Measuring α/ϕ_2 from $B \to \rho\rho$

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- Introduction
- Isospin analysis
  ... complications due to $\Gamma_\rho \neq 0$
  ... present constraints on $\alpha - \alpha_{\rm eff}$
- Other small corrections
  ... $\propto (1 - f_0)$ and EW penguins
- Summary

see: Falk, Z.L., Nir, Quinn, hep-ph/0310242, To appear in PRD

Introduction

- Want to determine CKM angle $\alpha \equiv \phi_2 \equiv \arg\left[-\left(V_{td}V_{tb}^*\right) / \left(V_{ud}V_{ub}^*\right)\right]$ from S_{+-}:

$$\frac{\Gamma(\overline{B}^0_{\rm phys}(t) \to \rho^+\rho^-) - \Gamma(B^0_{\rm phys}(t) \to \rho^+\rho^-)}{\Gamma(\overline{B}^0_{\rm phys}(t) \to \rho^+\rho^-) + \Gamma(B^0_{\rm phys}(t) \to \rho^+\rho^-)} = S_{+-}\sin(\Delta m\, t) - C_{+-}\cos(\Delta m\, t)$$

  If amplitudes with a single weak phase dominate, then $S_{+-} = \sin 2\alpha$

- Summer ’03 news: $B \to \rho\rho$ almost purely longitudinally polarized

$$\mathcal{B}(B \to \rho^0\rho^0)/\mathcal{B}(B \to \rho^-\rho^+) < 0.1 \quad (90\%\ \text{CL})$$

  [compare: $\mathcal{B}(B \to \pi^0\pi^0)/\mathcal{B}(B \to \pi^-\pi^+) \simeq 0.4$]

- $S_{\rho^+\rho^-}$ may soon give accurate model independent determination of α

  ... concentrate on differences compared to $B \to \pi\pi$

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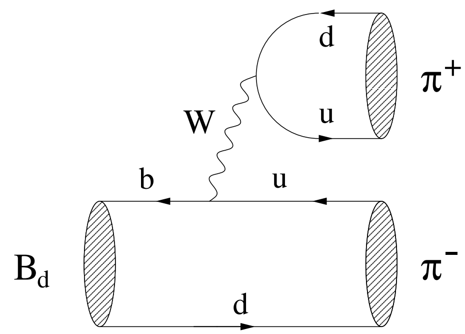
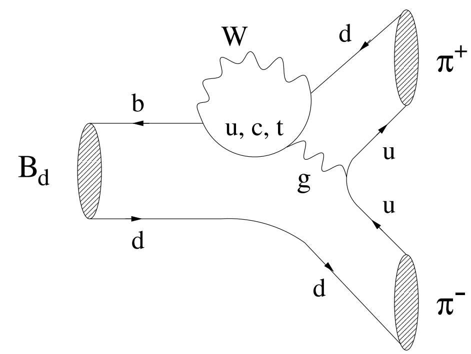
$B \to \pi\pi$: the problem

- There are tree and penguin amplitudes, just like in $B \to \psi K_S$





"Tree" ($b \to u\bar{u}d$): $\overline{A}_T = \overset{[\lambda^3]}{V_{ub}V_{ud}^*}\, A_{u\bar{u}d}$

"Penguin": $\overline{A}_P = \overset{[\lambda^3]}{V_{tb}V_{td}^*}\, P_t + \overset{[\lambda^3]}{V_{cb}V_{cd}^*}\, P_c + \overset{[\lambda^3]}{V_{ub}V_{ud}^*}\, P_u$

unitarity: $\overline{A}_{\pi^+\pi^-} = \underbrace{\overset{[\lambda^3]}{V_{ub}V_{ud}^*}}_{\text{same as Tree phase}} [A_{u\bar{u}d} + P_u - P_t] + \underbrace{\overset{[\lambda^3]}{V_{cb}V_{cd}^*}}_{\text{not suppressed}} [P_c - P_t]$

Define P and T by: $\overline{A}_{\pi^+\pi^-} = T_{+-}\, e^{-i\gamma} + P_{+-}\, e^{+i\beta}$

Two amplitudes with different weak- and possibly different strong phases; their values are not known model independently

- $\mathcal{B}(B \to K^-\pi^+) = (18.2 \pm 0.8) \times 10^{-6}$ to $\mathcal{B}(B \to \pi^-\pi^+) = (4.6 \pm 0.4) \times 10^{-6}$ ratio implies $|P/T| \sim 0.3$, so need $B \to \pi^0\pi^0$

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Isospin Symmetry

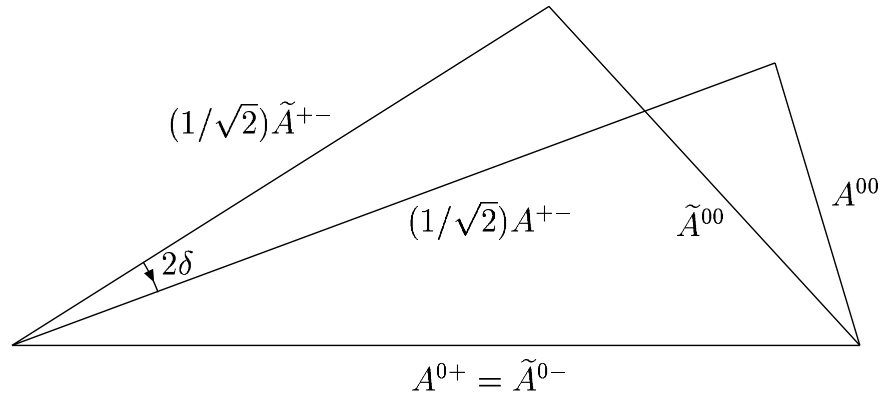
Isospin analysis

$\pi\pi$: Bose statistics $\Rightarrow\ I = 0, 2$

$$A_{ij} = T_{ij} e^{+i\gamma} + P_{ij} e^{-i\beta}$$
$$\overline{A}_{ij} = T_{ij} e^{-i\gamma} + P_{ij} e^{+i\beta}$$

A_{ij} [$\overline{A}_{ij}$] denote B^+, B^0 [B^-, $\overline{B}^0$] decays

$$\widetilde{A}^{ij} \equiv e^{2i\gamma} \bar{A}^{ij}$$



$\mathcal{B}(B \to \pi^0\pi^0) = (2.0 \pm 0.5) \times 10^{-6}$, so triangles are not squashed

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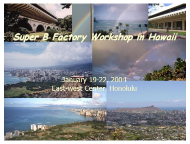
$\rho\rho$: Mixture of CP even/odd $(L = 0, 1, 2)$, but since B is spin-0, the combined space and spin wave function of the two ρ's is symmetric under particle exchange

Bose statistics: isospin of $\rho\rho$ symmetric under particle exchange $\Rightarrow I = 1$ absent

Same holds in transversity basis: isospin analysis applies for each $\sigma\ (= 0, \|, \perp)$



Complications due to $\Gamma_\rho \neq 0$

- Even for $\sigma = 0$ the possibility of $I = 1$ is reintroduced by finite Γ_ρ

  Can have antisymmetric dependence on both the two ρ mesons' masses and on their isospin indices $\Rightarrow\ I = 1$ (m_i = mass of a pion pair; B = Breit-Wigner)

$$
\begin{aligned}
A \sim B(m_1)B(m_2)\,\frac{1}{2}\Big[f(m_1,m_2)\,\rho^+(m_1)\rho^-(m_2) + f(m_2,m_1)\,\rho^+(m_2)\rho^-(m_1)\Big] \\
= B(m_1)B(m_2)\,\frac{1}{4}\Big\{\big[f(m_1,m_2)+f(m_2,m_1)\big]\underbrace{\big[\rho^+(m_1)\rho^-(m_2)+\rho^+(m_2)\rho^-(m_1)\big]}_{I=0,2} \\
+\big[f(m_1,m_2)-f(m_2,m_1)\big]\underbrace{\big[\rho^+(m_1)\rho^-(m_2)-\rho^+(m_2)\rho^-(m_1)\big]}_{I=1}\Big\}
\end{aligned}
$$

  If Γ_ρ vanished, then $m_1 = m_2$ and $I = 1$ part is absent

  E.g., no symmetry in factorization: $f(m_{\rho^-}, m_{\rho^+}) \sim f_\rho(m_{\rho^+})\,F^{B\to\rho}(m_{\rho^-})$

- Could not rule out $\mathcal{O}(\Gamma_\rho/m_\rho)$ contributions; no interference $\Rightarrow$ $\mathcal{O}(\Gamma_\rho^2/m_\rho^2)$ effects

  How would they show up...?

Constraining $I = 1$

- Leading $I = 1$ term can be parameterized as [e.g., from $B_i H_j^{kl}(\rho_k^i \partial^2 \rho_l^j - \rho_l^j \partial^2 \rho_k^i)$]

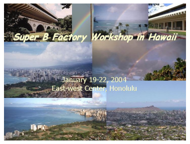
$$\left[c\,\frac{m_1 - m_2}{m_\rho}\right]^2 \left|B_\rho(m_1^2) B_\rho(m_2^2)\right|^2$$

Unfortunately, subleading $I =$ even contribution (cross-term) can have same form

$$\left[a + b\,\frac{(m_1 - m_2)^2}{m_\rho^2}\right]^2 \left|B_\rho(m_1^2) B_\rho(m_2^2)\right|^2$$

Expect a, b, c of the same order, so $ab/c^2 = \mathcal{O}(1)$

- To constrain them, either:
  - Add new term to fit and check for stability of the a^2 term, for which the isospin analysis should be carried out ($I = 1$ absent for $\rho^0\rho^0$)
  - Decrease the widths of the ρ bands or impose a cut on $|m_1 - m_2|$ to eliminate possible $I = 1$ term



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Bounds on $\delta (= \alpha - \alpha_{\rm eff})$

- Until the $\mathcal{B}[B^0 \to (\rho^0\rho^0)_\sigma]$ and $\mathcal{B}[\overline{B}^0 \to (\rho^0\rho^0)_\sigma]$ tagged rates are separately measured, one can bound δ_σ using Babar & Belle data

$$\mathcal{B}_{+-} = \frac{1}{2}\left(|A_{+-}|^2 + |\bar{A}_{+-}|^2\right) = (27 \pm 9) \times 10^{-6}\,, \qquad (f_0)_{+-} = 0.99^{+0.01}_{-0.07} \pm 0.03$$

$$\mathcal{B}_{+0} = \frac{1}{2}\left(|A_{+0}|^2 + |\bar{A}_{-0}|^2\right) = (26 \pm 6) \times 10^{-6}\,, \qquad (f_0)_{+0} = 0.97^{+0.03}_{-0.07} \pm 0.04$$

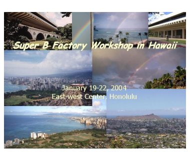
$$\mathcal{B}_{00} = \frac{1}{2}\left(|A_{00}|^2 + |\bar{A}_{00}|^2\right) = (0.6^{+0.8}_{-0.6}) \times 10^{-6}\,, \qquad [\mathcal{B}_{00} < 2.1 \times 10^{-6}\ (90\%\ \mathrm{CL})]$$

First two measured, and upper bound on $\mathcal{B}_{00}$ constrains $\mathcal{B}^0_{00} \ll \mathcal{B}^0_{+-}\,,\ \mathcal{B}^0_{+0}$

- Can bound δ_0 the same way as in $B \to \pi\pi$ [Grossman-Quinn / Gronau-London-Sinha-Sinha]

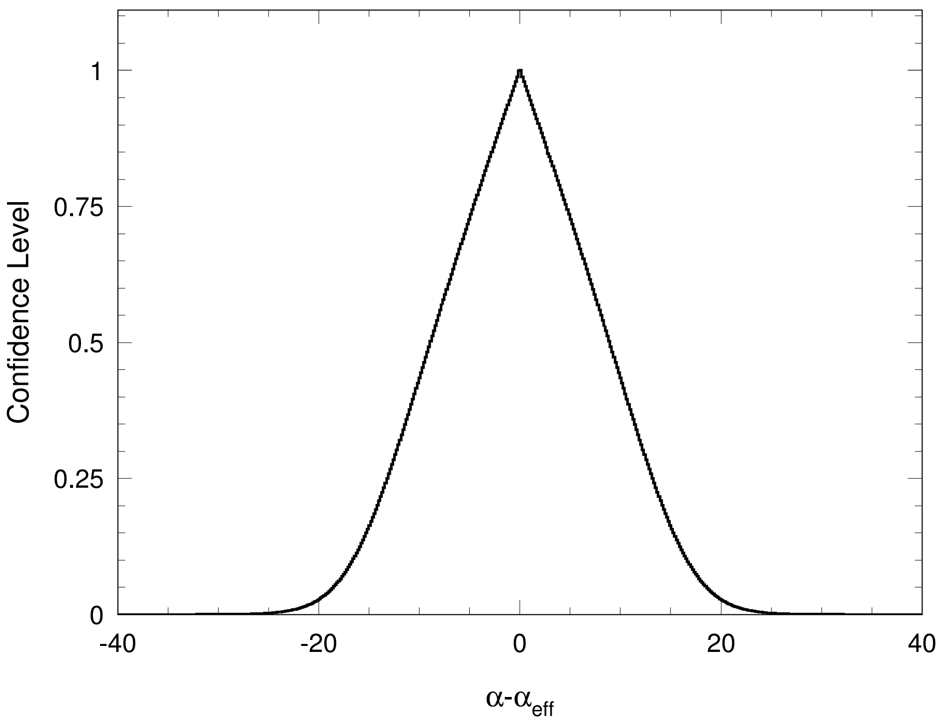
$$\cos 2\delta_0 \geq 1 - \frac{2\mathcal{B}^0_{00}}{\mathcal{B}^0_{+0}} + \frac{(\mathcal{B}^0_{+-} - 2\mathcal{B}^0_{+0} + 2\mathcal{B}_{00})^2}{4\mathcal{B}^0_{+-}\mathcal{B}^0_{+0}} + \ldots$$

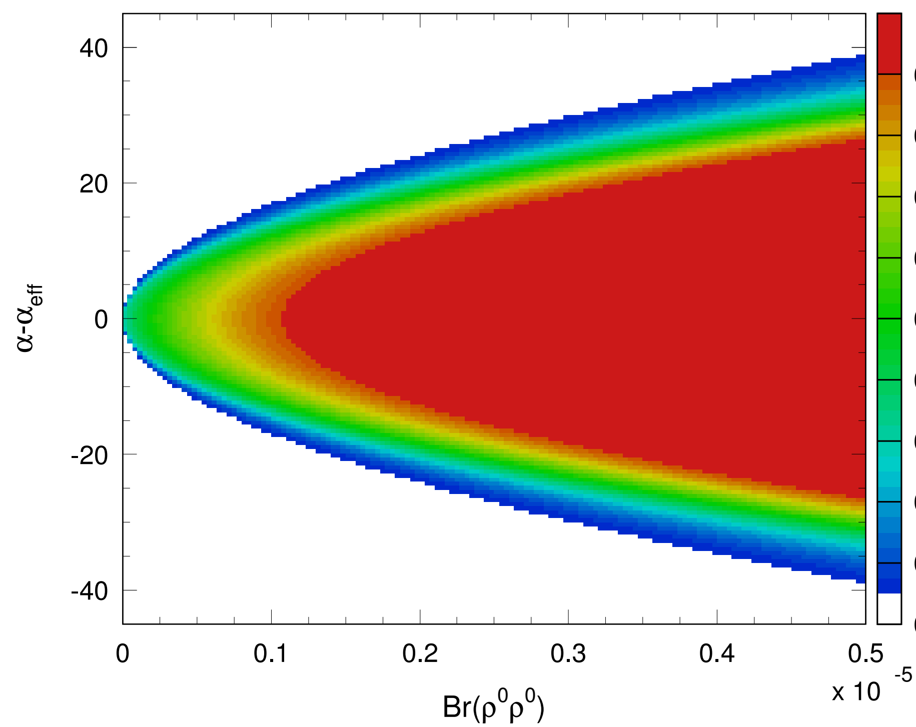
The bound also depends on experimental constraints on C_{+-} and C_{00}



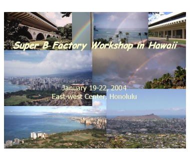
Resulting constraints

- Present data implies: $\cos 2\delta_0 > 0.83$ or $|\delta_0| < 17^\circ$ (90% CL)





Took $\mathcal{B}_{+-} = \mathcal{B}^0_{+-}$ and $\mathcal{B}_{+0} = \mathcal{B}^0_{+0}$ for simplicity [Fits done using CKMfitter package]



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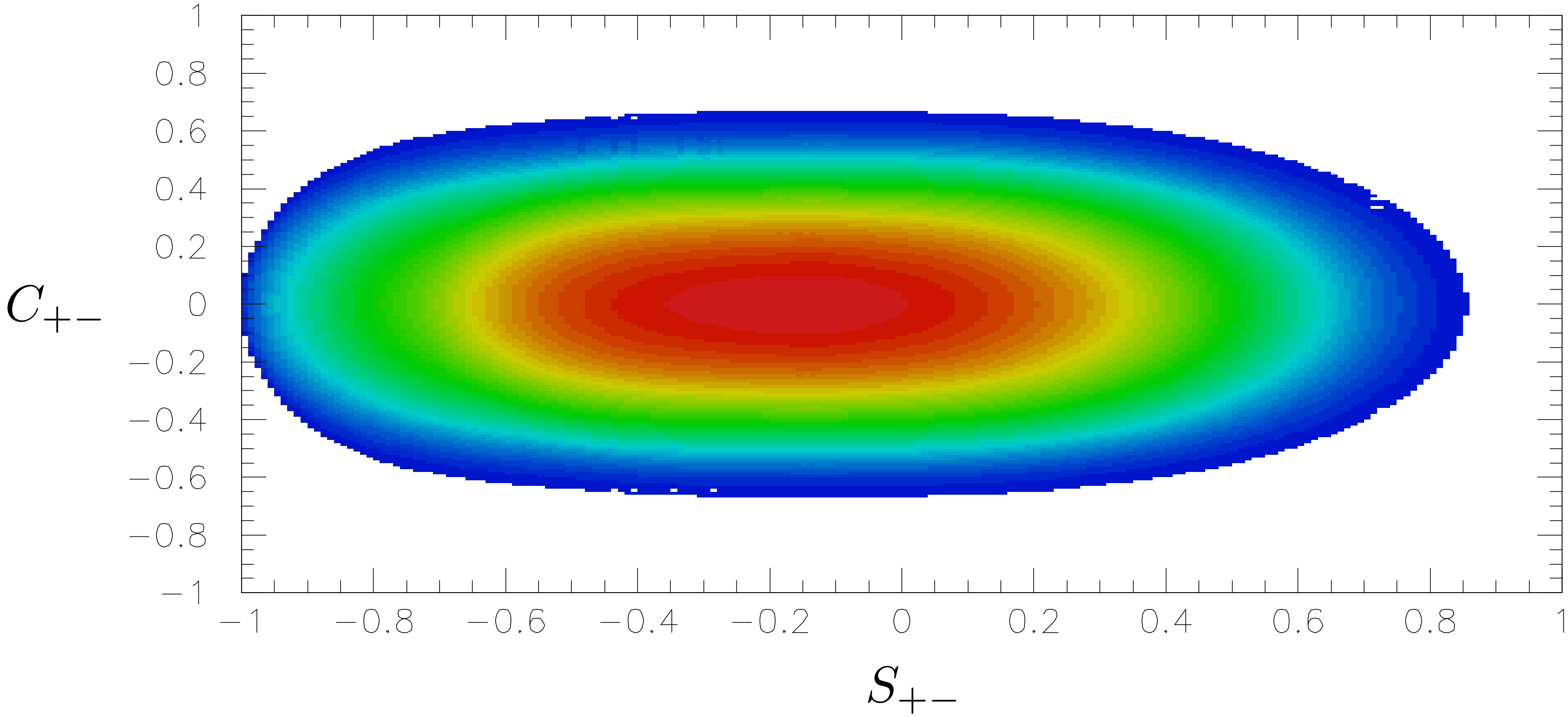
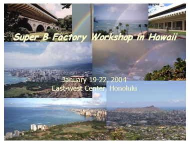
Presently allowed range of CP asymmetries

- Small $\mathcal{B}_{00}/\mathcal{B}_{+0}$ also bounds direct CPV:

$$|C^{\sigma}_{+-}| < 2\sqrt{\frac{\mathcal{B}^{\sigma}_{00}}{\mathcal{B}^{\sigma}_{+0}} - \left(\frac{\mathcal{B}^{\sigma}_{00}}{\mathcal{B}^{\sigma}_{+0}}\right)^2} \quad \Rightarrow \quad |C^{0}_{+-}| < 0.53 \ \ (90\%\ \mathrm{CL})$$





Corrections...

Corrections proportional to $1 - f_0$

- If S_{+-} not measured in longitudinal mode alone, use $S_{+-} = \sum_\sigma f_\sigma S^\sigma_{+-}$ to bound

$$|S^0_{+-} - S_{+-}| \le (1 - f_0)\left(1 + |S^0_{+-}|\right)$$

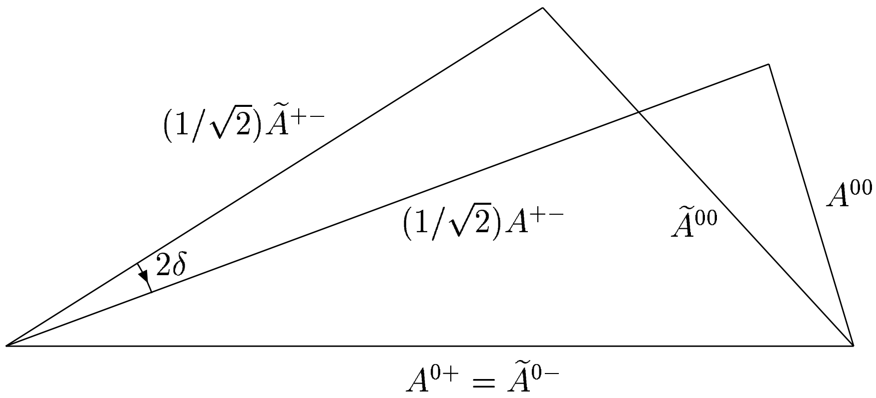
Expect the error in estimating S^0_{+-} to be smaller — to zeroth order in $|P^\sigma_{+-}/T^\sigma_{+-}|$ we have $S^{\parallel}_{+-} = -S^{\perp}_{+-} = S^0_{+-}$, so

$$S^0_{+-} - S_{+-} = (1 - f_0 - f_\parallel + f_\perp)\, S^0_{+-} + \mathcal{O}\big[(1 - f_0)\,|P_{+-}/T_{+-}|\big]$$

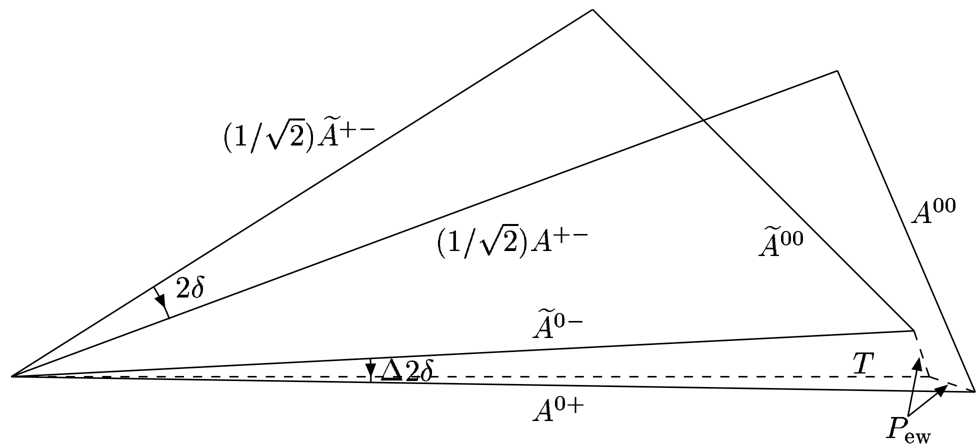
- Non-resonant $B \to 4\pi$ decays and other resonances that decay to 4π could have opposite CP than the dominant longitudinal mode

Contamination due to such contributions effectively included in the fit error of $1 - f_0$

Electroweak penguins



$\Rightarrow$



In $B \to \pi\pi$ isospin analysis, neglecting EWP: one more observable than unknown

Including EWP: 2 new unknowns, but in $B \to \rho\rho$ yet one more observable, $S_{\rho^0\rho^0}$

Insufficient: constrains a combination of $|P_{\rm ew}|$ and $\arg(P_{\rm ew})$, but does not fix $\Delta 2\delta$

For now, consistent to neglect them: $\mathcal{A}_{\mp 0} = \frac{|\bar{A}_{-0}|^2 - |A_{+0}|^2}{|\bar{A}_{-0}|^2 + |A_{+0}|^2} = -0.09 \pm 0.16$

Isospin violation due to $\rho - \omega - \phi$ mixing expected to be small

Conclusions

Summary

- Present measurements of the various $B \to \rho\rho$ rates already give significant limits on the uncertainty in the extraction of α from the CP asymmetry in $B \to \rho^+\rho^-$
- With higher precision, need to parameterize the data to allow for impact of possible $I = 1$ contributions that can affect results at the $\mathcal{O}(\Gamma_\rho^2/m_\rho^2)$ level
- $S_{\rho^+\rho^-}$ may give best model independent determination of α for some time to come
- Limit on theory error of α seems to be at the 5° level (data may tell us it's larger)