

Hints for new physics from radiative / electroweak B decays

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Hints for new physics in flavor decays
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- Introduction
- $b \rightarrow s\gamma$: Rate, asymmetries, inclusive & exclusive
- $b \rightarrow s\ell^+\ell^-$: Optimal observables to constrain short distance physics
Small and large q^2 regions; sensitivity to shape function, connections to $|V_{ub}|$
- $b \rightarrow s\nu\bar{\nu}$: The theoretically cleanest of all
- Conclusions

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I think it's more interesting to explore what's possible with ~ 100 times more data
(These measurements are of course important; chance of $> 3\sigma$ before upgrade?)



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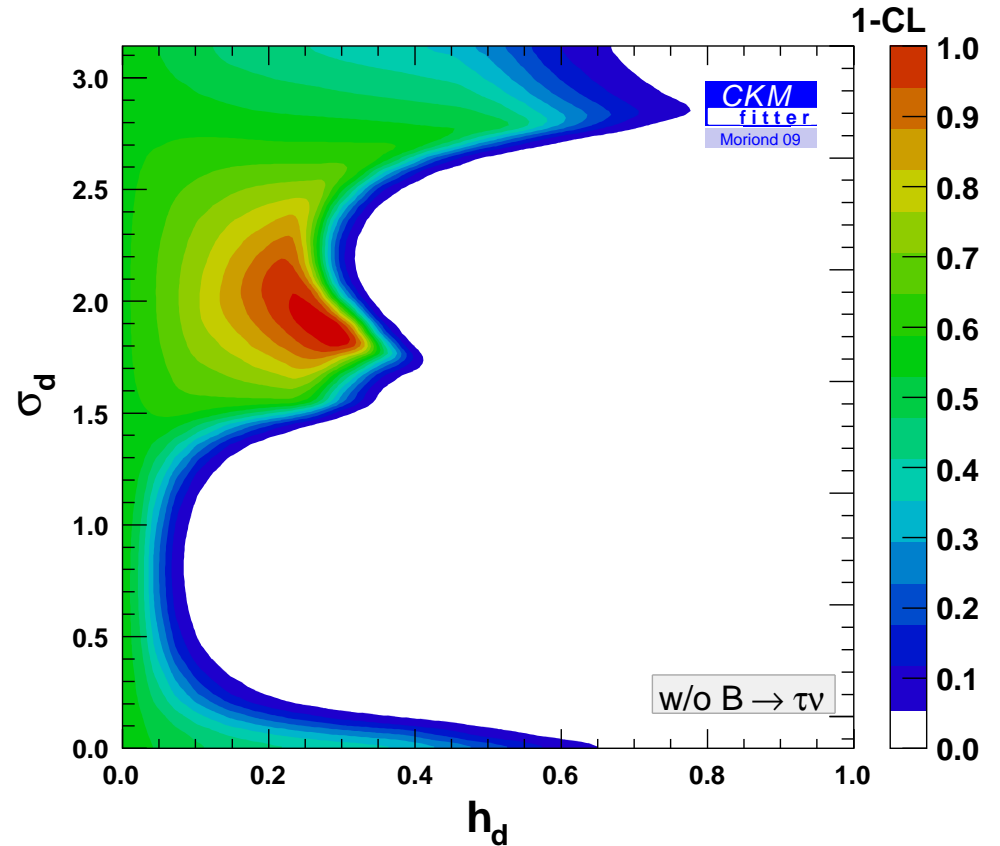
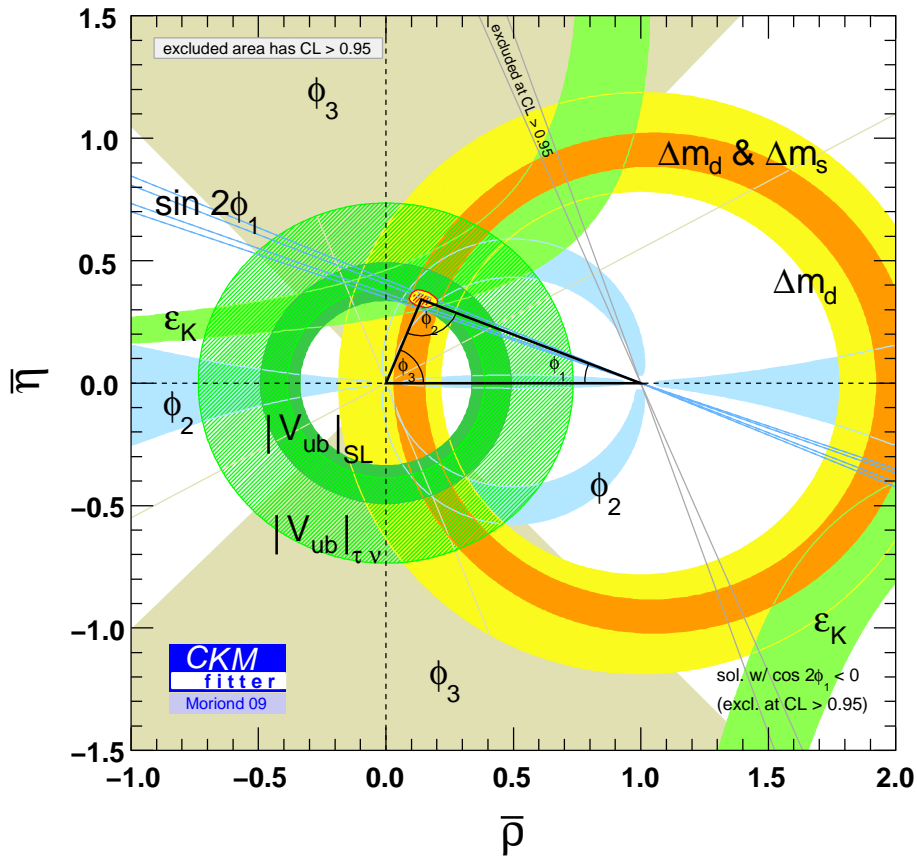
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- In my opinion, building a Super- B -factory is clearly justified

$2-3\sigma$ effects may be temporary, so let's concentrate on finding the best combinations of theoretical cleanliness and experimental feasibility



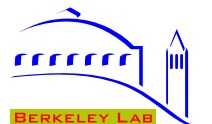
Main reason (for me) to continue



- Very impressive accomplishments
- Level of agreement between various measurements often misinterpreted

Parameterize: $M_{12} = M_{12}^{\text{SM}} (1 + h_d e^{2i\sigma_d})$

Most loop-mediated transitions may have 10–20% NP contributions w/o fine tuning



The rare B decay landscape

- Important probes of new physics (a crude guide, $\ell = e$ or μ)

Decay	\sim SM rate	present status	expected
$B \rightarrow X_s \gamma$	3.2×10^{-4}	$(3.52 \pm 0.25) \times 10^{-4}$	4%
$B \rightarrow \tau \nu$	1×10^{-4}	$(1.73 \pm 0.35) \times 10^{-4}$	5%
$B \rightarrow X_s \nu \bar{\nu}$	3×10^{-5}	$< 6.4 \times 10^{-4}$	only $K \nu \bar{\nu}$?
$B \rightarrow X_s \ell^+ \ell^-$	6×10^{-6}	$(4.5 \pm 1.0) \times 10^{-6}$	6%
$B_s \rightarrow \tau^+ \tau^-$	1×10^{-6}	$< \text{few } \%$	$\Upsilon(5S)$ run ?
$B \rightarrow X_s \tau^+ \tau^-$	5×10^{-7}	$< \text{few } \%$?
$B \rightarrow \mu \nu$	4×10^{-7}	$< 1.3 \times 10^{-6}$	6%
$B \rightarrow \tau^+ \tau^-$	5×10^{-8}	$< 4.1 \times 10^{-3}$	$\mathcal{O}(10^{-4})$
$B_s \rightarrow \mu^+ \mu^-$	3×10^{-9}	$< 5 \times 10^{-8}$	LHCb
$B \rightarrow \mu^+ \mu^-$	1×10^{-10}	$< 1.5 \times 10^{-8}$	LHCb

- Many interesting modes will first be seen at super- B (or LHCb)

Maintain ability for inclusive studies as much as possible (smaller theory errors)

- Some of the theoretically cleanest modes (ν , τ , inclusive) only possible at e^+e^-



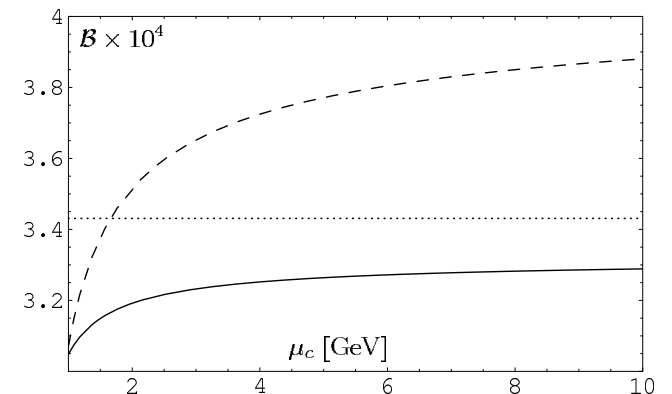
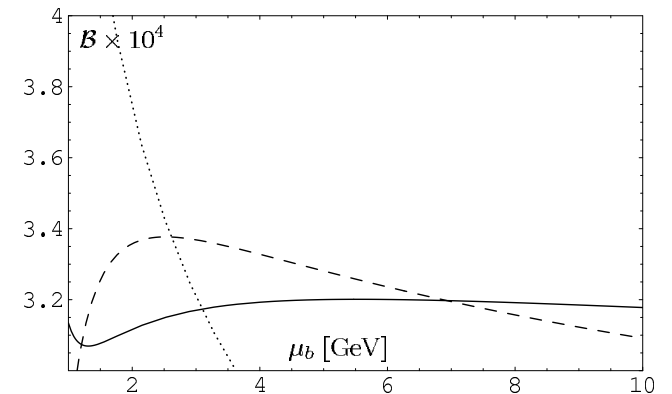
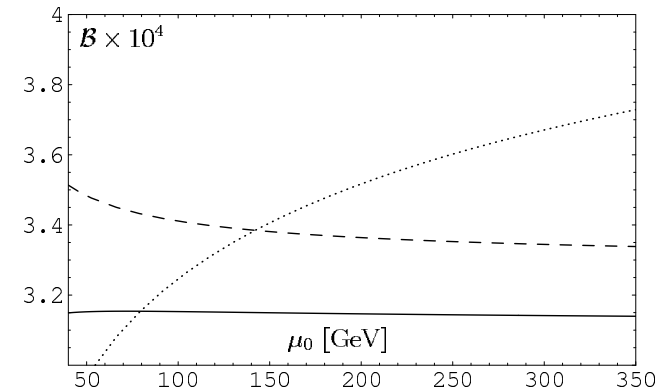
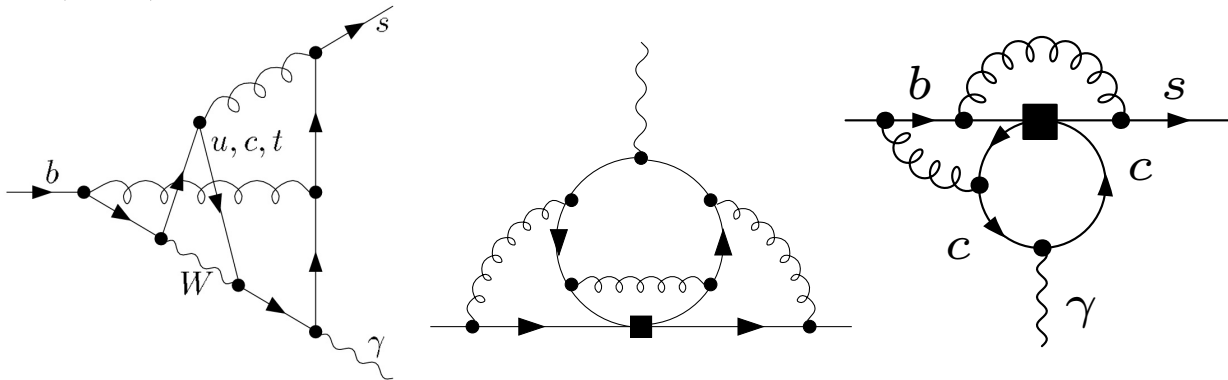
$$B \rightarrow X_s \gamma$$

Inclusive $B \rightarrow X_s \gamma$ calculations

- One (if not “the”) most elaborate SM calculations
Constrains many models: 2HDM, SUSY, LRSM, etc.
- NNLO practically completed [Misiak et al., hep-ph/0609232]
4-loop running, 3-loop matching and matrix elements

Scale dependencies significantly reduced \Rightarrow

- $\mathcal{B}(B \rightarrow X_s \gamma) \Big|_{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4}$
measurement: $(3.52 \pm 0.25) \times 10^{-4}$
- $\mathcal{O}(10^4)$ diagrams, e.g.:



The $B \rightarrow X_s \gamma$ photon spectrum

- World average: $\mathcal{B}(B \rightarrow X_s \gamma) = (3.52 \pm 0.25)10^{-4}$
Could have easily shown deviations from SM
- Exp. cut: $E_\gamma \gtrsim 1.9 \text{ GeV} \Rightarrow m_B - 2E_\gamma^{\text{cut}} \sim 1.5 \text{ GeV}$

Three cases:

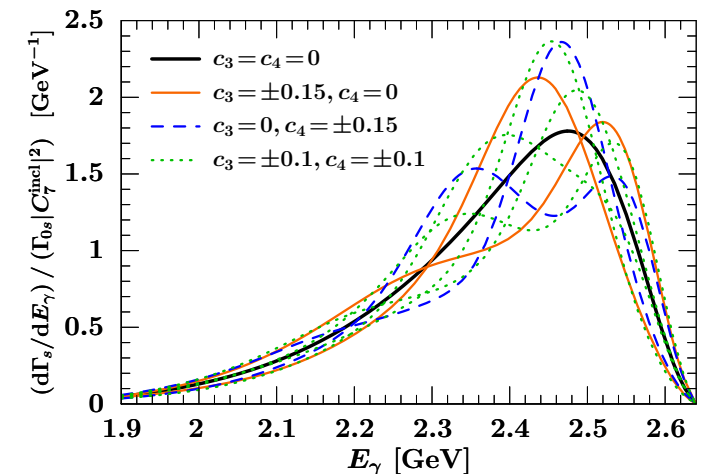
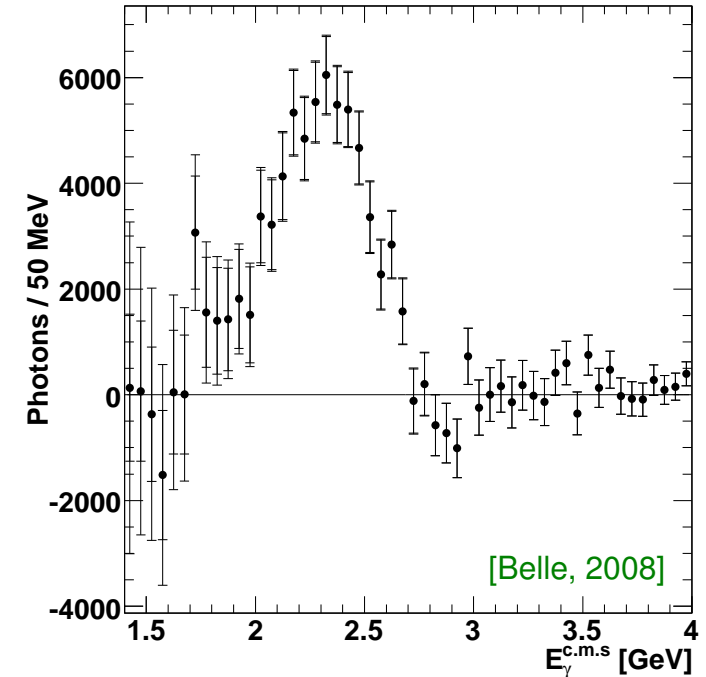
- 1) $\Lambda \sim m_B - 2E_\gamma \ll m_B$
- 2) $\Lambda \ll m_B - 2E_\gamma \ll m_B$
- 3) $\Lambda \ll m_B - 2E_\gamma \sim m_B$

Neither 1) nor 2) is appropriate

- Can combine 1–2 w/o expanding $\Lambda/(m_B - 2E_\gamma)$
[ZL, Stewart, Tackmann, 0807.1926]

9 models with the same 0th, 1st, 2nd moments \Rightarrow

Including all NNLL corrections, smaller shape function uncertainty for $E_\gamma \lesssim 2.1 \text{ GeV}$ than other studies

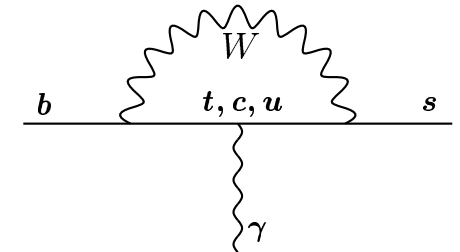


Photon polarization in $B \rightarrow X_s \gamma$

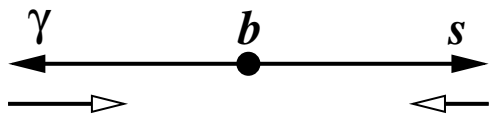
- Is $B \rightarrow X_s \gamma$ due to $O_7 \sim \bar{s} \sigma^{\mu\nu} F_{\mu\nu} (m_b P_R + m_s P_L) b$ or $\bar{s} \sigma^{\mu\nu} F_{\mu\nu} (m_b P_L + m_s P_R) b$?

SM: In $m_s \rightarrow 0$ limit, γ must be left-handed to conserve J_z

$O_7 \sim \bar{s} (m_b F_{\mu\nu}^L + m_s F_{\mu\nu}^R) b$, therefore $b \rightarrow s_L \gamma_L$ dominates



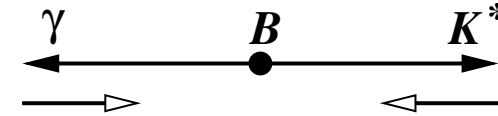
Inclusive $B \rightarrow X_s \gamma$



Assumption: 2-body decay

Does not apply for $b \rightarrow s \gamma g$

Exclusive $B \rightarrow K^* \gamma$



... quark model (s_L implies $J_z^{K^*} = -1$)

... higher K^* Fock states

- Had been expected to give $S_{K^* \gamma} = -2 (m_s/m_b) \sin 2\phi_1$ [Atwood, Gronau, Soni]

$$\frac{\Gamma[\bar{B}^0(t) \rightarrow K^* \gamma] - \Gamma[B^0(t) \rightarrow K^* \gamma]}{\Gamma[\bar{B}^0(t) \rightarrow K^* \gamma] + \Gamma[B^0(t) \rightarrow K^* \gamma]} = S_{K^* \gamma} \sin(\Delta m t) - C_{K^* \gamma} \cos(\Delta m t)$$

- Data: $S_{K^* \gamma} = -0.16 \pm 0.22$ — both the measurement and the theory can progress



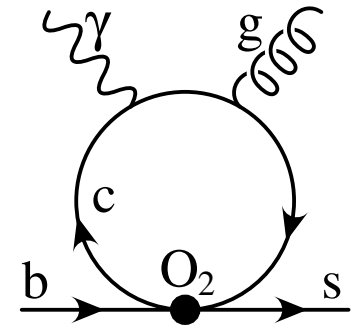
Right-handed photons in the SM

- Dominant source of “wrong-helicity” photons in the SM is O_2 [Grinstein, Grossman, ZL, Pirjol]

Equal $b \rightarrow s\gamma_L, s\gamma_R$ rates at $\mathcal{O}(\alpha_s)$; calculated to $\mathcal{O}(\alpha_s^2\beta_0)$

Inclusively only rates are calculable: $\Gamma_{22}^{(\text{brem})}/\Gamma_0 \simeq 0.025$

Suggests: $A(b \rightarrow s\gamma_R)/A(b \rightarrow s\gamma_L) \sim \sqrt{0.025/2} = 0.11$



- $B \rightarrow K^*\gamma$: At leading order in Λ_{QCD}/m_b , wrong helicity amplitude vanishes

Subleading order: no longer vanishes

[Grinstein, Grossman, ZL, Pirjol]

Order of magnitude: $\frac{A(\bar{B}^0 \rightarrow \bar{K}^{0*}\gamma_R)}{A(\bar{B}^0 \rightarrow \bar{K}^{0*}\gamma_L)} = \mathcal{O}\left(\frac{C_2}{3C_7} \frac{\Lambda_{\text{QCD}}}{m_b}\right) \sim 0.1$

Some additional suppression expected, but I don't find $\lesssim 0.02$ claims convincing

- Consider pattern in many modes, hope to build a case

[Atwood, Gershon, Hazumi, Soni]



Other observables

- Direct CP asymmetry:

$$A_{B \rightarrow X_s \gamma} = -0.012 \pm 0.028$$

$$A_{B \rightarrow X_{d+s} \gamma} = -0.011 \pm 0.012$$

$$A_{B \rightarrow K^* \gamma} = -0.010 \pm 0.028$$

Theoretical predictions < 0.01 , except $A_{B \rightarrow \rho \gamma}$ which is larger

- Isospin asymmetry: it seems to me that theoretical uncertainties would make it hard to argue for new physics
- If these observables don't show NP, I doubt higher K states, etc., could



$$B \rightarrow X_s l^+ l^-$$

Inclusive $b \rightarrow sl^+l^-$ calculations

- Complementary to $B \rightarrow X_s \gamma$
- Subtleties in power counting (as in $K \rightarrow \pi e^+e^-$)

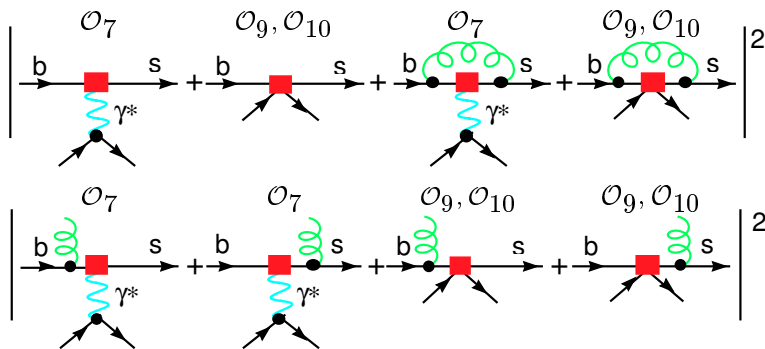
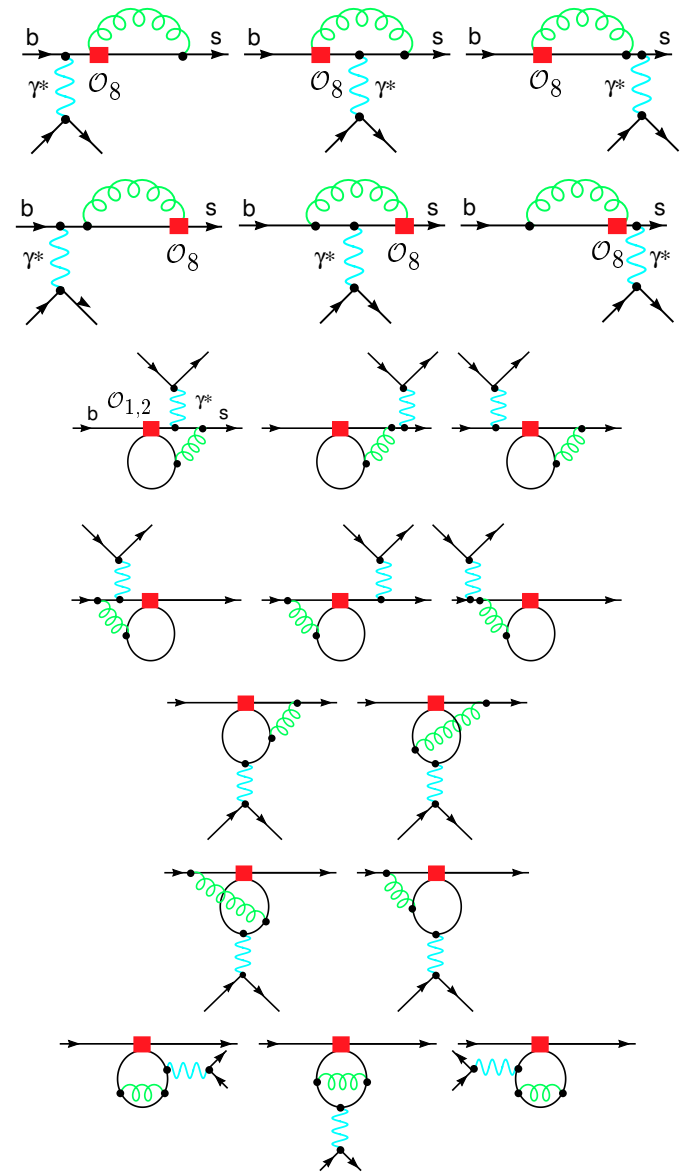
$$C_9(m_b) \sim C_9(m_W) + (\dots) \frac{C_2(m_W)}{\alpha_s(m_W)} \left\{ 1 - \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)} \right]^{(\dots)} \right\}$$

Scale & scheme dependence cancellation tricky

- NNLL: 2-loop matching, 2- and 3-loop running
2-loop matrix elements

$$\mathcal{B}(B \rightarrow X_s l^+ l^-) \Big|_{1 < q^2 < 6 \text{ GeV}^2} = (1.63 \pm 0.20) \times 10^{-6}$$

[Many authors: Bobeth, Misiak, Urban, Munz, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Bieri, Hovhannisyanyan, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]



The q^2 spectrum in $B \rightarrow X_s \ell^+ \ell^-$

- Rate depends (mostly) on

$$O_7 = \bar{m}_b \bar{s} \sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

$$O_9 = e^2 (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell),$$

$$O_{10} = e^2 (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$

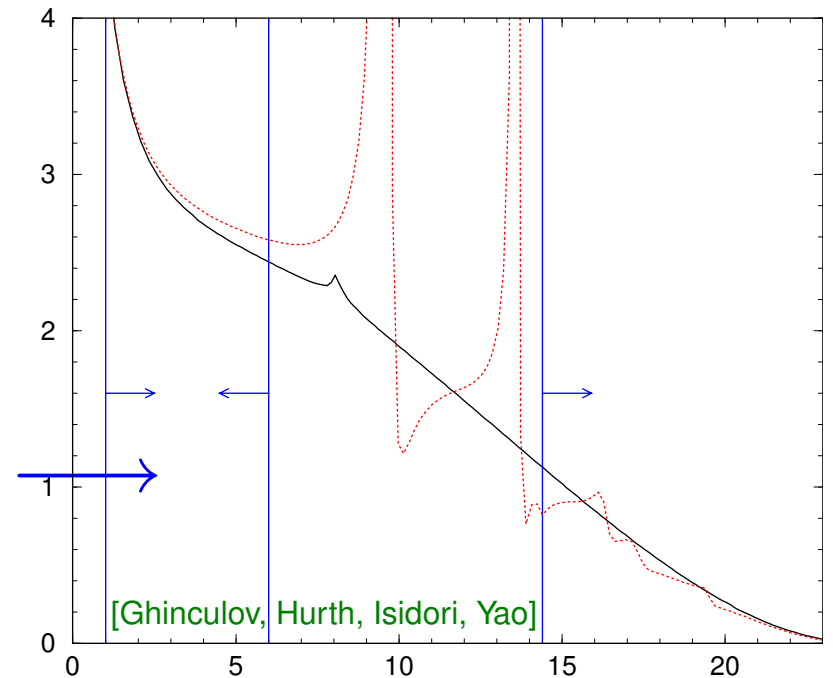
- NNLL $b \rightarrow s \ell^+ \ell^-$ perturbative calculations

Introduce $C_{7,9}^{\text{eff}}$ — complex with usual definition

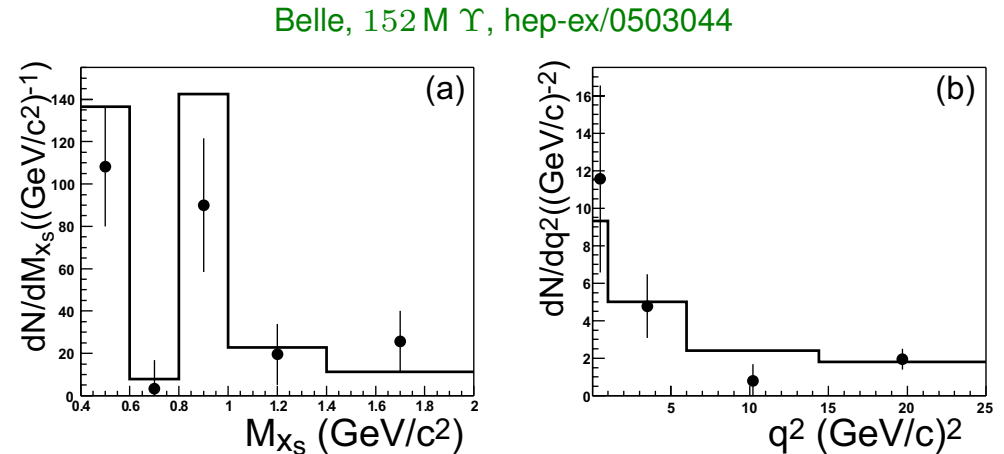
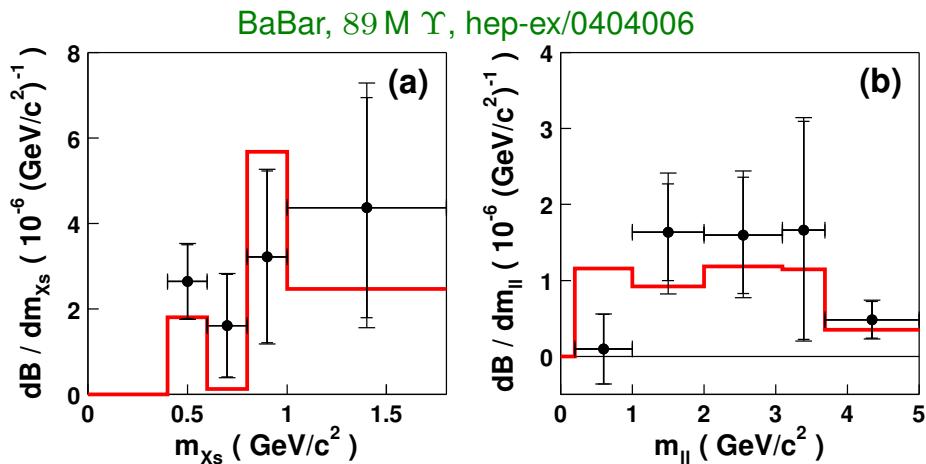
- Nonperturbative corrections $\propto 1/m_{b,c}^2$ [Falk, Luke, Savage; Ali, Hiller, Handoko, Morozumi; Buchalla, Isidori, Rey]

- In small q^2 region experiments need additional $m_{X_s} \lesssim 2 \text{ GeV}$ cut to suppress $b \rightarrow c(\rightarrow s \ell^+ \nu) \ell^- \bar{\nu} \Rightarrow$ additional nonperturbative effects

- Larger (smaller) rate, but more (less) background in the small (large) q^2 region



Inclusive $B \rightarrow X_s \ell^+ \ell^-$: wins in “neglectedness”



- Cut out J/ψ and ψ' regions, and impose an additional cut $m_X < 1.8 \text{ GeV}$ or 2 GeV to suppress large $b \rightarrow c \ell^- \bar{\nu} \rightarrow s \ell^- \ell^+ \nu \bar{\nu}$ background

Current measurements not really inclusive — sum $\sim 50\%$ of exclusive modes

- **World average:** $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-) = (4.5 \pm 1.0) \times 10^{-6}$ (with some black magic)
- **Small q^2 region:** $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{1 < q^2 < 6 \text{ GeV}^2} = (1.60 \pm 0.51) \times 10^{-6}$

- A key measurement that uses only a small fraction of the available data

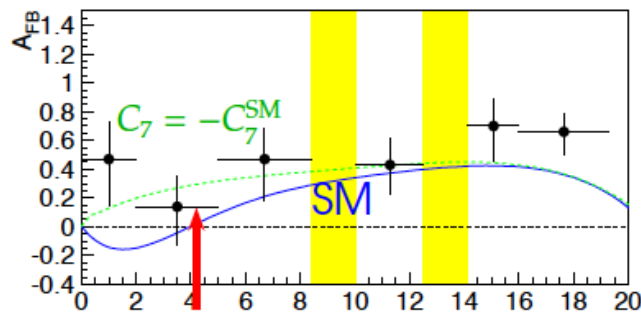


Exclusive $B \rightarrow K^{(*)}\ell^+\ell^-$ measurements

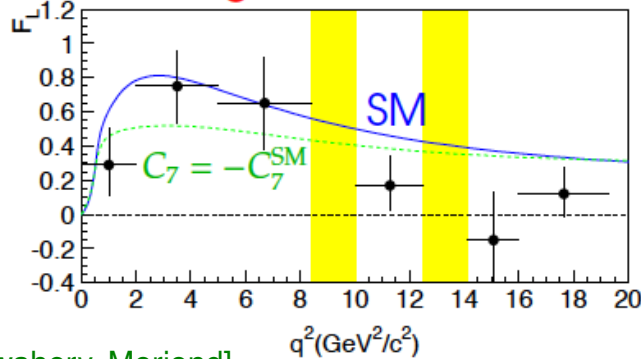
- Interesting recent $B \rightarrow K^*\ell^+\ell^-$ results — may be HINTS

Belle

A_{FB} (Belle arXiv:0810.0335, 657M $B\bar{B}$)



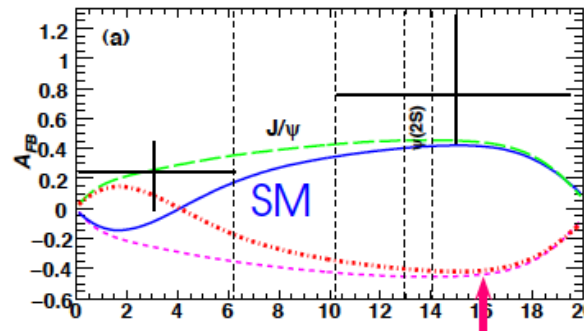
F_L No crossing (Opposite sign C_7)?
Not enough statistics



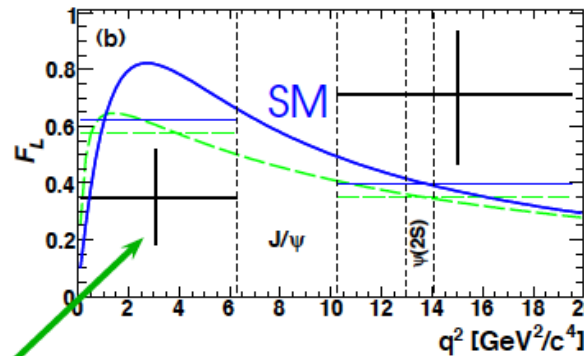
[Jawahery, Moriond]

BaBar

(BaBar arXiv:0804.4412, 384M $B\bar{B}$)



Opposite sign C_9C_{10} is disfavored



Anomaly? not in Belle

World averages:

$$\mathcal{B}(B \rightarrow K\ell^+\ell^-) = (0.43 \pm 0.04) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow K^*\ell^+\ell^-) = (1.00 \pm 0.11) \times 10^{-6}$$

- LHCb expects ($2, 10 \text{ fb}^{-1}$): $\sigma(q_{A_{FB}=0}^2) \approx 0.46, 0.27 \text{ GeV}^2 \Rightarrow \sigma(C_7^{\text{eff}}/C_9^{\text{eff}}) \sim 12, 7\%$



Standard approaches

- Previous analyses concentrated on two observables: $(s = q^2/m_b^2)$

$$\frac{d\Gamma}{ds} \sim \Gamma_0 (1-s)^2 \left[\left(|C_9|^2 + C_{10}^2 \right) (1+2s) + \frac{4}{s} |C_7|^2 (2+s) + 12 \operatorname{Re}(C_7 C_9^*) \right]$$

$$\frac{dA_{\text{FB}}}{ds} \sim -3\Gamma_0 (1-s)^2 s C_{10} \operatorname{Re} \left(C_9 + \frac{2}{s} C_7 \right)$$

$O_{1-6,8}$ contributions absorbed in $C_{7,9} \rightarrow C_{7,9}^{\text{eff}}(s)$, which are complex

- To look for new physics or to extract C_i :
 - Compute rate in SM (or any new physics model) and compare with data (redo for each model, hard to incorporate improvements in theory)
 - Extract C_i from fits to decay distributions (poor sensitivity, needs lots of data) (zero of A_{FB} near $-2C_7/C_9$ argued to be model independent in $B \rightarrow K^* \ell^+ \ell^-$)
- Want most effective ways to extract C_i from simple observables integrated over q^2



Angular decomposition

- Three (not two) terms with different sensitivity to C_i [Lee, ZL, Stewart, Tackmann, hep-ph/0612156]

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \Gamma_0 \left[(1+z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2) \right] \quad (s = q^2/m_b^2, z = \cos \theta)$$

$$H_T \sim 2(1-s)^2 s \left[\left(C_9 + \frac{2}{s} C_7 \right)^2 + C_{10}^2 \right] \quad [\Gamma = H_T + H_L]$$

$$H_L \sim (1-s)^2 \left[(C_9 + 2C_7)^2 + C_{10}^2 \right] \quad [\text{no } C_7/s \text{ pole}]$$

$$H_A \sim -4(1-s)^2 s C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \quad [H_A \equiv (4/3) A_{\text{FB}}]$$

θ : angle between \vec{p}_{ℓ^+} and $\vec{p}_{\bar{B}^0, B^-}$ [\vec{p}_{ℓ^-} and \vec{p}_{B^0, B^+}] in $\ell^+\ell^-$ center of mass frame

- Dependence on C_i : H_L is q^2 independent; $H_{T,A}$'s sensitivity to C_i depends on q^2
- Same structure for $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow K^* \ell^+ \ell^-$ — different at $\mathcal{O}(\alpha_s, 1/m_{c,b})$
 $B \rightarrow K^* \ell^+ \ell^-$: Two further angles (even more if ℓ^\pm polarizations considered)
- Three terms sensitive to different combinations of Wilson coefficients

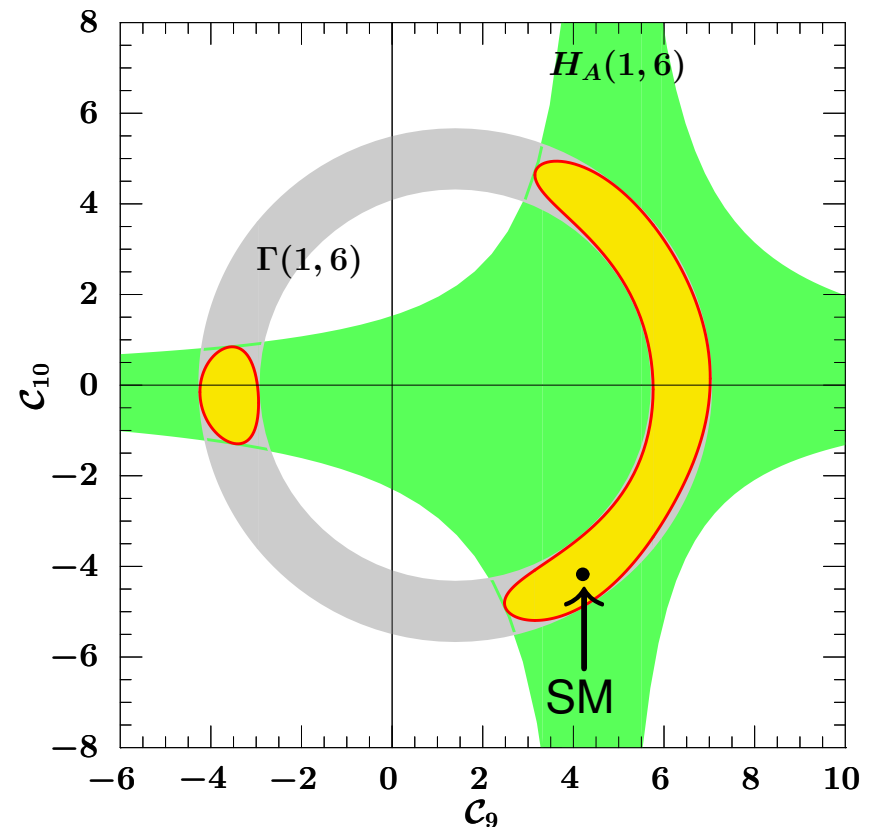


An illustrative toy analysis

- Inclusive, with guesstimated error for 1 ab^{-1}

Define: $H_i(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} dq^2 H_i(q^2)$

- Small q^2 -dependence

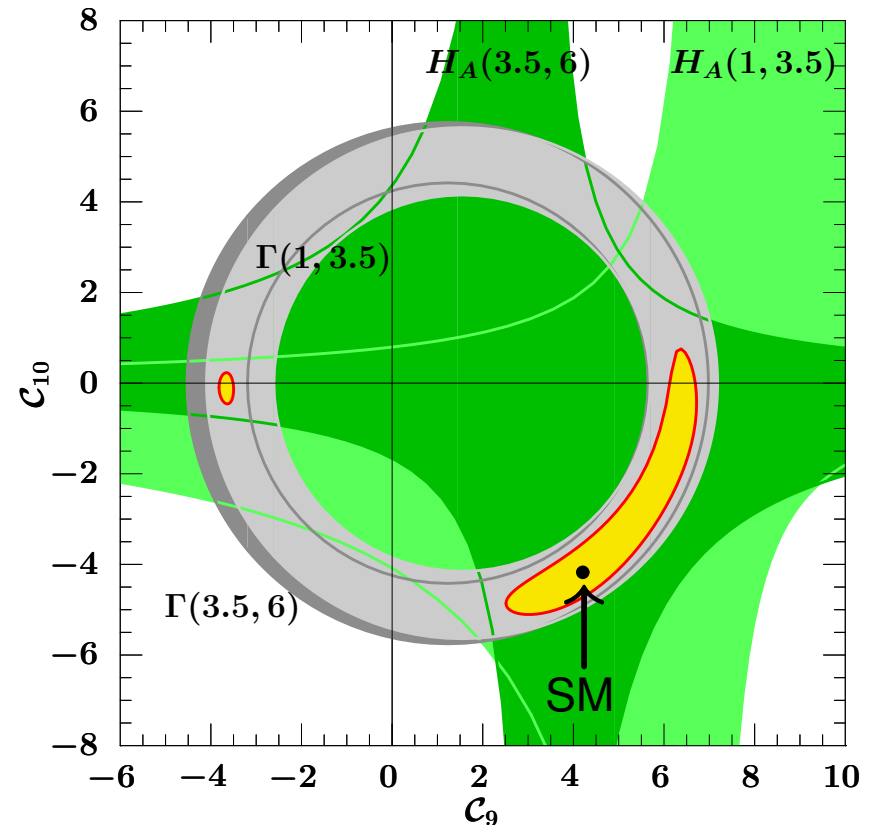


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- Small q^2 -dependence \Rightarrow splitting Γ in two regions not useful (splitting $H_A \equiv A_{\text{FB}}$ is!)

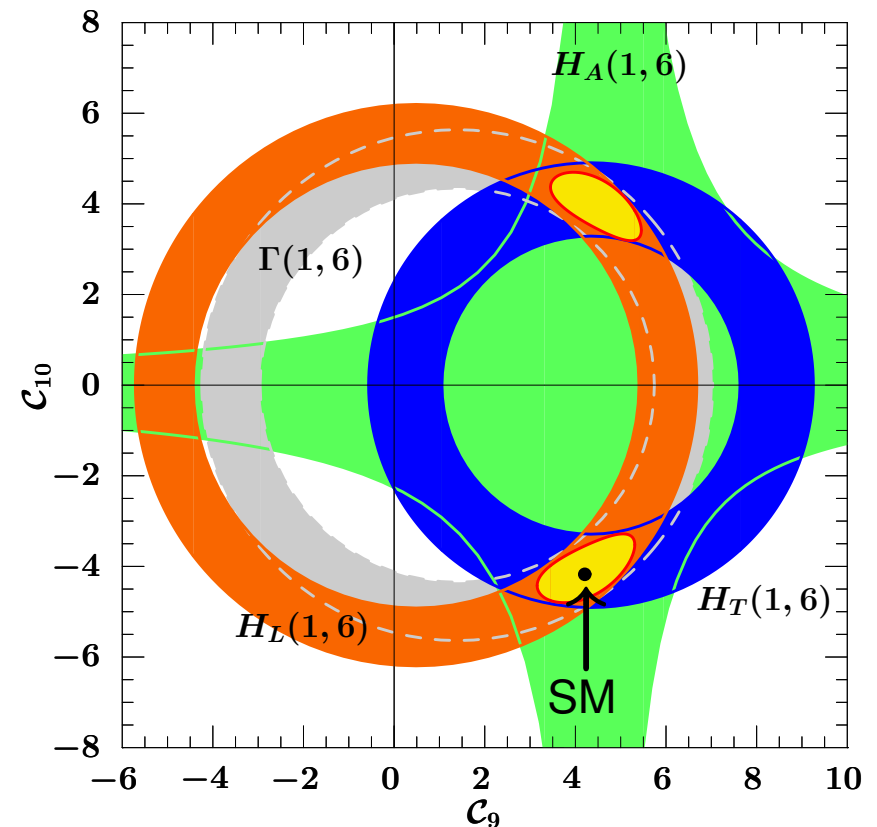


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- $H_L \propto q^2$ -independent combination of C_i 's
 \Rightarrow integrate over as large region as possible



- Separating H_T and H_L is very powerful

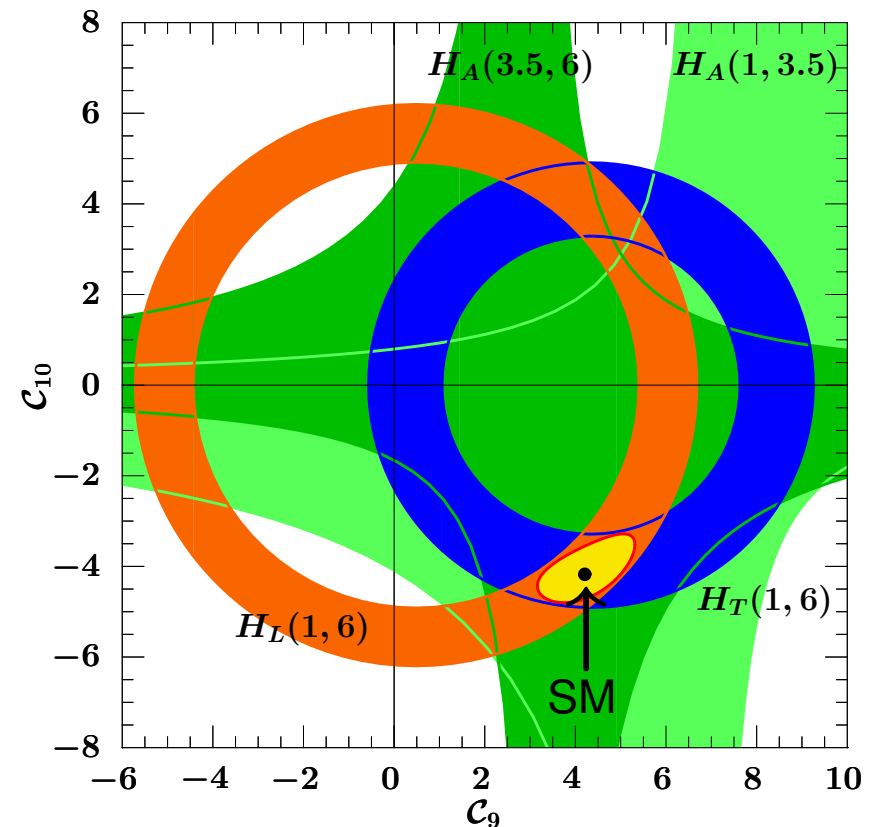


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- $H_L \propto q^2$ -independent combination of \mathcal{C}_i 's
 \Rightarrow integrate over as large region as possible
- H_T and H_A : different q^2 regions sensitive to different combinations of \mathcal{C}_i 's
 Separating $H_A(1, 3.5)$ vs $H_A(3.5, 6)$ and/or $H_T(1, 3.5)$ vs $H_T(3.5, 6)$ appears promising
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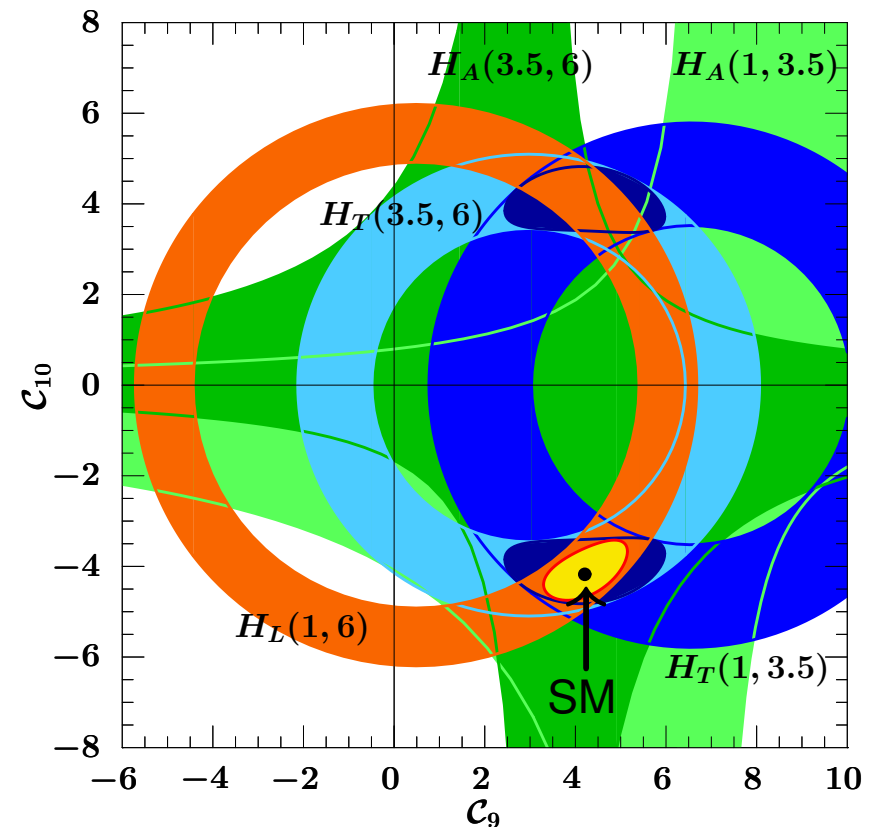
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- Separating H_T and H_L is very powerful
- Can extract all information from a few integrated rates



Effects of m_X^{cut} at small q^2

Details: K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0512191

$B \rightarrow X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two independent kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

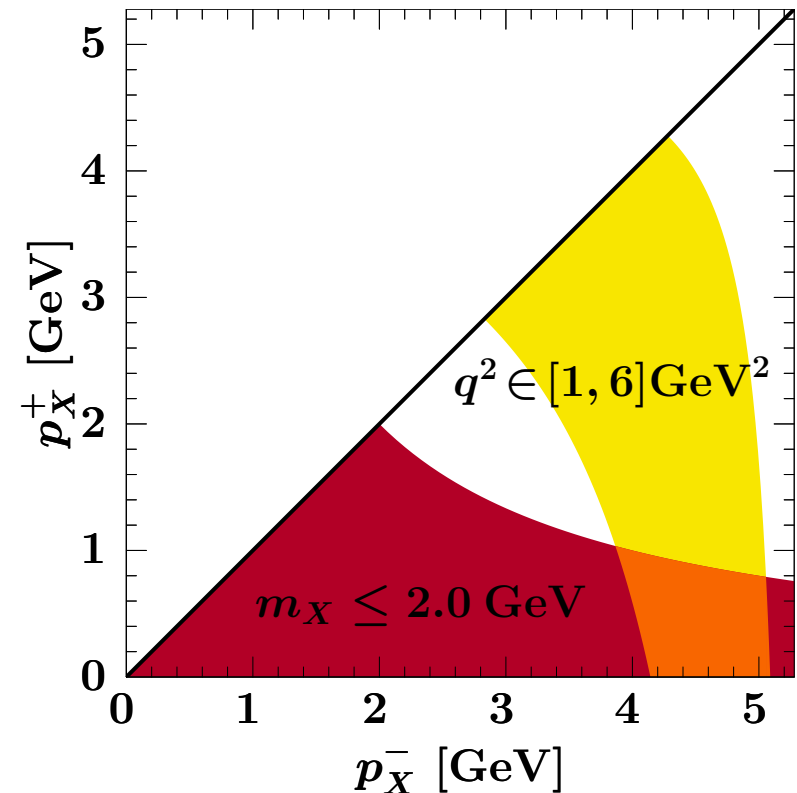
q^2 not large & $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

- Jet-like hadronic final state, $p_X^+ \ll p_X^-$

$$p_X^+ = E_X - |\vec{p}_X| = \mathcal{O}(\Lambda_{\text{QCD}})$$

$$p_X^- = E_X + |\vec{p}_X| = \mathcal{O}(m_B)$$

- Nonperturbative physics is important
- Described by same shape function as spectra in $B \rightarrow X_s \gamma$, $X_u \ell \bar{\nu}$; use to reduce uncertainties



Effects of m_X cut at lowest order

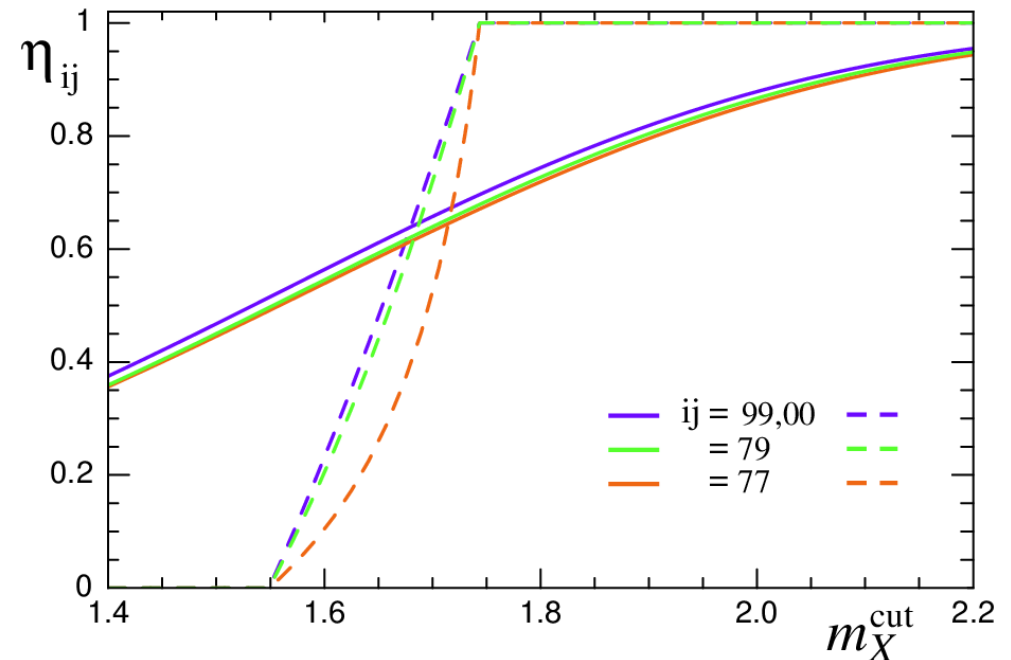
- Define:

$$\eta_{ij} = \frac{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \int_0^{m_X^{\text{cut}}} dm_X^2 \frac{d\Gamma_{ij}}{dq^2 dm_X^2}}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\Gamma_{ij}}{dq^2}}$$

ij : C_9^2 and C_{10}^2 , $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- η_{ij} determine fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\text{cut}} = 1.8 \text{ GeV}$ may be hadronic physics, not new physics

Experiments use Fermi-motion model to incorporate m_X^{cut} effect [Earlier work: Ali & Hiller, '98]



Effects of m_X cut at lowest order

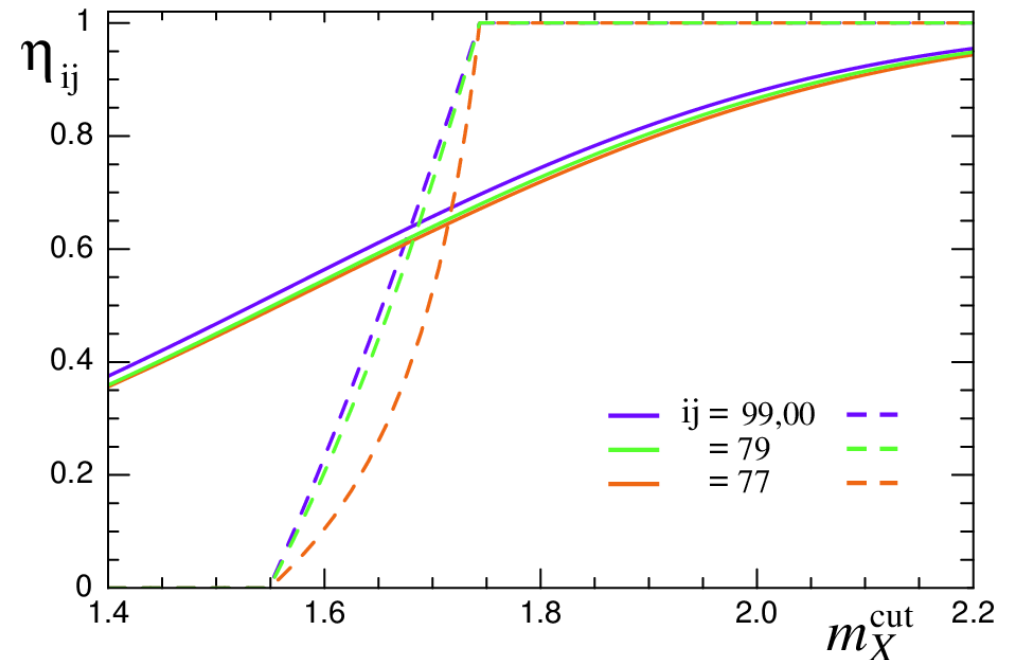
- Define:

$$\eta_{ij} = \frac{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \int_0^{m_X^{\text{cut}}} dm_X^2 \frac{d\Gamma_{ij}}{dq^2 dm_X^2}}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\Gamma_{ij}}{dq^2}}$$

ij : C_9^2 and C_{10}^2 , $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- Strong m_X^{cut} dependence: Raising it (if possible) would reduce uncertainty

If $1 - \eta$ is sizable, so is its uncertainty

- Approximate universality of η_{ij} : since shape function varies on scale $p_X^+/\Lambda_{\text{QCD}}$, while $\Gamma_{ij}^{\text{parton}}$ varies on scale $p_X^+/m_b \Rightarrow \eta \approx \eta_{ij}$



Including NLL corrections

- Universality maintained; estimate shape function uncertainties using $B \rightarrow X_s \gamma$

- Find for $\mathcal{B}(1 < q^2 < 6 \text{ GeV}^2)/10^{-6}$

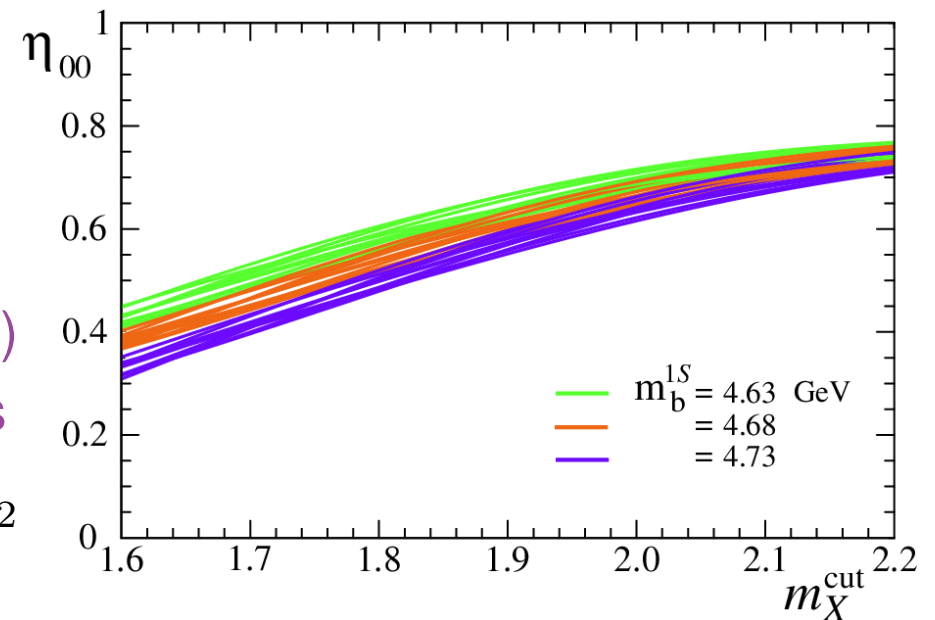
$$m_X^{\text{cut}} = 1.8 \text{ GeV}: 1.20 \pm 0.15$$

$$m_X^{\text{cut}} = 2.0 \text{ GeV}: 1.48 \pm 0.14$$

$$\text{NNLL, no } m_X \text{ cut}: 1.57 \pm 0.11$$

- A_{FB} only slightly affected (a-priori nontrivial)
Find $q_0^2 \sim 3 \text{ GeV}^2$, lower than earlier results

- NNLL reduces μ dependence, effect on q^2 spectrum small \Rightarrow expect $\eta^{(\text{NLL})} \approx \eta^{(\text{NNLL})}$



- If increasing m_X^{cut} above 2 GeV hard \Rightarrow keep $m_X^{\text{cut}} < m_D$, normalize to $B \rightarrow X_u \ell \bar{\nu}$ with same cuts:

$$R = \Gamma^{\text{cut}}(B \rightarrow X_s \ell^+ \ell^-) / \Gamma^{\text{cut}}(B \rightarrow X_u \ell \bar{\nu})$$

Both shape function (m_X^{cut}) and m_b dependence drastically reduced

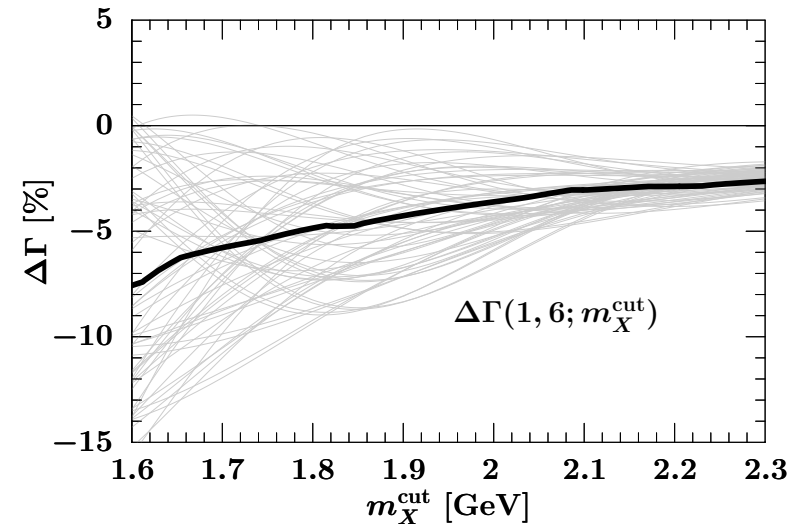


Subleading shape functions

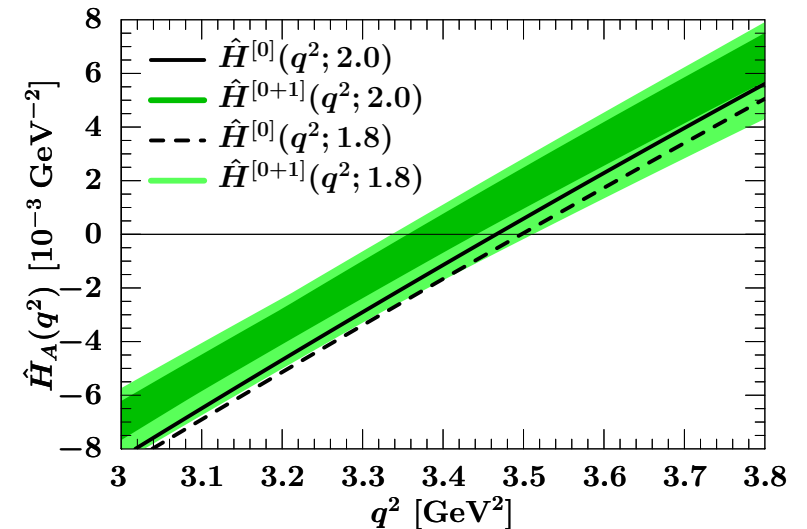
- Rate for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$: uncertainty increases with decreasing m_X^{cut}

Same holds for H_L, H_T, H_A components individually as well

- Forward-backward asymmetry: shows that the location where $A_{\text{FB}}(q_0^2) = 0$ is not really special
- Uncertainty of q_0^2 similar to the perturbative one
Not obvious that the zero of A_{FB} has advantage
- There are power corrections to $B \rightarrow K^* \ell^+ \ell^-$ form factor relations relevant to determine q_0^2



[Lee & Tackmann, 0812.0001]



Large q^2 region ($q^2 > m_{\psi'}^2$)

Details: ZL & F. Tackmann, 0707.1694

Large q^2 region: complementary with small q^2

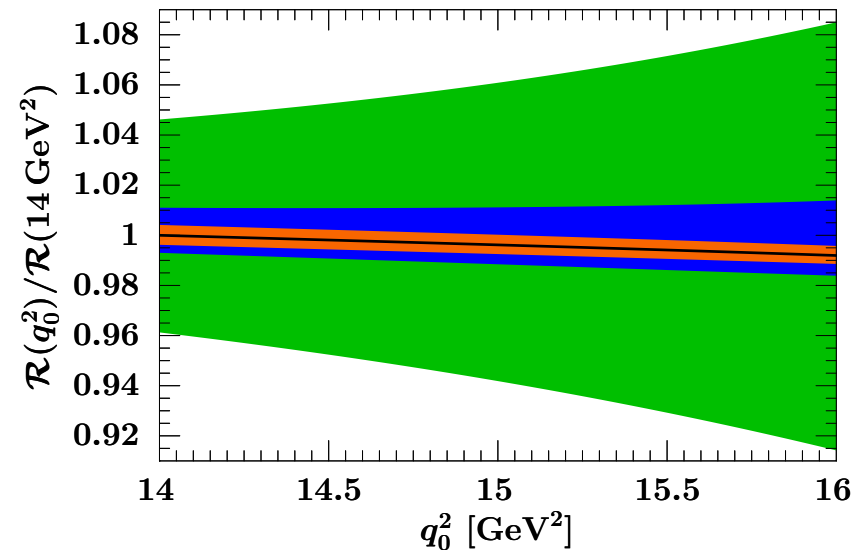
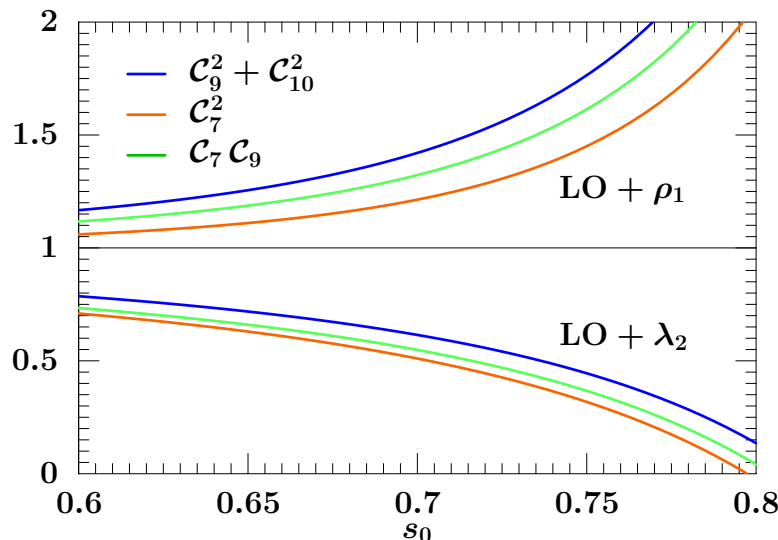
- Theory: largest errors (i) expansion in $\Lambda_{\text{QCD}}/(m_b - \sqrt{q^2})$; (ii) huge m_b dependence
Experiment: smaller rate, but higher efficiency

- Both can be reduced / eliminated \Rightarrow uncertainty $\sim 5\%$ (missing NNLL at large q^2)

$$\frac{\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{dq^2} dq^2}{\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B^0 \rightarrow X_u \ell \bar{\nu})}{dq^2} dq^2} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2} \frac{\alpha_{\text{em}}^2}{8\pi^2} \mathcal{R}(q_0^2)$$

uncertainties suppressed by:

$$1 - \frac{(C_9 + 2C_7)^2 + C_{10}^2}{C_9^2 + C_{10}^2} \simeq 0.12$$



Large q^2 region measured in $B \rightarrow X_u \ell \bar{\nu}$

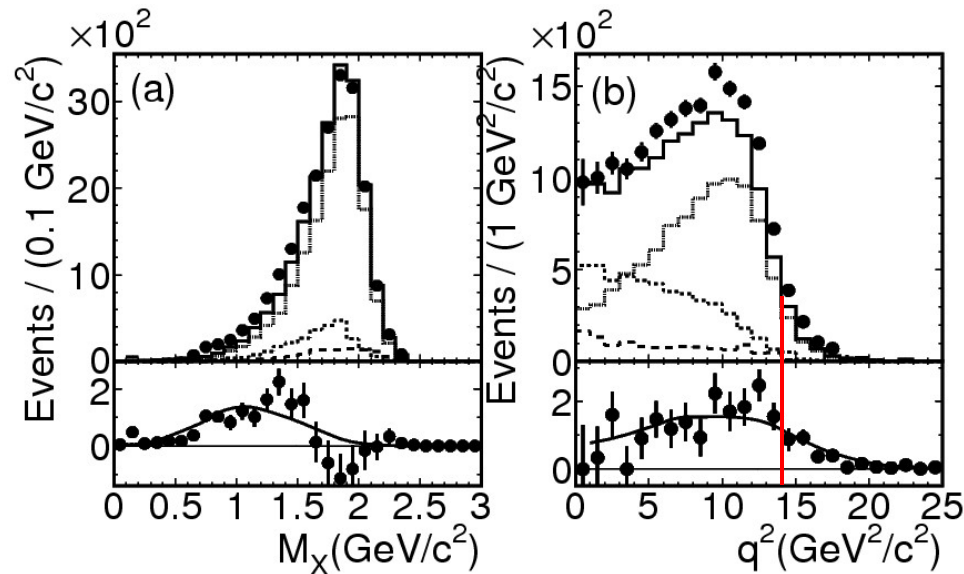
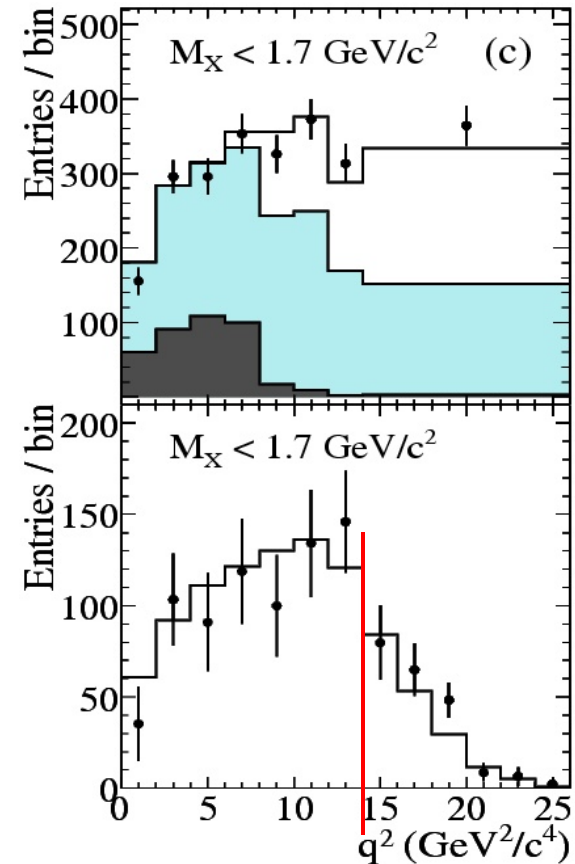


FIG. 4: (a) M_X distribution for $q^2 > 8.0 \text{ GeV}^2/c^2$. (b) q^2 distribution for $M_X < 1.7 \text{ GeV}/c^2$. Points are the data and histograms are backgrounds from $D^* \ell \nu$ (dotted), $D \ell \nu$ (short dashed), others (long dashed), and total background contribution (solid). Lower plots show the data after background subtraction. Solid curves show the inclusive MC predictions for $B \rightarrow X_u \ell \bar{\nu}$.

Belle, 87 fb^{-1} , PRL **92** (2004) 101801 [hep-ex/0311048]



BaBar, $383 \text{ m } \Upsilon$, arXiv:0708.3702

- The $m_X > 1.7 \text{ GeV}$ cut is irrelevant for $q^2 > 12.8 \text{ GeV}^2$ (up to resolution effects)
- Separating B^0 vs. B^\pm can control 4-quark operator contributions (weak annihil.)



$$B \rightarrow X_s \nu \bar{\nu}$$

Theoretically cleanest $b \rightarrow s$ decays

- Noticed that ALEPH $B \rightarrow X_c \tau \nu$ search via large E_{miss} also bounds $B \rightarrow X_s \nu \bar{\nu}$
[Grossman, ZL, Nardi, hep-ph/9510378]
Subsequent ALEPH bound $\mathcal{B}(B \rightarrow X_s \nu \bar{\nu}) < 6.4 \times 10^{-4}$ is the best to date
- Can also bound $B_{(s)} \rightarrow \tau^+ \tau^- (X)$ at few % level
[Grossman, ZL, Nardi, hep-ph/9607473]
BaBar established: $\mathcal{B}(B \rightarrow \tau^+ \tau^-) < 4.1 \times 10^{-3}$
- Models with unrelated couplings in each channel, e.g., SUSY without R-parity¹
Models with enhanced 3332 generation couplings: $B \rightarrow X_s \nu \bar{\nu}$, $X_s \tau \tau$, $B_s \rightarrow \tau \tau$
- Even in 2020, we'll have (exp. bound)/(SM prediction) $\gtrsim 10^3$ in some channels
E.g.: $B_{(s)} \rightarrow \tau^+ \tau^-$, $B_{(s)} \rightarrow e^+ e^-$, maybe more...

¹“Can do everything except make coffee” — Babar Physics Book



Experimental possibilities

- $B \rightarrow K\nu\bar{\nu}$: Existing studies suggest that even at Super- B only this mode is measurable with decent $\sim 20\%$ precision

Only $\sim 10\%$ of the inclusive rate; expected rate from lattice QCD (recoil range?)

- $B \rightarrow K^*\nu\bar{\nu}$: can use “Grinstein-type double ratio”, only few % uncertainty

$$\frac{B \rightarrow K^*\nu\bar{\nu}}{B \rightarrow \rho\ell\bar{\nu}} \times \frac{D \rightarrow \rho\ell\bar{\nu}}{D \rightarrow K^*\ell\bar{\nu}} = 1 + \mathcal{O}\left(\frac{m_s}{\Lambda_{\text{QCD}}} \times \frac{\Lambda_{\text{QCD}}}{m_{c,b}}\right)$$

[ZL, Wise, hep-ph/9512225]

- **Inclusive:** A careful study seems warranted; very precise theory predictions for $\mathcal{B}(B \rightarrow X_s\nu\bar{\nu})/\mathcal{B}(B \rightarrow X_u\ell\bar{\nu})$ or $\mathcal{B}(B \rightarrow X_s\nu\bar{\nu})/\mathcal{B}(B \rightarrow X_s\ell^+\ell^-)$ (in not too small parts of phase space)



Conclusions

Looking for unknown unknowns*

- Will NP be seen in the quark sector?

B : Semileptonic $|V_{ub}|$ and $B \rightarrow \tau\nu$ agree, in conflict with $\sin 2\phi_1$?

D : CPV in $D^0-\bar{D}^0$ mixing?

B_s : large β_s or $B_s \rightarrow \mu^+\mu^-$?

- Will NP be seen in the lepton sector?

$\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow \mu\mu\mu$, ...?

- Will LHC see new particles beyond a Higgs?

SUSY, something else, understand in detail?

- I don't know, but I'm sure it's worth finding out...!

*unknown unknowns:

“There are known knowns. There are things we know that we know.

There are known unknowns. That is to say, there are things that we now know we don't know.

But there are also unknown unknowns. There are things we do not know we don't know.”

[Rumsfeld, DOD briefing, Feb 12, 2002]



Conclusions

- Consistency of precision flavor measurements with SM is a problem for NP @ TeV
- Inclusive decays will remain important (theoretical cleanliness)
- Both in the large- q^2 and in small- q^2 regions, combined analysis with $B \rightarrow X_u \ell \bar{\nu}$ and $B \rightarrow X_s \gamma$ will give best sensitivity (smallest hadronic uncertainty)
- To achieve maximal sensitivity to NP in $B \rightarrow X_s \ell^+ \ell^-$, separate rate not only to $d\Gamma/dq^2$ and A_{FB} , but terms proportional to $1 + \cos^2 \theta$, $1 - \cos^2 \theta$, $\cos \theta$
- Few integrated rates may give as good info as fit to 2-d distribution & zero of A_{FB}
Sensitivity to NP survives both in small- and large- q^2 regions ($\sim 5\%$ uncertainties)
- Many important modes to probe new FCNC from TeV scale are only doable in e^+e^- machine: final states with τ 's and ν 's, B reconstruction ability, hermeticity





Backup slides

$B \rightarrow X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

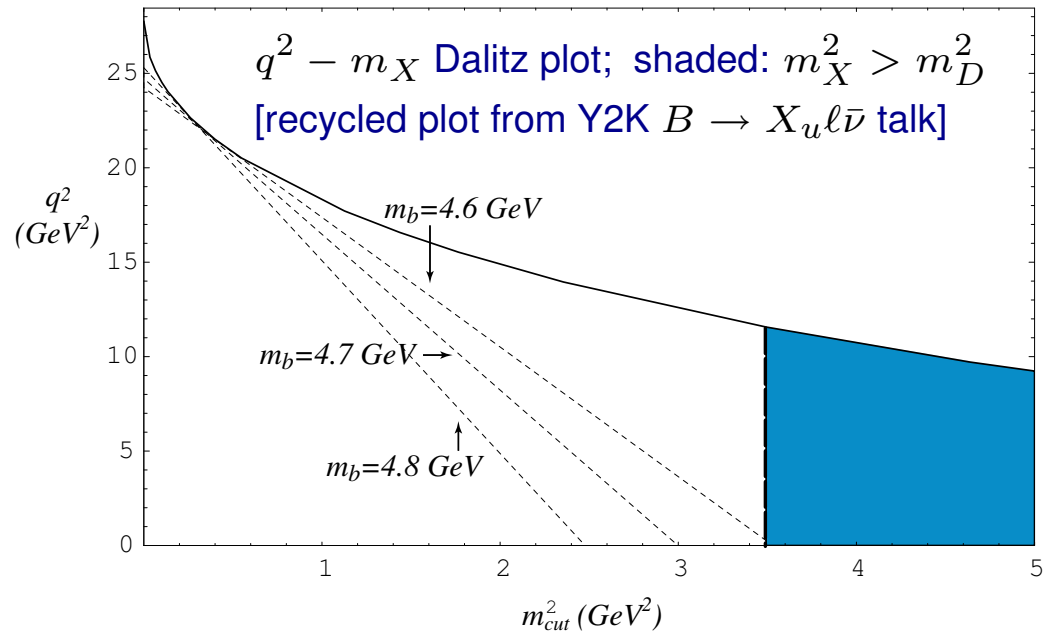
$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\text{QCD}}) \quad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \quad n, \bar{n} = (1, \pm \vec{p}_X / |\vec{p}_X|)$$

- $p_X^+ \ll p_X^-$: jet-like hadronic final state

- Parton level: $\Gamma \propto f(q^2) \delta[(m_b v - q)^2]$
 $m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$

rate vanishes left of the dashed lines

- Nonperturbative physics is important
 Same shape fn as in $B \rightarrow X_s \gamma, X_u \ell \bar{\nu}$



$B \rightarrow X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

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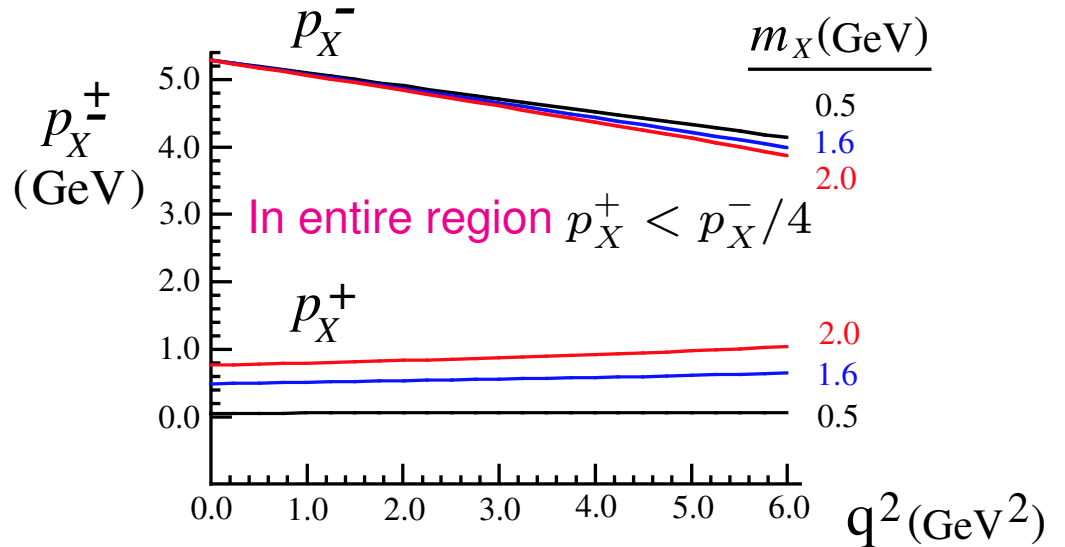
$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\text{QCD}}) \quad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \quad n, \bar{n} = (1, \pm \vec{p}_X / |\vec{p}_X|)$$

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 $m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$

rate vanishes left of the dashed lines

- Nonperturbative physics is important
 Same shape fn as in $B \rightarrow X_s \gamma, X_u \ell \bar{\nu}$



Including higher order corrections

- Introduce a scheme to separate terms sensitive to new physics from four-quark operator contributions (for which the SM is assumed)
- Define $C_{7,9}$ as μ - and q^2 -independent constants, **real** in the SM

$$C_{7,9}^{\text{incl}}(q^2) = C_{7,9} + \underbrace{F_{7,9}(q^2)}_{\alpha_s} + \underbrace{G_{7,9}(q^2)}_{1/m_c^2} \quad (F_{7,9} \text{ include NNLL})$$

- Use m_b^{1S} to improve perturbation series; do not normalize to $\Gamma(B \rightarrow X \ell \bar{\nu})$

Keep $\bar{m}_b(\mu)C_7(\mu)$ together and unexpanded — no reason to expand $\bar{m}_b(\mu)$

- Numerically small Λ^2/m_c^2 correction can be simply included:

$$G_9(q^2) = \frac{10}{1-2s} G_7(q^2) = -\frac{5}{6} \frac{\lambda_2}{m_c^2} C_2 \frac{\mathcal{F}[q^2/(4m_c^2)]}{1 - q^2/(4m_c^2)}$$

Blows up as $(4m_c^2 - q^2)^{-1/2}$ as $q^2 \rightarrow 4m_c^2$; assume OK for $q^2 \lesssim 3m_c^2 \sim 6 \text{ GeV}^2$



Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s , treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$
OK in local OPE region: include small nonpert. corrections ($\lambda_{1,2}$, etc.) at the end
 - Shape function region: only the rate is calculable, $\Gamma \sim \text{Im} \langle B|T\{O_i^\dagger(x)O_j(0)\}|B\rangle$
 $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ “enhancement”, but $|C_9(m_b)| \sim C_{10}$
 - Need to take it seriously to cancel scheme- and scale-dependence in running
 - Don’t want power counting: $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2) \sim \langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level
-
- “Split matching” in SCET: separate μ -dependence in matrix element which cancels that in $m_{\text{weak}} \rightarrow m_b$ running from dependencies on scales $\mu_i \sim \sqrt{m_b \Lambda_{\text{QCD}}}$ and $\mu_\Lambda \sim 1 \text{ GeV}$ — can work to different orders



Aside: long distance effects

- A worry (at least, for me) that will be ignored in this talk:

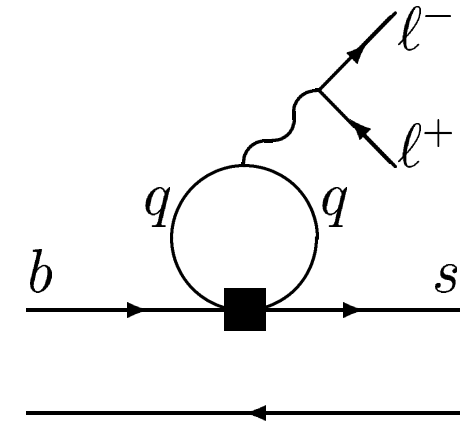
$$\mathcal{B}(B \rightarrow \psi X_s) \sim 4 \times 10^{-3}$$

↓

$$\mathcal{B}(\psi \rightarrow \ell^+ \ell^-) \sim 6 \times 10^{-2}$$

Combined rate: $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-) \sim 2 \times 10^{-4}$

This is ~ 30 times the short distance contribution!



- Averaged over a large region of q^2 , the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \dots$. This is what happens in $e^+e^- \rightarrow$ hadrons, in τ decay, etc., but NOT here
- Is it consistent to “cut out” the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)



Is $C_7(m_b) = -C_7^{\text{SM}}(m_b)$ excluded?

- Inclusive:** rate in small q^2 region, in units of 10^{-6} (world average: 1.60 ± 0.51)

[Gambino, Haisch, Misiak, hep-ph/0410155]

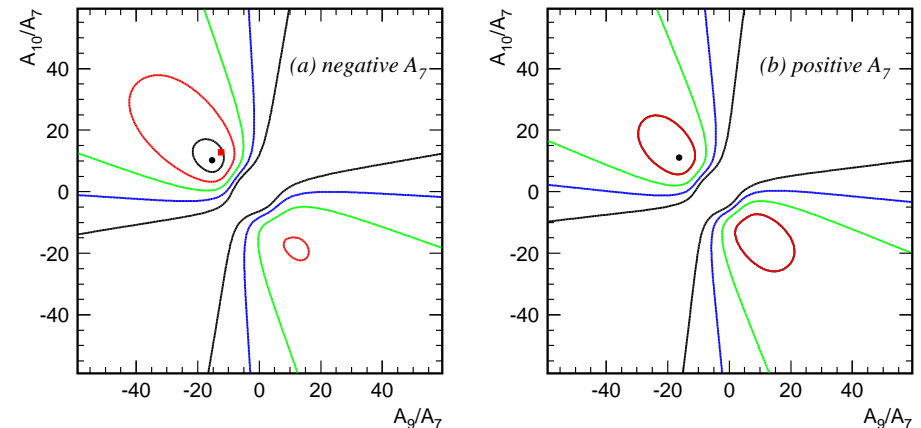
TABLE II: Predictions for $B(\bar{B} \rightarrow X_s l^+ l^-)$ [10^{-6}] in the Standard Model and with reversed sign of \tilde{C}_7^{eff} for the same ranges of q^2 as in Tab. I.

Range	SM	$\tilde{C}_7^{\text{eff}} \rightarrow -\tilde{C}_7^{\text{eff}}$
(a)	4.4 ± 0.7	8.8 ± 1.0
(b)	1.57 ± 0.16	3.30 ± 0.25

$\tilde{C}_7^{\text{eff}} \rightarrow -\tilde{C}_7^{\text{eff}}$ is not the best way to proceed

“Preliminary”	m_X^{cut}	rate (C_7^{SM})	rate ($C_7^{\text{non-SM}}$)
NNLL “GHM”	—	1.57	3.18
NNLL “us”	—	1.57	2.99
NLL	—	1.74	3.61
NLL	2.0 GeV	1.35	3.09
NLL	1.8 GeV	1.10	2.49

- Exclusive:** with some model dependence, Belle’s A_{FB} measurement fixes sign of C_9/C_{10} , but not sign of C_7 relative to $C_{9,10}$



- I also think $C_7 > 0$ is unlikely, but probably disfavored only about the 2σ level



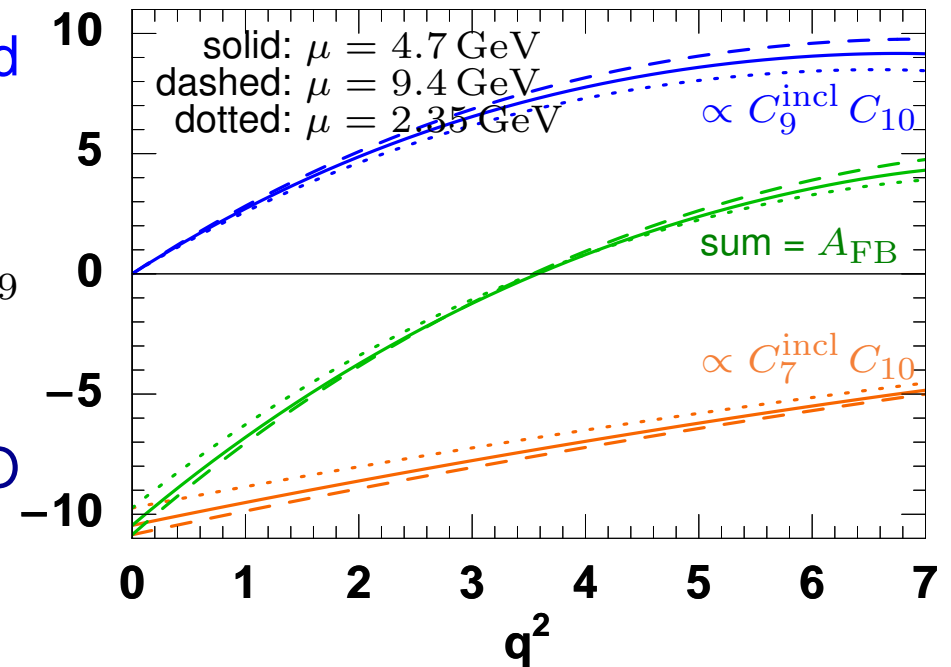
The μ dependence of A_{FB}

- Zero of A_{FB} , $A_{\text{FB}}(q_0^2) = 0$ sometimes said to be particularly clean in inclusive as well
- μ -dep. smaller than for rate, linear in C_7, C_9 (C_{10} is μ independent, rate is quadratic)

Cancellations reduce μ -dep of zero @NLO

Some terms tend to cancel even at NNLO

- Uncertainty of q_0^2 not relevant; the physical question is sensitivity to C_7/C_9 , for which it's not obvious that the zero of A_{FB} has an advantage
- Whether uncertainty from q_0^2 is parametrically reduced in $B \rightarrow K^* \ell^+ \ell^-$ depends on relative size of factorizable / nonfactorizable contributions to form factors



Exclusive $B \rightarrow K^* \ell^+ \ell^-$ with SCET

- Angular decomposition involves: $\zeta_{\parallel, \perp}(s)$ and $\zeta_{\parallel, \perp}^J(s) \sim$ (non-)factorizable parts

$$H_T \sim 2s\lambda^3 \left\{ C_{10}^2 [\zeta_{\perp}(s)]^2 + \left| C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right|^2 \right\}$$

$$H_A \sim -4s\lambda^3 C_{10} \zeta_{\perp}(s) \operatorname{Re} \left\{ C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right\}$$

$$H_L \sim \frac{1}{2} \lambda^3 \left(C_{10}^2 + \left| C_9 + 2C_7 \frac{m_b}{m_B} \right|^2 \right) [\zeta_{\parallel}(s) - \zeta_{\parallel}^J(s)]^2 \quad (\lambda = \sqrt{(1-s)^2 - 2\rho(1+s) + \rho^2})$$

- Form factors: reduce to a few numbers using asymptotic dependence

$$\zeta_{\perp}^{(J)}(s) = \frac{\zeta_{\perp}^{(J)}(0)}{(1-s)^2} \left[1 + \mathcal{O}\left(\alpha_s, \frac{\Lambda}{E}\right) \right] \quad (1.9 < E < 2.7 \text{ GeV})$$

- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B \rightarrow K^* \ell^+ \ell^-)}$ & $B \rightarrow K \ell^+ \ell^-$



Exclusive $B \rightarrow K^* \ell^+ \ell^-$ with SCET

- Angular decomposition involves: $\zeta_{\parallel, \perp}(s)$ and $\zeta_{\parallel, \perp}^J(s) \sim$ (non-)factorizable parts

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$$H_A \sim -4s\lambda^3 C_{10} \zeta_{\perp}(s) \operatorname{Re} \left\{ C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right\}$$

$$H_L \sim \frac{1}{2} \lambda^3 \left(C_{10}^2 + \left| C_9 + 2C_7 \frac{m_b}{m_B} \right|^2 \right) [\zeta_{\parallel}(s) - \zeta_{\parallel}^J(s)]^2 \quad (\lambda = \sqrt{(1-s)^2 - 2\rho(1+s) + \rho^2})$$

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- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B \rightarrow K^* \ell^+ \ell^-)}$ & $B \rightarrow K \ell^+ \ell^-$

$$\Gamma(B \rightarrow K^* \gamma) = \frac{G_F^2}{8\pi^3} \frac{\alpha_{\text{em}}}{4\pi} |V_{tb} V_{ts}^*|^2 m_B^3 (m_b^{1S})^2 (1-\rho)^3 |C_7(0)|^2 [\zeta_{\perp}(0) + \zeta_{\perp}^J(0)]^2$$



Exclusive $B \rightarrow K^* \ell^+ \ell^-$ with SCET

- Angular decomposition involves: $\zeta_{\parallel, \perp}(s)$ and $\zeta_{\parallel, \perp}^J(s) \sim$ (non-)factorizable parts

$$H_T \sim 2s\lambda^3 \left\{ C_{10}^2 [\zeta_{\perp}(s)]^2 + \left| C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right|^2 \right\}$$

$$H_A \sim -4s\lambda^3 C_{10} \zeta_{\perp}(s) \operatorname{Re} \left\{ C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right\}$$

$$H_L \sim \frac{1}{2} \lambda^3 \left(C_{10}^2 + \left| C_9 + 2C_7 \frac{m_b}{m_B} \right|^2 \right) [\zeta_{\parallel}(s) - \zeta_{\parallel}^J(s)]^2 \quad (\lambda = \sqrt{(1-s)^2 - 2\rho(1+s) + \rho^2})$$

- Form factors: reduce to a few numbers using asymptotic dependence

$$\zeta_{\perp}^{(J)}(s) = \frac{\zeta_{\perp}^{(J)}(0)}{(1-s)^2} \left[1 + \mathcal{O}\left(\alpha_s, \frac{\Lambda}{E}\right) \right] \quad (1.9 < E < 2.7 \text{ GeV})$$

- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B \rightarrow K^* \ell^+ \ell^-)}$ & $B \rightarrow K \ell^+ \ell^-$

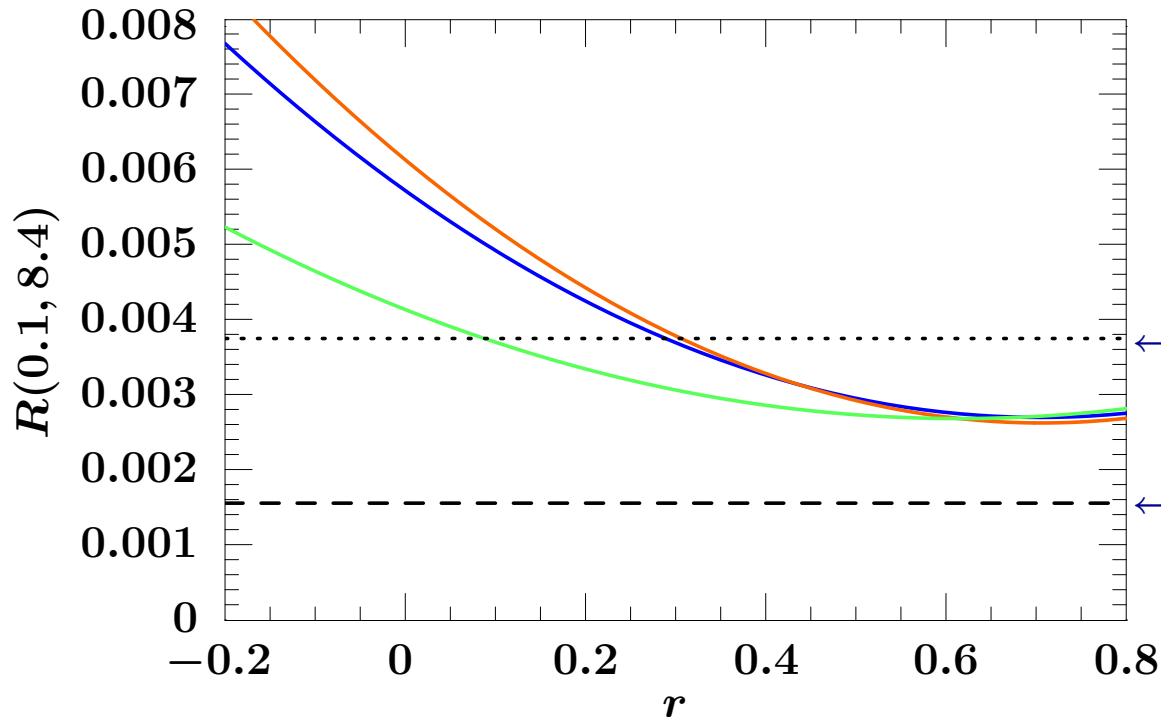
- Three ratios of: $\Gamma(B \rightarrow K^* \gamma)$, $H_T(0, 8)$, $H_A(0, 4)$, $H_A(4, 8)$

Determine: C_{10}/C_7 , C_9/C_7 , and hadronic parameter $\zeta_{\perp}^J(0)/[\zeta_{\perp}(0) + \zeta_{\perp}^J(0)]$



Constraining hadronic physics

$$R(q_1^2, q_2^2) \equiv \frac{H_T(q_1^2, q_2^2)}{\Gamma(B \rightarrow K^* \gamma)} = \frac{\alpha_{\text{em}} m_B^2}{12\pi m_b^2} \int_{q_1^2/m_B^2}^{q_2^2/m_B^2} ds \frac{\lambda^3 s}{(1-\rho)^3 (1-s)^4} \times \left\{ \frac{C_{10}^2}{C_7^2} (1-r)^2 + \left[\frac{C_9}{C_7} (1-r) + \frac{2 m_b}{s m_B} (1-sr) \right]^2 \right\}$$



Plot $R(0.1, 8.41)$ and BaBar data

$$r \equiv \frac{\zeta_{\perp}^J(0)}{\zeta_{\perp}(0) + \zeta_{\perp}^J(0)}$$

← 1σ upper bound

← central value

Too early to tell...

- BaBar & Belle have already a lot more data (expect / predict H_T to increase)

