$|V_{ub}|$ and $|V_{cb}|$: theoretical developments

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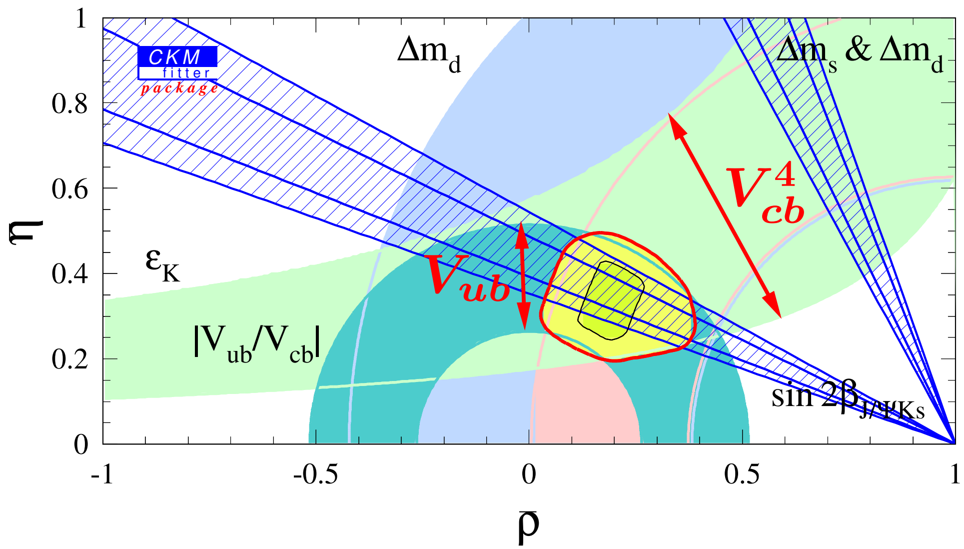
- Introduction
- $|V_{cb}|$ — exclusive, inclusive
- $|V_{ub}|$ — exclusive, inclusive
- Conclusions

Why care about $|V_{ub}|$ and $|V_{cb}|$?

$|V_{ub}|$: dominant uncertainty of the side opposite to $\beta \equiv \phi_1$

$|V_{cb}|$: large part of the uncertainty in ϵ_K constraint, and in $K \to \pi \nu \bar{\nu}$ in the future

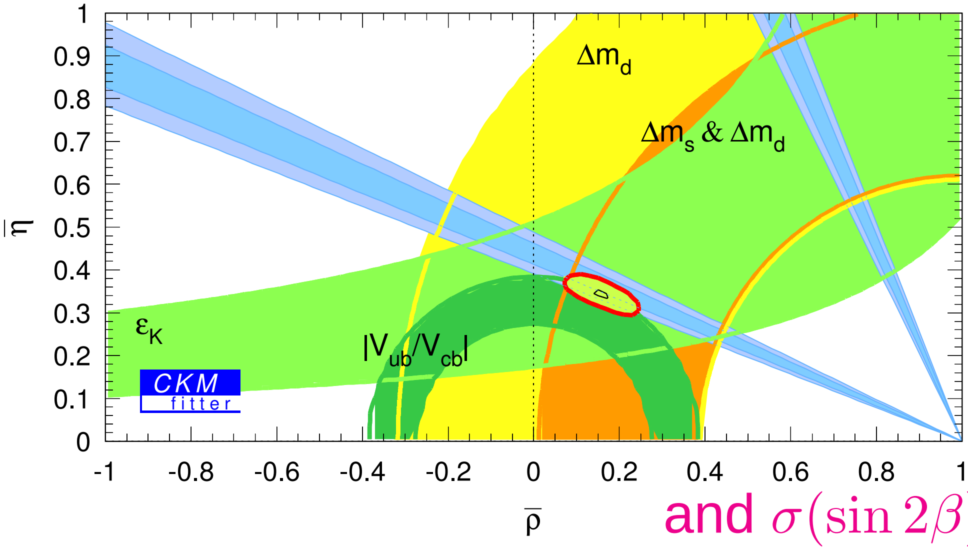


Look for New Physics: compare (i) angles with sides; (ii) tree and loop processes ... semileptonic decays crucial for this

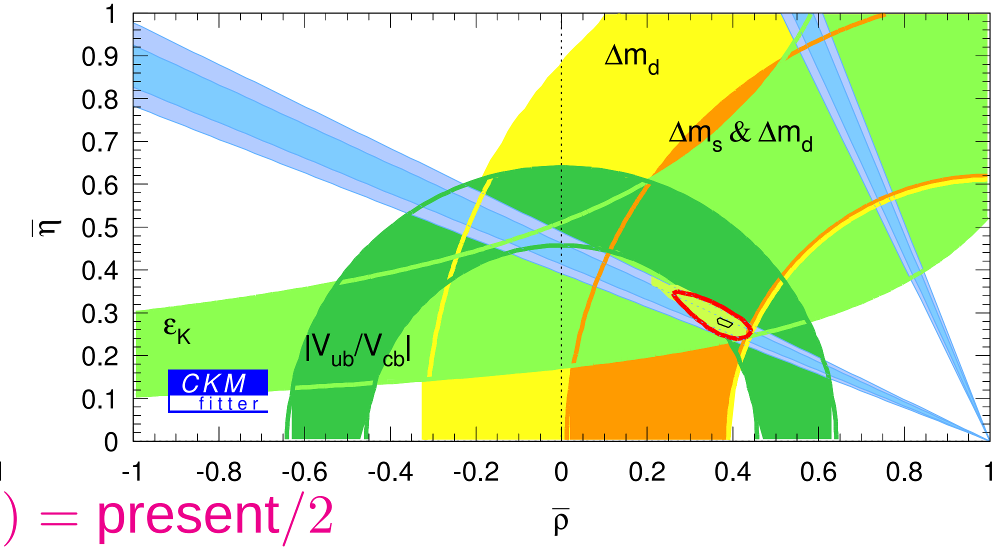
$b \to q\gamma$, $b \to q\,\ell^+\ell^-$, and $b \to q\,\nu\bar{\nu}$ ($q = s, d$) are sensitive probes of the SM theoretical tools same as for $|V_{xb}|$ — accuracy of theory limits sensitivity to NP

Some “extreme” scenarios for $|V_{ub}|$

$|V_{ub}| = (3.0 \pm 0.15 \pm 0.15) \times 10^{-3}$



$|V_{ub}| = (5.0 \pm 0.25 \pm 0.25) \times 10^{-3}$



and $\sigma(\sin 2\beta) = \text{present}/2$

(Not realistic, by this time B_s mixing should be measured)

Recent incl. [excl.] measurements of $|V_{ub}|$ high [low], overlap smaller than before

Both fits less good than with average $|V_{ub}|$

Central values: difference of γ above $25°$; require Δm_s near min / max

$\Rightarrow$ Must aim at $\sigma(|V_{ub}|) \sim 5\%$

Hadronic uncertainties

- To believe that a small discrepancy is due to new physics, need model independent predictions

Define: [strong interaction] model independent $\equiv$ theoretical uncertainty suppressed by small parameters
... so theorists argue about (small parameters)$\times\mathcal{O}(1)$ instead of $\mathcal{O}(1)$ effects

Most of the recent progress comes from expanding in Λ/m_Q and $\alpha_s(m_Q)$
... a priori not known whether $\Lambda \sim 200$MeV or ~ 2GeV ($f_\pi, m_\rho, m_K^2/m_s$)
... need experimental guidance to see which cases work how well

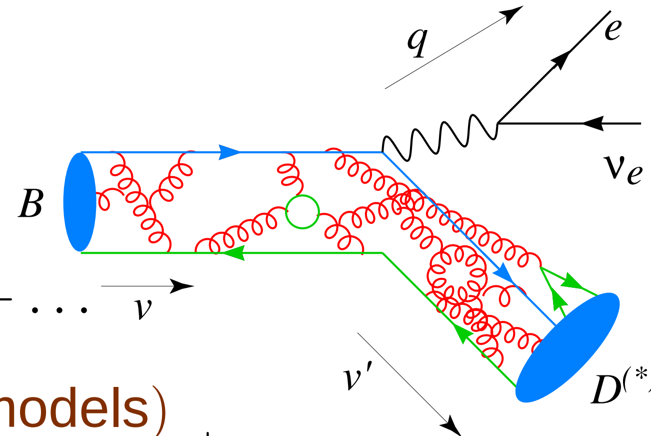
$|V_{cb}|$ — exclusive

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar{\nu}$

- Heavy Quark Symmetry: brown muck only feels $v \to v'$ (not $m_b \to m_c$ or $\vec{s}_b \to \vec{s}_c$)

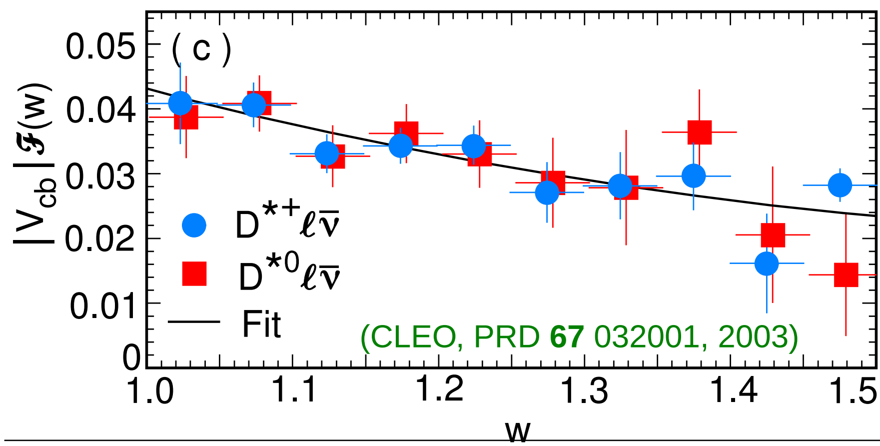
$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar{\nu})}{\mathrm{d}w} = (\dots)\,(w^2-1)^{3(1)/2}\; |V_{cb}|^2\, \mathcal{F}^2_{(*)}(w)$$

$w \equiv v \cdot v'$ Isgur-Wise function $+ \dots$



$$\mathcal{F}(1) = 1_{\rm Isgur-Wise} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = 1_{\rm Isgur-Wise} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\rm Luke}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$



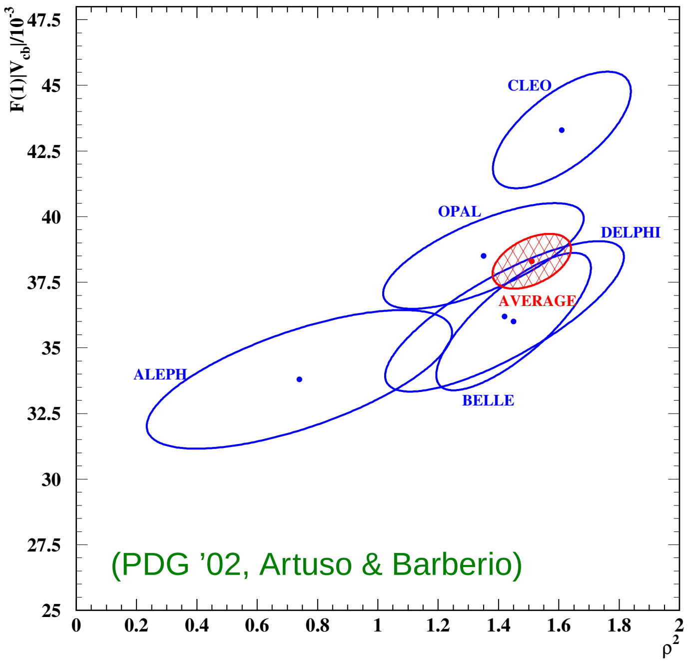
Experiments measure: $|V_{cb}| \times \mathcal{F}_*(w)$

Theory issues: (i) $\mathcal{F}_*(1)$, (ii) shape

Theory predicts: $\mathcal{F}_*(1) = 0.91 \pm 0.04$

[$1 - \mathcal{F}_*(1)$: lattice, sum rules, models]

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar{\nu}$ (cont.)



$|V_{cb}|$ sensitive to shape of $\mathcal{F}_*(w)$: fits use analyticity constraint (slope vs. curvature at $w=1$)

(Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert)

$\Rightarrow |V_{cb}| = (42.1 \pm 1.1_{\rm exp} \pm 1.9_{\rm th}) \times 10^{-3}$ (hep-ph/0304132)

... HQS relates $B \to D$ and D^* shapes (Grinstein, ZL)

... Sum rule relations to $B \to D^{**}\ell\bar{\nu}$

- New bounds on derivatives of Isgur-Wise function (Le Yaouanc, Oliver, Raynal, PLB **557** 207, 2003)

$$(-1)^n\,\xi^{(n)}(1) \geq \frac{2n+1}{4}\left[(-1)^{n-1}\,\xi^{(n-1)}(1)\right] \quad \Rightarrow \quad (-1)^n\,\xi^{(n)}(1) \geq \frac{(2n+1)!!}{2^{2n}}$$

- Questions: (i) how to best use constraints on shape?

  (ii) if $0^+, 1^+$ D states were $\sim 2.22, 2.36$ GeV with $\Gamma \sim 300$ MeV, could it affect $|V_{cb}|$?

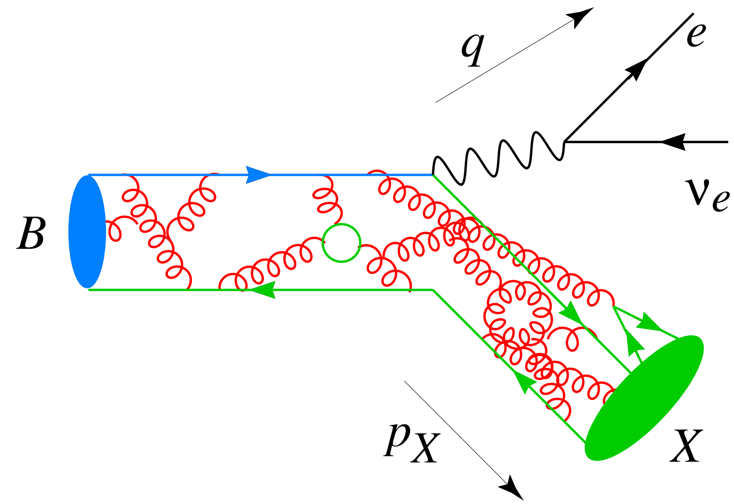
$|V_{cb}|$ — inclusive

Why inclusive decays?

- Sum over hadronic final states, subject to constraints determined by short distance physics

  Decay: short distance (calculable)

  Hadronization: long distance (nonperturbative), but probability to hadronize somehow is unity



- Rates calculable in an OPE, expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$:

$$\mathrm{d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

In "most" of phase space, details of b quark wavefunction unimportant, only averages matter: $\lambda_1 \sim \langle k^2 \rangle$ not well-known, $\lambda_2 \sim \langle \sigma_{\mu\nu} G^{\mu\nu} \rangle = (m_{B^*}^2 - m_B^2)/4$, ...

Interesting quantities computed to order α_s, $\alpha_s^2\beta_0$, and $1/m^3$

Issues relevant for $B \to X_c \ell \bar\nu$

- Total semileptonic rate precisely calculable:

$$|V_{cb}| \sim \left[42 \pm (\text{error mostly in } m_b \;\&\; \lambda_1)\right] \times 10^{-3} \left(\frac{\mathcal{B}(B \to X_c \ell \bar\nu)}{0.105} \, \frac{1.6\,\mathrm{ps}}{\tau_B} \right)^{1/2}$$

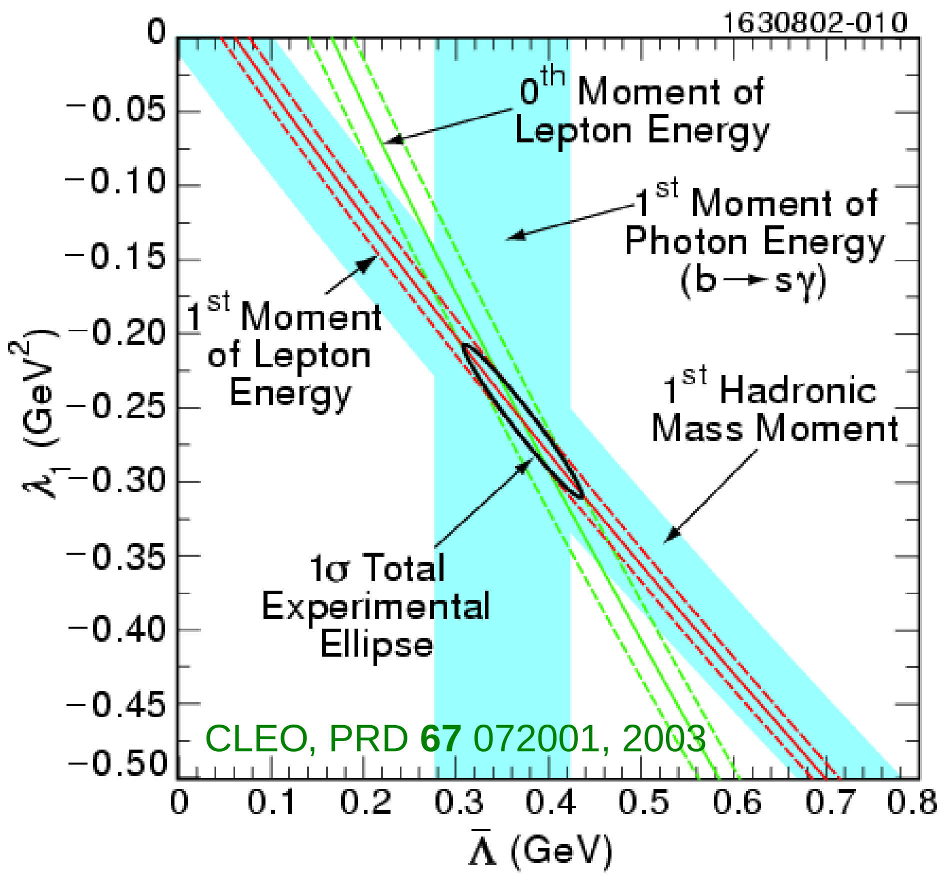
  - Values of m_b and λ_1?
  - Four more nonperturbative parameters at $\mathcal{O}(\Lambda^3_{\mathrm{QCD}}/m_b^3)$
  - Theoretical uncertainties (perturbation theory, masses)
  - In restricted regions, OPE can break down (especially relevant for $|V_{ub}|$)
  - Implicit assumption: quark-hadron duality

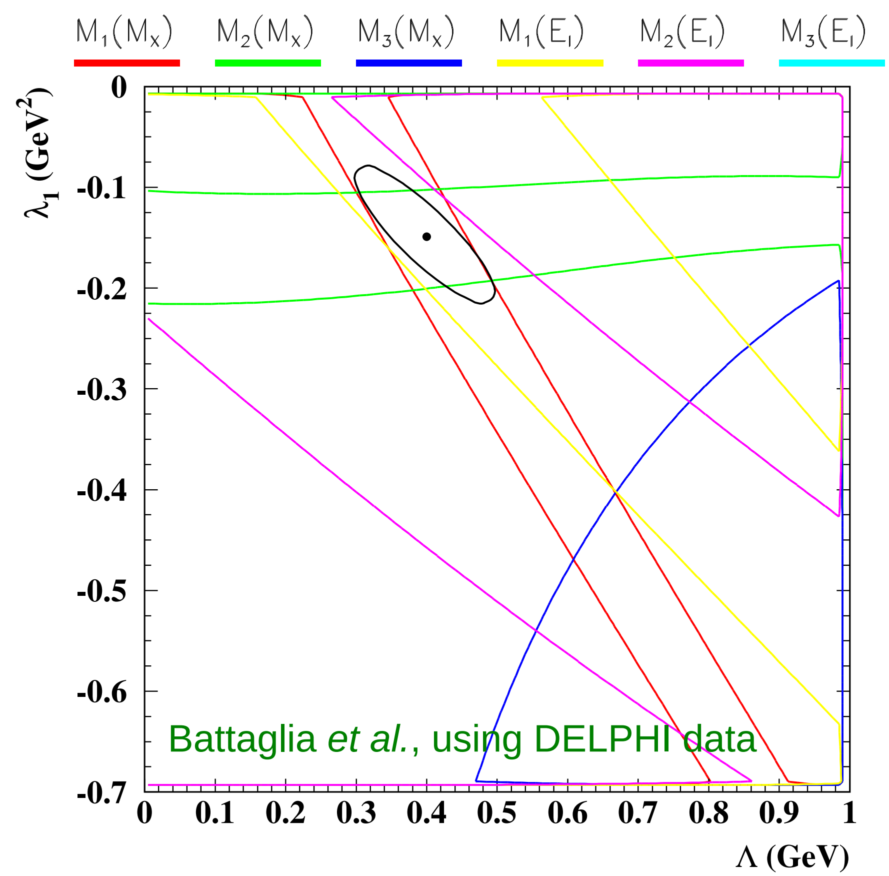
- Address these and determine unknown param's and $|V_{cb}|$ from shape variables:

"Moments:" $$\langle X \rangle = \langle X \rangle_{\mathrm{parton}} + \frac{0}{m_b} F_\Lambda + \frac{\lambda_i}{m_b^2} F_{\lambda_i} + \frac{\rho_i}{m_b^3} F_{\rho_i} + \dots$$

$\langle X \rangle_{\mathrm{parton}}$ and each F_i has an expansion in α_s and depends on m_c/m_b

Many shape variables measured...

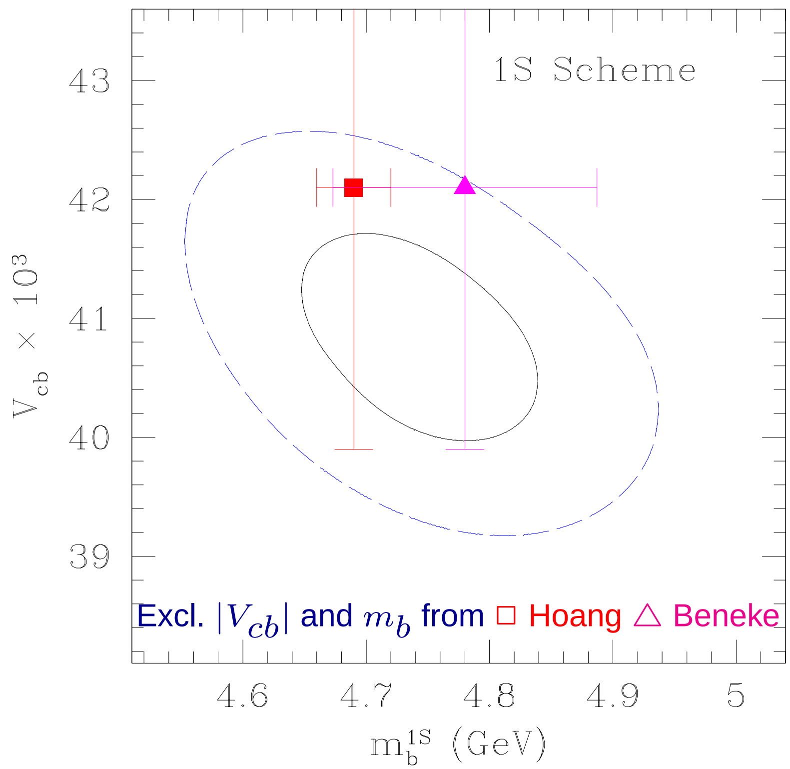




They allow: (i) precision extractions of m_b and HQET matrix elements

(ii) testing validity of the whole approach

Global fit as of Fall '02



Results: (Bauer, ZL, Luke, Manohar, PRD **67** 054012, 2003)

$$|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$$

$$m_b^{1S} = (4.74 \pm 0.10)\ \text{GeV}$$

$$\overline{m}_b(\overline{m}_b) = (4.22 \pm 0.09)\ \text{GeV}$$

Similar fits: (Battaglia *et al.*, PLB **556** 41, 2003)

$$|V_{cb}| = (41.9 \pm 1.1) \times 10^{-3}$$

$$m_b(1\ \text{GeV}) = (4.59 \pm 0.08)\ \text{GeV}$$

$$\Rightarrow\ m_b^{1S} \simeq 4.69\ \text{GeV}$$

Theoretical uncertainties dominate $\Rightarrow$ their correlations are essential when many observables determine hadronic parameters and $|V_{cb}|$

Theoretical limitations: setting all experimental errors to zero, we would obtain

$$\sigma(|V_{cb}|) = 0.35 \times 10^{-3} \qquad \sigma(m_b^{1S}) = 35\ \text{MeV}$$

Bauer-Trott moments

- Constructed to suppress (enhance) sensitivity to certain matrix elements

(fractional moments of E_ℓ spectrum)

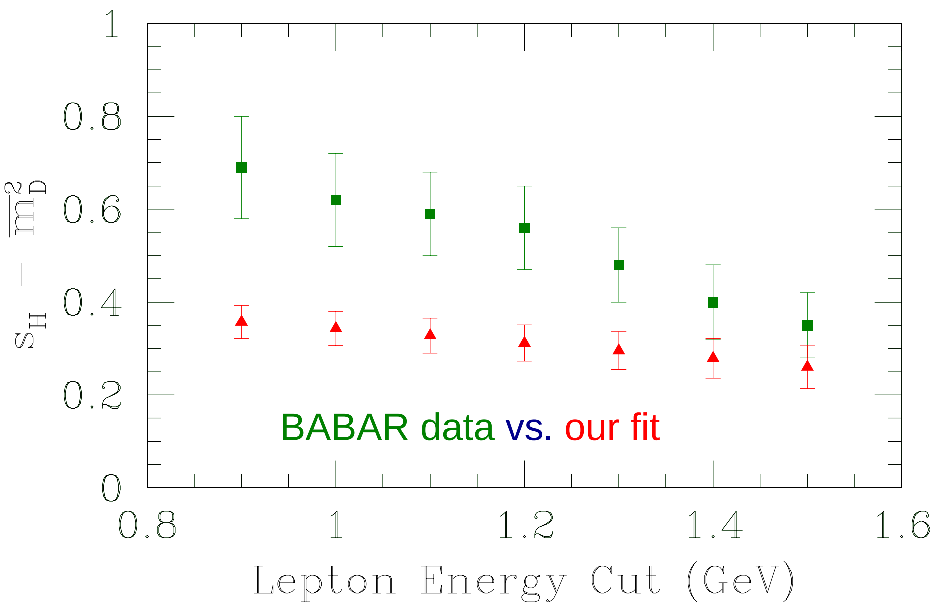
R_{3a}	R_{3b}	R_{4a}	R_{4b}	D_3	D_4
0.302 ± 0.003	2.261 ± 0.013	2.127 ± 0.013	0.684 ± 0.002	0.520 ± 0.002	0.604 ± 0.002
above was our prediction, below is CLEO measurement					
0.3016 ± 0.0007	2.2621 ± 0.0031	2.1285 ± 0.0030	0.6833 ± 0.0008	0.5193 ± 0.0008	0.6036 ± 0.0006

Data and theory beautifully consistent (for $E_\ell \geq 1.5$ GeV)

NB: excited D states make small contribution in this region

Two possible caveats and the D_{sJ}^*

Hadronic moments for $E_\ell < 1.5\,$GeV



Difference seems significant

– Eliminate implicit model dependence in measurements

"Gremm-Kapustin puzzle" ('97)

If no X_c between D^* and $D_1(2420)$...

$\langle m_X^2 \rangle$ implies $\leq 25\%$ excited charm in $B \to X_c \ell \bar\nu$ decay, while:

$\mathcal{B}(B \to X_c \ell \bar\nu) - \mathcal{B}(B \to D^{(*)} \ell \bar\nu) \sim 35\%$

$\Rightarrow$ assumption / theory / data wrong?

May be a disappearing problem

– BELLE: 0^+ D_0^* at $2290\,$MeV, well below predictions (ICHEP'02)

– BABAR's $D_{sJ}^*(2317)$: corresponding non-strange D should be < 2290

$\Rightarrow$ Precise $D_{u,d,s}$ spectroscopy crucial

Summary for $|V_{cb}|$

- Current precision is already at the $4-5\%$ level
- Limiting theory errors — inclusive: m_b and matrix elements
  exclusive: $\mathcal{F}_{(*)}(1)$ and shape
- "Duality" hard to quantify — cross-checks are important
- Inclusive and exclusive determinations both important
- If all caveats resolved, $\sigma(|V_{cb}|)$ may be reduced to $1-2\%$ level

Possible improvements:

– better consistenty and precision of shape variables ($B \to X_c \ell \bar{\nu}$ and $X_s \gamma$)

– full α_s^2 calculation of spectra (surprises unlikely)

– better understanding of $B \to D^{(*)} \ell \bar{\nu}$ shapes; unquenched lattice form factors

$|V_{ub}|$ — exclusive

Exclusive $b \to u$ decays

- Less constraints from heavy quark symmetry than in $b \to c$

$\Rightarrow$ $B \to \ell\bar{\nu}$ measures $f_B \times |V_{ub}|$ — need to rely on lattice f_B

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$\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ (Grinstein, '93)

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$$\frac{B \to \rho\ell\bar\nu}{B \to K^*\ell^+\ell^-} \times \frac{D \to K^*\ell\bar\nu}{D \to \rho\ell\bar\nu}$$ — accessible soon? (ZL & Wise, ’96)

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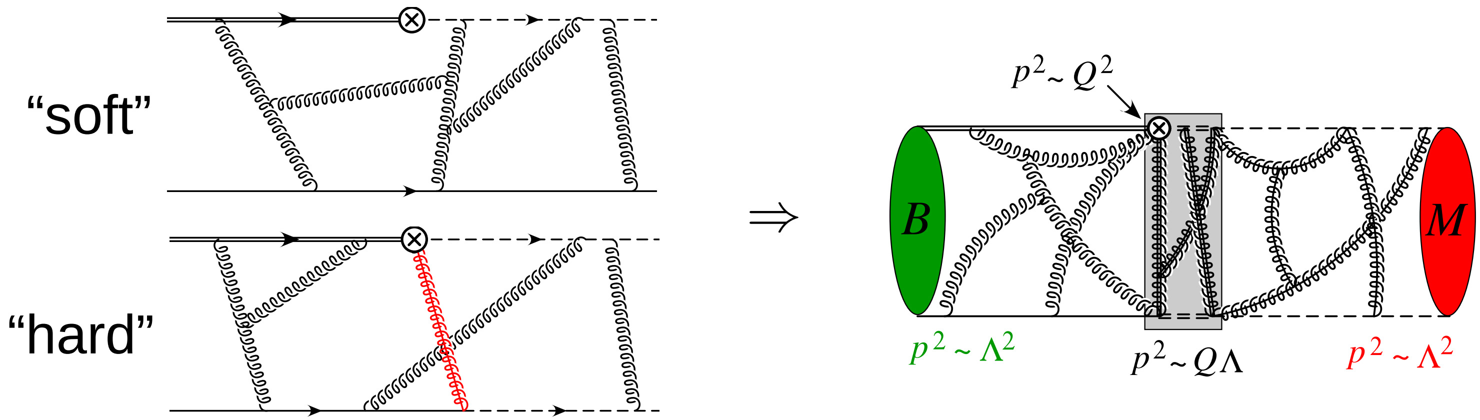
$\dfrac{B \to \ell\bar\nu}{B_s \to \ell^+\ell^-} \times \dfrac{D_s \to \ell\bar\nu}{D \to \ell\bar\nu}$ — very clean... in a decade (Ringberg workshop, lots of beer, ’03)

Soft-collinear effective theory

(Talks by Fleming & Pirjol)

- A new EFT to describe the interactions of energetic but low invariant mass particles with soft quanta ["the" connection between heavy quarks and jet physics?]
  ... Operator formulation instead of studying regions of Feynman diagrams
  ... Simplified and new proofs ($B \to D\pi$) of factorization theorems (Bauer, Pirjol, Stewart)

- E.g., $B \to \pi\ell\bar{\nu}$ form factor: Issues: tails of wave fn's, Sudakov suppression, etc.

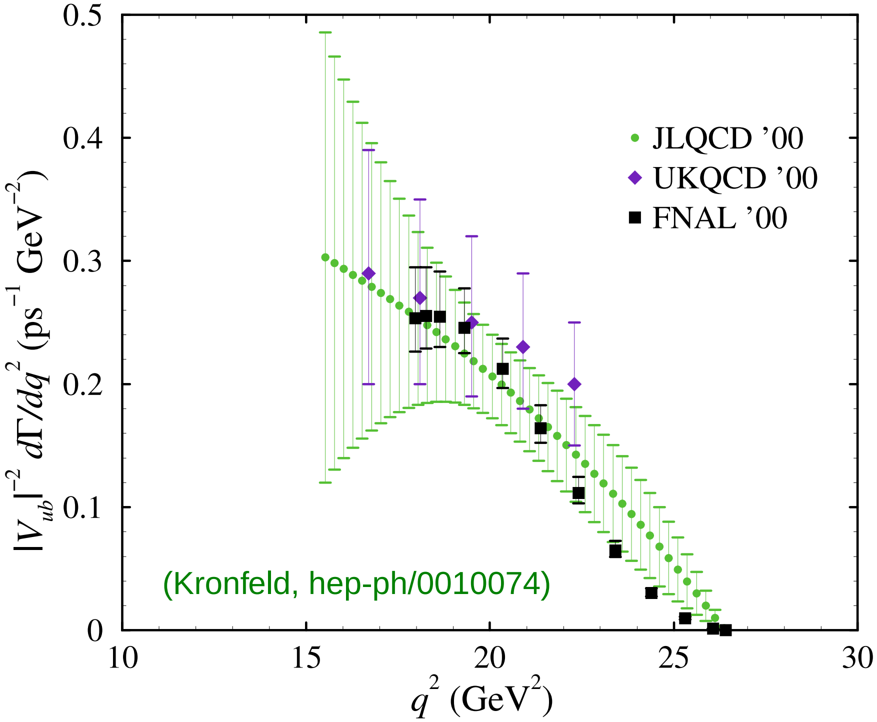


Recently proven: $F(Q) = f^{\text{non-fact.}}(Q) + f^{\text{fact.}}(Q)$ (Bauer, Pirjol, Stewart)

Hope to understand accuracy of form factor relations in low q^2 region (Charles *et al.*)

$B \to \pi\ell\bar{\nu}$ from lattice QCD

(Talks by Becirevic & Davies)



Present calculations are quenched
Need unquenched to be model independent

Few – $10\,\%$ errors seem to be achievable

Calculations in larger/full q^2 range may become possible (presently low p_π)

$B \to \rho$ harder due to sizable Γ_ρ / m_ρ

$|V_{ub}|$ — inclusive

The problem for $B \to X_u \ell \bar\nu$

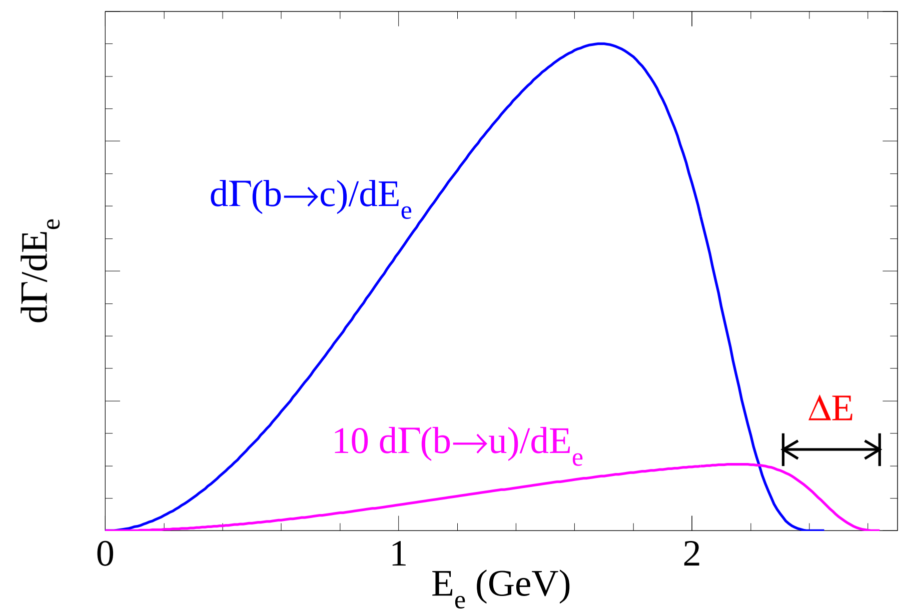
- Total rate known at $\sim 5\%$ level, similar to $\Gamma(B \to X_c \ell \bar\nu)$ (Hoang, ZL, Manohar)

$$|V_{ub}| \sim \left[3.04 \pm 0.08_{m_b} \pm 0.08_{\rm pert}\right] \times 10^{-3} \left(\frac{\mathcal{B}(B \to X_u \ell \bar\nu)}{0.001} \, \frac{1.6\,\mathrm{ps}}{\tau_B}\right)^{1/2}$$

Can huge charm background ($|V_{cb}/V_{ub}| \sim 10$) be removed w/o phase space cuts?

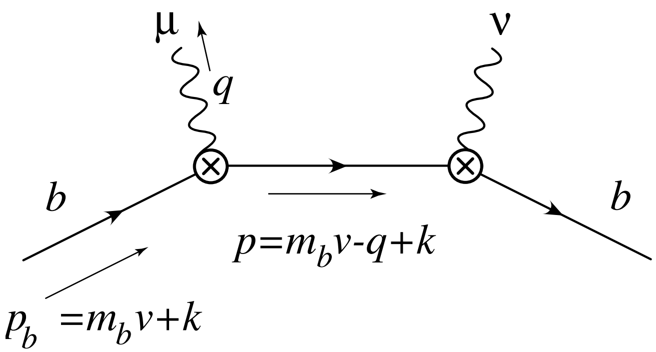
- If cuts needed, life gets more complicated: perturbative and nonperturbative corrections can get a lot larger

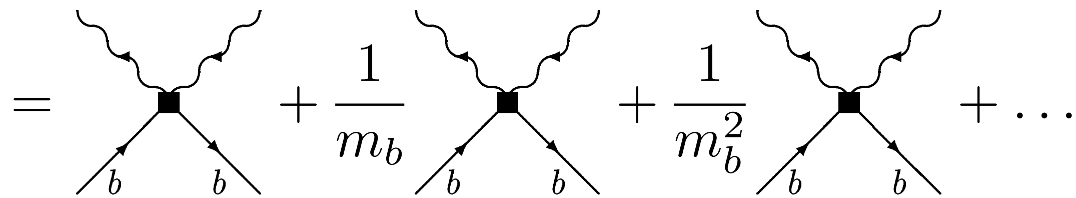
  E.g.: purely nonperturbative effects shift endpoint from $m_b/2$ to $m_B/2$



Back to the OPE: when should it converge?

- Can think of the OPE as expansion of forward scattering amplitude in $k \sim \Lambda_{\rm QCD}$





$\sim$ field theoretic version of multipole expansion

Time ordered product short distance dominated if expansion in k converges:

$$\frac{1}{(m_b v - q + k)^2} = \frac{1}{(m_b v - q)^2 + 2k \cdot (m_b v - q) + k^2}$$

Need to allow: $m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$

OPE breaks down: m_X restricted to few $\times\, \Lambda_{\rm QCD}$ (trivial — resonances)

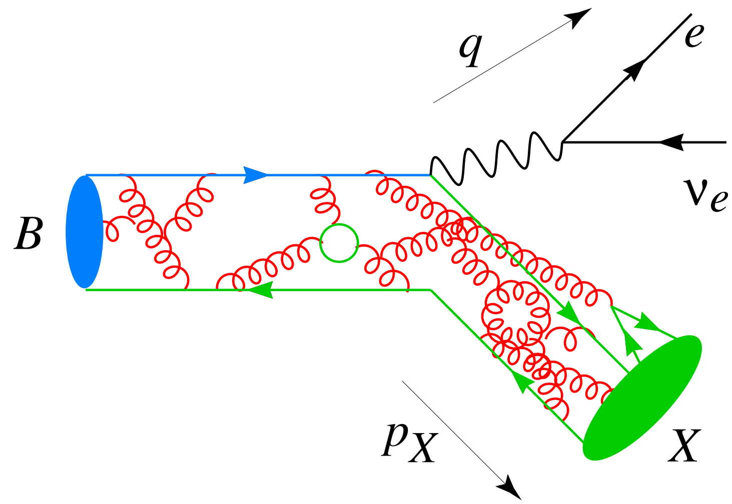
$m_X^2 \sim E_X \Lambda_{\rm QCD}$ but $E_X \gg \Lambda_{\rm QCD}$ (nontrivial — many states)

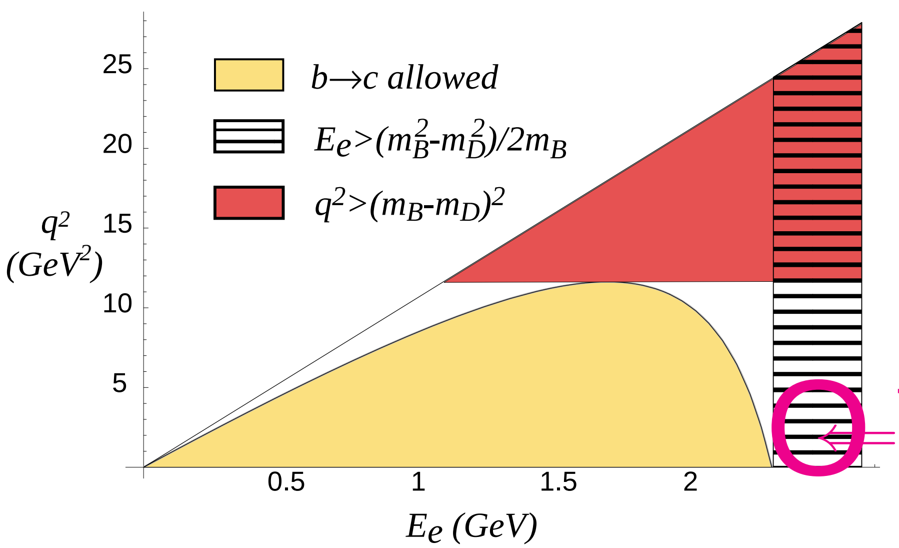
$\Rightarrow$ Design cuts to avoid these regions

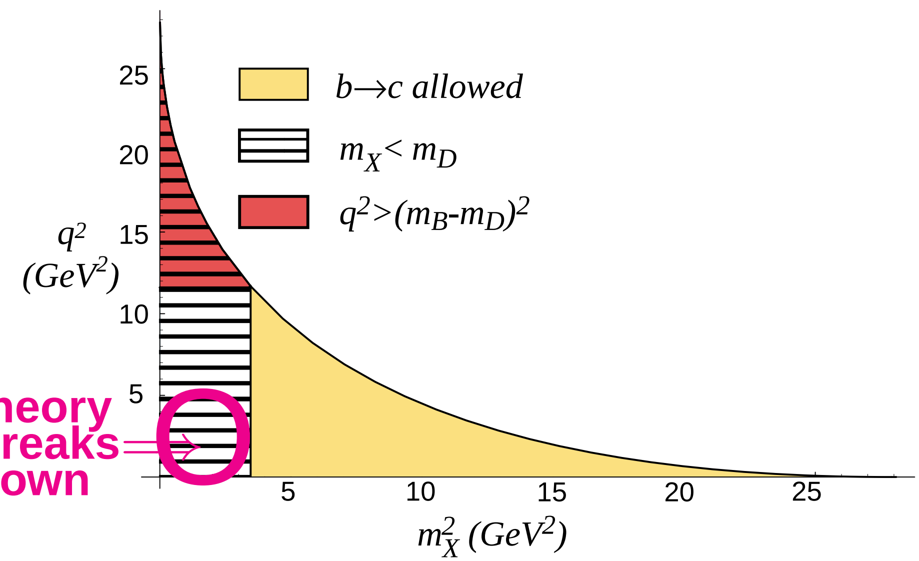
Inclusive $B \to X_u \ell \bar{\nu}$ phase space

Possible cuts to eliminate $B \to X_c \ell \bar{\nu}$ background:

- Lepton spectrum: $E_\ell > (m_B^2 - m_D^2)/2m_B$
- Hadronic mass spectrum: $m_X < m_D$
- Dilepton mass spectrum: $q^2 > (m_B - m_D)^2$
- Combinations of cuts



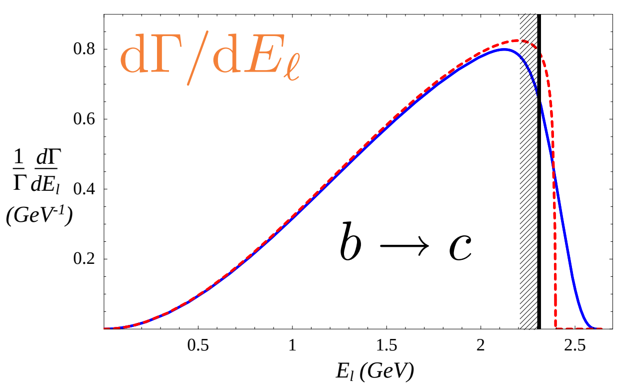


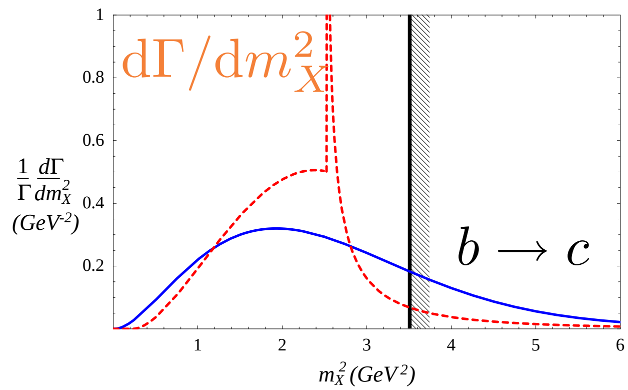


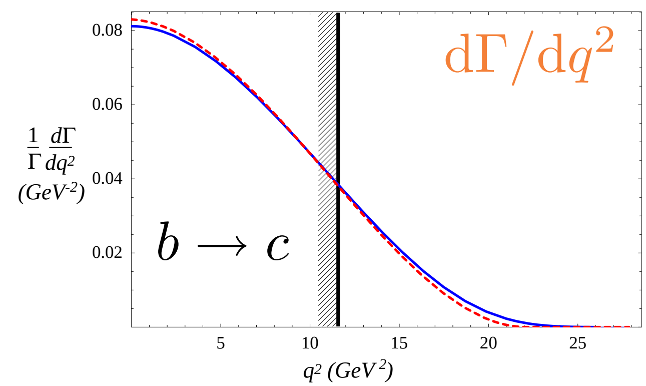
$B \to X_u \ell \bar{\nu}$ spectra

- Troubles come from the coincidence: $m_c^2 \approx m_b \times 400\,\text{MeV}$

$E_\ell > (m_B^2 - m_D^2)/2m_B$ or $m_X < m_D$ include $E_X \sim m_b/2 \;\Rightarrow\; m_X^2 \not\gg E_X \Lambda_{\text{QCD}}$







— b quark decay to $\mathcal{O}(\alpha_s)$
— incl. "Fermi-motion" (model)

Theory happy →

← Experiment happy

Exp:	"easy"	need neutrino reconstruction	
Rate:	$\sim 10\%$	$\sim 80\%$	$\sim 20\%$
OPE:	infinite set of terms equally important		first few terms converge

Large E_ℓ and small m_X regions

Bad: infinite set of terms in OPE equally important (shape function)

Good: Fermi motion effects universal at leading order in $\Lambda_{\rm QCD}/m_b$
related to $B \to X_s\gamma$ photon spectrum (Neubert; Bigi, Shifman, Uraltsev, Vainshtein)

- $E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$: NLO Sudakov logs resummed (Leibovich, Low, Rothstein)

  Operators other than O_7 in $B \to X_s\gamma$ (Neubert)

  Terms unrelated to $B \to X_s\gamma$ sizable (Leibovich, ZL, Wise; Bauer, Luke, Mannel)

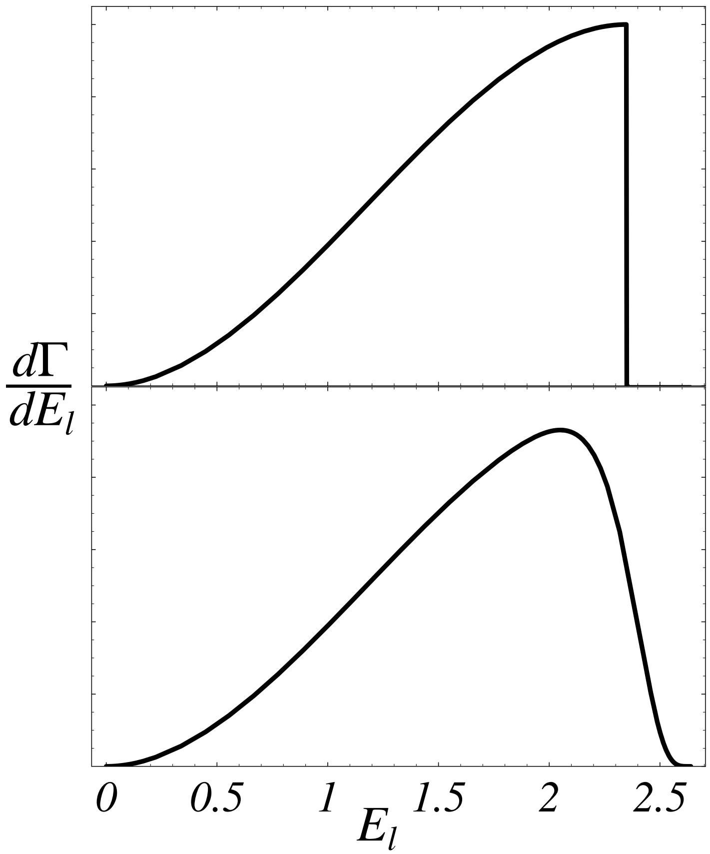
- $m_X < m_D$: lot more rate, but nonperturbative input formally still $\mathcal{O}(1)$ corrections smaller and inclusive description should be valid, but model dependence increases rapidly as $m_X^{\rm cut}$ lowered (Barger *et al.*; Falk, ZL, Wise; Bigi, Dikeman, Uraltsev)

NB: $\bar\Lambda$ & λ_1 (HQET) $\neq$ $\bar\Lambda$ & λ_1 (shape function models), e.g., De Fazio & Neubert
best would be to use $B \to X_s\gamma$ spectrum directly

Lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for Fermi motion



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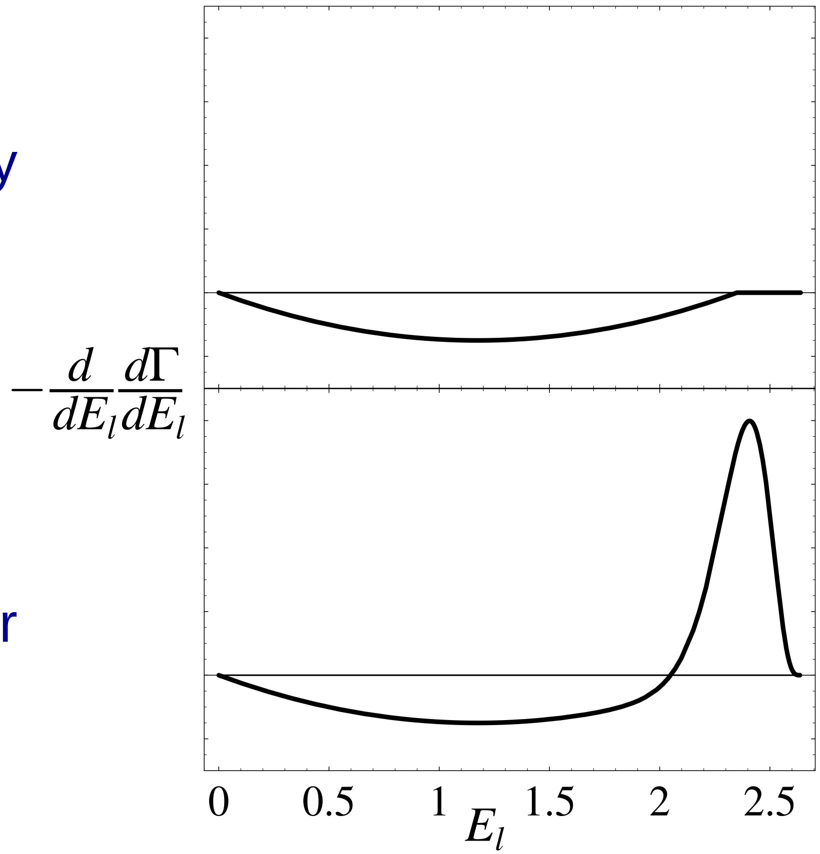

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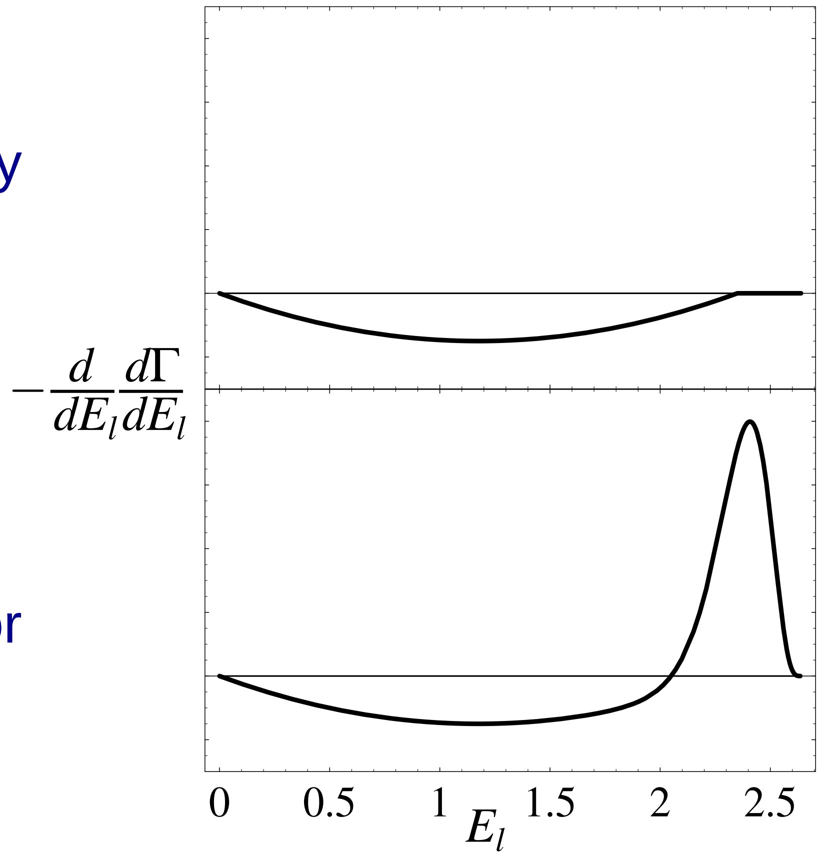
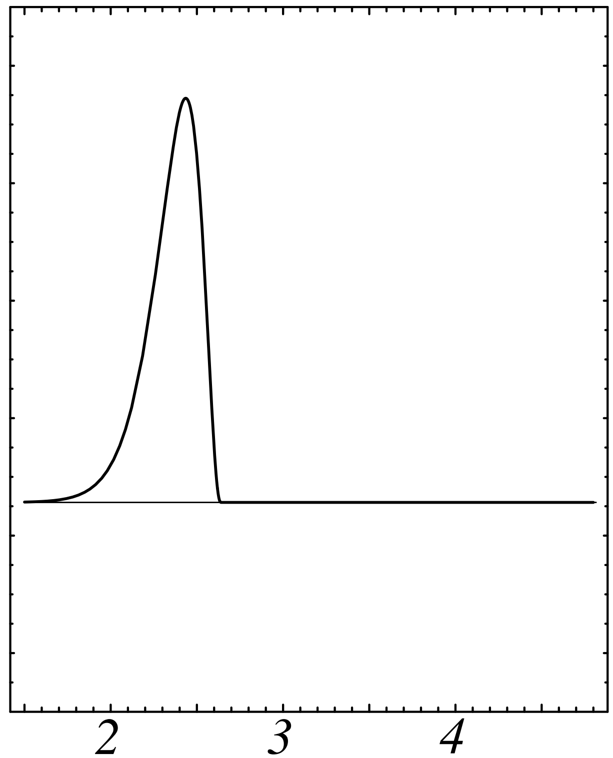


difference:



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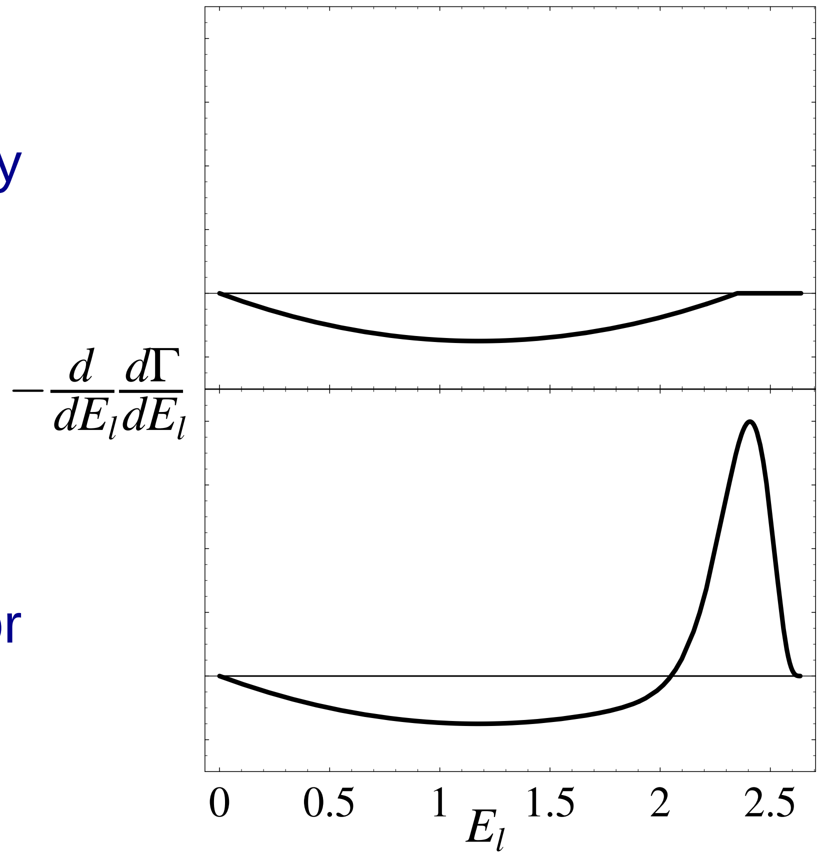
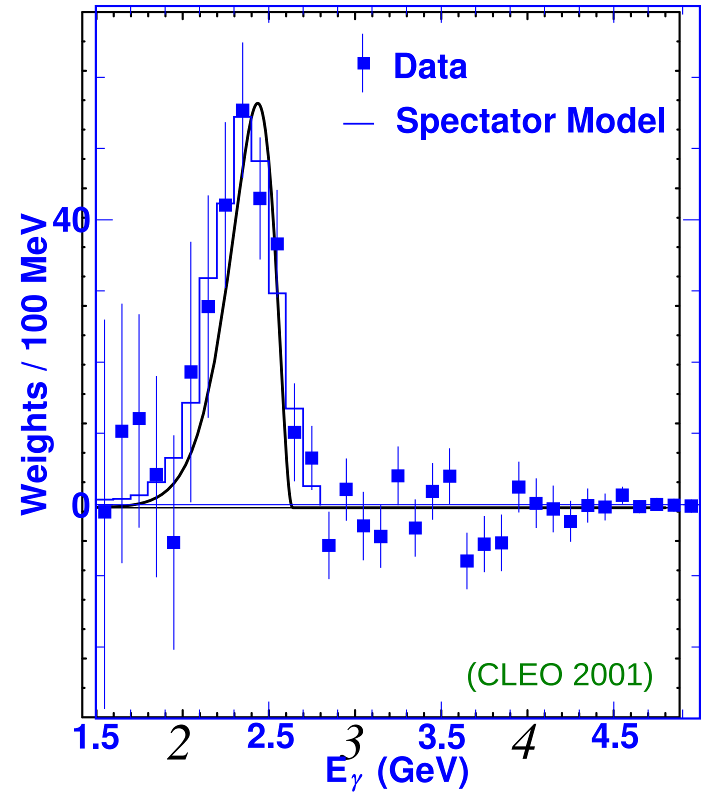
b quark decay spectrum

with a model for Fermi motion



difference:

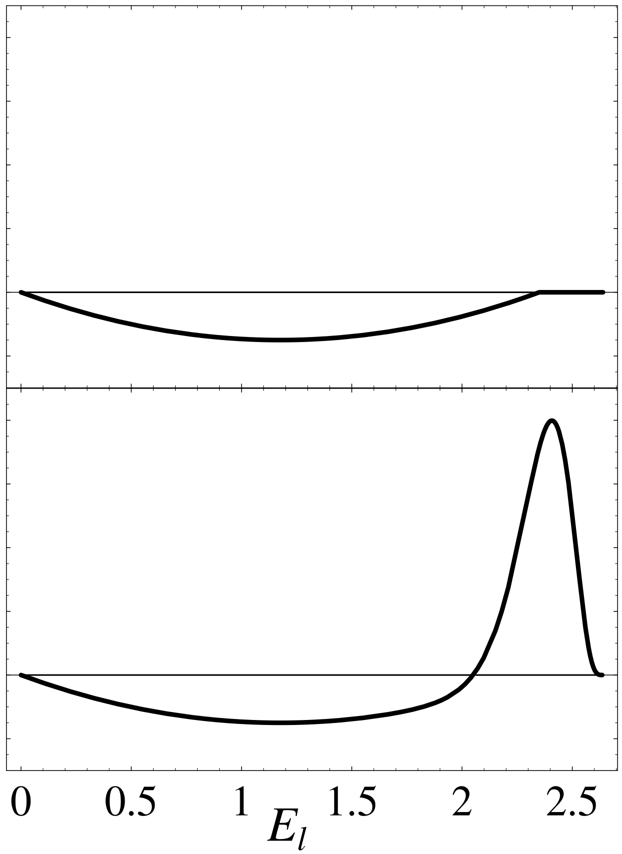


Lepton endpoint vs. $B \to X_s\gamma$

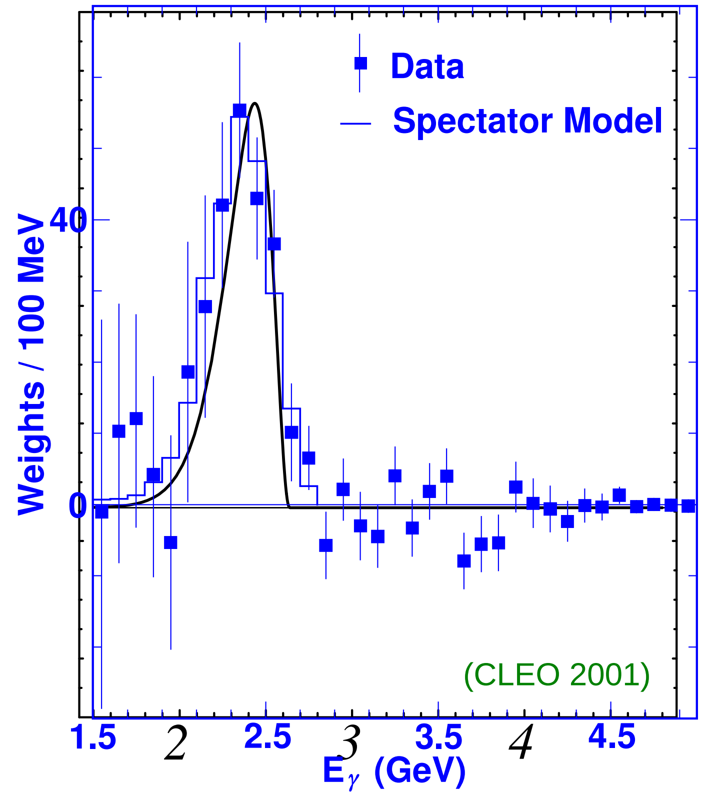
b quark decay spectrum

with a model for Fermi motion

$-\frac{d}{dE_l}\frac{d\Gamma}{dE_l}$



difference:



Limiting uncertainties: subleading corrections?
inclusive enough?

$\Downarrow$ (CLEO 2002)

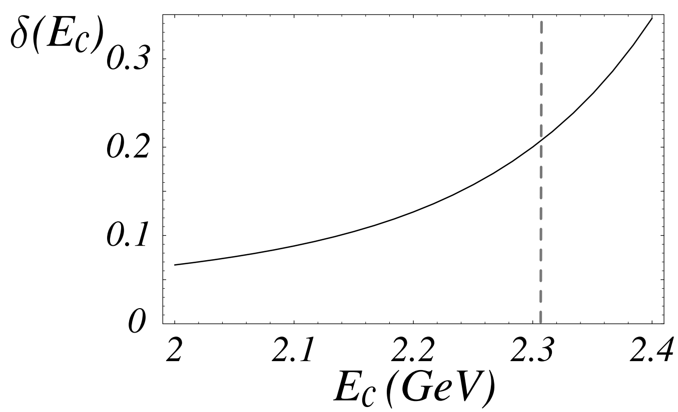
$$|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$$

Sizable subleading twist effects

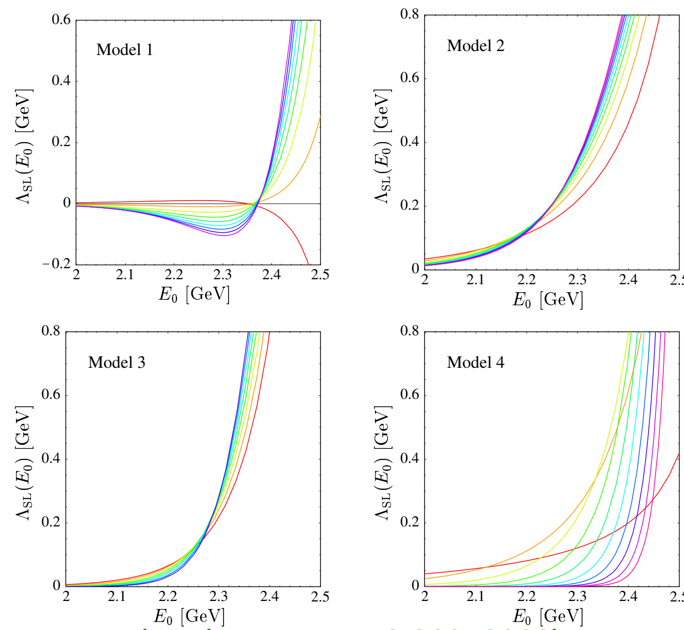
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \frac{G_F^2\, m_b^5\, |V_{ub}|^2}{192\,\pi^3} \bigg\{ y^2(3-2y)\, 2\theta(1-y) - \frac{\lambda_2}{m_b^2}\Big[11\,\delta(1-y) - 2y^2(6+5y)\theta(1-y)\Big] - \frac{\lambda_1}{m_b^2}\Big[\frac{1}{3}\,\delta'(1-y) + \frac{1}{3}\,\delta(1-y) - \frac{10}{3}\,y^3\theta(1-y)\Big] + \ldots \bigg\}$$

Coefficient corresponding to **11** is **3** in $B \to X_s\gamma$ (Leibovich, ZL, Wise, PLB **539** 242, 2002)

Models: $\sim 15\%$ effect in $|V_{ub}|$ for $E_\ell^{\mathrm{cut}} = 2.3$ GeV, decrease with E_ℓ^{cut}



(Bauer, Luke, Mannel, PLB **543** 261, 2002)



(Neubert, PLB **543** 269, 2002)

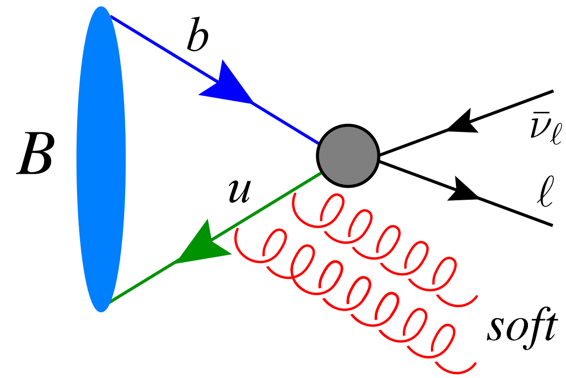
What part is “calculable”, what is the “uncertainty”?

Weak annihilation (sub-subleading)

- Bad news: $\mathcal{O}(\Lambda^3_{\rm QCD}/m_b^3)$ in rate, enhanced by $16\pi^2$

  ... concentrated at large E_ℓ, q^2, and small m_X^2

  $\Rightarrow$ enters all $|V_{ub}|$ extractions



(Bigi & Uraltsev; Voloshin; Leibovich, ZL, Wise)

  Cancellation between: $\langle B|(\bar b\gamma^\mu P_L u)\,(\bar u\gamma_\mu P_L b)|B\rangle$

  $$\langle B|(\bar b P_L u)\,(\bar u P_L b)|B\rangle$$

  Estimated, with large uncertainty, as:

  $$\mathcal{O}\left[16\pi^2 \times \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \times \left(\begin{array}{c}\text{factorization}\\ \text{violation}\end{array}\right)\right] \sim 0.03\left(\frac{f_B}{200\,\text{MeV}}\right)^2 \frac{B_2 - B_1}{0.1}$$

  If $\sim 3\%$ uncertainty in total rate, then $\sim 15\%$ in $|V_{ub}|$ from lepton endpoint, $\lesssim 10\%$ in $|V_{ub}|$ from large q^2 region, less for $m_X < m_D$ (more rate included)

- Constrain WA: compare D^0 vs. D_s SL widths, or V_{ub} from $B^\pm$ vs. B^0 decay

Large q^2 region

- Good: first few terms in OPE can be trusted (Bauer, ZL, Luke '00)

  full $\mathcal{O}(\alpha_s^2)$ result known (Czarnecki & Melnikov '01)

  Bad: expansion is more like in $\Lambda_{\rm QCD}/m_c$ and $\alpha_s(m_c)$ than at scale m_b (Neubert '00)

- Combined q^2 & m_X cuts: more rate, scale goes up $m_c \Rightarrow \frac{m_b^2 - q_{\rm cut}^2}{m_b \Lambda_{\rm QCD}}$ (Bauer, ZL, Luke '01)

Cuts on $(q^2,\ m_X)$	included fraction of $b \to u \ell \bar\nu$ rate	error of $\lvert V_{ub}\rvert$ $\delta m_b = 80/30$ MeV
6 GeV^2, m_D	46%	$8\%/5\%$
8 GeV^2, 1.7 GeV	33%	$9\%/6\%$
$(m_B - m_D)^2, m_D$	17%	$15\%/12\%$

Strategy: (i) reconstruct $p_\nu \Rightarrow q^2, m_X$; make cut on m_X as large as possible

(ii) for a given m_X cut, reduce q^2 cut to minimize overall uncertainty

Can get $30-40\%$ of events, even with cuts away from $b \to c$ region

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Summary for $|V_{ub}|$

- Total $B \to X_u \ell \bar{\nu}$ rate known precisely; phase space cuts seem unavoidable
- $E_\ell > (m_B^2 - m_D^2)/2m_B$: simplest experimentally
  ... even using $B \to X_s \gamma$ spectrum, corrections are $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$
  ... only $\sim 10\%$ of phase space — inclusive enough?
- $m_X^2 < m_D^2$: lots of rate but still sensitive to shape function
  ... uncertainties increase rapidly if cut is significantly below m_D
- $q^2 > (m_B - m_D)^2$: no (leading) shape funtion, expansion formally in $\Lambda_{\rm QCD}/m_c$
- combined q^2 and m_X cuts: less rate than pure m_X cut, good theoretical control

$\Rightarrow$ Tricky business, need to measure $|V_{ub}|$ in as many clean ways as possible, confidence will be gained by convergence of extractions

Wishlist for $|V_{ub}|$

Experiment:

- get the cuts as close to the charm threshold as possible
- improve measurement of $B \to X_s\gamma$ photon spectrum (lower cut) and try to use it directly instead of through parameterizations
- constrain WA by comparing $|V_{ub}|$ from $B^\pm$ vs. B^0, or D^0 vs. D_s SL widths

Theory:

- full α_s^2 corrections (beyond $\alpha_s^2\beta_0$) known only for total rate and q^2 spectrum, not for other distributions

Both:

- precise determination of m_b — rate $\propto m_b^5$, even stronger sensitivity with cuts

Conclusions

Conclusions

- $|V_{cb}|$ is known at the $\sim 5\%$ level, error may soon become half of this
  inclusive: consistency of moments; exclusive: $\mathcal{F}_*(1)$ from unquenched lattice

- Model independent $\sim 10\%$ $|V_{ub}|$ seems posible, ultimately similar to present $|V_{cb}|$
  inclusive: neutrino reconstruction crucial; exclusive: needs unquenched lattice

- For both $|V_{cb}|$ and $|V_{ub}|$, important to pursue both inclusive and exclusive

- Progress in understanding exclusive heavy $\to$ light form factors for $q^2 \ll m_B^2$
  $B \to \pi/\rho\, \ell\bar{\nu},\ K^*\gamma,\ K^{(*)}\ell^+\ell^-$ below the $\psi \Rightarrow$ increase sensitivity to new physics
  ... related to issues in factorization in charmless decays

- Theoretical limit for inclusive $|V_{cb}|$ and $|V_{ub}|$ appear to be about $\sim 1\%$ and $\sim 5\%$

Extra slides

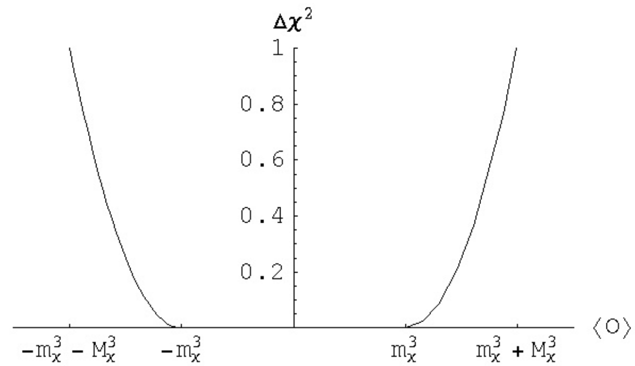
“Moments” — theoretical uncertainties

Define theoretical uncertainties, so it is not judged case-by-case and a posteriori

Avoid large weight to an accurate measurement that cannot be computed reliably

- Unknown $1/m_b^3$ matrix elements — $\mathcal{O}(\Lambda_{\rm QCD}^3)$ but no preferred value $\Rightarrow$ add in fit:

$$\Delta\chi^2(m_\chi, M_\chi) = \begin{cases} 0\,, & |\langle\mathcal{O}\rangle| \le m_\chi^3 \\ \left[|\langle\mathcal{O}\rangle| - m_\chi^3\right]^2 / M_\chi^6\,, & |\langle\mathcal{O}\rangle| > m_\chi^3 \end{cases}$$



Take $M_\chi = 0.5\,$GeV, and vary $0.5\,$GeV $< m_\chi < 1\,$GeV

- Uncomputed higher order terms — estimate using naive dimensional analysis:
  - $(\alpha_s/4\pi)^2 \sim 0.0003$
  - $(\alpha_s/4\pi)(\Lambda_{\rm QCD}^2/m_b^2) \sim 0.0002$
  - $\Lambda_{\rm QCD}^4/(m_b^2 m_c^2) \sim 0.001$

Use relative error: $\sqrt{(0.001)^2 + (\text{last-computed}\,/\,2)^2}$

Results in $1S$ scheme

- Do fits both excluding (top) and including (bottom) BABAR data

m_χ [GeV]	χ^2	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	5.0	40.8 ± 0.9	4.74 ± 0.10
1.0	3.5	41.1 ± 0.9	4.74 ± 0.11
0.5	12.9	40.8 ± 0.7	4.74 ± 0.10
1.0	8.5	40.9 ± 0.8	4.76 ± 0.11

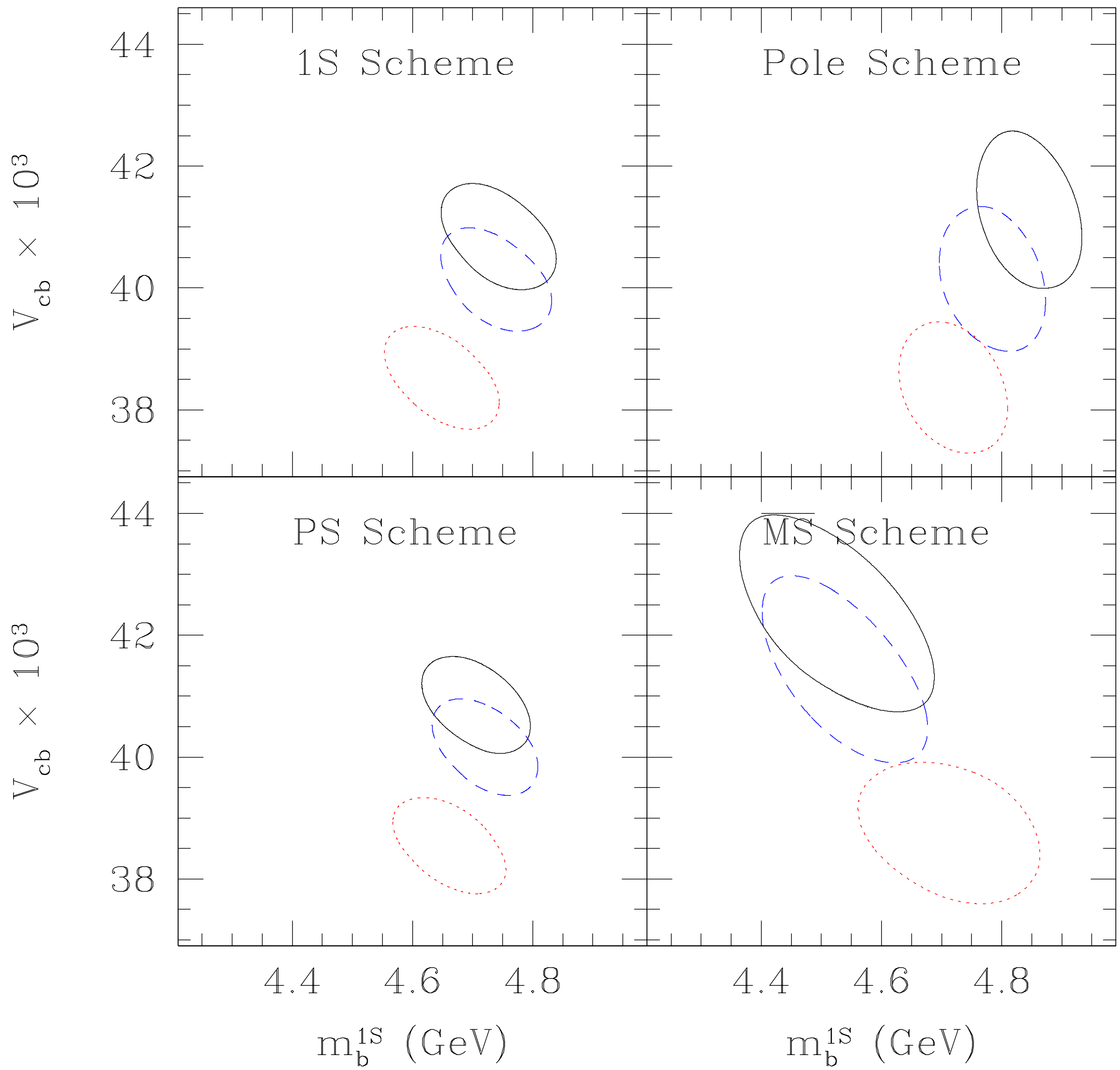
Sensitivity to m_χ is small ($1/m^3$ errors significant, but so are their correlations)

BABAR data increases χ^2/d.o.f. significantly — more later

Theoretical uncertainties important — neglecting them gives $\chi^2 = 81$ for 9 d.o.f.
Including only $1/m^3$ terms gives $\chi^2 = 21$ for 5 d.o.f.; much better (but still bad) fit

Results in different mass schemes

tree level, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2\beta_0)$



better convergence in 1S and PS schemes than in pole or $\overline{\text{MS}}$