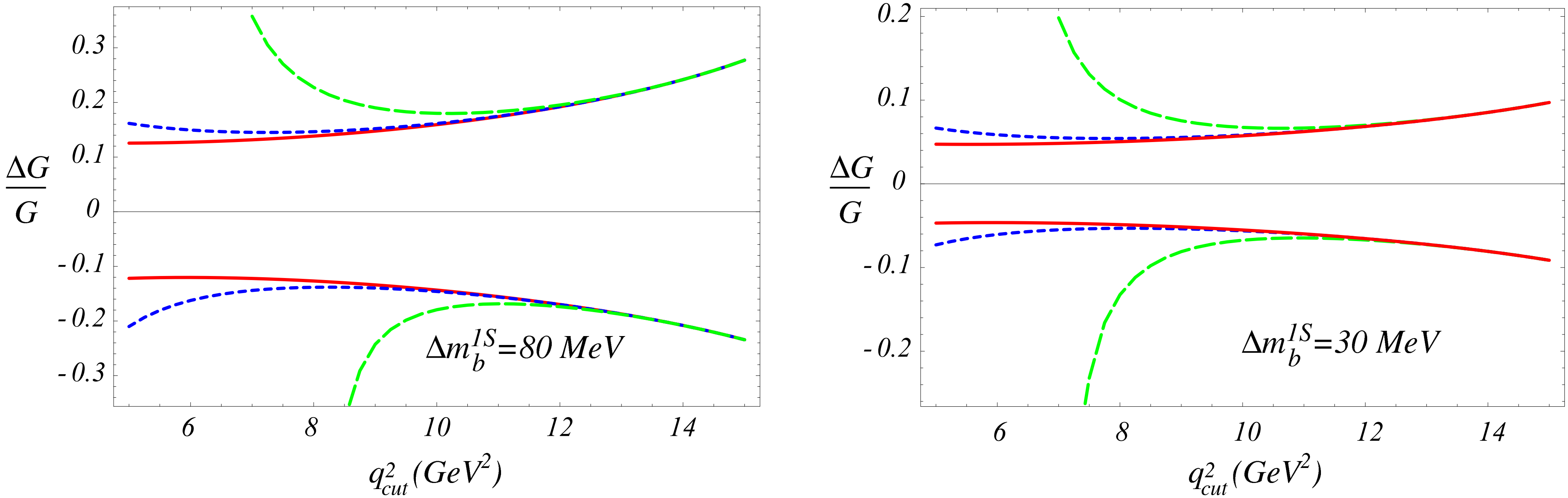
included fraction:

$1.21\, G(q^2_{\rm cut}, m_{\rm cut})$

(a) The $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2_{\rm BLM})$ contributions to $G(q^2_{\rm cut}, m_{\rm cut})$, normalized to tree level result, for $m_{\rm cut} = 1.86$ GeV (solid), 1.7 GeV (short dashed), 1.5 GeV (long dashed)

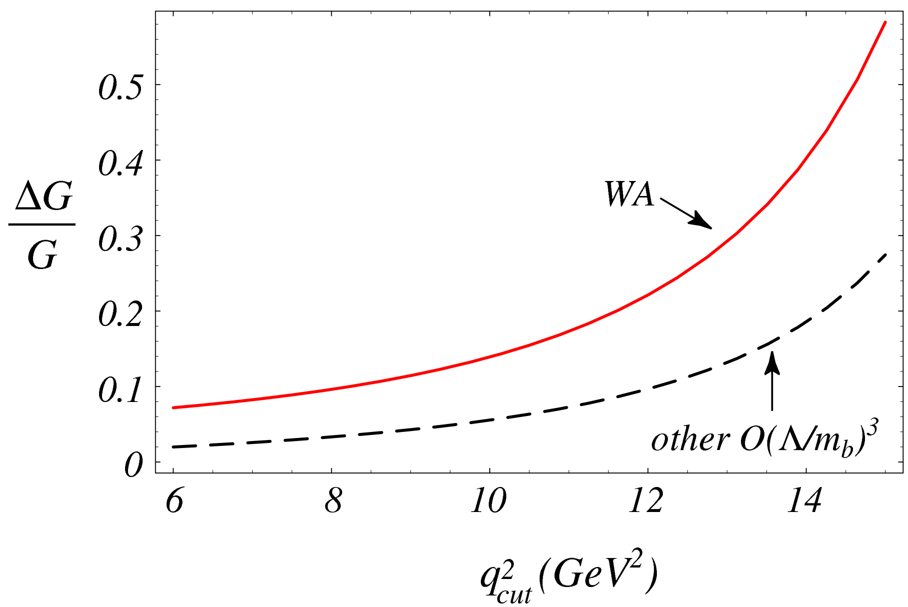
(b) Scale variation: difference between perturbative corrections to $G(q^2_{\rm cut}, m_{\rm cut})$, normalized to the tree level result, for $\mu = 4.7$ GeV and $\mu = 1.6$ GeV

Uncertainties (2): b quark mass

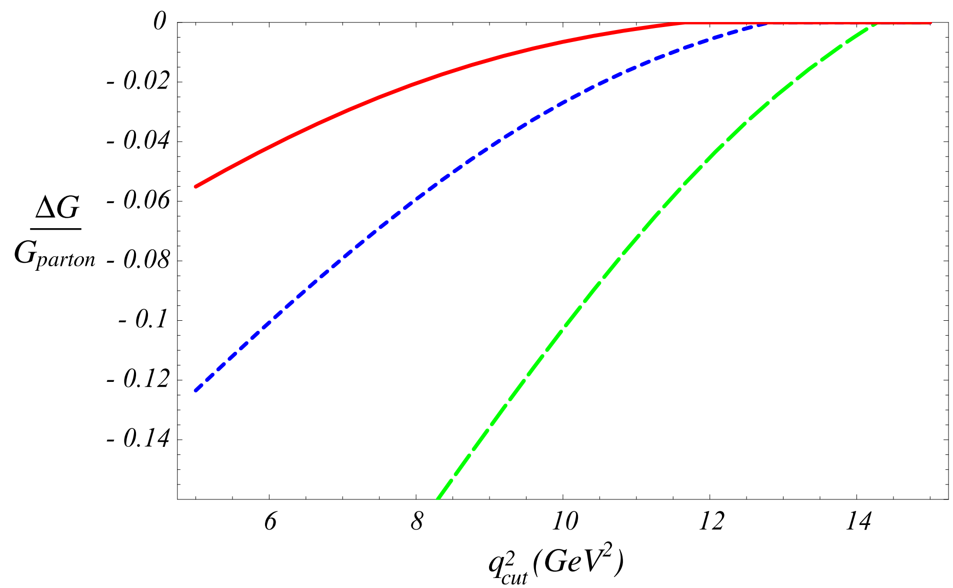


Fractional effect of ± 80 MeV (left) and ± 30 MeV (right) uncertainty in m_b^{1S} on $G(q^2_{\rm cut}, m_{\rm cut})$ for $m_{\rm cut} = 1.86$ GeV (solid), 1.7 GeV (short dashed), and 1.5 GeV (long dashed)

Uncertainties (3): higher dimension operators



Estimate of the uncertainties due to dimension-six terms in the OPE as a function of $q^2_{\rm cut}$ from weak annihilation (solid) and other operators (dashed)



Effect of a model structure function on $G(q^2_{\rm cut}, m_{\rm cut})$ as a function of $q^2_{\rm cut}$ for $m_{\rm cut} = 1.86$ GeV (solid), 1.7 GeV (short dashed) and 1.5 GeV (long dashed)

$$f(k_+) = \frac{a^a}{\Gamma(a)}\,(1-x)^{a-1}\,e^{-a(1-x)}$$

a and x related to $\bar\Lambda$ and λ_1

Aside: exclusive $b \to u$ decays

- Less constraints from heavy quark symmetry than in $b \to c$

$\Rightarrow$ $B \to \ell\bar\nu$ measures $f_B \times |V_{ub}|$ — need to rely on lattice f_B

$\Rightarrow$ Useful constraints from unitarity/analyticity + SCET may also help

$\Rightarrow$ Ratios $= 1$ in heavy quark or chiral symmetry limit ($+$ study corrections)

- Deviations of "Grinstein-type double ratios" from unity are more suppressed:

$\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ (Grinstein, '93)

$\dfrac{B \to \rho\ell\bar\nu}{B \to K^*\ell^+\ell^-} \times \dfrac{D \to K^*\ell\bar\nu}{D \to \rho\ell\bar\nu}$ — accessible soon? (ZL & Wise, '96; Ginstein & Pirjol, '02-04)

$\dfrac{B \to \ell\bar\nu}{B_s \to \ell^+\ell^-} \times \dfrac{D_s \to \ell\bar\nu}{D \to \ell\bar\nu}$ — very clean... in a decade (Ringberg workshop, lots of beer, '03)

Summary: $|V_{ub}|$

- Model independent $\sim 10\%$ $|V_{ub}|$ is posible, ultimately similar to $|V_{cb}|$ 1–2 years ago
- Theoretical limit for (inclusive) $|V_{ub}|$ appears to be around the 5% level

  Such precision can be achieved even with cuts away from the $b \to c$ threshold
- Need to measure $|V_{ub}|$ in as many clean ways as possible, confidence will be gained by convergence of extractions

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Wishlist:

- get the cuts as close to the charm threshold as possible
- constrain WA by comparing $|V_{ub}|$ from $B^\pm$ vs. B^0, or D^0 vs. D_s SL widths
- improve measurement of $B \to X_s\gamma$ photon spectrum (and lower cut)

$|V_{cb}|$ — global fit

Goals of a global fit

- Well-known that semileptonic B decay rate gives a precise determination of $|V_{cb}|$

  The devil is in the details:

  – Size of theoretical uncertainties? Investigate them (incl. duality) experimentally

    NB: even before semileptonic data from B factories, inclusive $|V_{cb}|$ looked more precise than exclusive, but duality was not well-constrained directly from data

  – What are the values of m_b, λ_1, etc.? Determine them in same analysis as $|V_{cb}|$

  – Theoretical correlations between different observables $\Rightarrow$ Include them

  – All observables fit using a consistent scheme $\Rightarrow$ study sheme dependence

  – Optimal use of data $\Rightarrow$ reduce uncertainties

The players

1.) Inclusive semileptonic $B \to X_c \ell \bar\nu$ branching ratio, B lifetime, and R_0 below

2.) Shape variables (largely) independent of CKM elements

– Lepton energy moments in $B \to X_c \ell \bar\nu$ [use: BABAR, BELLE, CLEO, DELPHI]

$$R_n(E_{\rm cut}) = \int_{E_{\rm cut}} E_\ell^n \frac{{\rm d}\Gamma}{{\rm d}E_\ell}\, {\rm d}E_\ell\,, \qquad \langle E_\ell^n \rangle_{E_{\rm cut}} = \frac{R_n(E_{\rm cut})}{R_0(E_{\rm cut})}\,, \qquad \langle (E_\ell - \langle E_\ell \rangle)^n \rangle_{E_{\rm cut}}$$

– Hadronic invariant mass moments in $B \to X_c \ell \bar\nu$ [use: BABAR, BELLE, CDF, CLEO, DELPHI]

$$\langle m_X^{2n} \rangle_{E_{\rm cut}} = \int_{E_{\rm cut}} (m_X^2)^n \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\, {\rm d}m_X^2 \bigg/ \int_{E_{\rm cut}} \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\, {\rm d}m_X^2\,, \qquad \langle (m_X^2 - \langle m_X^2 \rangle)^n \rangle$$

– Photon energy moments in $B \to X_s \gamma$ [use: BABAR, BELLE, CLEO]

$$\langle E_\gamma^n \rangle_{E_{\rm cut}} = \int_{E_{\rm cut}} E_\gamma^n \frac{{\rm d}\Gamma}{{\rm d}E_\gamma}\, {\rm d}E_\gamma \bigg/ \int_{E_{\rm cut}} \frac{{\rm d}\Gamma}{{\rm d}E_\gamma}\, {\rm d}E_\gamma\,, \qquad \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle$$

Include all available correlations (published or not)

Ingredients of the analysis

- Required for doing fits: (multi-loop calculations use pole mass and α_s in $\overline{\rm MS}$)

1. Scheme for m_b — expansion in $\Lambda_{\rm QCD}/m_b$ common to all schemes
2. Scheme for m_c — may or may not expand $m_b - m_c$ in HQET
3. HQET matrix elements
4. Work consistently to a given order ($\Lambda_{\rm QCD}^3/m_b^3,\ \alpha_s\Lambda_{\rm QCD}/m_b,\ \alpha_s^2\beta_0$)

- New compared to our analysis in 2002

1. For m_X moments, include E_ℓ cut dependence at $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b)$ [Trott, hep-ph/0402120]
2. Compare expanding or not expanding $m_b - m_c$ in HQET matrix elements
3. Study $\langle m_X\rangle$ and $\langle m_X^3\rangle$ moments
4. Slightly different error estimates

b-quark mass schemes

- Use 5 mass schemes for comparison — do all fits completely in each

Pole mass:

- renormalon ambiguity of order $\Lambda_{\rm QCD}$
- perturbation series poorly behaved
- these problems may be related — asymptotic nature of perturbation series related to nonperturbative corrections

$\overline{\rm MS}$ mass still poorly behaved perturbation series; best to use a “threshold mass”:

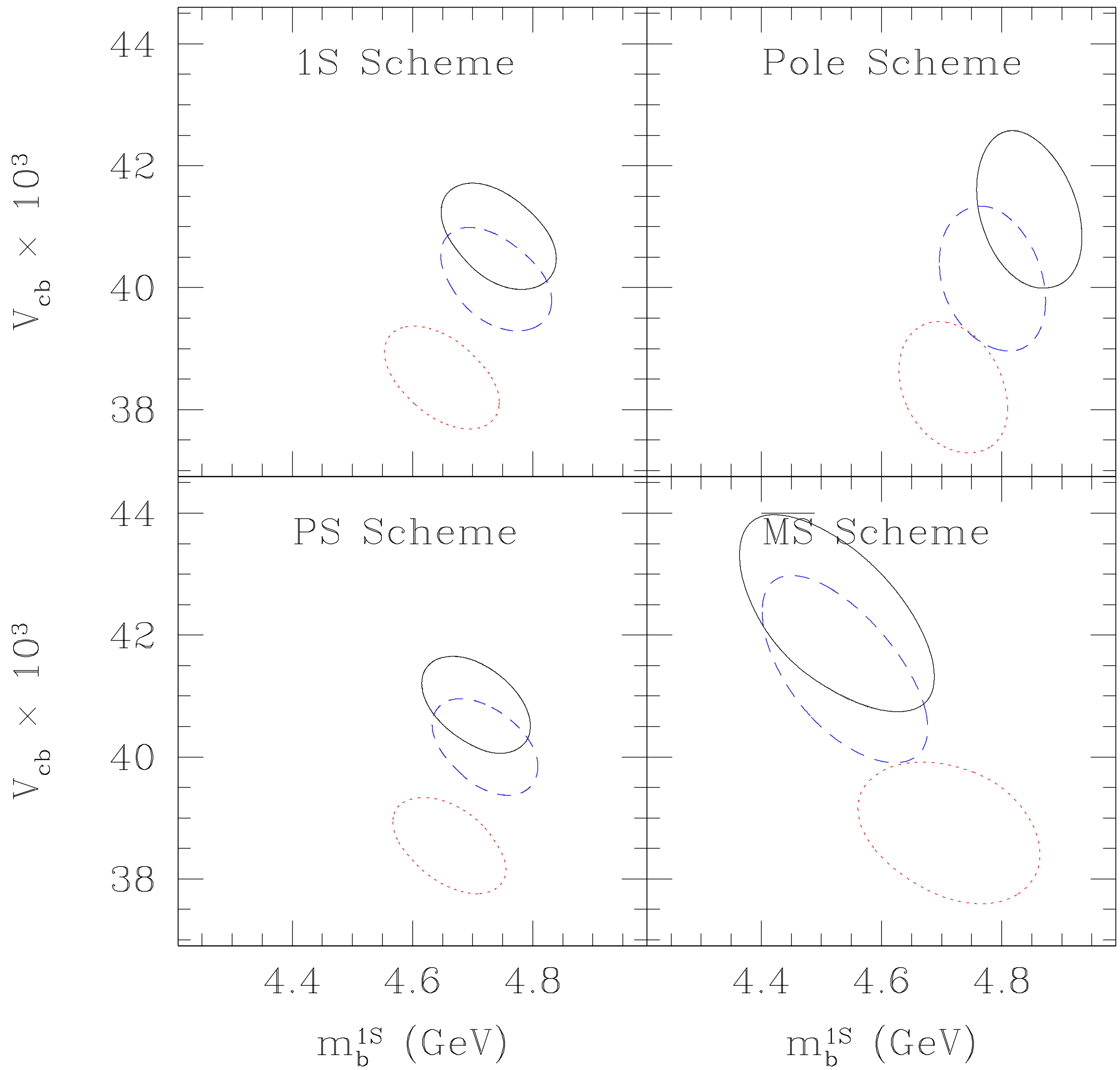
$1S$ mass using the upsilon expansion

PS mass and kinetic mass require introducing a factorization scale μ_f that enters linearly, e.g.: $m_{\rm pole} = m_{\rm PS} + \ldots + \mathcal{O}(\alpha_s \mu_f)$

b-quark mass scheme dependence

tree level, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2\beta_0)$

[plot from 2002]



Better convergence in 1S and PS schemes than in pole or $\overline{\text{MS}}$

c-quark mass schemes

- To expand or not to expand...?

  1.) Expand $m_b - m_c$ in HQET, then 4 time ordered products appear; or

  2.) Consider m_c as an independent parameter

- Parameter counting (in addition to $|V_{cb}|$):

  1.) $m_b,\ \lambda_1,\ \rho_1,\ \mathcal{T}_{1-3}\ \Rightarrow\ 6$
  $m_c,\ \lambda_2,\ \rho_2$ eliminated using $m_{B^{(*)}} - m_{D^{(*)}}$; a linear combination of $\mathcal{T}_{1-4}$ absent

  2.) $m_b,\ m_c,\ \lambda_{1,2},\ \rho_{1,2}\ \Rightarrow\ 6$
  then it is still a separate question what scheme to use for m_c

- A priori cannot tell which is better (could use full QCD, only two parameters: $m_{b,c}$)

$\Rightarrow$ How well can experimentally measured quantities be reliably computed?

Aside: limits of the theory

- If $m_{b,c} \gg \Lambda_{\rm QCD}$ and $m_b/m_c = \mathcal{O}(1)$:
  - Expand $m_b - m_c$ in HQET matrix elements
  - Fewest number of parameters at lower orders in $\Lambda_{\rm QCD}/m_{c,b}$
  - The usual philosophy of effective theories: move unknowns to higher orders

- If $m_b \gg \Lambda_{\rm QCD}$ and $m_b/m_c \gg \mathcal{O}(1)$:
  - Do not expand $m_b - m_c$ in HQET to avoid expansion in $\Lambda_{\rm QCD}/m_c$
  - Use $\overline{m}_c(m_b)$, similar to $Z, h \to q\bar{q}$, and not a threshold charm mass
  - The way to include finite m_s effects in $B \to X_s\gamma$

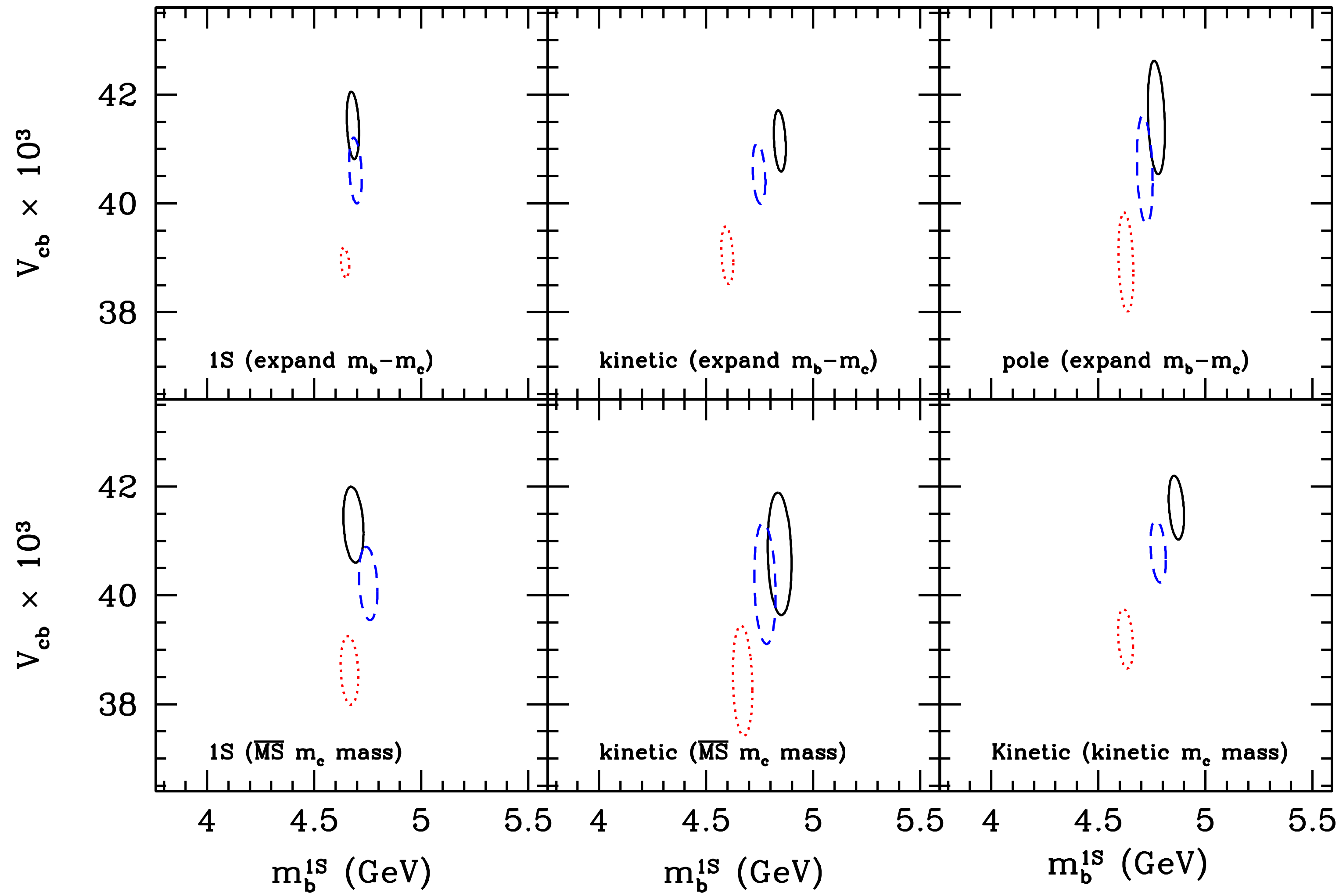
Using the kinetic mass for m_c reintroduces expansion in $\Lambda_{\rm QCD}/m_c$ and perturbation theory in terms of $\alpha_s(m_c)$ (not a small coupling)

c-quark mass scheme dependence

tree level

$\mathcal{O}(\alpha_s)$

$\mathcal{O}(\alpha_s^2\beta_0)$



- No evidence that not expanding $m_b - m_c$ is better than expanding it

Half-integer (odd) m_X moments

- Recently half-integer moments, $(m_X^2)^{n/2}$ ($n =$ odd) received some attention

  Square root introduces branch cut, analytic structure differs from other moments

  Proposed formulae involve expansions in $m_b\Lambda_{\rm QCD}/m_c^2 = \mathcal{O}(1)$ [Gambino & Uraltsev; Trott]

  NB: expansion well-behaved in SV limit, $m_{b,c} \gg m_b - m_c \gg \Lambda_{\rm QCD}$, then the "natural" scheme is to expand $m_b - m_c$ in HQET

  For these moments, order $\beta_0(\alpha_s/4\pi)^2$ terms unknown (to us [Uraltsev, hep-ph/0403166])

- We do not use these moments in main fit, but compare fit results with the data

  (Here I'll also show result with these moments included in the fit)

Theoretical uncertainties

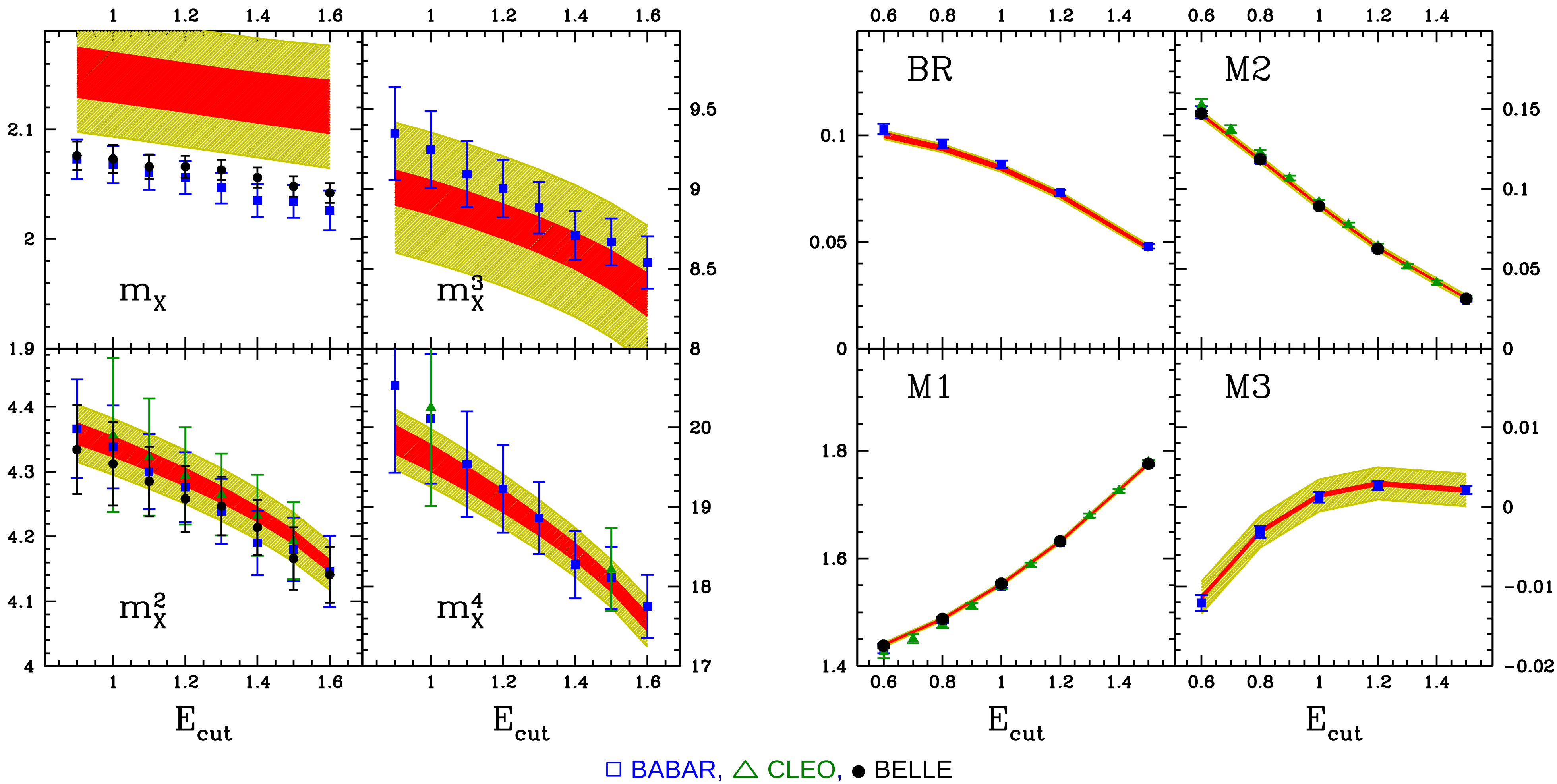
- Define theoretical uncertainties, so it is not judged case-by-case and a posteriori
  Avoid large weight to an accurate measurement that cannot be computed reliably

  Uncomputed higher order terms — estimate using naive dimensional analysis:

  - $\Lambda_{\rm QCD}^4/(m_b^2 m_c^2) \sim 0.001$ in $1S_{\rm EXP}$ & kin$_{\rm EXP}$, $\Lambda_{\rm QCD}^4/m_b^4 \sim 0.0001$ in $1S_{\rm NO}$ & kin$_{\rm NO}$
  - $(\alpha_s/4\pi)^2 \sim 0.0003$ $\otimes$ half of the last computed term
    for non-integer hadronic moments $\beta_0(\alpha_s/4\pi)^2 \sim 0.003$ not known
  - $(\alpha_s/4\pi)(\Lambda_{\rm QCD}^2/m_b^2) \sim 0.0002$
  - extracted m_b from $\langle E_\gamma \rangle$ less reliable for larger $E_{\rm cut}$ $\Rightarrow$ increase error
  - assume correlation of theory errors similar to those of the experimental errors

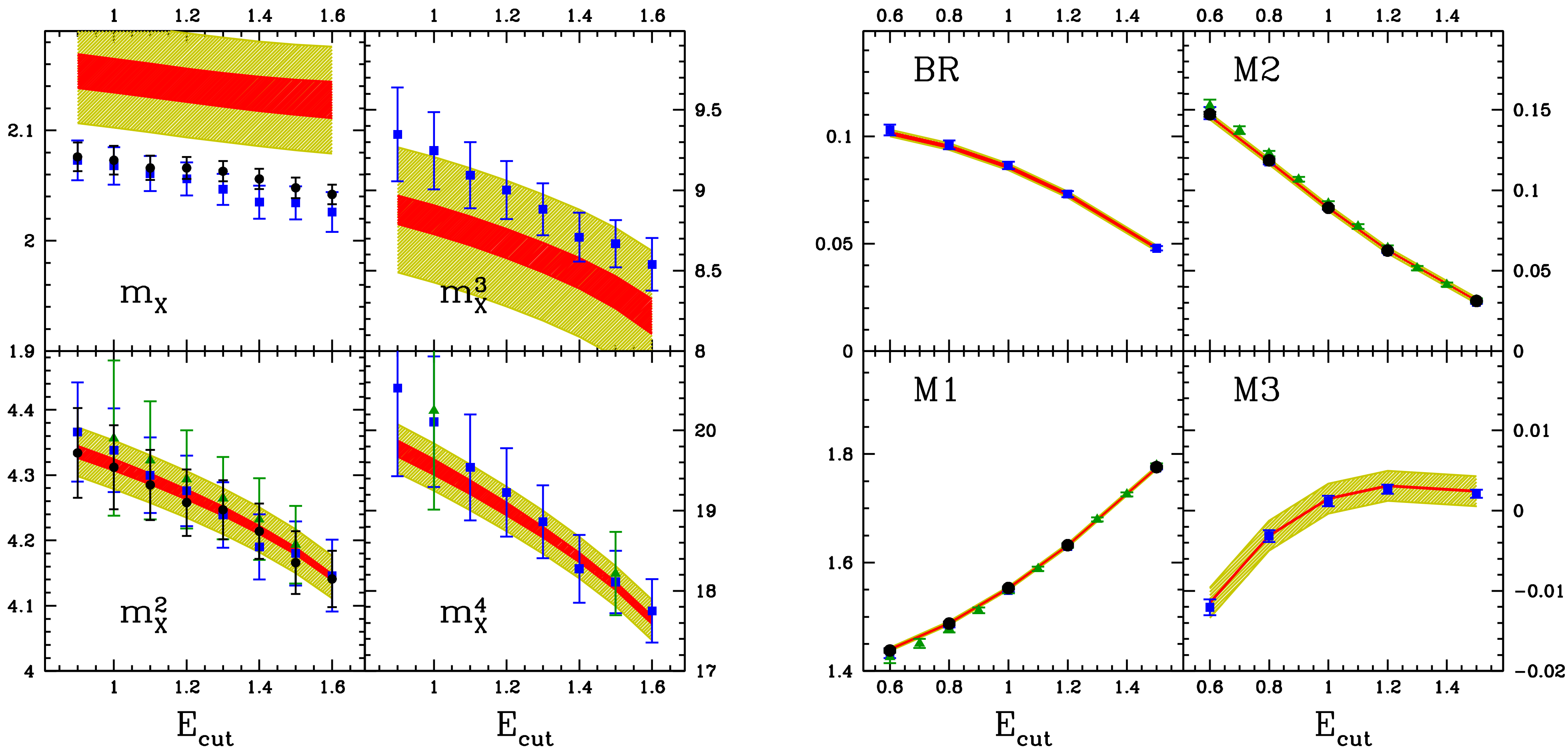
- Theory error estimate: above combined in quadrature scaled with m_B^n or $(m_B/2)^n$
  and maximal values of moments on [0,1]: $f_0 = f_1 = 1$, $f_2 = 1/4$, $f_3 = 1/(6\sqrt{3})$

Data and fit results (with theory errors)



Red shaded: fit error; Yellow shaded: estimated theoretical uncertainties

Data and fit results (without theory errors)



□ BABAR, △ CLEO, ● BELLE

Red shaded: fit error; Yellow shaded: estimated theoretical uncertainties

More on fit results

Scheme	σ^2_{theory}	χ^2/ν	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]	λ_1 [GeV^2]
$1S_{\text{EXP}}$	yes	$50.9/86$	41.4 ± 0.6	4.68 ± 0.03	-0.27 ± 0.04
kin$_{\text{EXP}}$	yes	$52.6/86$	$41.2 \pm 0.6 \pm 0.1$	$4.70 \pm 0.03 \pm 0.03$	$-0.19 \pm 0.04 \pm 0.04$
$1S_{\text{EXP}}$	no	$148.4/86$	41.5 ± 0.3	4.69 ± 0.02	-0.31 ± 0.03
kin$_{\text{EXP}}$	no	$238.8/86$	$41.1 \pm 0.3 \pm 0.7$	$4.74 \pm 0.01 \pm 0.11$	$-0.33 \pm 0.03 \pm 0.11$

NB: BABAR [hep-ex/0404017] obtained: $|V_{cb}| = (41.4 \pm 0.4_{\text{exp}} \pm 0.4_{\text{HQE}} \pm 0.6_{\text{th}}) \times 10^{-3}$

Can fit $1/m^3$ matrix elements consistently, but they are not well-determined

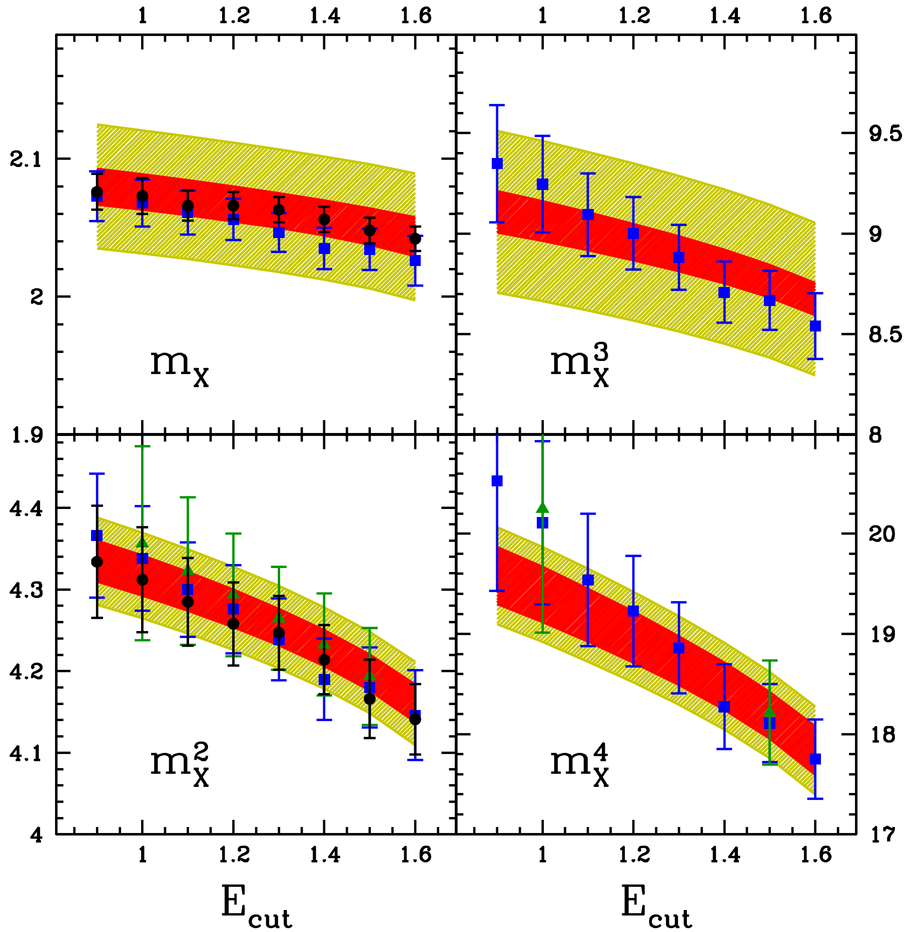
[$1/m^3$ errors significant, but so are their correlations]

Correlations critical: separate results for parameters give much larger $|V_{cb}|$ error

- Theoretical errors important: they are probably overestimated (χ^2/ν too small)

  Evidence that higher order terms matter (fit w/o theory errors gives too large χ^2/ν)

What if we fit $\langle m_X \rangle$ and $\langle m_X^3 \rangle$?



Fit with theory errors (without Belle data)

Results compatible with fits w/o $\langle m_X^{1,3} \rangle$

$$|V_{cb}| = (41.0 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3}$$

$$m_b^{1S} = (4.76 \pm 0.03)\ \text{GeV}$$

Future limitations

- Setting all experimental errors to zero, we would obtain

$$\sigma(|V_{cb}|) \times 10^3 = 0.35\,, \qquad \sigma(m_b^{1S}) = 35\,\text{MeV}$$

- Bauer-Trott moments: suppress (enhance) sensitivity to certain matrix elements

(fractional moments of E_ℓ spectrum for $E_\ell \geq 1.5$ GeV)

R_{3a}	R_{3b}	R_{4a}	R_{4b}	D_3	D_4
0.302 ± 0.003	2.261 ± 0.013	2.127 ± 0.013	0.684 ± 0.002	0.520 ± 0.002	0.604 ± 0.002
above was our prediction (2002), below is subsequent CLEO measurement					
0.3016 ± 0.0007	2.2621 ± 0.0031	2.1285 ± 0.0030	0.6833 ± 0.0008	0.5193 ± 0.0008	0.6036 ± 0.0006

Data and theory beautifully consistent; no evidence for theory getting less reliable

(NB: excited D states make small contribution in this region)

Summary: $|V_{cb}|$

- Shape variables allow: (i) precision extractions of m_b and HQET matrix elements
  (ii) testing validity of the whole approach
- Many schemes give compatible results; in $1S_{\mathrm{EXP}}$ scheme we obtain
  $|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3}$, $\quad m_b^{1S} = (4.68 \pm 0.03)$ GeV
- Kinetic m_c mass suspect; Half-integer hadronic mass moments less reliable
- Since theoretical uncertainties dominate, their correlations are essential when fitting many observables to determine hadronic parameters and $|V_{cb}|$
- Quark-hadron duality seems to be woking at better than 1% level
- Theoretical limit for (inclusive) $|V_{cb}|$ appears to be around the 1% level

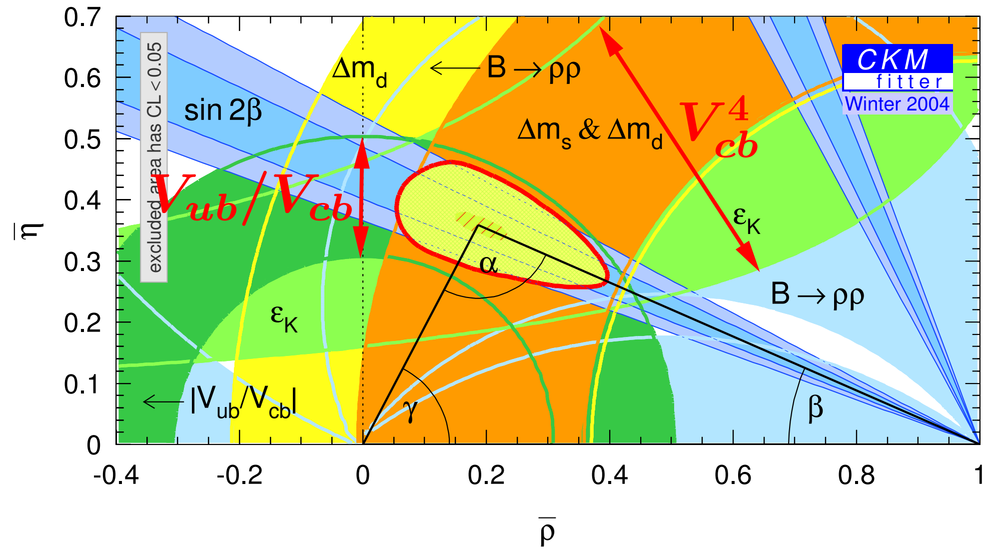
Backup slides

Why care about $|V_{ub}|$ and $|V_{cb}|$?

$|V_{ub}|$: determines side opposite to β

$|V_{cb}|$: large part of the uncertainty in ϵ_K

error of $K \to \pi\nu\bar{\nu} \propto \sigma(|V_{cb}^4|)$



How well OPE works for $b \to c$ spectra may affect what we believe about accuracy of $|V_{ub}|$ using phase space cuts

Inclusive decays mediated by $b \to s\gamma$, $b \to s\,\ell^+\ell^-$, and $b \to s\,\nu\bar{\nu}$ transitions are sensitive probes of the SM; theoretical tools for semileptonic and rare decays are similar — understanding accuracy of theory affects sensitivity to new physics

Aside: perturbation theory at m_c

- Schemes with $m_b - m_c$ expanded in HQET determine $m_b - m_c$ very precisely; e.g., in $1S_{\text{EXP}}$ scheme: $m_b - m_c = 3.41 \pm 0.01$ GeV

  Converting this to $\overline{\text{MS}}$ mass:

  $$\overline{m}_c(\overline{m}_c) = 0.90 \pm 0.04\,\text{GeV}, \qquad \overline{m}_c(\overline{m}_c) = 1.07 \pm 0.04\,\text{GeV}$$

  depending on whether the perturbative conversion factor is reexpanded or not

  Large difference between dividing by $1 + a_1\alpha_s + a_2\alpha_s^2$ and multiplying by $1 - a_1\alpha_s + (a_1^2 - a_2)\alpha_s^2$, because α_s not small at the scale $\overline{m}_c$

Forms of expansions

Expand $m_b - m_c$ in HQET:

$$\begin{aligned} X_{E_{\rm cut}} &= X^{(1)} + X^{(2)}\Lambda + X^{(3)}\Lambda^2 + X^{(4)}\Lambda^3 + X^{(5)}\lambda_1 \\ &+ X^{(6)}\lambda_2 + X^{(7)}\lambda_1\Lambda + X^{(8)}\lambda_2\Lambda + X^{(9)}\rho_1 \\ &+ X^{(10)}\rho_2 + X^{(11)}\mathcal{T}_1 + X^{(12)}\mathcal{T}_2 + X^{(13)}\mathcal{T}_3 \\ &+ X^{(14)}\mathcal{T}_4 + X^{(15)}\epsilon + X^{(16)}\epsilon^2_{\rm BLM} + X^{(17)}\epsilon\Lambda \end{aligned}$$

Do not expand $m_b - m_c$ in HQET:

$$\begin{aligned} Y_{E_{\rm cut}} &= Y^{(1)} + Y^{(2)}\Lambda + Y^{(3)}\Lambda_c + Y^{(4)}\Lambda^2 + Y^{(5)}\Lambda\Lambda_c \\ &+ Y^{(6)}\Lambda_c^2 + Y^{(7)}\Lambda^3 + Y^{(8)}\Lambda^2\Lambda_c + Y^{(9)}\Lambda\Lambda_c^2 \\ &+ Y^{(10)}\Lambda_c^3 + Y^{(11)}\lambda_1 + Y^{(12)}\lambda_2 + Y^{(13)}\lambda_1\Lambda \\ &+ Y^{(14)}\lambda_2\Lambda + Y^{(15)}\lambda_1\Lambda_c + Y^{(16)}\lambda_2\Lambda_c + Y^{(17)}\rho_1 \\ &+ Y^{(18)}\rho_2 + Y^{(19)}\epsilon + Y^{(20)}\epsilon^2_{\rm BLM} + Y^{(21)}\epsilon\Lambda + Y^{(22)}\epsilon\Lambda_c \end{aligned}$$

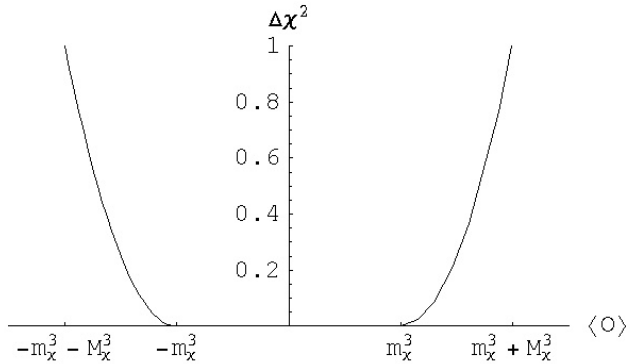
Size of matrix elements

- More data points than unknowns, but no strong constraint on $1/m_b^3$ mx elements

  Unknown $1/m_b^3$ matrix elements — $\mathcal{O}(\Lambda_{\rm QCD}^3)$, but not well determined

  $\Rightarrow$ Add to χ^2 function in the fit:

$$\Delta\chi^2(m_\chi, M_\chi) = \begin{cases} 0\,, & |\langle\mathcal{O}\rangle| \le m_\chi^3 \\ \left[|\langle\mathcal{O}\rangle| - m_\chi^3\right]^2 / M_\chi^6\,, & |\langle\mathcal{O}\rangle| > m_\chi^3 \end{cases}$$



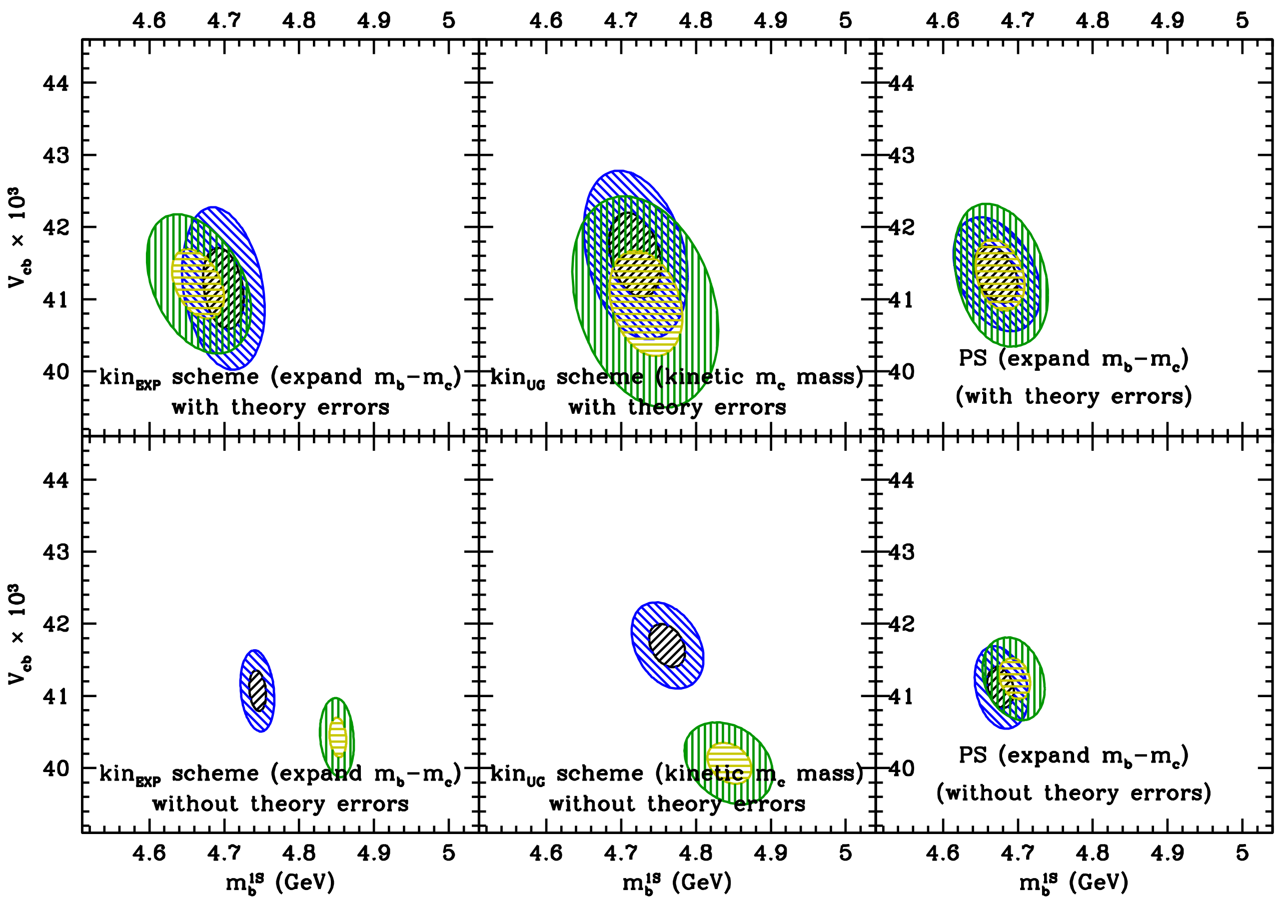
  Take $M_\chi = 0.5\,$GeV, and vary $0.5\,$GeV $< m_\chi < 1\,$GeV

  Does not affect fit results significantly; more later

- Not included in theory error estimate: uncertainties from “duality violation”

  $\Rightarrow$ Use the data and the fit to constrain it

Sensitivity to μ_b (kin) and to μ_f (PS)



black & yellow: $\Delta\chi^2 = 1$

blue & green: $\Delta\chi^2 = 4$

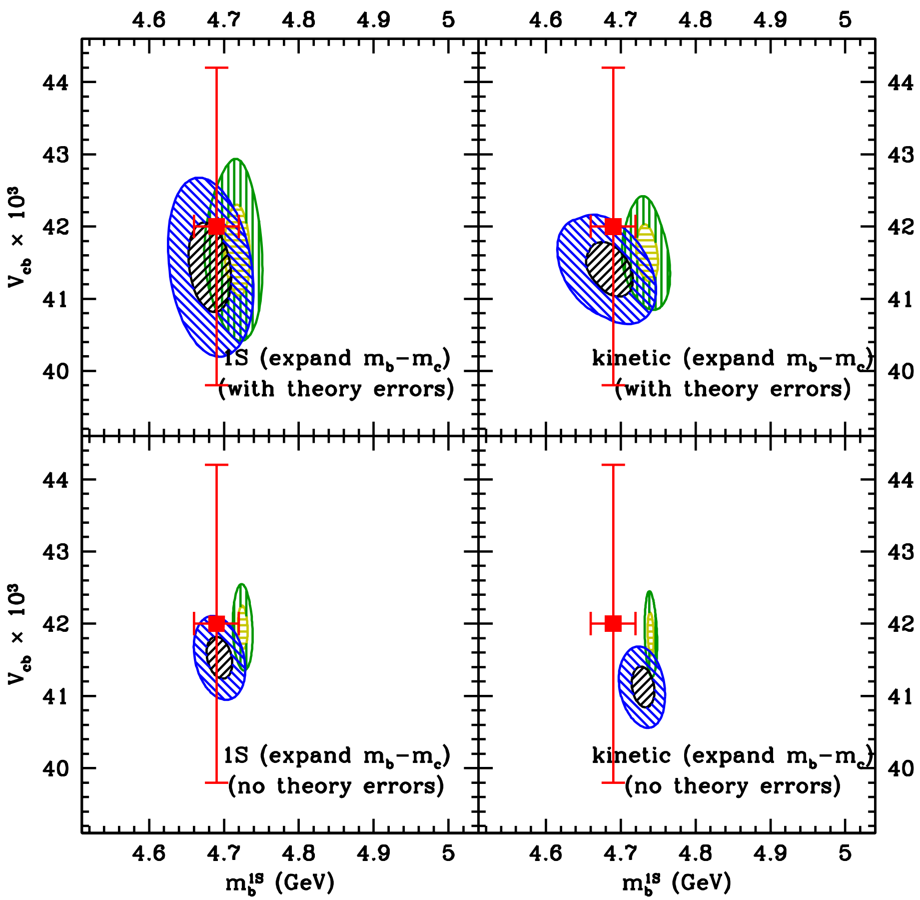
Without theory errors, fit result can move more than what χ^2 indicates (almost flat directions)

PS scheme results insensitive to choice of μ_f

black & blue: $\mu_b = 1\,$GeV $\qquad \mu_f = 2\,$GeV

yellow & green: $\mu_b = 1.5\,$GeV $\qquad \mu_f = 1.5\,$GeV

Results for $|V_{cb}|$ and m_b



black & yellow: $\Delta\chi^2 = 1$

blue & green: $\Delta\chi^2 = 4$

yellow & green: omit restriction on range of Λ^3_{QCD} matrix elements

Compare: m_b^{1S} [Hoang] & $|V_{cb}|_{\text{excl}}$ [PDG]

$1S_{\text{EXP}}$ fit, including theory errors:

$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3},$$
$$m_b^{1S} = (4.68 \pm 0.03)\ \text{GeV}$$