Who needs SCET in $B \to X\ell^+\ell^-$?

Zoltan Ligeti

CERN, Nov. 9, 2005

- Introduction

  Calculations and measurements of $B \to X_s \ell^+\ell^-$

- Small q^2 region in presence of q^2 and m_X cuts

  Ingredients of the calculation

  Results, universality of ϵ, implications

Details: K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0511nnn

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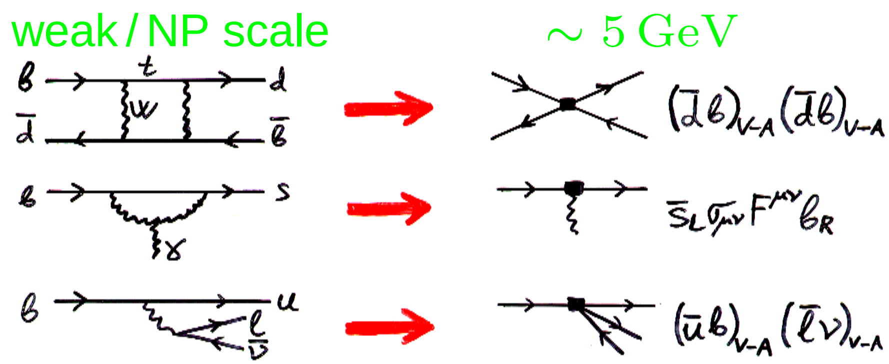
(Michelangelo said: “Avoid just telling us about your last paper”)

Questions for flavor physics

- At scale m_b, $\mathcal{O}(100)$ higher dimensional flavor changing operators

  Depend on a few param's in SM $\Rightarrow$ intricate correlations between s, c, b, t decays



E.g.: $\dfrac{\Delta m_d}{\Delta m_s}$, $\dfrac{b \to d\gamma}{b \to s\gamma}$, $\dfrac{b \to d\ell^+\ell^-}{b \to s\ell^+\ell^-}$ all $\propto \left|\dfrac{V_{td}}{V_{ts}}\right|$ in SM, but test different S.D. physics

- Question: does the SM (i.e., integrating out virtual W, Z, and quarks in tree and loop diagrams) explain all flavor changing interactions? Right coeff's? Right op's?

- $\mathcal{B}(B \to X_s\gamma) = (3.4 \pm 0.3) \times 10^{-4}$ — great triumph; major effort toward NNLO Expected error $\lesssim 5\%$ (4-loop running, 3-loop matching and matrix elements)

- $\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}$ also agrees with SM; NNLO calculation practically completed, theory error $\sim 10\%$

Status of $B \to X_s \ell^+ \ell^-$

- NNLO $b \to s\ell^+\ell^-$ perturbative calculation

  [Bobeth, Misiak, Urban, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]

  Nonperturbative corrections to rate

  [Falk, Luke, Savage, Ali, Hiller, Handoko, Morozumi, Buchalla, Isidori, Rey]

- Rate depends on (mostly)

$$O_7 = m_b\, \bar{s}\sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

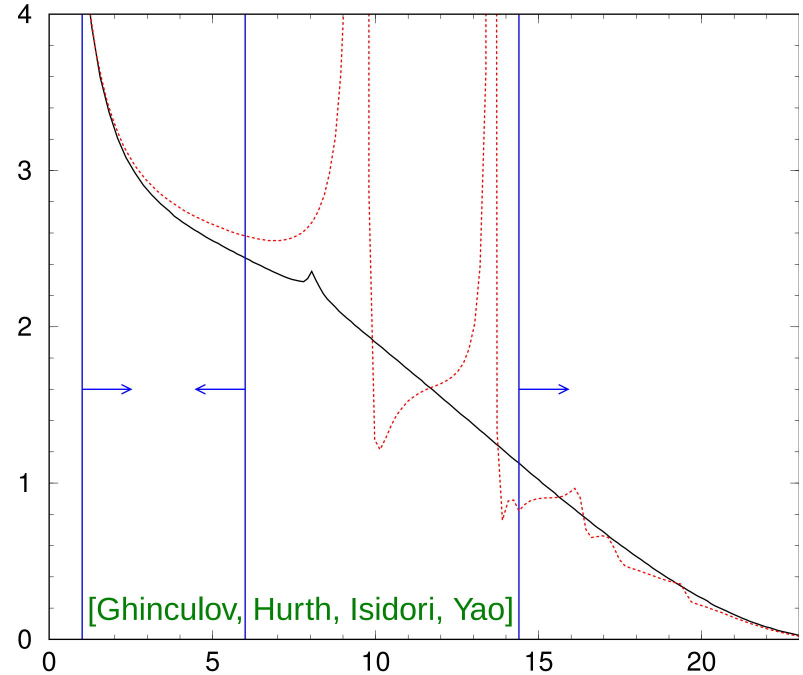
$$O_9 = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell),$$

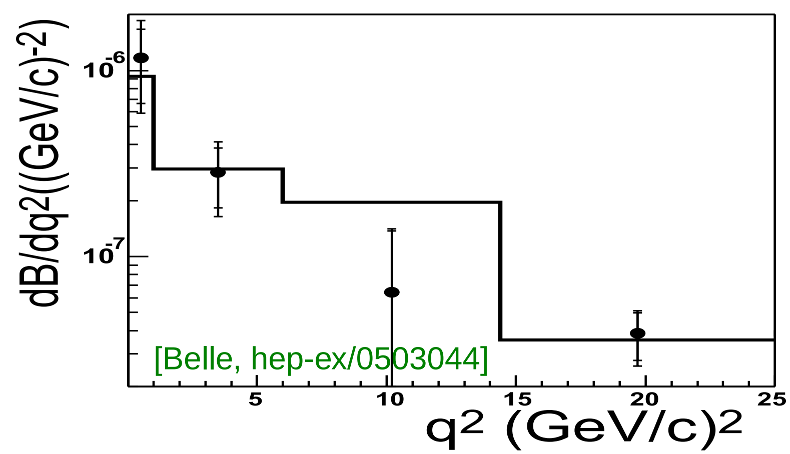
$$O_{10} = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

  Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$

- Experiments use additional cut, $m_{X_s} \lesssim 2\,\mathrm{GeV}$

  ($2\,\mathrm{GeV}$ [Belle, hep-ex/0503044]; $1.8\,\mathrm{GeV}$ [Babar, hep-ex/0404006])





$B \to X_s \ell^+ \ell^-$ kinematics

- There are only two kinematic variables symmetric in p_{ℓ^+} and p_{ℓ^-}

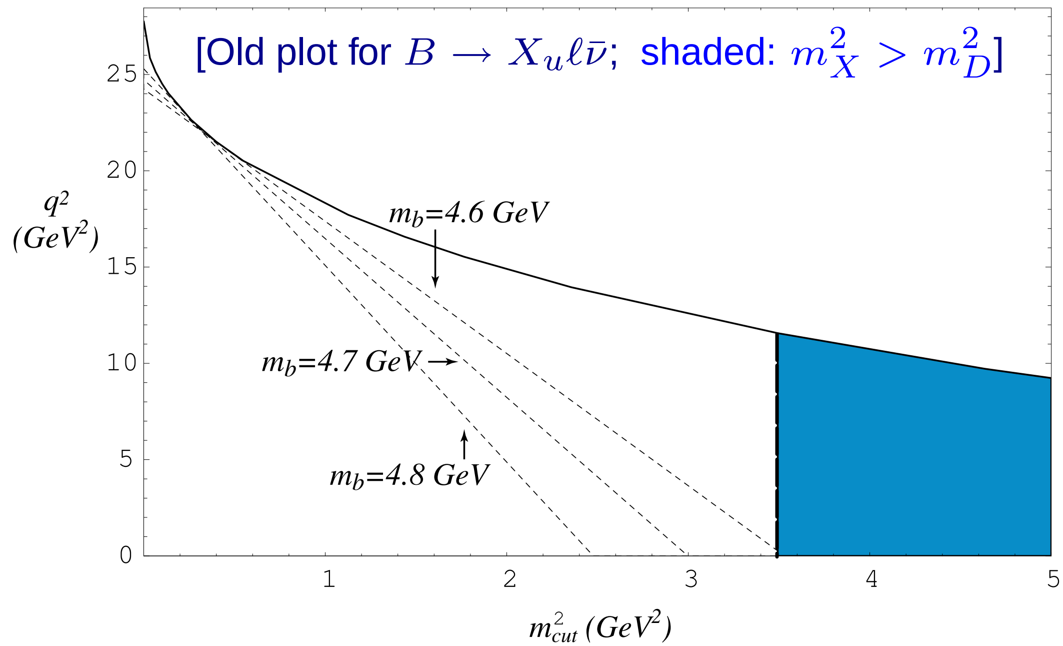
$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

$m_X^2 \ll m_B^2$ & $m_B^2 - q^2 \not\ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B)$ & $E_X^2 \gg m_X^2 \Rightarrow p_X$ near light-cone

$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\rm QCD}) \qquad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \qquad n, \bar{n} = (1, \pm \vec{p}_X / |\vec{p}_X|)$$

- $p_X^+ \ll p_X^-$: jet-like hadronic final state

  Theoretical issues similar to measurement of $|V_{ub}|$ from $B \to X_u \ell \bar{\nu}$



- Parton level: $\Gamma \propto f(q^2)\, \delta[(m_b v - q)^2]$

$$m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$$

  rate vanishes left of the dashed lines

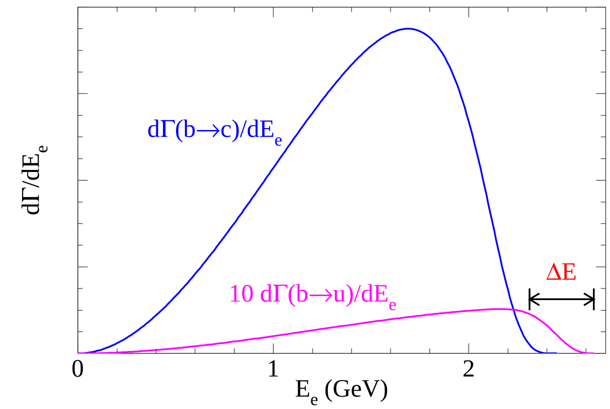
  $\Rightarrow$ nonperturbative physics important

Reminder: inclusive decays

- $|V_{cb}|$: hadronic param's (m_b, Λ, $\lambda_{1,2}$, etc.) fitted from $\sim$90 observables; tests theory
  $\Rightarrow |V_{cb}| = (41.5 \pm 0.7) \times 10^{-3},\ m_b^{1S} = 4.68 \pm 0.03\,\text{GeV},\ \overline{m}_c(m_c) = 1.22 \pm 0.06\,\text{GeV}$

- $|V_{ub}|$: rate known to $\sim 5\%$; phase space cuts to remove $B \to X_c \ell \bar\nu$ (essentially all but q^2) introduce $\mathcal{O}(1)$ dependence on nonperturbative b quark distribution function

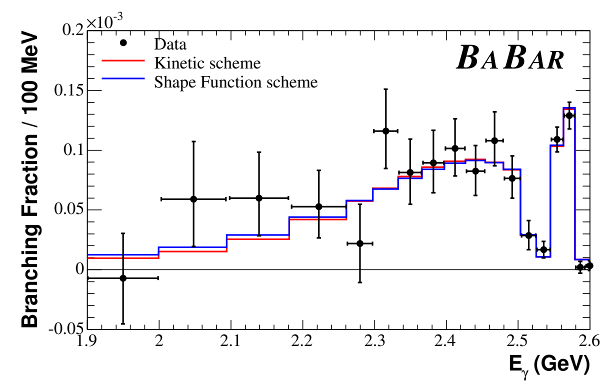
  Hadronic parameters become functions, not constants
  Leading order: universal and related to $B \to X_s\gamma$; but several new unknown functions at $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$



- $\mathcal{B}(B \to X_s\gamma) = (3.4 \pm 0.3) \times 10^{-4}$ — triumph for SM

  Major effort toward NNLO: pert. theory error $\lesssim 5\%$

  Crucial to measure with as low $E_\gamma^{\rm cut}$ as possible



Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s, treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$

  This is OK in local OPE region (e.g., rate or q^2 spectrum) where nonperturbative corrections ($\lambda_{1,2}$, etc.) are small and can be included at the end

- Shape function region: only the rate is calculable, $\Gamma \sim \mathrm{Im}\, \langle B|\, T\{O_i^\dagger(x)\, O_j(0)\}|B\rangle$

  $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ "enhancement", but $|C_9(m_b)| \sim C_{10}$

  – Need to take it seriously to cancel scheme- and scale-dependence in running

  – Do not want power counting to imply that $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2)$ is of same order as $\langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level

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- Matching onto SCET, can separate μ-dependence in matrix element that cancels that in running from $\mathcal{O}(m_W)$ to $\mathcal{O}(m_b)$, and dependence on scales $\sqrt{m_b \Lambda_{\mathrm{QCD}}}$ and $\mu_{\mathrm{hadr}} \sim 1\,\mathrm{GeV}$ — can work to different orders

Matching and running below m_b

- Match $\mathcal{H}_w(\mu_h)$ on SCET at $\mu_h \sim m_b$

- Run down to $\mu_i \sim \sqrt{m_b \Lambda_{\rm QCD}}$

$$\mathrm{d}^3\Gamma^{(0)} = H \int \mathrm{d}k\, J(k)\, f^{(0)}(k)$$

H and J perturbative, $f^{(0)}$ nonperturbative

- Take $f^{(0)}(k)$ from $B \to X_s\gamma$, or run model from μ_0 to μ_i [Bosch, Lange, Neubert, Paz]
(recall: $\Lambda_{\rm QCD}/m_b$ suppressed shape functions are non-universal)

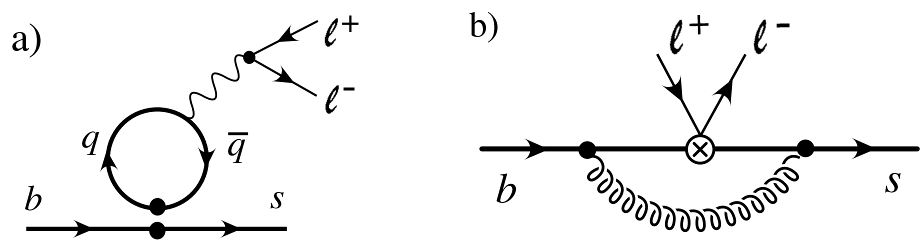
$$f^{(0)}(\hat\omega, \mu_i) = \frac{e^{V_S(\mu_i,\mu_0)}}{\Gamma(1+\eta)} \left(\frac{\hat\omega}{\mu_0}\right)^\eta \int_0^1 \mathrm{d}t\, f^{(0)}\Big[\hat\omega(1-t^{1/\eta}),\, \mu_0\Big] \qquad \eta = \frac{16}{27}\, \ln \frac{\alpha_s(\mu_0)}{\alpha_s(\mu_i)}$$

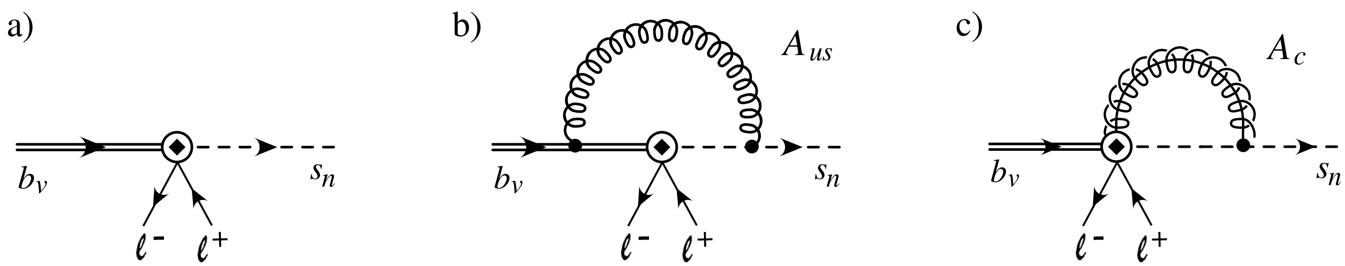
Matching onto SCET

- SCET operators: $J_{\ell\ell}^{(0)} = \sum_{i=a,b,c} C_{9i}(s)\Big(\bar\chi_{n,p}\,\Gamma_i^\mu\,\mathcal{H}_v\Big)(\bar\ell\gamma_\mu\ell)$ + similar $C_{10,7}$ terms

$$\mathcal{H}_v = Y^\dagger h_v\,, \quad \chi_n = W^\dagger \xi_n\,, \qquad \Gamma^\mu_{a-c} = P_R\left\{\gamma^\mu, v^\mu, \frac{n^\mu}{n\cdot v}\right\}$$

[Lee and Stewart, hep-ph/0511???]





- Wilson Coefficients: $C_{9a} = \widetilde C_9^{\rm eff}[1+\mathcal{O}(\alpha_s)] \qquad C_{9b,c} = \mathcal{O}(\alpha_s)$

  Some parts of the "usual" NLL $\mathcal{O}(\alpha_s)$ corrections included in $\widetilde C_9^{\rm eff}$ [Misiak, Buras, Munz]

  now contribute to the jet function, J, some others to the shape function, $f^{(0)}(k)$

Effects of m_X cut at lowest order

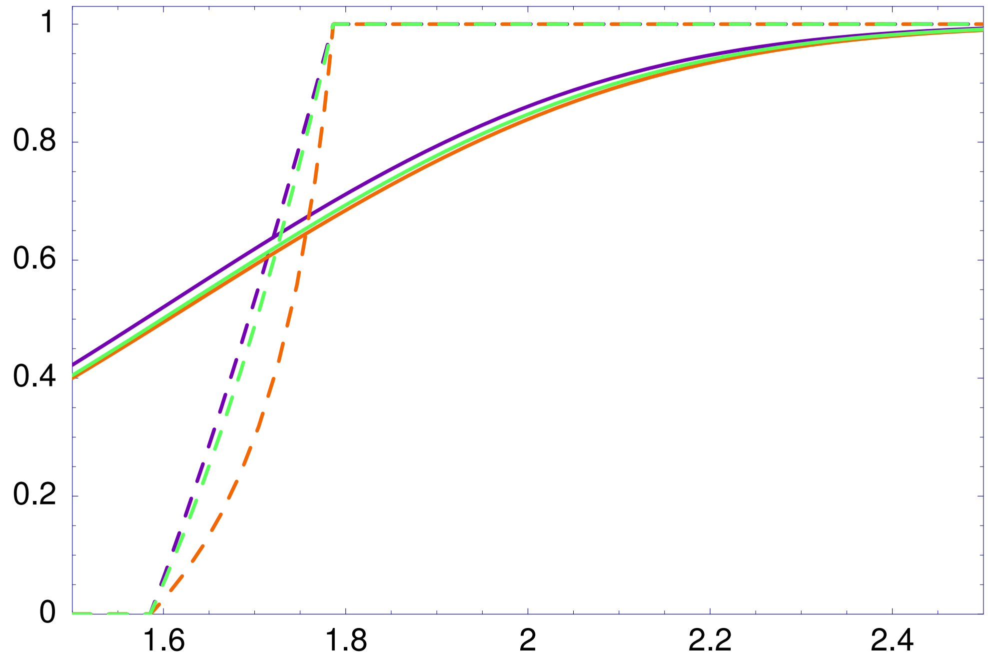
- Define:

$$\epsilon_i = \frac{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2 \, \frac{\mathrm{d}\Gamma_i}{\mathrm{d}q^2\,\mathrm{d}m_X^2}}{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma_i}{\mathrm{d}q^2}}$$

$i = C_9^2$ and C_{10}^2, C_7C_9, C_7^2 — different functionally for each contribution

dashed: tree level

solid: with a fixed shape function model



- ϵ determines fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ may be hadronic physics, not new physics

[Experimental papers use ACCMM model to describe $m_X > 1.1\,\mathrm{GeV}$ region]

Effects of m_X cut at lowest order

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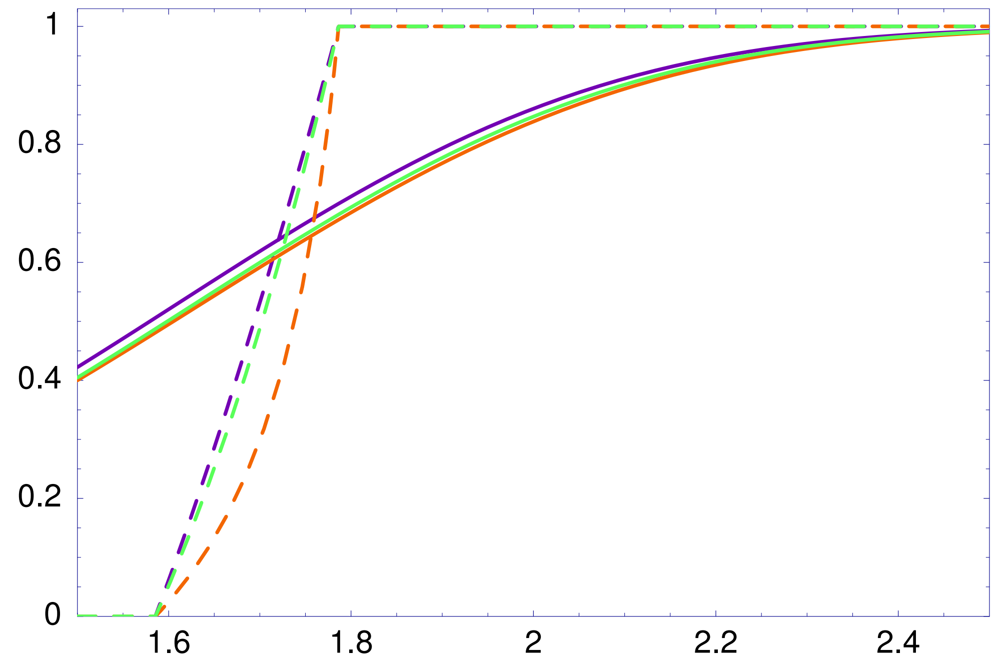


- Strong m_X^{cut} dependence: important to raise it above $\sim 2.2\,\mathrm{GeV}$
  Once $1-\epsilon$ is sizable, so will be its uncertainty

- Approximate universality of ϵ_i: because shape function varies on scale $p_X^+/\Lambda_{\mathrm{QCD}}$, while $\Gamma_i^{\mathrm{parton}}$ varies on scale $p_X^+/m_b \;\Rightarrow\; \epsilon \approx \epsilon_i$

Comments

- Modest q^2-dependence of C_9 for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ can be included trivially

  Shape function uncertainties estimated using $B \to X_s\gamma$ spectrum

  Since largest effect of NNLO is to reduce μ-dependence, while not significantly affecting q^2 distribution, ϵ at NNLO is approximately the same as at NLO

- If increasing m_X^{cut} above $\sim 2.2\,\mathrm{GeV}$ is very hard experimentally, can keep it below m_D and normalize to $B \to X_u\ell\bar\nu$ rate with same cuts to minimize uncertainties

- Sensitivity to NP survives, must take hadronic effects into account correctly

Conclusions

- To achieve theoretical limits in sensitivity to NP in $B \to X\ell^+\ell^-$, small q^2 region is important
- Experimentally used m_X cuts make observed rate sensitive to the shape function
- SF region: expansion for rate, not the amplitude, reorganize perturbation theory
- Approximate universality of ϵ_i for different contributions
- Using $B \to X_s\gamma$ and/or $B \to X_u\ell\bar{\nu}$ data, sensitivity to NP not reduced

Who needs SCET in $B \to X\ell^+\ell^-$?

One-page introduction to SCET

- Effective theory for processes involving energetic hadrons, $E \gg \Lambda$

[Bauer, Fleming, Luke, Pirjol, Stewart, + ...]

Introduce distinct fields for relevant degrees of freedom, power counting in λ

modes	fields	$p = (+, -, \perp)$	p^2
collinear	$\xi_{n,p},\, A^\mu_{n,q}$	$E(\lambda^2, 1, \lambda)$	$E^2\lambda^2$
soft	$q_q,\, A^\mu_s$	$E(\lambda, \lambda, \lambda)$	$E^2\lambda^2$
usoft	$q_{us},\, A^\mu_{us}$	$E(\lambda^2, \lambda^2, \lambda^2)$	$E^2\lambda^4$

SCET$_{\rm I}$: $\lambda = \sqrt{\Lambda/E}$ — jets $(m{\sim}\Lambda E)$

SCET$_{\rm II}$: $\lambda = \Lambda/E$ — hadrons $(m{\sim}\Lambda)$

Match QCD $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$

- Can decouple ultrasoft gluons from collinear Lagrangian at leading order in λ

$$\xi_{n,p} = Y_n\, \xi^{(0)}_{n,p} \qquad A_{n,q} = Y_n\, A^{(0)}_{n,q}\, Y_n^\dagger \qquad Y_n = \mathrm{P}\exp\Big[ig \int_{-\infty}^{x} \mathrm{d}s\, n\cdot A_{us}(ns)\Big]$$

Nonperturbative usoft effects made explicit through factors of Y_n in operators

New symmetries: collinear / soft gauge invariance

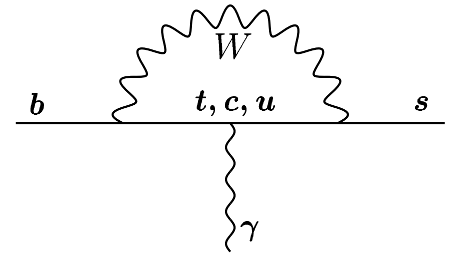
- Simplified / new $(B \to D\pi, \pi\ell\bar\nu)$ proofs of factorization theorems [Bauer, Pirjol, Stewart]

Photon polarization in $B \to X_s\gamma$

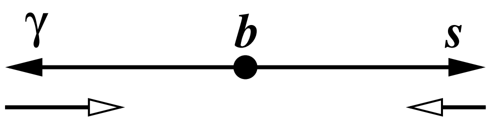
- Is $B \to X_s\gamma$ due to $O_7 \sim \bar{s}\,\sigma^{\mu\nu} F_{\mu\nu}(m_b P_R + m_s P_L) b$ or $\bar{s}\,\sigma^{\mu\nu} F_{\mu\nu}(m_b P_L + m_s P_R) b$?

SM: In $m_s \to 0$ limit, γ must be left-handed to conserve J_z

$O_7 \sim \bar{s}\,(m_b\, F^L_{\mu\nu} + m_s\, F^R_{\mu\nu})\, b$, therefore $b \to s_L \gamma_L$ dominates



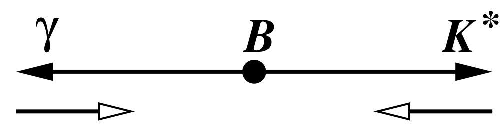
Inclusive $B \to X_s\gamma$



Assumption: 2-body decay

Does not apply for $b \to s\gamma g$

Exclusive $B \to K^*\gamma$



... quark model (s_L implies $J_z^{K^*} = -1$)

... higher K^* Fock states

- One measurement so far; had been expected to give $S_{K^*\gamma} = -2\,(m_s/m_b)\sin 2\beta$

[Atwood, Gronau, Soni]

$$\frac{\Gamma[\overline{B}^0(t) \to K^*\gamma] - \Gamma[B^0(t) \to K^*\gamma]}{\Gamma[\overline{B}^0(t) \to K^*\gamma] + \Gamma[B^0(t) \to K^*\gamma]} = S_{K^*\gamma}\sin(\Delta m\, t) - C_{K^*\gamma}\cos(\Delta m\, t)$$

- What is the SM prediction? What limits the sensitivity to new physics?

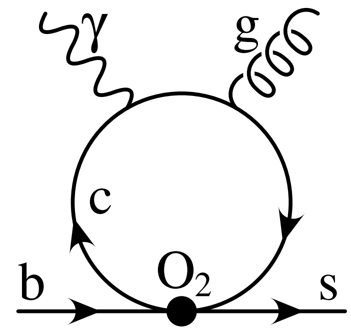
Right-handed photons in the SM

- Dominant source of “wrong-helicity” photons in the SM is O_2 [Grinstein, Grossman, ZL, Pirjol]

  Equal $b \to s\gamma_L,\ s\gamma_R$ rates at $\mathcal{O}(\alpha_s)$; calculated to $\mathcal{O}(\alpha_s^2\beta_0)$

  Inclusively only rates are calculable: $\Gamma_{22}^{(\mathrm{brem})}/\Gamma_0 \simeq 0.025$

  Suggests: $A(b \to s\gamma_R)/A(b \to s\gamma_L) \sim \sqrt{0.025/2} = 0.11$



- Exclusive $B \to K^*\gamma$: factorizable part contains an operator that could contribute at leading order in $\Lambda_{\mathrm{QCD}}/m_b$, but its $B \to K^*\gamma$ matrix element vanishes

  Subleading order: several contributions to $\overline{B}{}^0 \to \overline{K}{}^{0*}\gamma_R$, no complete study yet

  We estimate: $$\frac{A(\overline{B}{}^0 \to \overline{K}{}^{0*}\gamma_R)}{A(\overline{B}{}^0 \to \overline{K}{}^{0*}\gamma_L)} = \mathcal{O}\left(\frac{C_2}{3C_7}\,\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) \sim 0.1$$

- Data: $S_{K^*\gamma} = -0.13 \pm 0.32$ — both the measurement and the theory can progress