Learning from $b \to s\ell^+\ell^-$ effectively

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2nd Workshop on Flavor Dynamics
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- Introduction — status of measurements and theory
- Optimal observables to constrain short distance physics
  There is a third independent observable besides $\mathrm{d}\Gamma/\mathrm{d}q^2$ and $\mathrm{d}A_{\rm FB}/\mathrm{d}q^2$
- $B \to X_s\ell^+\ell^-$ in the small q^2 region
  Cuts on q^2 & $m_X \Rightarrow$ nonperturbative shape function effects as in $B \to X_u\ell\bar\nu$
- $B \to X_s\ell^+\ell^-$ in the large q^2 region
  Can eliminate m_b-dependence and most of $1/m_b^n$ uncertainty

Details: K. Lee, ZL, I. Stewart, F. Tackmann: hep-ph/0612156 & hep-ph/0512191
ZL, F. Tackmann: arXiv:0707.1694

Perturbative $b \to s\ell^+\ell^-$ calculations

- Complementary to $B \to X_s\gamma$
- Subtleties in power counting (as in $K \to \pi e^+ e^-$)

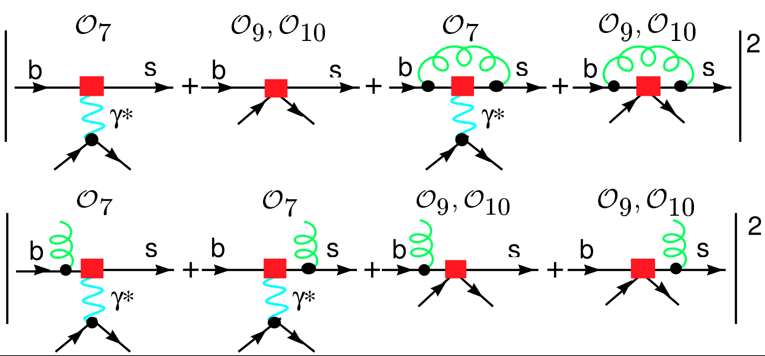
$$C_9(m_b) \sim C_9(m_W) + (\ldots)\frac{C_2(m_W)}{\alpha_s(m_W)}\left\{1 - \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{(\cdots)}\right\}$$

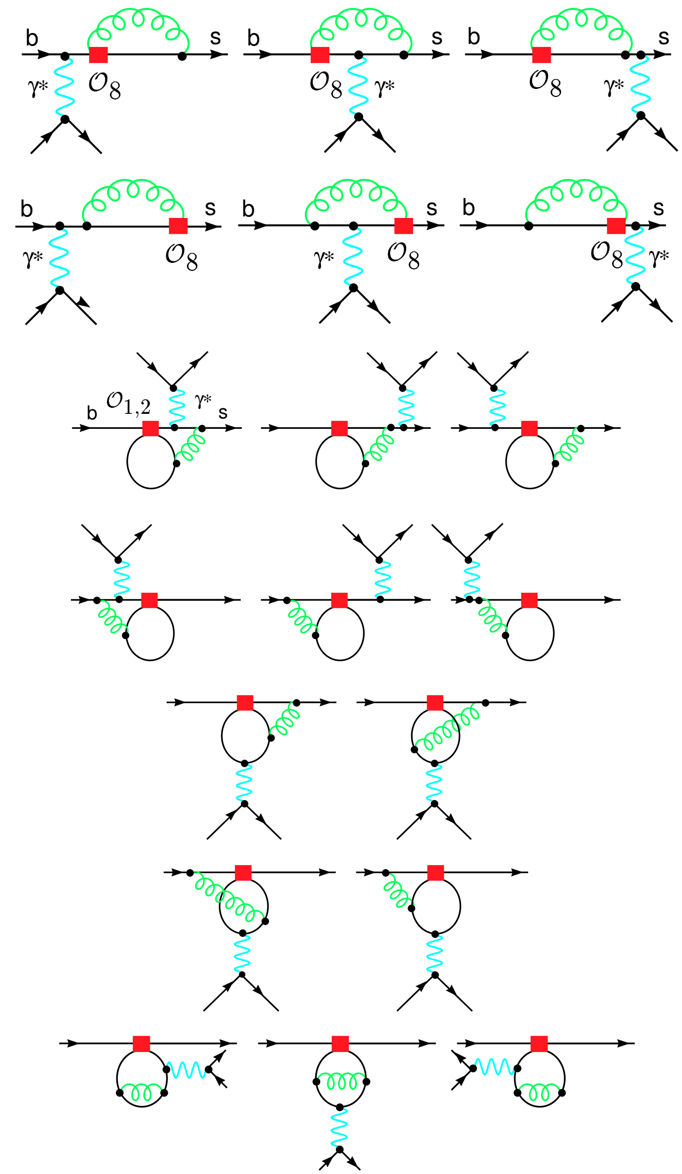
Scale & scheme dependence cancellation tricky

- NNLL: 2-loop matching, 2- and 3-loop running
  2-loop matrix elements

$$\mathcal{B}(B \to X_s \ell^+\ell^-)\Big|_{1<q^2<6\,\mathrm{GeV}^2} = (1.63 \pm 0.20) \times 10^{-6}$$

[Many authors: Bobeth, Misiak, Urban, Munz, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Bieri, Hovhannisyan, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]





The q^2 spectrum in $B \to X_s \ell^+ \ell^-$

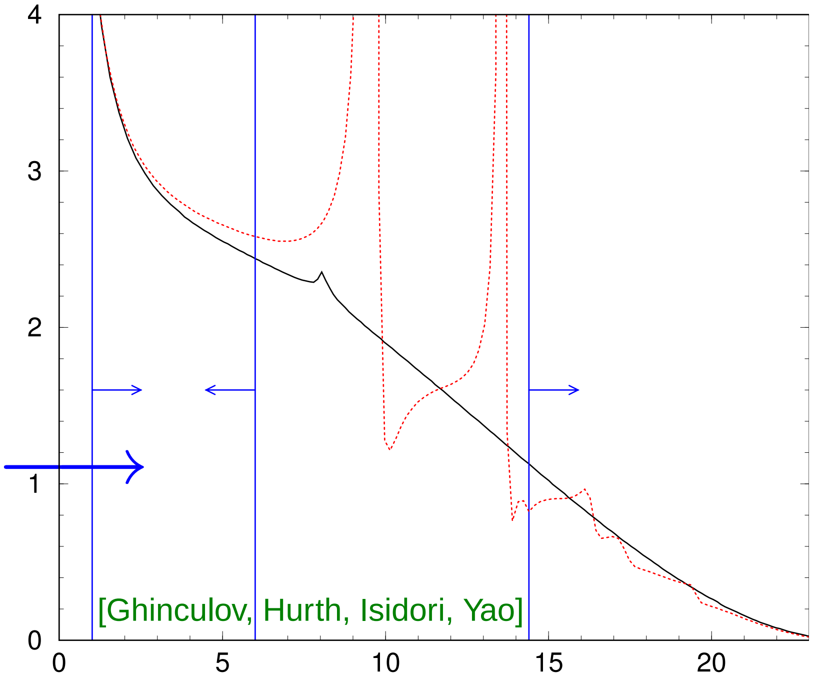
- Rate depends (mostly) on

$$O_7 = \overline{m}_b\, \bar{s}\sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

$$O_9 = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell),$$

$$O_{10} = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$



- NNLL $b \to s \ell^+ \ell^-$ perturbative calculations

  Introduce $C_{7,9}^{\mathrm{eff}}$ — complex with usual definition

- Nonperturbative corrections to q^2 spectrum ($1/m_b^2$ and $1/m_c^2$)

  [Falk, Luke, Savage; Ali, Hiller, Handoko, Morozumi; Buchalla, Isidori, Rey]

- Small q^2 region also has the larger rate $\Rightarrow$ smaller experimental errors

Standard approaches

- Previous analyses concentrated on two observables: $(s = q^2/m_b^2)$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}s} \sim \Gamma_0\,(1-s)^2 \bigg[\Big(|C_9|^2 + C_{10}^2 \Big) (1+2s) + \frac{4}{s}\,|C_7|^2\,(2+s) + 12\,\mathrm{Re}(C_7 C_9^*) \bigg]$$

$$\frac{\mathrm{d}A_{\mathrm{FB}}}{\mathrm{d}s} \sim -3\Gamma_0\,(1-s)^2 s\, C_{10}\,\mathrm{Re}\Big(C_9 + \frac{2}{s}\,C_7 \Big)$$

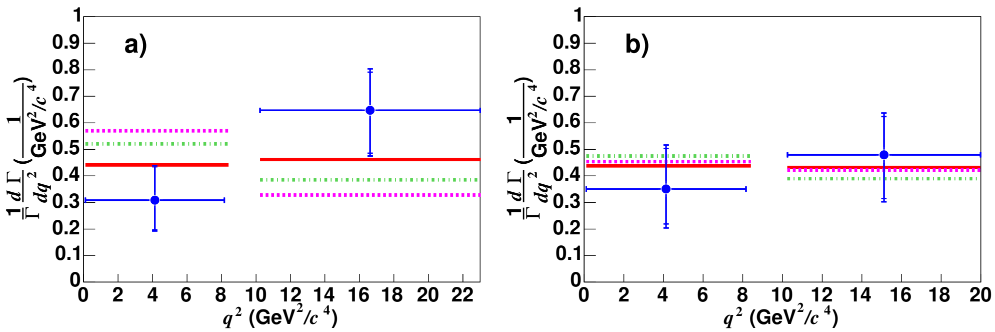
$O_{1-6,8}$ contributions absorbed in $C_{7,9} \to C_{7,9}^{\mathrm{eff}}(s)$, which are complex

- To look for NP or to extract C_i:
  - Compute rate in SM (or in a NP model) and compare with data (have to be redone for each model, hard to incorporate improvements in theory)
  - Extract C_i from fits to decay distributions (poor sensitivity, needs lots of data) (zero of A_{FB} near $-2C_7/C_9$ argued to be model independent in $B \to K^* \ell^+ \ell^-$)

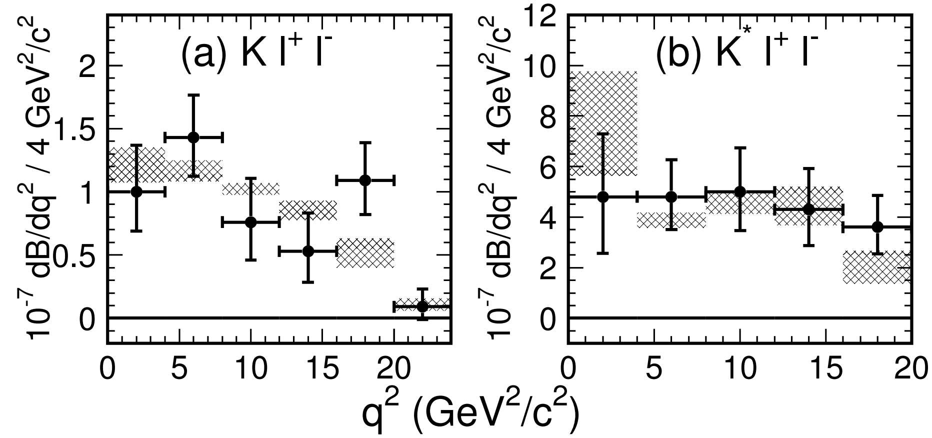
- Want most effective ways to extract C_i from simple observables integrated over q^2

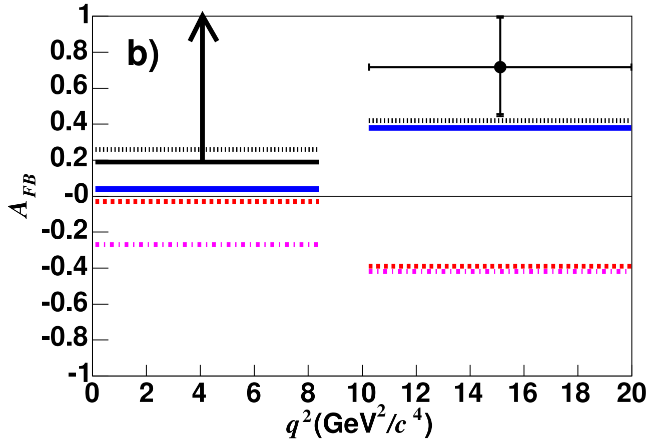
Exclusive $B \to K^{(*)}\ell^+\ell^-$ measurements

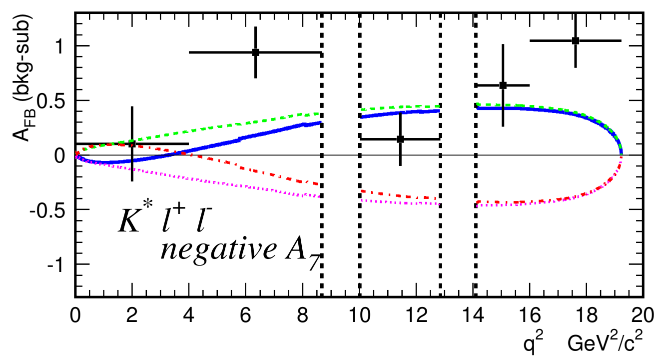
BaBar, $229\,\mathrm{m}\ \Upsilon$

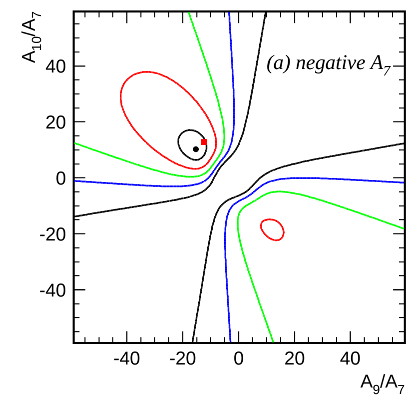


Belle, $253\,\mathrm{m}$ and $386\,\mathrm{m}\ \Upsilon$





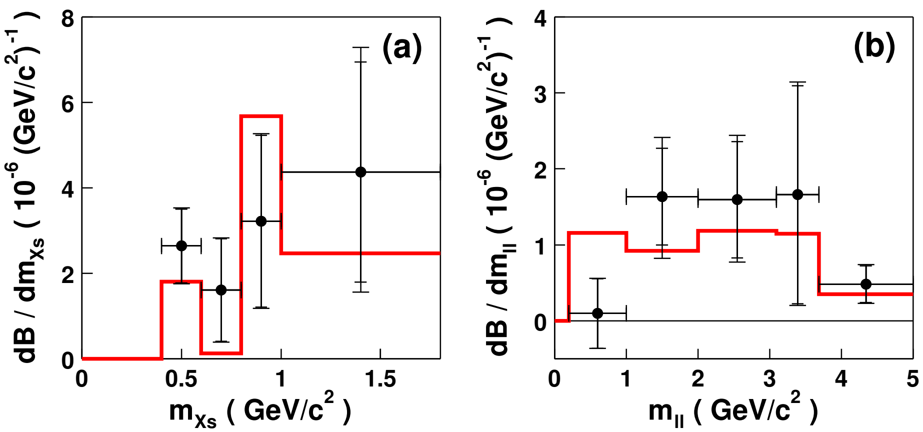




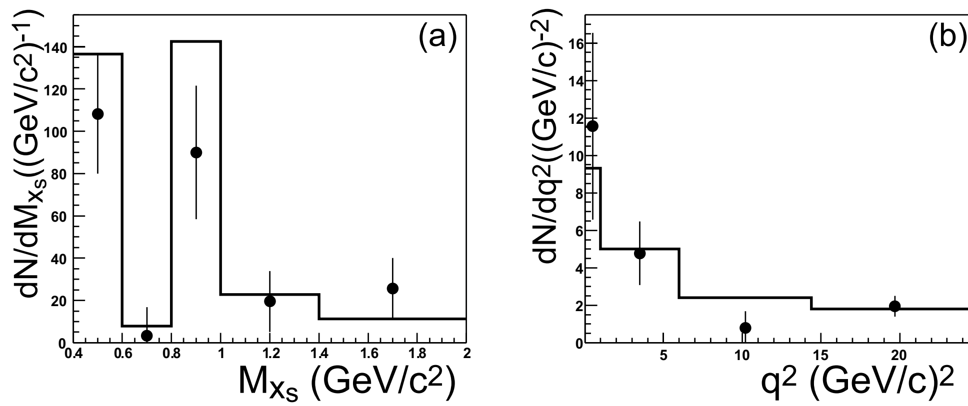
- World averages: $\mathcal{B}(B \to K^{(*)}\ell^+\ell^-) = (0.44 \pm 0.05) \times 10^{-6},\ (1.17 \pm 0.16) \times 10^{-6}$
- LHCb expects $(2, 10\,\mathrm{fb}^{-1})$: $\sigma(q^2_{A_{\rm FB}=0}) \approx 0.46, 0.27\,\mathrm{GeV}^2 \Rightarrow \sigma(C_7^{\rm eff}/C_9^{\rm eff}) \sim 12, 7\,\%$

Inclusive $B \to X_s \ell^+ \ell^-$ measurements

BaBar, $89\,$m Υ



Belle, $152\,$m Υ



- Cut out J/ψ and ψ' regions, and impose an additional cut $m_X < 1.8\,\text{GeV}$ or $2\,\text{GeV}$ to suppress huge $b \to c\ell^-\bar\nu \to s\ell^-\ell^+\nu\bar\nu$ background

  Current measurements not really inclusive — sum $\sim 50\%$ of exclusive modes

- World average: $\mathcal{B}(B \to X_s \ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}$ (with some black magic)

  Small q^2 region: $\mathcal{B}(1 < q^2 < 6\,\text{GeV}^2) = (1.60 \pm 0.51) \times 10^{-6}$

- A key measurement that utilizes only a small fraction of the available data

Aside: long distance effects

- A worry (at least, for me) that will be ignored in this talk:

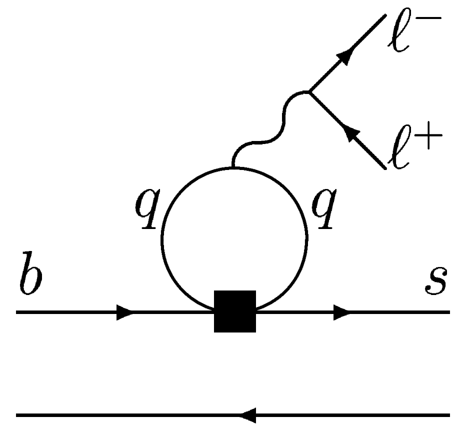
$$\mathcal{B}(B \to \psi X_s) \sim 4 \times 10^{-3}$$

$$\downarrow$$

$$\mathcal{B}(\psi \to \ell^+\ell^-) \sim 6 \times 10^{-2}$$

Combined rate: $\mathcal{B}(B \to X_s \ell^+\ell^-) \sim 2 \times 10^{-4}$

This is ~ 30 times the short distance contribution!



- Averaged over a large region of q^2, the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \ldots$
This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but NOT here

- Is it consistent to “cut out” the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)

Optimal observables

Angular decomposition

- Three (not two) components with different sensitivity to C_i $\quad (s = q^2/m_b^2,\ z = \cos\theta)$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}z} = \frac{3}{8}\Gamma_0\Big[(1+z^2)H_T(q^2) + 2z\,H_A(q^2) + 2(1-z^2)H_L(q^2)\Big]$$

$$H_T \sim 2\,(1-s)^2\,s\left[\left(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\right)^2 + \mathcal{C}_{10}^2\right] \qquad [\Gamma = H_T + H_L]$$

$$H_L \sim (1-s)^2\Big[(\mathcal{C}_9 + 2\mathcal{C}_7)^2 + \mathcal{C}_{10}^2\Big] \qquad [\text{no } \mathcal{C}_7/s \text{ pole}]$$

$$H_A \sim -4\,(1-s)^2\,s\,\mathcal{C}_{10}\left(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\right) \qquad [H_A \equiv (4/3)A_{\mathrm{FB}}]$$

θ: angle between $\vec{p}_{\ell^+}$ and $\vec{p}_{\bar{B}^0,\,B^-}$ [$\vec{p}_{\ell^-}$ and $\vec{p}_{B^0,\,B^+}$] in $\ell^+\ell^-$ center of mass frame

- Dependence on C_i: H_L is q^2 independent; $H_{T,A}$'s sensitivity to C_i depends on q^2

- Same structure for $B \to X_s\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ — different at $\mathcal{O}(\alpha_s, 1/m_{c,b})$
  $B \to K^*\ell^+\ell^-$: Two further angles (even more if $\ell^\pm$ polarizations considered)

- Three terms sensitive to different combinations of Wilson coefficients

Higher order corrections

- Introduce a scheme to separate NP-sensitive terms from four-quark operator contributions (for which the SM is assumed)

- Define $\mathcal{C}_{7,9}$ as μ- and q^2-independent constants, real in the SM

$$C_{7,9}^{\rm incl}(q^2) = \mathcal{C}_{7,9} + \underbrace{F_{7,9}(q^2)}_{\alpha_s} + \underbrace{G_{7,9}(q^2)}_{1/m_c^2}$$

($F_{7,9}$ include NNLL)

- Use m_b^{1S} to improve perturbation series; do not normalize to $\Gamma(B \to X \ell \bar\nu)$

  Keep $\overline{m}_b(\mu) C_7(\mu)$ together and unexpanded — no reason to expand $\overline{m}_b(\mu)$

- Numerically small Λ^2/m_c^2 correction can be simply included:

$$G_9(q^2) = \frac{10}{1-2s}\, G_7(q^2) = -\frac{5}{6}\,\frac{\lambda_2}{m_c^2}\, C_2\, \frac{\mathcal{F}[q^2/(4m_c^2)]}{1-q^2/(4m_c^2)}$$

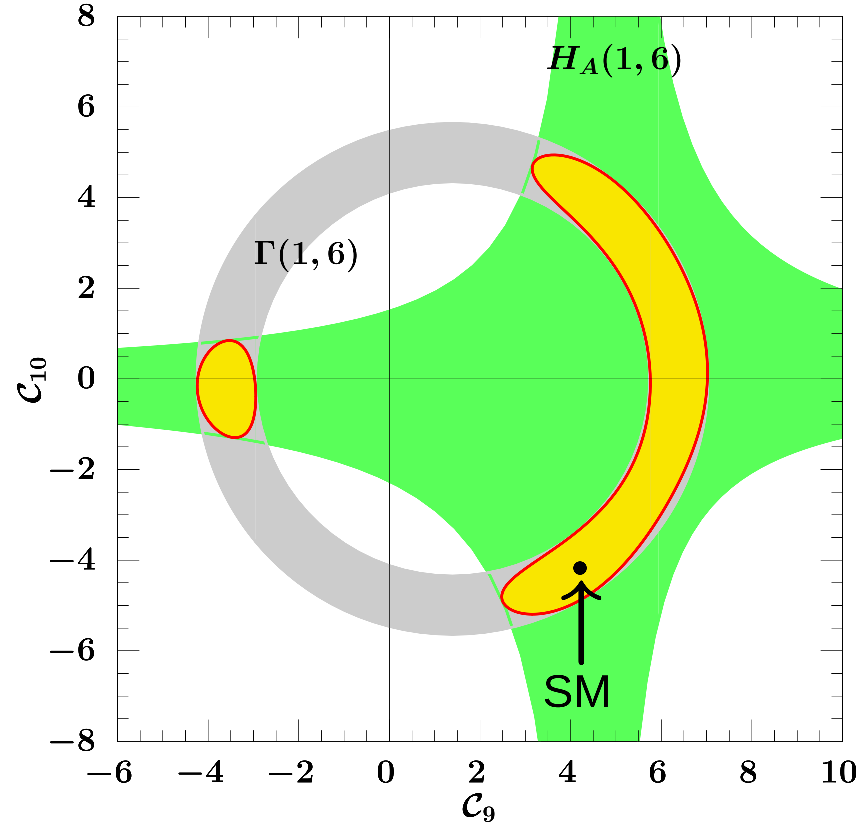
Blows up as $(4m_c^2 - q^2)^{-1/2}$ as $q^2 \to 4m_c^2$; assume OK for $q^2 \lesssim 3m_c^2 \sim 6\,{\rm GeV}^2$

An illustrative toy analysis

- Inclusive, with guesstimated error for $1\,\mathrm{ab}^{-1}$

  Define: $H_i(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} \mathrm{d}q^2\, H_i(q^2)$

- Small q^2-dependence



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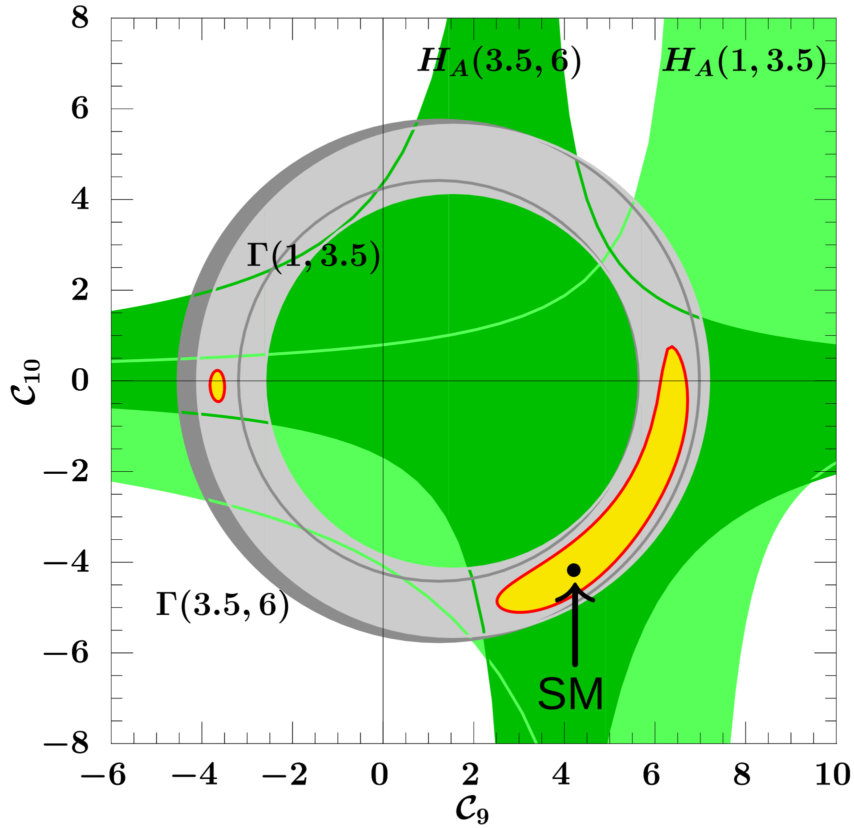
- Small q^2-dependence $\Rightarrow$ splitting Γ in two regions not useful (splitting $H_A \equiv A_{\rm FB}$ is!)



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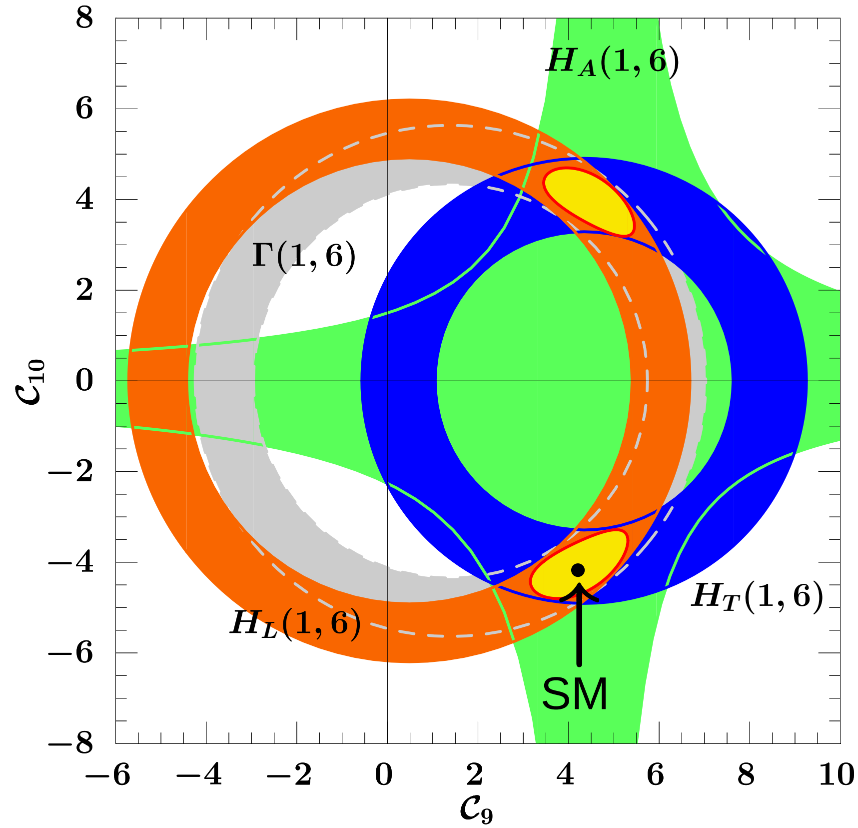
- $H_L \propto q^2$-independent combination of $\mathcal{C}_i$'s
  $\Rightarrow$ integrate over as large region as possible



- Separating H_T and H_L is very powerful

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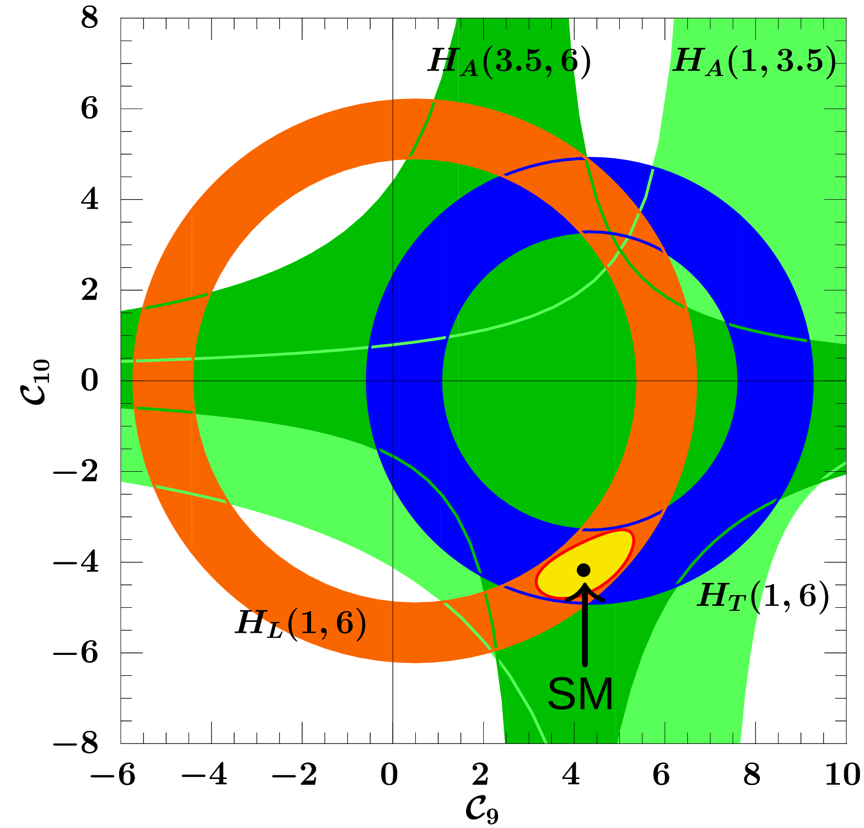
- H_T and H_A: different q^2 regions sensitive to different combinations of $\mathcal{C}_i$'s

  Separating $H_A(1, 3.5)$ vs $H_A(3.5, 6)$ and/or $H_T(1, 3.5)$ vs $H_T(3.5, 6)$ appears promising



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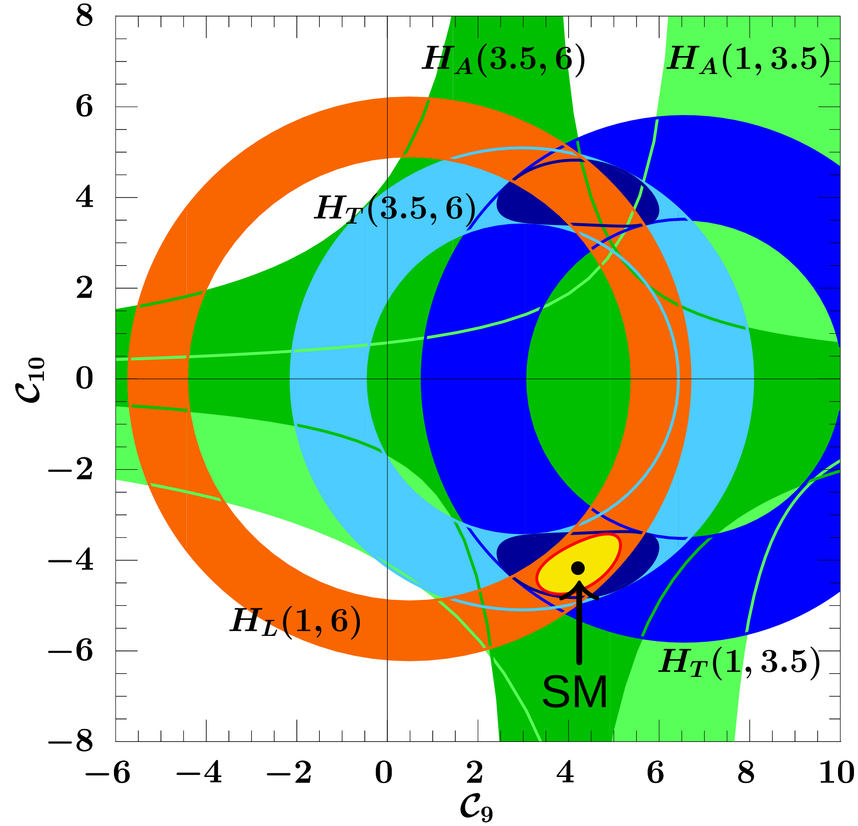

- Separating H_T and H_L is very powerful

- Can extract all information from a few integrated rates

Exclusive $B \to K^*\ell^+\ell^-$ with SCET

- Seven $B \to K^*\ell^+\ell^-$ form factors $\Rightarrow$ fewer functions in the $E \gg \Lambda_{\rm QCD}$ limit

  Angular decomposition: $\zeta(s)$ and $\zeta^J(s)$ $\sim$ non-factorizable and factorizable parts

$$H_T \sim 2s\lambda^3 \bigg\{ \mathcal{C}_{10}^2\, [\zeta_\perp(s)]^2 + \bigg| \mathcal{C}_9\, \zeta_\perp(s) + \frac{2\mathcal{C}_7}{s}\, \frac{m_b}{m_B} \Big[\zeta_\perp(s) + (1-s)\zeta_\perp^J(s)\Big] \bigg|^2 \bigg\}$$

$$H_A \sim -4s\lambda^3\, \mathcal{C}_{10}\, \zeta_\perp(s)\, \mathrm{Re}\bigg\{ \mathcal{C}_9\, \zeta_\perp(s) + \frac{2\mathcal{C}_7}{s}\, \frac{m_b}{m_B} \Big[\zeta_\perp(s) + (1-s)\zeta_\perp^J(s)\Big] \bigg\}$$

$$H_L \sim \frac{1}{2}\, \lambda^3 \bigg(\mathcal{C}_{10}^2 + \bigg| \mathcal{C}_9 + 2\mathcal{C}_7\, \frac{m_b}{m_B} \bigg|^2 \bigg) \Big[\zeta_\parallel(s) - \zeta_\parallel^J(s)\Big]^2 \qquad \Big(\lambda = \sqrt{(1-s)^2 - 2\rho\,(1+s) + \rho^2}\,\Big)$$

- Form factors $\Rightarrow$ few numbers: $\zeta_\perp^{(J)}(s) = \dfrac{\zeta_\perp^{(J)}(0)}{(1-s)^2} \Big[1 + \mathcal{O}\Big(\alpha_s, \dfrac{\Lambda}{E}\Big)\Big]$ $\quad (1.9 < E < 2.7\mathrm{GeV})$

- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B\to K^*\ell^+\ell^-)}$ & $B \to K\ell^+\ell^-$

- $\Gamma(B \to K^*\gamma) = \dfrac{G_F^2}{8\pi^3}\, \dfrac{\alpha_{\rm em}}{4\pi}\, |V_{tb}V_{ts}^*|^2\, m_B^3 (m_b^{1S})^2 (1-\rho)^3\, |\mathcal{C}_7(0)|^2\, \Big[\zeta_\perp(0) + \zeta_\perp^J(0)\Big]^2$

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- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B\to K^*\ell^+\ell^-)}$ & $B \to K\ell^+\ell^-$

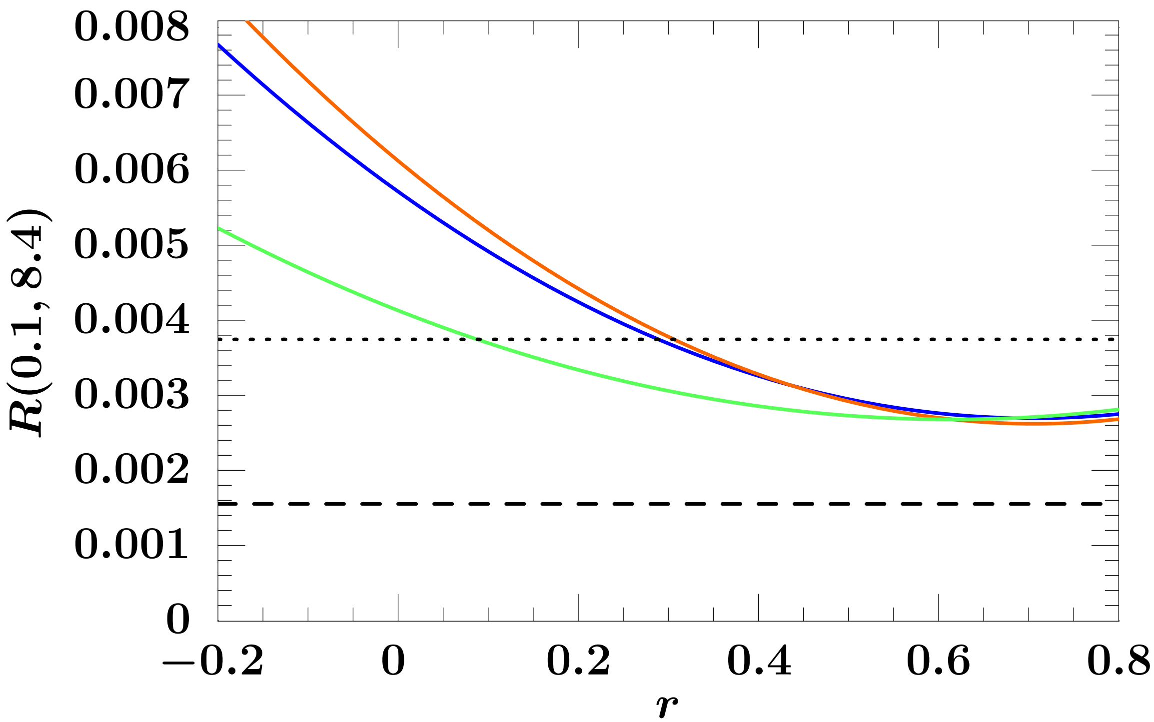
- Three ratios of: $\Gamma(B \to K^*\gamma)$, $H_T(0,8)$, $H_A(0,4)$, $H_A(4,8)$

  Determine: $\mathcal{C}_{10}/\mathcal{C}_7$, $\mathcal{C}_9/\mathcal{C}_7$, and hadronic parameter $\zeta_\perp^J(0)/[\zeta_\perp(0) + \zeta_\perp^J(0)]$

Constraining hadronic physics

$$R(q_1^2, q_2^2) \equiv \frac{H_T(q_1^2, q_2^2)}{\Gamma(B \to K^*\gamma)} = \frac{\alpha_{\rm em}}{12\pi} \frac{m_B^2}{m_b^2} \int_{q_1^2/m_B^2}^{q_2^2/m_B^2} {\rm d}s\, \frac{\lambda^3\, s}{(1-\rho)^3\,(1-s)^4}$$

$$\times \left\{ \frac{\mathcal{C}_{10}^2}{\mathcal{C}_7^2}\,(1-r)^2 + \left[\frac{\mathcal{C}_9}{\mathcal{C}_7}\,(1-r) + \frac{2}{s}\frac{m_b}{m_B}\,(1-sr)\right]^2 \right\}$$



Plot $R(0.1, 8.41)$ and BaBar data

$$r \equiv \frac{\zeta_\perp^J(0)}{\zeta_\perp(0) + \zeta_\perp^J(0)}$$

1σ upper bound

central value

Some expect r to be small

- Will be interesting with more data (expect / predict central value of H_T to increase)

Small q^2 region

Recall: NNLL for $B \to X_s \ell^+ \ell^-$

- Rate depends (mostly) on

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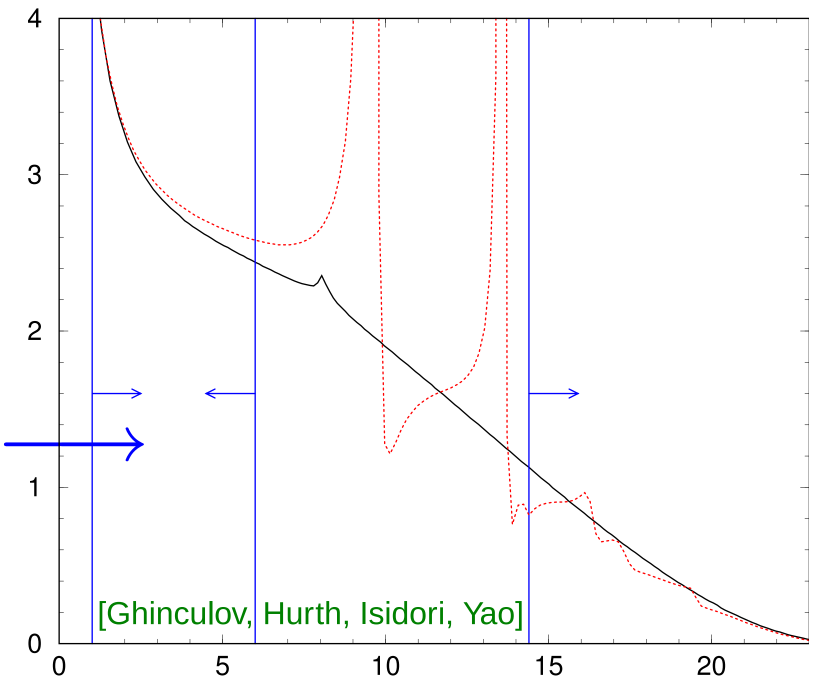
Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$

– NNLL $b \to s \ell^+ \ell^-$ perturbative calculations

– Nonperturbative corrections to q^2 spectrum



- In small q^2 region experiments need additional $m_{X_s} \lesssim 2\,\mathrm{GeV}$ cut to suppress $b \to c(\to s \ell^+ \nu) \ell^- \bar{\nu}$ $\Rightarrow$ nonperturbative effects

- Theory similar to that for phase space cuts in inclusive $|V_{ub}|$ measurements

$B \to X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two kinematic variables are symmetric in $p_{\ell+}$ and $p_{\ell-}$

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\rm QCD}) \qquad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \qquad n, \bar{n} = (1, \pm \vec{p}_X / |\vec{p}_X|)$$

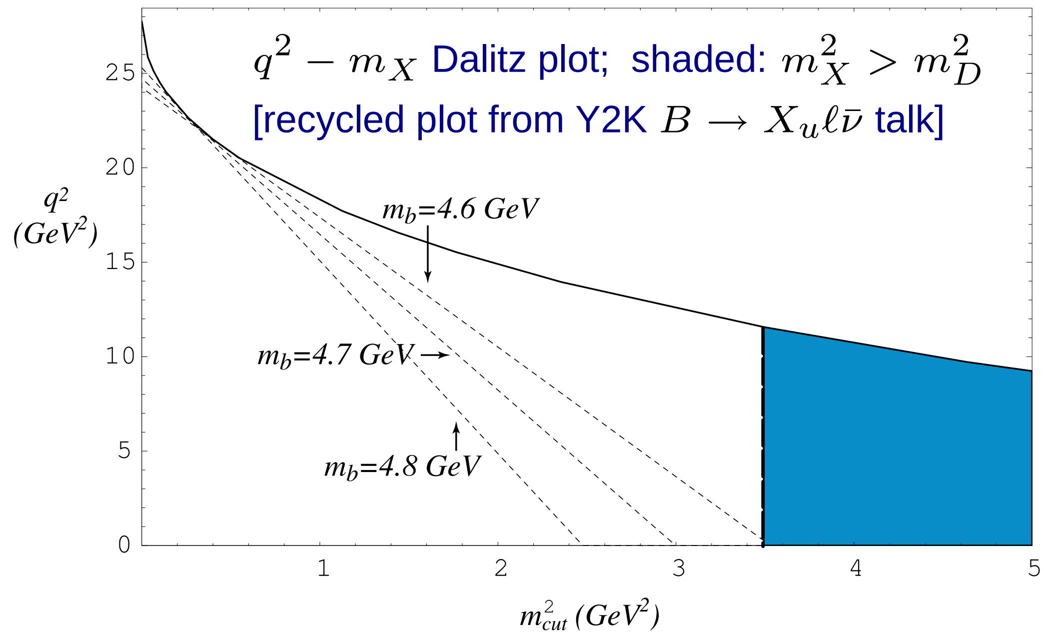
- $p_X^+ \ll p_X^-$: jet-like hadronic final state
- Parton level: $\Gamma \propto f(q^2)\, \delta[(m_b v - q)^2]$

$$m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$$

rate vanishes left of the dashed lines

- Nonperturbative physics is important

Same shape fn as in $B \to X_s \gamma$, $X_u \ell \bar{\nu}$



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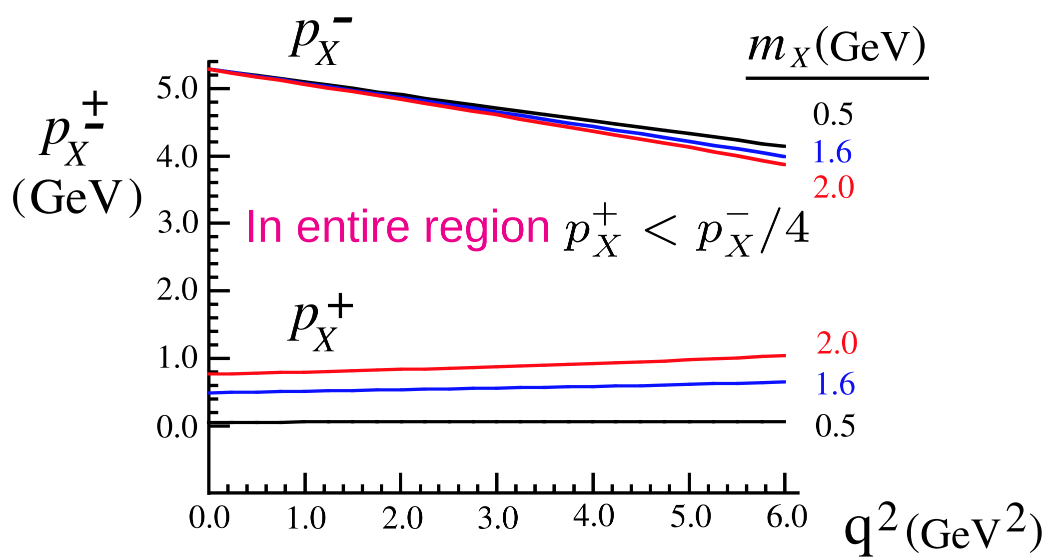
- Nonperturbative physics is important
  Same shape fn as in $B \to X_s \gamma,\ X_u \ell \bar{\nu}$



Effects of m_X cut at lowest order

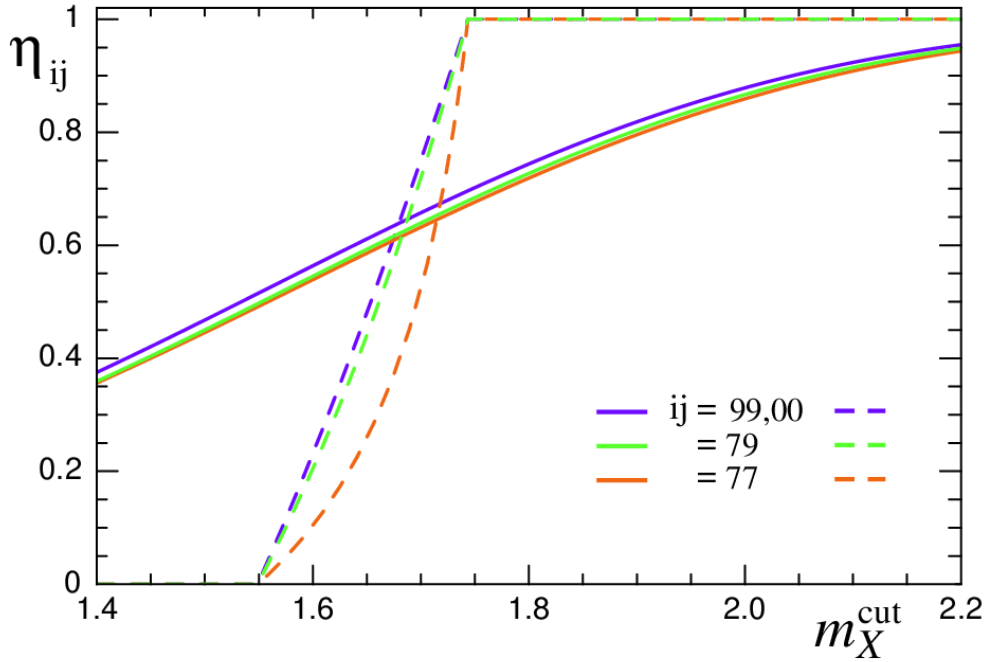
- Define:

$$\eta_{ij} = \frac{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2\,\mathrm{d}m_X^2}}{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2}}$$

ij: C_9^2 and C_{10}^2, $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- η_{ij} determine fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ may be hadronic physics, not new physics

Experiments use Fermi-motion model to incorporate m_X^{cut} effect [Earlier work: Ali & Hiller, '98]

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Effects of m_X cut at lowest order

- Define:

$$\eta_{ij} = \frac{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2\,\mathrm{d}m_X^2}}{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2}}$$

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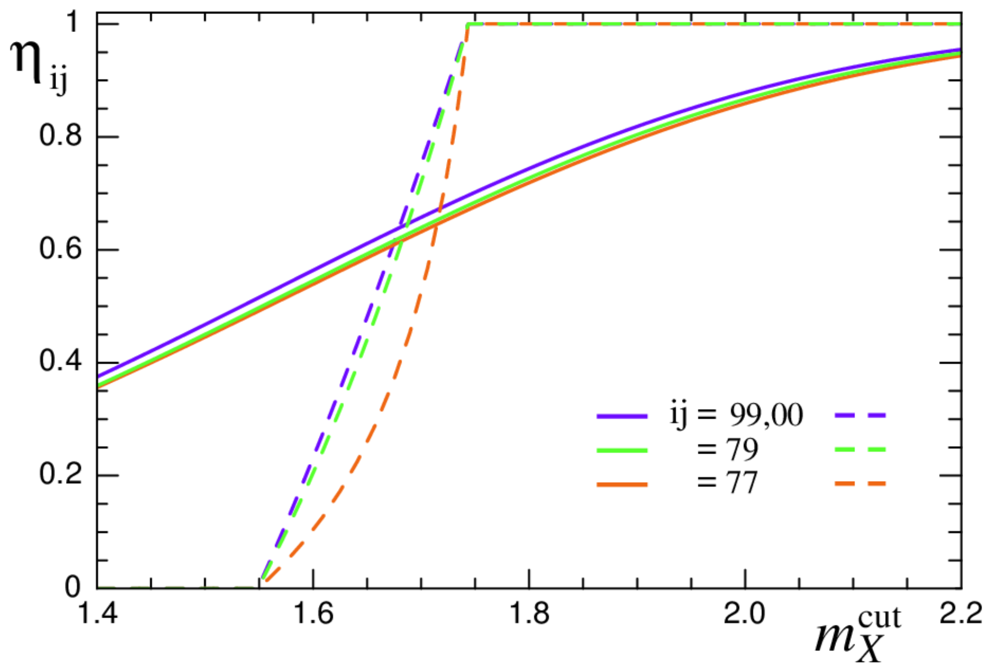

- Strong m_X^{cut} dependence: Raising it (if possible) would reduce uncertainty
  If $1-\eta$ is sizable, so is its uncertainty

- Approximate universality of η_{ij}: since shape function varies on scale $p_X^+/\Lambda_{\mathrm{QCD}}$, while $\Gamma_{ij}^{\mathrm{parton}}$ varies on scale p_X^+/m_b $\Rightarrow$ $\eta \approx \eta_{ij}$

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Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s, treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$

  OK in local OPE region: include small nonpert. corrections ($\lambda_{1,2}$, etc.) at the end

- Shape function region: only the rate is calculable, $\Gamma \sim \mathrm{Im}\,\langle B|\, T\{O_i^\dagger(x)\, O_j(0)\}|B\rangle$

  $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ “enhancement”, but $|C_9(m_b)| \sim C_{10}$

  – Need to take it seriously to cancel scheme- and scale-dependence in running

  – Don’t want power counting: $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2) \sim \langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level

---

- Matching onto SCET: separate μ-dependence in matrix element which cancels that in $m_{\rm weak} \to m_b$ running from dependencies on scales $\mu_i \sim \sqrt{m_b\Lambda_{\rm QCD}}$ and $\mu_\Lambda \sim 1\,\mathrm{GeV}$ — can work to different orders

“Split matching” and running below m_b

- Match H_w at $\mu_0 \sim m_b$ onto scale invariant operators [Lee & Stewart]

$$C_9^{\rm mix}(\mu_0)\,(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\ell) + C_{10}(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\gamma_5\ell) - C_7^{\rm mix}(\mu_0)\,\frac{2\overline{m}_b q_\nu}{q^2}\,(\bar s_L\sigma^{\mu\nu}b_R)_{\mu=m_b}(\bar\ell\gamma_\mu\ell)$$

- Match onto SCET at $\mu_b \sim m_b$ [$\mu_b = \mu_0$ in practice; common anom. dim. below m_b]

- Run down to $\mu_i \sim \sqrt{m_b\Lambda_{\rm QCD}}$, calculate

$$\mathrm{d}^3\Gamma^{(0)} = H\int \mathrm{d}k\,J(k)\,f^{(0)}(k)$$

H and J perturbative, $f^{(0)}$ nonperturbative

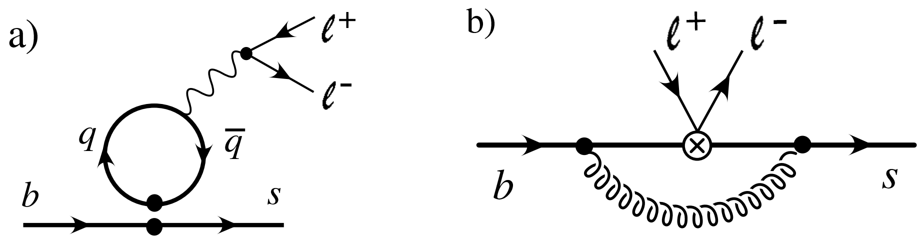
- Take $f^{(0)}(k)$ from $B \to X_s\gamma$, or run model from μ_Λ to μ_i [Bosch, Lange, Neubert, Paz]
(recall: $\Lambda_{\rm QCD}/m_b$ suppressed shape functions are non-universal)

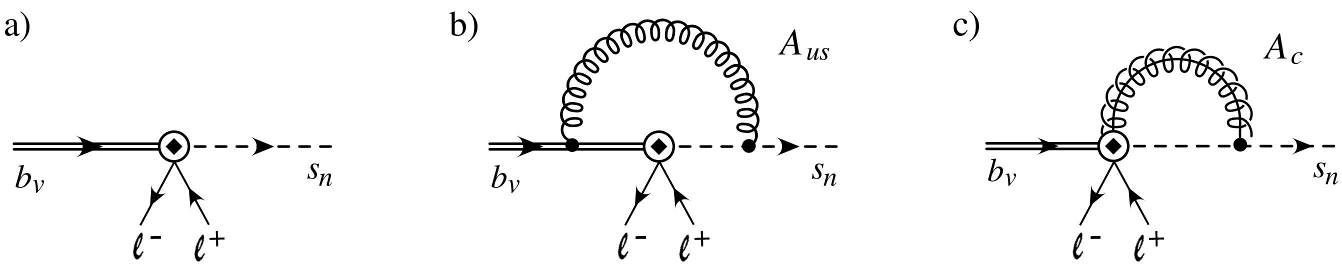
$$f^{(0)}(\hat\omega,\mu_i) = \frac{e^{V_S(\mu_i,\mu_\Lambda)}}{\Gamma(1+\eta)}\left(\frac{\hat\omega}{\mu_\Lambda}\right)^\eta \int_0^1 \mathrm{d}t\,f^{(0)}\Big[\hat\omega(1-t^{1/\eta}),\,\mu_\Lambda\Big] \qquad \eta = \frac{16}{25}\ln\frac{\alpha_s(\mu_\Lambda)}{\alpha_s(\mu_i)}$$

Matching onto SCET

- SCET operators: $J_{\ell\ell}^{(0)} = \sum_{i=a,b,c} C_{9i}(s)\left(\bar\chi_{n,p}\,\Gamma_i^\mu\,\mathcal{H}_v\right)(\bar\ell\gamma_\mu\ell)$ + similar $C_{10,7}$ terms

$$\mathcal{H}_v = Y^\dagger h_v\,, \quad \chi_n = W^\dagger \xi_n\,, \qquad \Gamma^\mu_{a-c} = P_R\left\{\gamma^\mu, v^\mu, \frac{n^\mu}{n\cdot v}\right\}$$





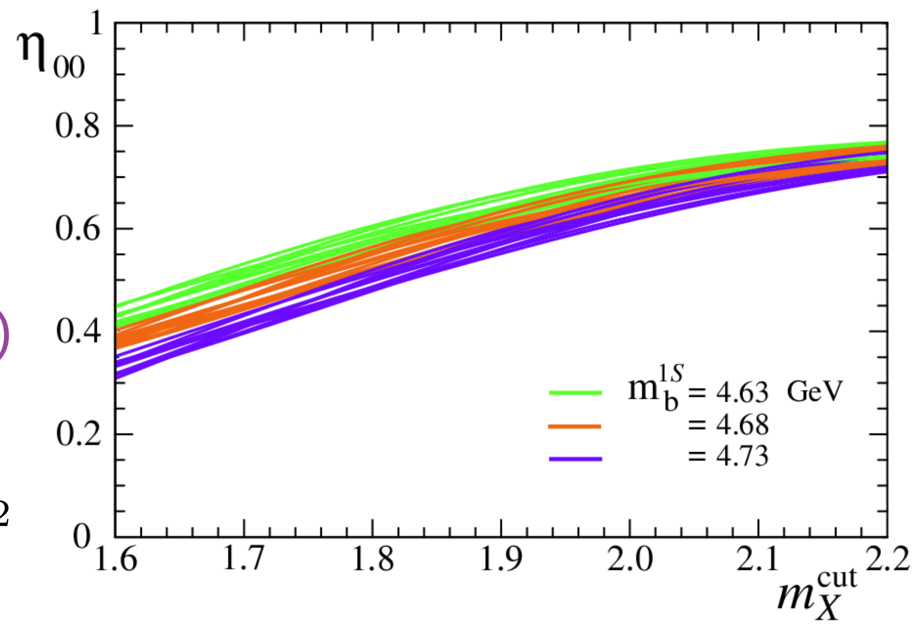
SCET Wilson coefficients: $C_{9a} = \widetilde{C}_9^{\rm eff}[1+\mathcal{O}(\alpha_s)]$ $\qquad C_{9b,c} = \mathcal{O}(\alpha_s)$

- Some parts of the "usual" NLL $\mathcal{O}(\alpha_s)$ corrections included in $\widetilde{C}_9^{\rm eff}$ [Misiak; Buras, Munz] now contribute to the jet function, J, some others to the shape function, $f^{(0)}(k)$

Including NLL corrections

- Universality maintained; estimate shape function uncertainties using $B \to X_s\gamma$
- Find for $\mathcal{B}(1 < q^2 < 6\,\mathrm{GeV}^2)/10^{-6}$

$$m_X^{\rm cut} = 1.8\,\mathrm{GeV}:\ \ 1.20 \pm 0.15$$
$$m_X^{\rm cut} = 2.0\,\mathrm{GeV}:\ \ 1.48 \pm 0.14$$
NNLL, no m_X cut: 1.57 ± 0.11



- $A_{\rm FB}$ only slightly affected (a-priori nontrivial)
  Find $q_0^2 \sim 3\,\mathrm{GeV}^2$, lower than earlier results
- NNLL reduces μ dependence, effect on q^2 spectrum small $\Rightarrow$ expect $\eta^{(\rm NLL)} \approx \eta^{(\rm NNLL)}$

- If increasing $m_X^{\rm cut}$ above $2\,\mathrm{GeV}$ hard $\Rightarrow$ keep $m_X^{\rm cut} < m_D$, normalize to $B \to X_u \ell\bar\nu$ with same cuts:

$$R = \Gamma^{\rm cut}(B \to X_s \ell^+\ell^-)/\Gamma^{\rm cut}(B \to X_u \ell\bar\nu)$$

Both shape function ($m_X^{\rm cut}$) and m_b dependence drastically reduced

Is $C_7(m_b) = -C_7^{\rm SM}(m_b)$ excluded?

- Inclusive: rate in small q^2 region, in units of 10^{-6} (world average: 1.60 ± 0.51)

[Gambino, Haisch, Misiak, hep-ph/0410155]

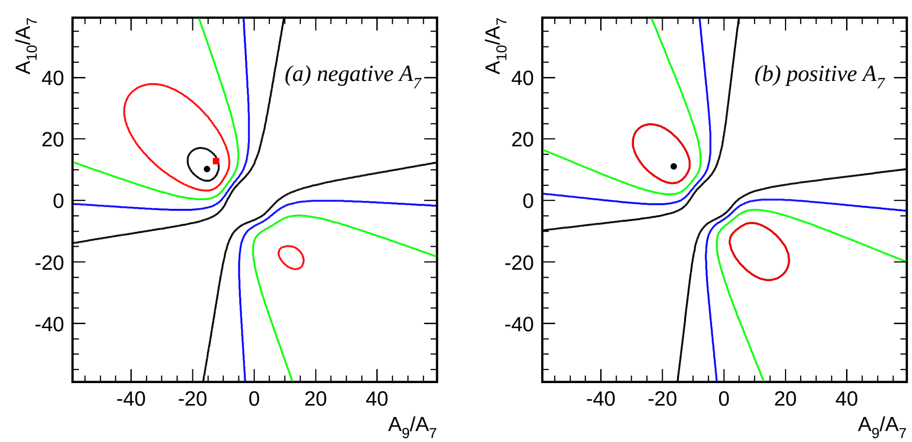
TABLE II: Predictions for $\mathcal{B}(\bar B \to X_s l^+ l^-)$ $[10^{-6}]$ in the Standard Model and with reversed sign of $\widetilde C_7^{\rm eff}$ for the same ranges of q^2 as in Tab. I.

Range	SM	$\widetilde C_7^{\rm eff} \to -\widetilde C_7^{\rm eff}$
(a)	4.4 ± 0.7	8.8 ± 1.0
(b)	1.57 ± 0.16	3.30 ± 0.25

$\widetilde C_7^{\rm eff} \to -\widetilde C_7^{\rm eff}$ is not the best way to proceed

"Preliminary"	$m_X^{\rm cut}$	rate $(\mathcal{C}_7^{\rm SM})$	rate $(\mathcal{C}_7^{\rm non-SM})$
NNLL "GHM"	—	1.57	3.18
NNLL "us"	—	1.57	2.99
NLL	—	1.74	3.61
NLL	$2.0\,{\rm GeV}$	1.35	3.09
NLL	$1.8\,{\rm GeV}$	1.10	2.49

- Exclusive: with some model dependence, Belle's $A_{\rm FB}$ measurement fixes sign of C_9/C_{10}, but not sign of C_7 relative to $C_{9,10}$



- I also think $C_7 > 0$ is unlikely, but probably disfavored only about the 2σ level

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Large q^2 region

Large q^2 region: complementary with small q^2

- Theory: largest errors (i) expansion in $\Lambda_{\rm QCD}/(m_b - \sqrt{q^2})$; (ii) huge m_b dependence

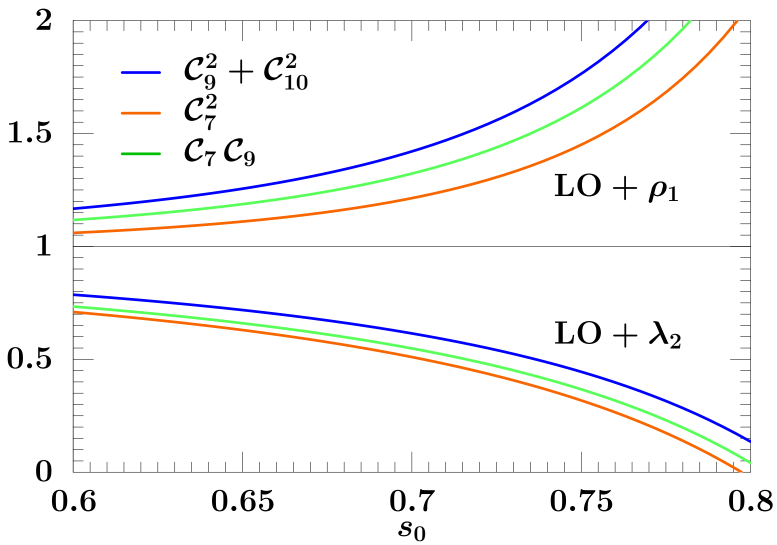
  Experiment: smaller rate, but higher efficiency

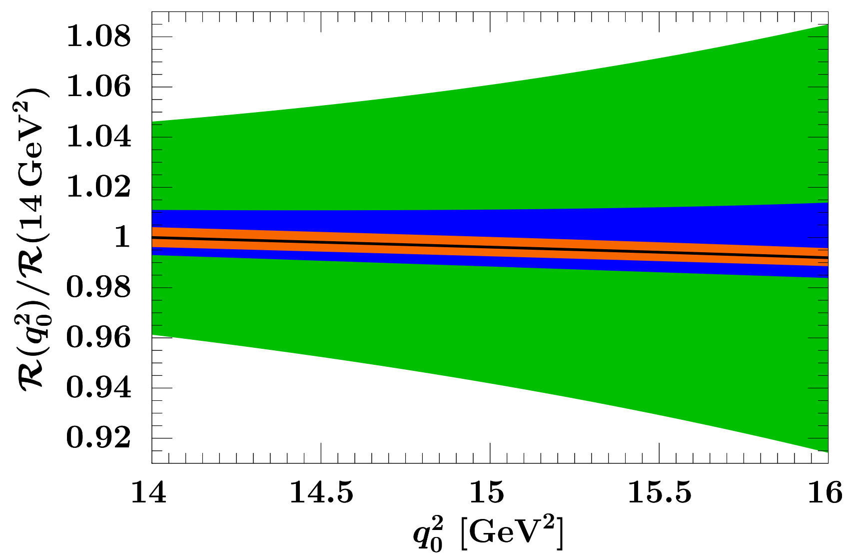
- Both can be reduced / eliminated $\Rightarrow$ uncertainty $\sim 5\%$ (missing NNLL at large q^2)

$$\frac{\displaystyle\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B \to X_s \ell^+\ell^-)}{dq^2}\, dq^2}{\displaystyle\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B^0 \to X_u \ell\bar\nu)}{dq^2}\, dq^2} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2}\, \frac{\alpha_{\rm em}^2}{8\pi^2}\, \mathcal{R}(q_0^2)$$

uncertainties suppressed by:

$$1 - \frac{(\mathcal{C}_9 + 2\mathcal{C}_7)^2 + \mathcal{C}_{10}^2}{\mathcal{C}_9^2 + \mathcal{C}_{10}^2} \simeq 0.12$$





Large q^2 region measured in $B \to X_u \ell \bar\nu$

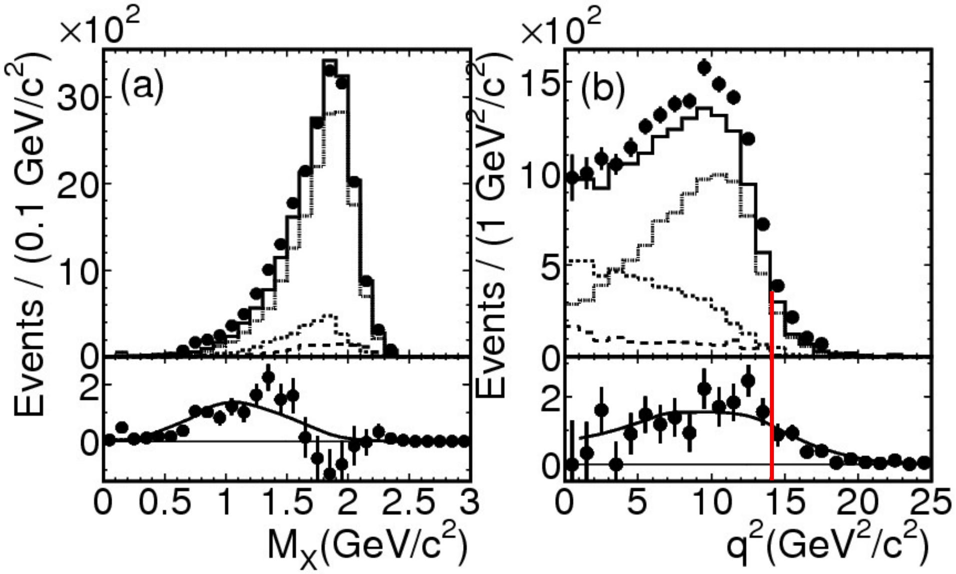
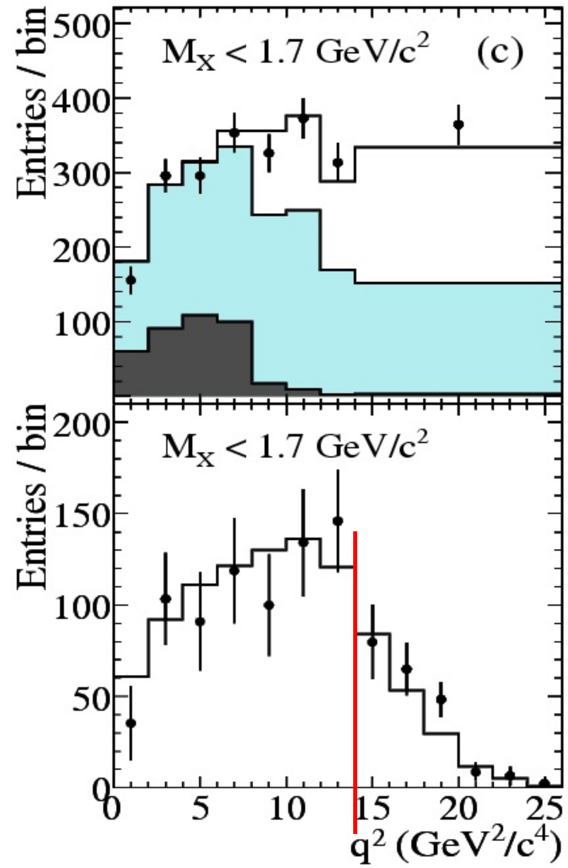


FIG. 4: (a) M_X distribution for $q^2 > 8.0$ GeV$^2/c^2$. (b) q^2 distribution for $M_X < 1.7$ GeV$/c^2$. Points are the data and histograms are backgrounds from $D^*\ell\nu$ (dotted), $D\ell\nu$ (short dashed), others (long dashed), and total background contribution (solid). Lower plots show the data after background subtraction. Solid curves show the inclusive MC predictions for $B \to X_u \ell\nu$.

Belle, $87\,\mathrm{fb}^{-1}$, PRL **92** (2004) 101801 [hep-ex/0311048]



BaBar, $383\,\mathrm{m}\ \Upsilon$, arXiv:0708.3702

- The $m_X > 1.7\,\mathrm{GeV}$ cut is irrelevant for $q^2 > 12.8\,\mathrm{GeV}^2$ (up to resolution effects)
- Separating B^0 from $B^\pm$ is important to control 4-quark operator contribution (WA)

Summary and conclusions

- To achieve maximal sensitivity to NP, separate rate: $1+\cos^2\theta$, $1-\cos^2\theta$, $\cos\theta$ (both in inclusive & exclusive; latter can be analyzed without form factor models)
- Fits to 2-dim distributions and zero of $A_{\rm FB}$ not essential, just a few integrated rates
- Small q^2: measured rate depends on shape function (local OPE is not sufficient)
  SF region: expansion for rate, not the amplitude, reorganize perturbation theory
- First model independent calculation of $q^2 < 6\,\text{GeV}^2$ rate in the presence of $m_X^{\rm cut}$
- Theoretical uncertainty at large q^2 can approach 5% by combining measurements
  BaBar & Belle should quote $\mathcal{B}(B\to X_u\ell\bar\nu)\big|_{q^2>14,\,15\text{GeV}^2}$, try to separate B^0 / $B^\pm$
- Must take hadronic effects into account using $B\to X_s\gamma$ and / or $B\to X_u\ell\bar\nu$ data
  Sensitivity to NP can survive both in small and large q^2 regions