# Inclusive $|V_{ub}|$ unlimited

**Zoltan Ligeti**

**SCET Workshop, MIT, March 24–26, 2009**

# Outline

- Introduction
- Complete description of $B \to X_s\gamma$ [ZL, Stewart, Tackmann, arXiv:0807.1926]
- Complete description of $B \to X_u\ell\bar{\nu}$ [ZL, Stewart, Tackmann, to appear]
- A glimpse at SIMBA [Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, to appear]
- Conclusions

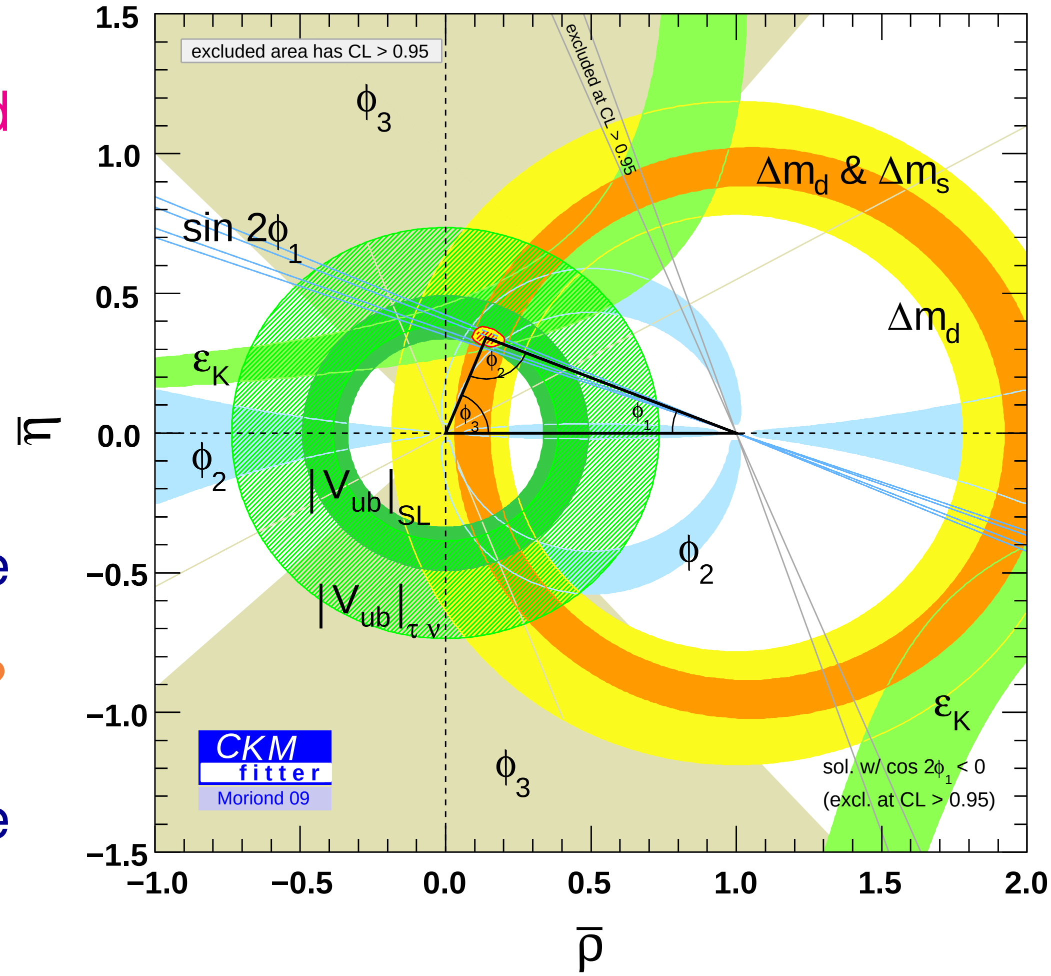
# The importance of $|V_{ub}|$

- $|V_{ub}|$ determined by tree-level decays
  Crucial for comparing tree-dominated and loop-mediated processes

- $|V_{ub}|_{\pi\ell\bar{\nu}-\mathrm{LQCD}} = (3.5 \pm 0.5) \times 10^{-3}$
  $|V_{ub}|_{\mathrm{incl-BLNP}} = (4.32 \pm 0.35) \times 10^{-3}$
  $|V_{ub}|_{\tau\nu} = (5.2 \pm 0.5 \pm 0.4_{f_B}) \times 10^{-3}$
  SM CKM fit, $\sin 2\phi_1$, favors small value

- Fluctuation, bad theory, new physics?

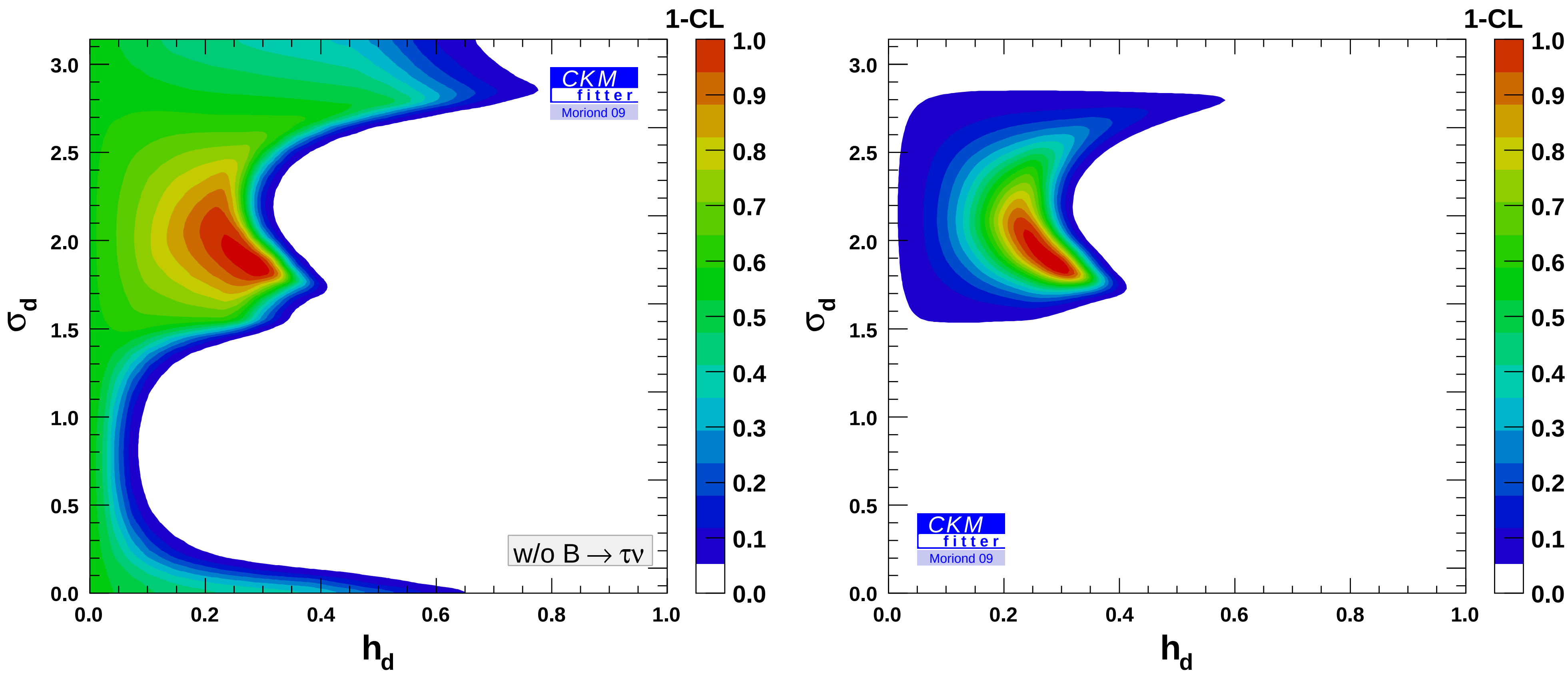
- The level of agreement between the measurements often misinterpreted




- $10$–$20\%$ non-SM contributions to most loop-mediated transitions are still possible

# $|V_{ub}|$ matters...

- Including $B \to \tau\bar{\nu}$, the SM is "disfavored" at $> 2\sigma$




Parameterize NP in $B^0$–$\overline{B}^0$ mixing: $M_{12} = M_{12}^{\rm SM}\,(1 + h_d\, e^{2i\sigma_d})$

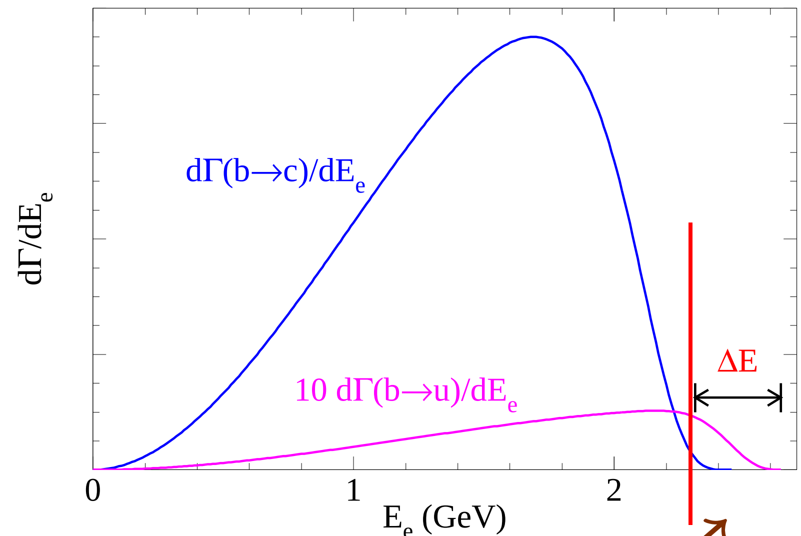
- $10$–$20\%$ non-SM contributions to most loop-mediated transitions are still possible

# The challenge of inclusive $|V_{ub}|$ measurements

- Total rate known with $\sim 4\%$ accuracy, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$ [Hoang, ZL, Manohar]

- To remove the huge charm background $(|V_{cb}/V_{ub}|^2 \sim 100)$, need phase space cuts

  Phase space cuts can enhance perturbative and nonperturbative corrections drastically




  Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ and determine shape

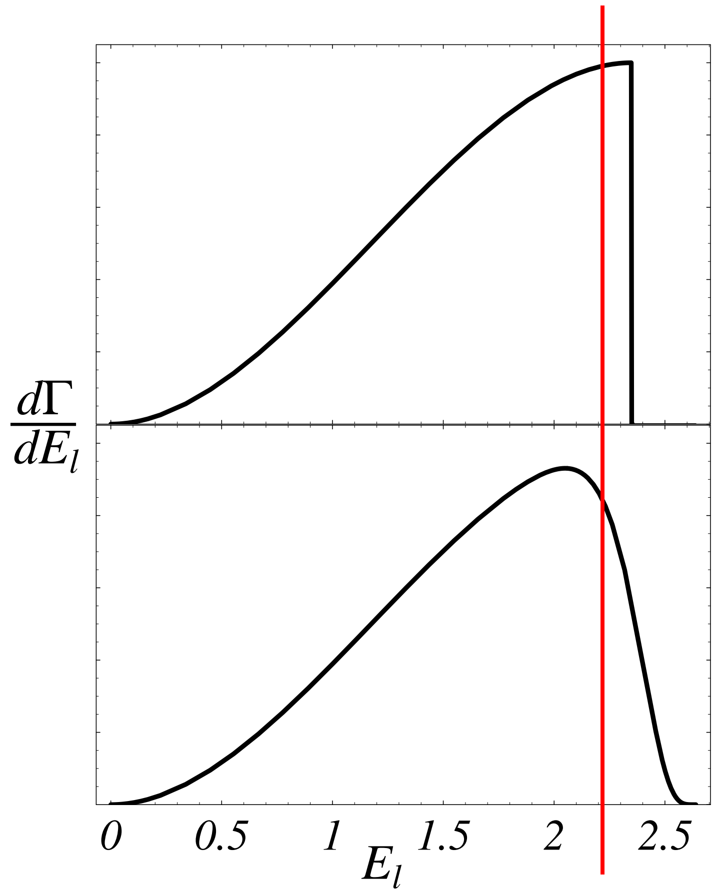
- Endpoint region determined by $b$ quark PDF in $B$; want to extract it from data to make predictions — at lowest order $\propto B \to X_s \gamma$ photon spectrum

[Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

# $|V_{ub}|$: lepton endpoint vs. $B \to X_s\gamma$

$b$ quark decay spectrum

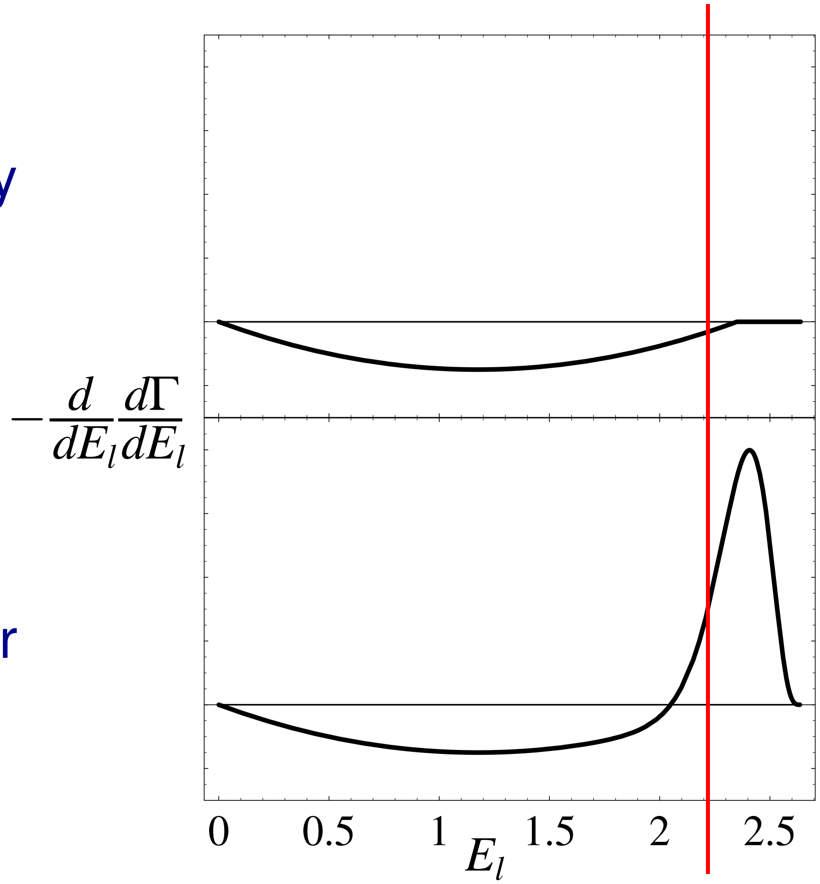
with a model for $b$ quark PDF



# $|V_{ub}|$: lepton endpoint vs. $B \to X_s\gamma$

$b$ quark decay spectrum

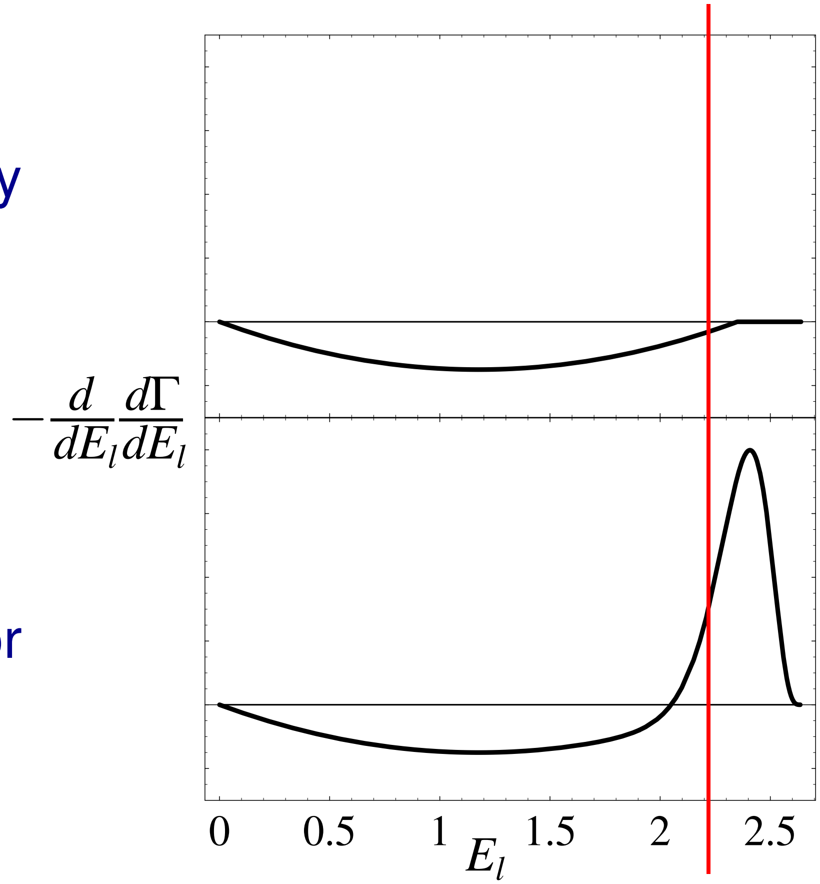
with a model for $b$ quark PDF



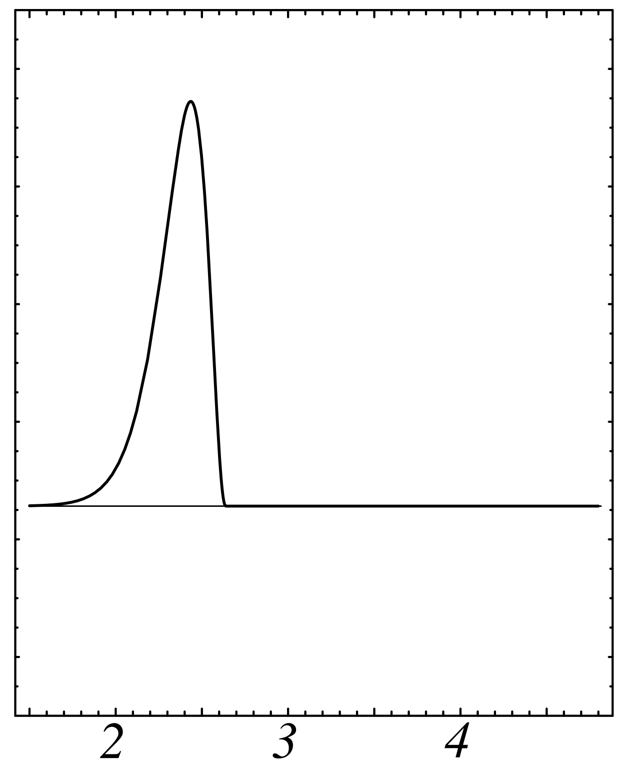
# $|V_{ub}|$: lepton endpoint vs. $B \to X_s\gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF



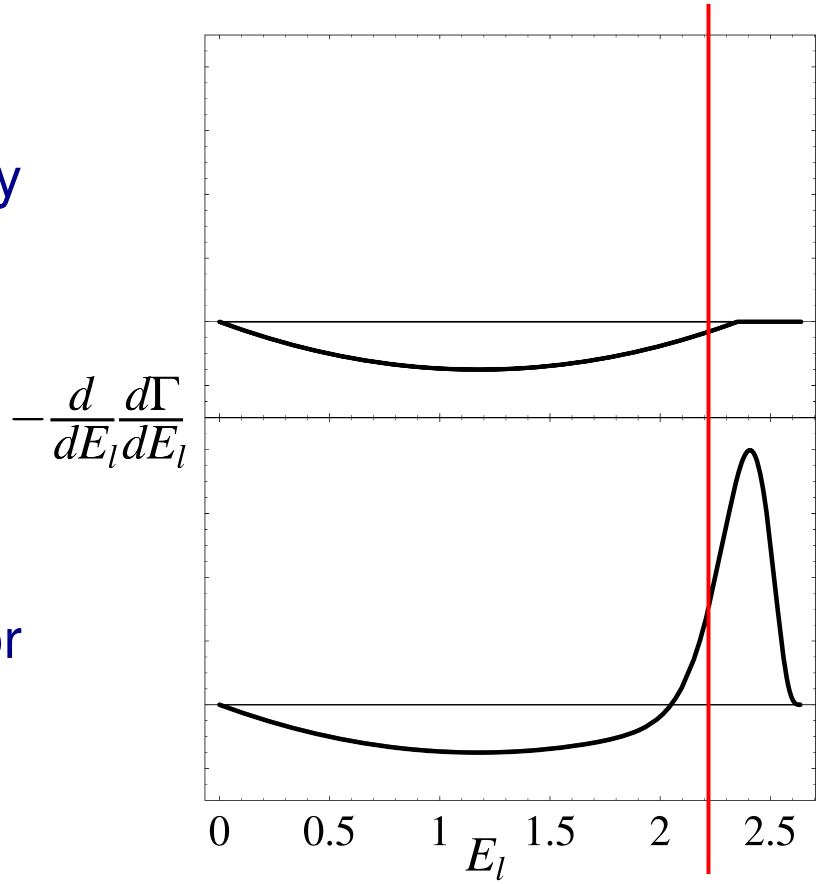

difference:



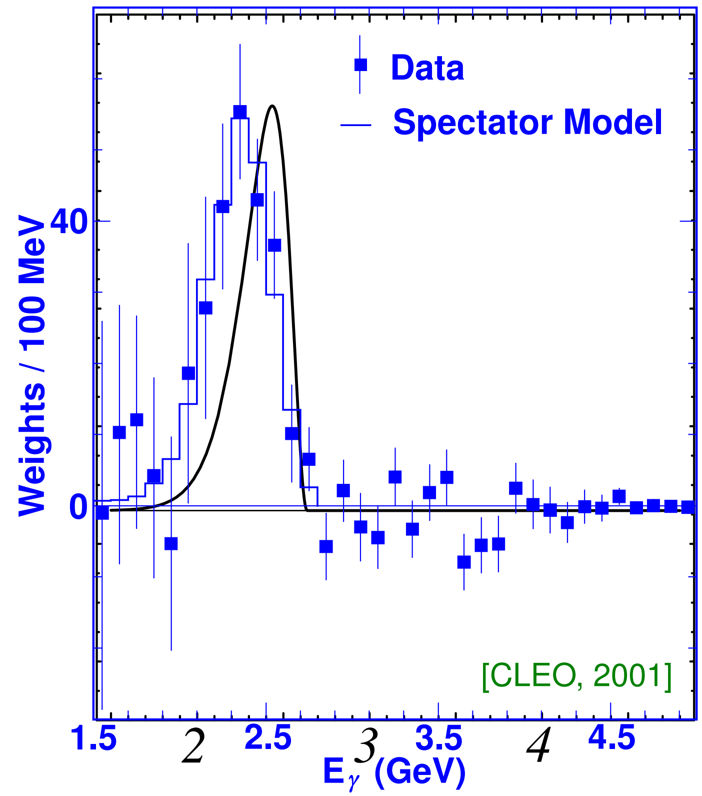
# $|V_{ub}|$: lepton endpoint vs. $B \to X_s \gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF



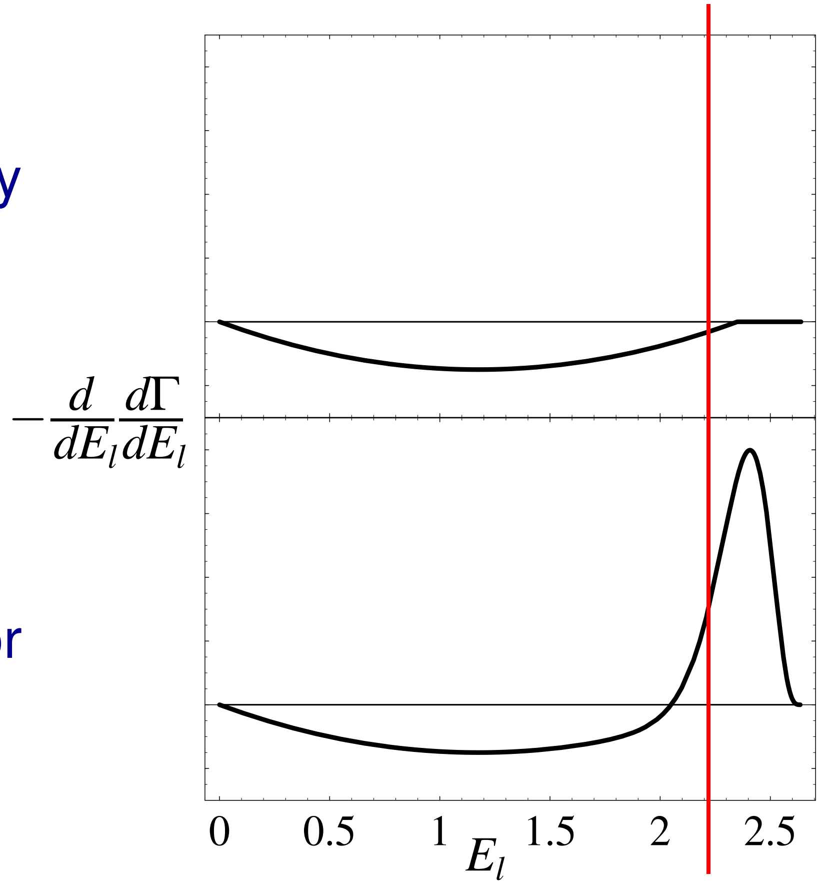

difference:



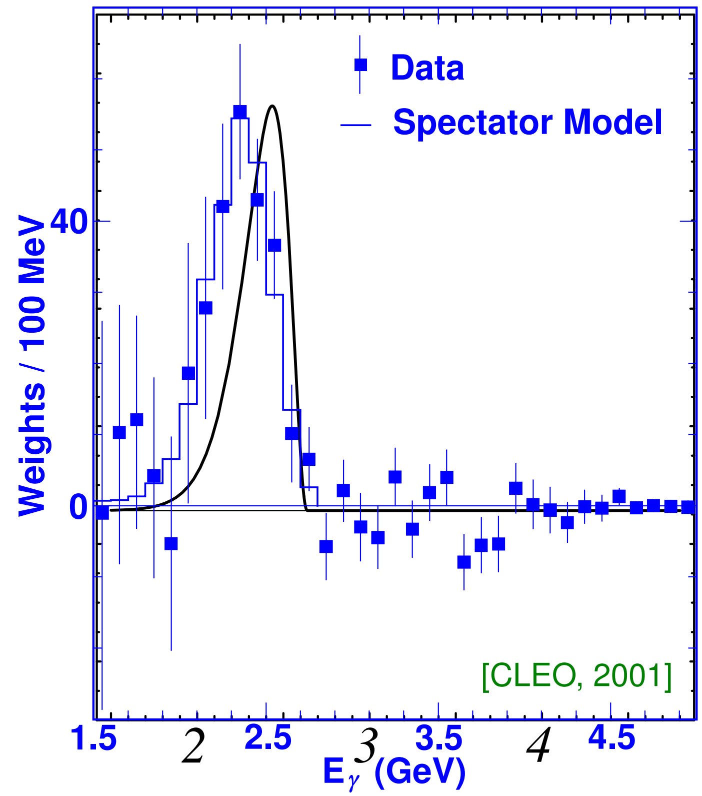
# $|V_{ub}|$: lepton endpoint vs. $B \to X_s\gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF




difference:




- Both of these spectra are determined at lowest order by the $b$ quark PDF in the $B$
- Lots of work toward extending beyond leading order; many open issues...

# Past efforts: BLNP (best so far)

- Treated factorization & resummation in shape function region correctly
- Use fixed functional forms to model shape function (similar to PDF's)
- Analysis tied to shape function scheme for $m_b$, $\lambda_1$ (One scheme for each approach)
- Need for "tail gluing" to get correct perturbative tail (other approaches don't even do this [DGE, GGOU, ADFR])

$\Rightarrow$ Hard to assess uncertainties

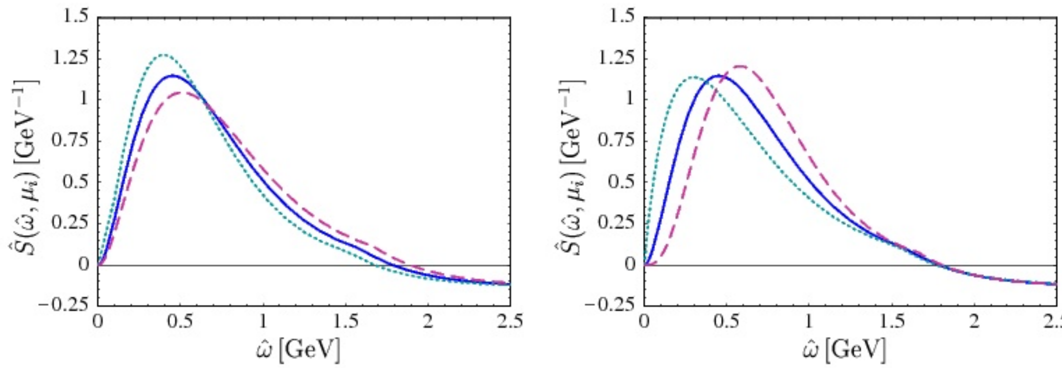



Figure 6: Various models for the shape function at the intermediate scale $\mu_i = 1.5$ GeV, corresponding to different parameter settings in Table 1. Left: Functions S1, S5, S9 with "correlated" parameter variations. Right: Functions S3, S5, S7 with "anti-correlated" parameter variations.

[Bosch, Lange, Neubert, Paz]

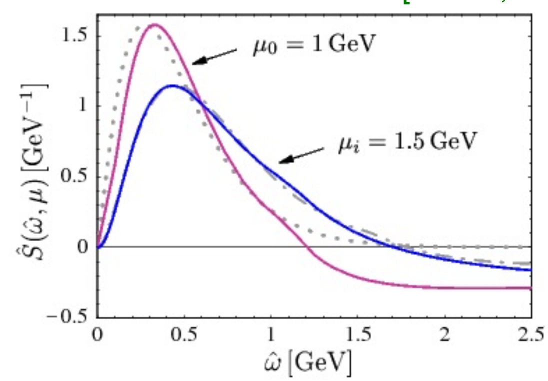



Figure 7: Renormalization-group evolution of a model shape function from a low scale $\mu_0$ (sharply peaked solid curve) to the intermediate scale $\mu_i$ (broad solid curve). See the text for an explanation of the other curves.

## Start with $B \to X_s \gamma$

[ZL, Stewart, Tackmann, PRD 78 (2008) 114014, arXiv:0807.1926]

# Regions of phase space

- $B \to X_s\gamma$ gives one of the strongest bounds on NP
- $p_X^+ \equiv m_B - 2E_\gamma \lesssim 2\,\mathrm{GeV}$, and $< 1\,\mathrm{GeV}$ at the peak

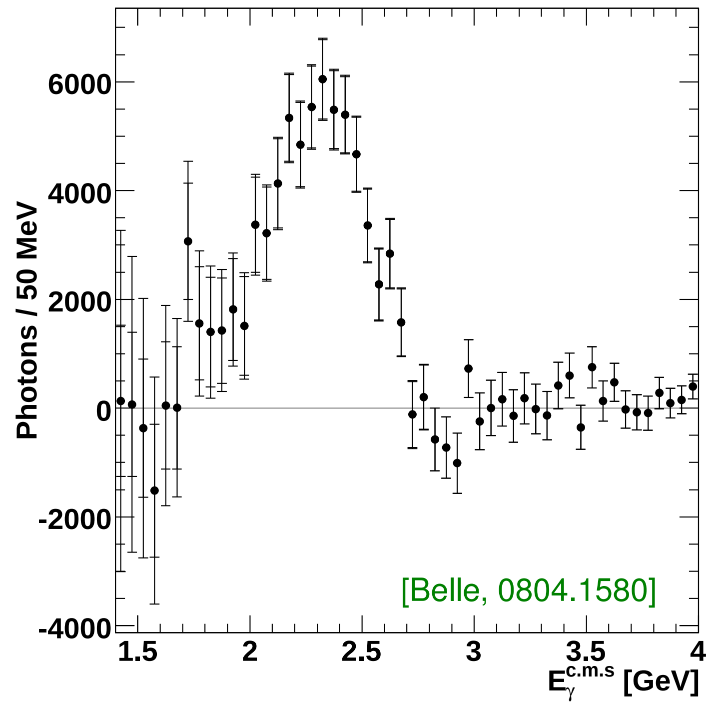
Three cases: 1) $\Lambda \sim m_B - 2E_\gamma \ll m_B$

2) $\Lambda \ll m_B - 2E_\gamma \ll m_B$

3) $\Lambda \ll m_B - 2E_\gamma \sim m_B$

[Sometimes called 1) SCET and 2) MSOPE]

- Neither 1) nor 2) is fully appropriate




- Include all known results in regions 1) – 2)

| | | | |
|---|---|---|---|
| LL: | 1–loop $\Gamma_{\rm cusp}$, | tree-level matching | |
| NLL: | 2–loop $\Gamma_{\rm cusp}$, | 1–loop matching, | 1–loop $\gamma_x$ |
| NNLL: | 3–loop $\Gamma_{\rm cusp}$, | 2–loop matching, | 2–loop $\gamma_x$ |

We can combine 1) – 2) without expanding shape function in $\Lambda/p_X^+$

# The shape function ($b$ quark PDF in $B$)

- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

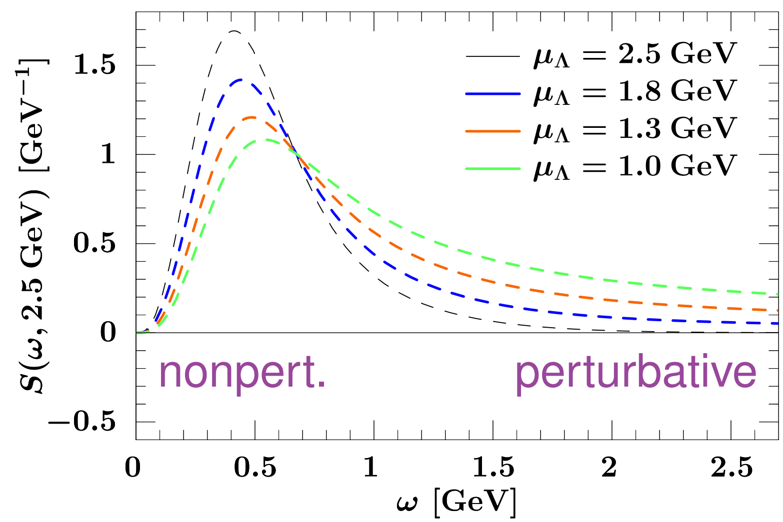
If $S(\omega, \mu_\Lambda)$ has exponentially small tail, any small running gives a long tail and divergent moments

[Balzereit, Mannel, Kilian]

$$S(\omega, \mu_i) = \int \mathrm{d}\omega' \, U_S(\omega - \omega', \mu_i, \mu_\Lambda) \, S(\omega', \mu_\Lambda)$$

Constraint: moments (OPE) + $B \to X_s \gamma$ shape

How to combine these?




Model $\begin{cases} S & \text{(dash)} \\ & \end{cases}$ run to $2.5\mathrm{GeV}$

# The shape function ($b$ quark PDF in $B$)

- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

If $S(\omega, \mu_\Lambda)$ has exponentially small tail, any small running gives a long tail and divergent moments [Balzereit, Mannel, Kilian]

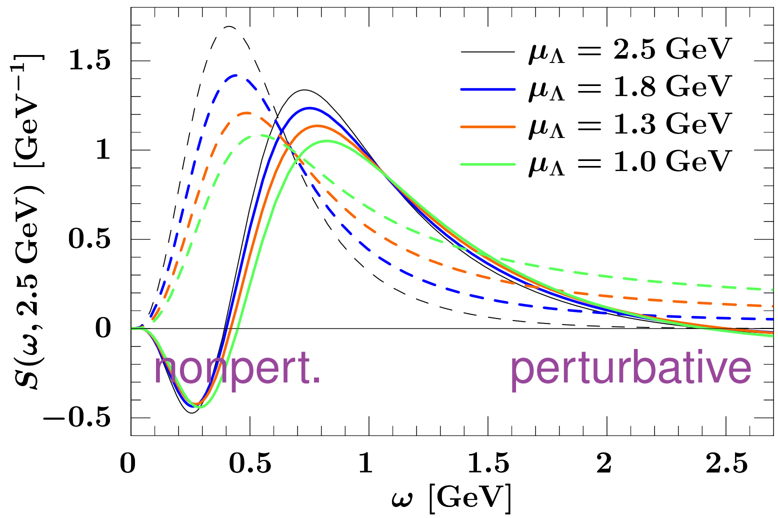
$$S(\omega, \mu_i) = \int d\omega'\, U_S(\omega - \omega', \mu_i, \mu_\Lambda)\, S(\omega', \mu_\Lambda)$$

Constraint: moments (OPE) + $B \to X_s \gamma$ shape

How to combine these?

Derive: [ZL, Stewart, Tackmann, 0807.1926]

$$S(\omega, \mu_\Lambda) = \int dk\, C_0(\omega - k, \mu_\Lambda) F(k)$$




– Consistent setup at any order, in any scheme

– Stable results for varying $\mu_\Lambda$ (SF modeling scale, must be part of uncert.)

– Similar to how all matrix elements are defined e.g., $B_K(\mu) = \widehat{B}_K\, [\alpha_s(\mu)]^{2/9}(1 + \ldots)$

Model $\begin{cases} S & \text{(dash)} \\ F & \text{(solid)} \end{cases}$ run to $2.5\mathrm{GeV}$

- Consistent to impose moment constraints on $F(k)$, but not on $S(\omega, \mu_\Lambda)$ w/o cutoff

# Derivation of the magic formula (1)

- The shape function is the matrix element of a nonlocal operator:

$$S(\omega,\mu) = \langle B|\, \underbrace{\bar b_v\, \delta(iD_+ - \delta + \omega)\, b_v}_{O_0(\omega,\mu)} \,|B\rangle , \qquad \delta = m_B - m_b$$

Integrated over a large enough region, $0 \le \omega \le \Lambda$, one can expand $O_0$ as

$$O_0(\omega,\mu) = \sum C_n(\omega,\mu)\ \underbrace{\bar b_v\, (iD_+ - \delta)^n\, b_v}_{Q_n} + \ldots = \sum C_n(\omega-\delta,\mu)\ \underbrace{\bar b_v\, (iD_+)^n\, b_v}_{\widetilde Q_n} + \ldots$$

The $C_n$ are the same for $Q_n$ and $\widetilde Q_n$ (since $O_0$ only depends on $\omega - \delta$)

- Matching: $\langle b_v| O_0(\omega{+}\delta,\mu) |b_v\rangle = \sum C_n(\omega,\mu)\, \langle b_v|\widetilde Q_n|b_v\rangle = C_0(\omega,\mu), \quad \langle b_v|\widetilde Q_n|b_v\rangle = \delta_{0n}$

$$\langle b_v(k_+)| O_0(\omega+\delta,\mu) |b_v(k_+)\rangle = C_0(\omega + k_+,\mu) = \sum \frac{k_+^n}{n!}\, \frac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$$

$$\langle b_v(k_+)| O_0(\omega+\delta,\mu) |b_v(k_+)\rangle = \sum C_n(\omega,\mu) \langle b_v|\widetilde Q_n|b_v\rangle = \sum C_n(\omega,\mu)\, k_+^n$$

- Comparing last two lines: $C_n(\omega,\mu) = \dfrac{1}{n!}\, \dfrac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$ [Bauer & Manohar]

# Derivation of the magic formula (2)

- Define the nonperturbative function $F(k)$ by: [ZL, Stewart, Tackmann; Lee, ZL, Stewart, Tackmann]

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega - k, \mu_\Lambda)\, F(k), \qquad C_0(\omega, \mu) = \langle b_v | O_0(\omega + \delta, \mu) | b_v \rangle$$

uniquely defines $F(k)$: $\widetilde{F}(y) = \widetilde{S}(y, \mu) / \widetilde{C}_0(y, \mu)$

- Expand in $k$: $S(\omega, \mu) = \sum_n \frac{1}{n!} \frac{\mathrm{d}^n C_0(\omega, \mu)}{\mathrm{d}\omega^n} \int \mathrm{d}k\, (-k)^n F(k)$

Compare with previous page $\Rightarrow$ $\int \mathrm{d}k\, k^n F(k) = (-1)^n \langle B | Q_n | B \rangle$

$$\langle B | Q_0 | B \rangle = 1\,, \quad \langle B | Q_1 | B \rangle = -\delta\,, \quad \langle B | Q_2 | B \rangle = -\frac{\lambda_1}{3} + \delta^2$$

More complicated situation for higher moments, so stop here

- This treatment is fully consistent with the OPE

# Changing schemes: $m_b$

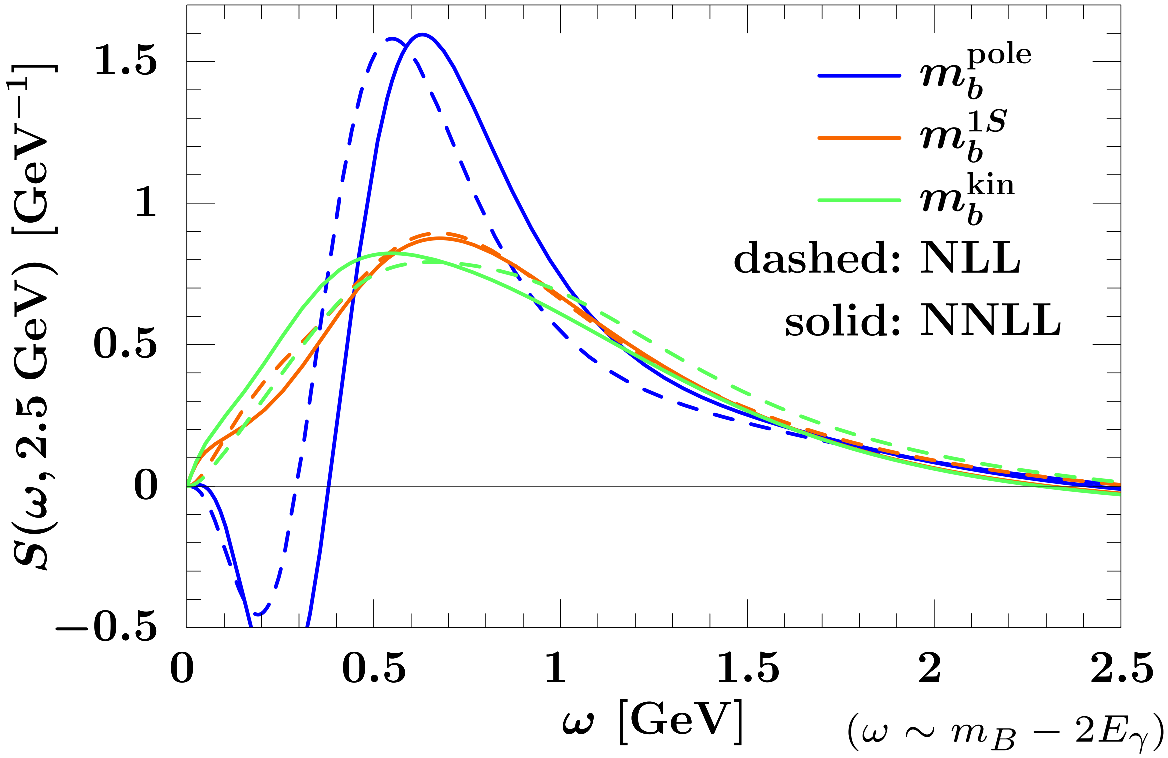
- Converting results to a short distance mass scheme removes dip at small $\omega$:

- Want to define short distance (hatted) quantities such that:

$$S(\omega) = \int \mathrm{d}k\, C_0(\omega - k)\, F(k) = \int \mathrm{d}k\, \widehat{C}_0(\omega - k)\, \widehat{F}(k)$$

Switch from pole to short distance scheme:

$$m_b = \widehat{m}_b + \delta m_b$$
$$\lambda_1 = \widehat{\lambda}_1 + \delta\lambda_1$$



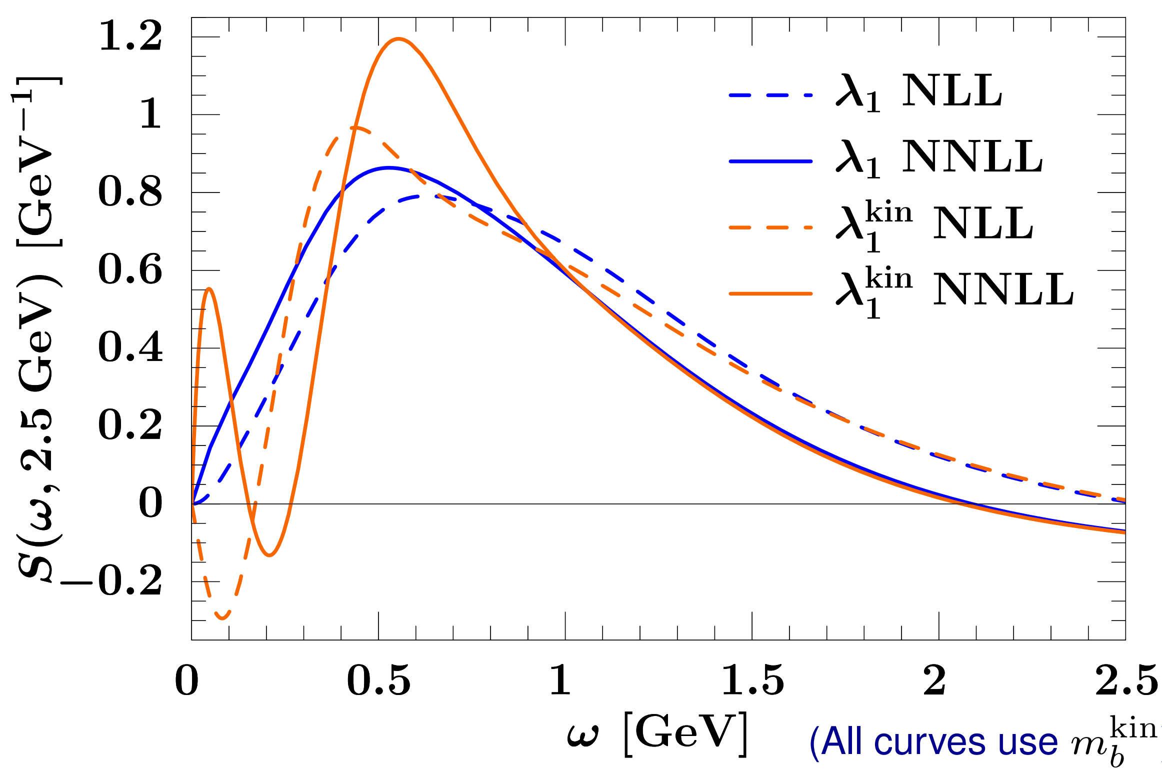

$(\omega \sim m_B - 2E_\gamma)$

$$\widehat{C}_0(\omega) = C_0(\omega + \delta m_b) - \frac{\delta\lambda_1}{6}\frac{\mathrm{d}^2}{\mathrm{d}\omega^2} C_0(\omega) = \left[1 + \delta m_b \frac{\mathrm{d}}{\mathrm{d}\omega} + \left(\frac{(\delta m_b)^2}{2} - \frac{\delta\lambda_1}{6}\right)\frac{\mathrm{d}^2}{\mathrm{d}\omega^2}\right] C_0(\omega)$$

- Can use any short distance mass scheme ($1S$, kinetic, PS, shape function, ...)

# Changing schemes: $\lambda_1$

- We find that kinetic scheme, $\mu_\pi^2 \equiv -\lambda_1^{\rm kin}$, oversubtracts; similar to $\overline{m}_b(\overline{m}_b)$ issues
- Introduce "invisible" scheme: $\lambda_1^{\rm i} = \lambda_1 - 0\alpha_s - R^2 \frac{\alpha_s^2(\mu)}{\pi^2} \frac{C_F C_A}{4} \left(\frac{\pi^2}{3} - 1\right) \quad (R = 1\,{\rm GeV})$

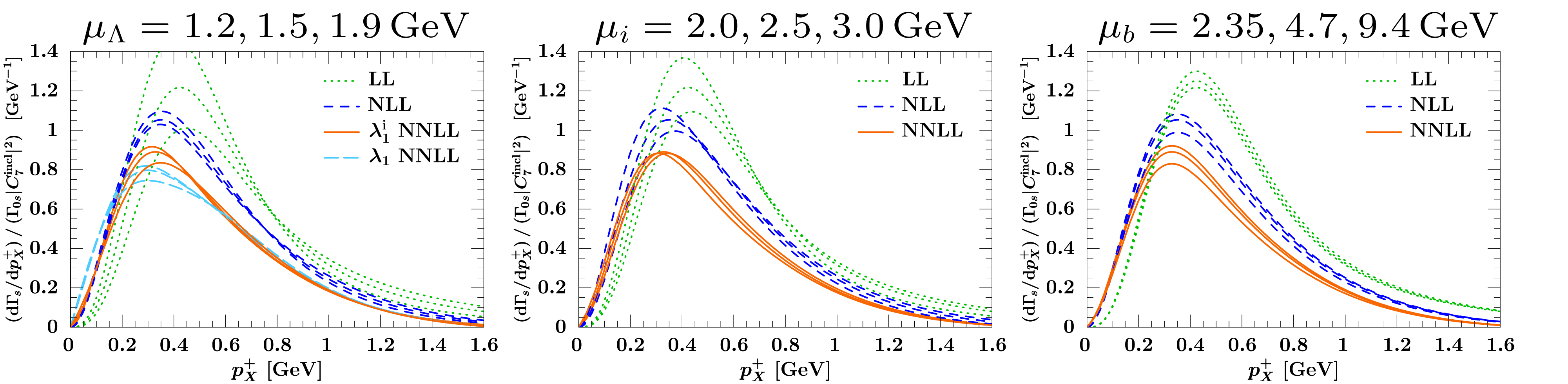


(All curves use $m_b^{\rm kin}$)

- Yet to be seen if shape function scheme for $\lambda_1$ gives good behavior

# Scale (in)dependence of $B \to X_s\gamma$ spectrum

- Dependence on 3 scales in the problem:




$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}p_X^+} = \Gamma_{0s}\, H_s(p_X^+, \mu_b)\, U_H(m_b, \mu_b, \mu_i) \int \mathrm{d}k\, \widehat{P}(m_b, k, \mu_i)\, \widehat{F}(p_X^+ - k) \qquad \left(p_X^+ = m_B - 2E_\gamma\right)$$

$\widehat{P}, \widehat{F}$ indicate use of short distance schemes: $m_b^{1S}$ and $\lambda_1^{\mathrm{i}}$

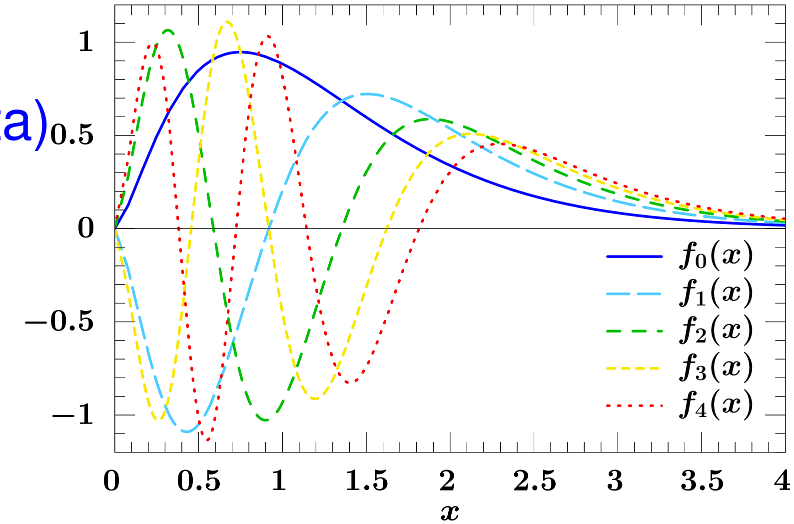
- In other approaches, using models for $S(\omega, \mu_\Lambda)$ run up to $\mu_i$, dependence on $\mu_\Lambda$ ignored so far, but it must be considered an uncertainty $\Rightarrow$ This is how to solve it

# Designer orthonormal functions

- Devise suitable orthonormal basis functions (earlier: fit parameters of model functions to data)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum c_n f_n(x)\Big]^2,$$ $n$ th moment $\sim \Lambda_{\rm QCD}^n$

$$f_n(x) \sim P_n[y(x)]$$ $\leftarrow$ Legendre polynomials

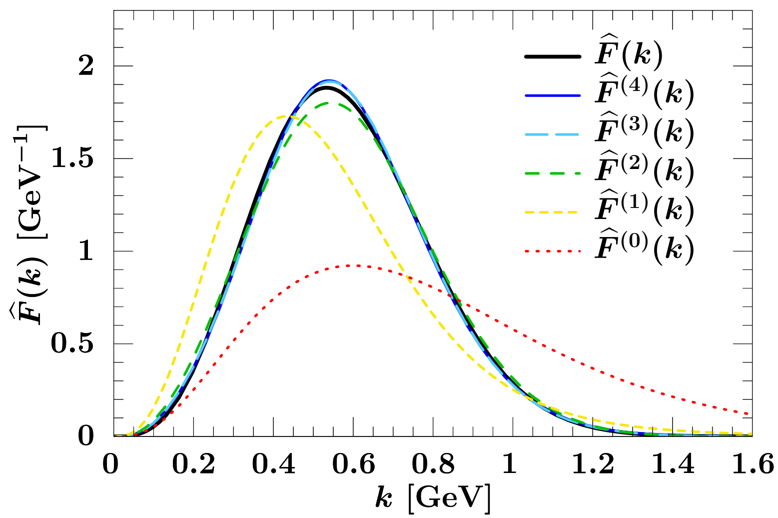



- Approximating a model shape function

Better to add a new term in an orthonormal basis than a new parameter to a model:

– less parameter correlations

– errors easier to quantify




"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

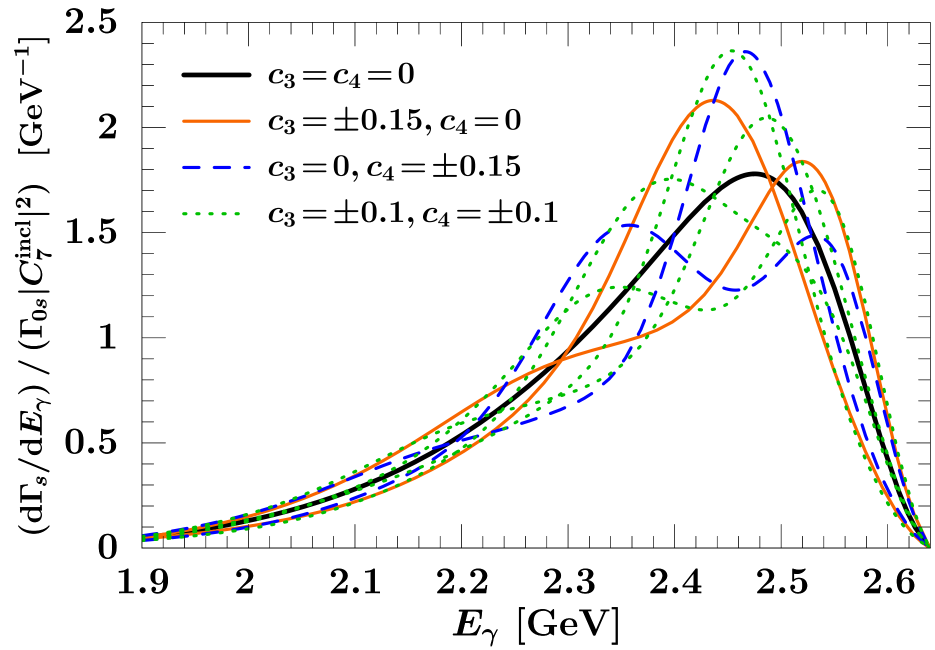
# Back to the $B \to X_s\gamma$ spectrum

- 9 models: same 0th, 1st, 2nd moments

  Including all NNLL contributions, find:

  – Shape in peak region not determined at all by first few moments

  – Smaller shape function uncertainty for $E_\gamma \lesssim 2.1\,\text{GeV}$ than earlier studies




- Not shown in this plot: subleading shape functions
  subleading corrections not in $C_7^{\rm incl}(0)$
  kinematic power corrections
  boost to $\Upsilon(4S)$ frame

- Same analysis can also be used for: $B \to X_s\ell^+\ell^-$, $|V_{ub}|$

# More complicated: $B \to X_u \ell \bar{\nu}$

[ZL, Stewart, Tackmann, to appear]

# Regions of phase space (again)

- "Natural" kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$ — "jettyness" of hadronic final state

  $B \to X_s\gamma$: $p_X^+ = m_B - 2E_\gamma$ & $p_X^- \equiv m_B$, but independent variables in $B \to X_u \ell\bar\nu$

- Three cases: 1) $\Lambda \sim p_X^+ \ll p_X^-$

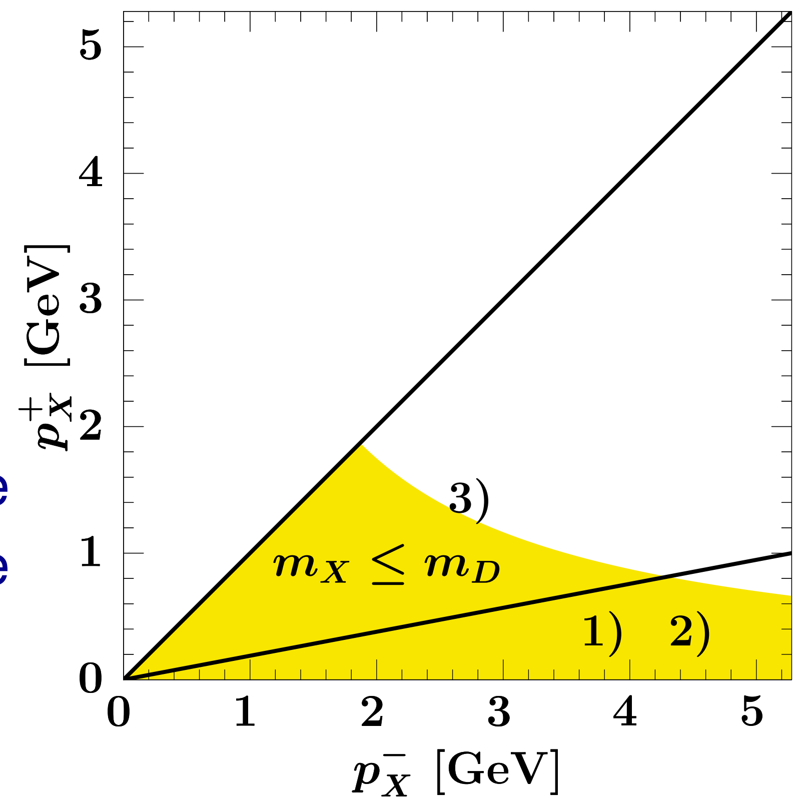
  2) $\Lambda \ll p_X^+ \ll p_X^-$

  3) $\Lambda \ll p_X^+ \sim p_X^-$

  Make no assumptions how $p_X^-$ compares to $m_B$

- $B \to X_u \ell\bar\nu$: 3-body final state, appreciable rate in region 3), where hadronic final state not jet-like

  E.g., $m_X^2 < m_D^2$ does not imply $p_X^+ \ll p_X^-$




- Can combine 1) – 2) just as we did in $B \to X_s\gamma$, by not expanding in $\Lambda/p_X^+$

# Charmless $B \to X_u \ell \bar\nu$ made charming

- To combine all $3$ regions: do not expand in $\Lambda/p_X^+$ nor in $p_X^+/p_X^-$ (nontrivial!)
- Want to have: result accurate to NNLL and $\Lambda_{\rm QCD}/m_b$ in regions 1) – 2) and to order $\alpha_s^2\beta_0$ and $\Lambda_{\rm QCD}^2/m_b^2$ when phase space limits are in region 3)

---

Start with triple differential rate (involves a delta-fn at the parton level at $\mathcal{O}(\alpha_s^0)$, which is smeared by the shape function)

The $p_X^+/p_X^-$ terms, which are not suppressed in local OPE region, start at $\mathcal{O}(\alpha_s)$

Recently completed $\mathcal{O}(\alpha_s^2)$ matching computations [Ben's & Guido's talks]

[Bonciani & Ferroglia, 0809.4687; Asatrian, Greub, Pecjak, 0810.0987; Beneke, Huber, Li, 0810.1230; Bell, 0810.5695]

**SIMBA**

SIMBA – Analysis of B-Meson Inclusive Spectra –

[Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, to appear]

# Fitting charmless inclusive decay spectra

- Fit strategy: $\widehat{F}(k)$ enters the spectra linearly $\Rightarrow$ can calculate independently the contribution of $f_m\, f_n$ in the expansion of $\widehat{F}(k)$:

$$\mathrm{d}\Gamma = \sum \underbrace{c_m\, c_n}_{\text{fit}}\ \underbrace{\mathrm{d}\Gamma_{mn}}_{\text{compute}}$$

$$\mathrm{d}\Gamma_{mn} = \Gamma_0\, H(p^-) \int_0^{p_X^+} \mathrm{d}k\, \frac{\widehat{P}(p^-, k)}{\lambda}\ \underbrace{f_m\!\left(\frac{p_X^+ - k}{\lambda}\right) f_n\!\left(\frac{p_X^+ - k}{\lambda}\right)}_{\text{basis functions}}$$

Fit the $c_i$ coefficients from the measured (binned) spectra

- What we hope to achieve:
  - Correlation and error propagation of SF uncertainties
  - Simultaneous fit using all available information
  - Realistic estimate of model uncertainties (fit parameters $c_{0,\ldots,N}$ constrained by data; vary $N$, the number of orthonormal basis functions in fit)

# A preliminary $B \to X_s\gamma$ fit

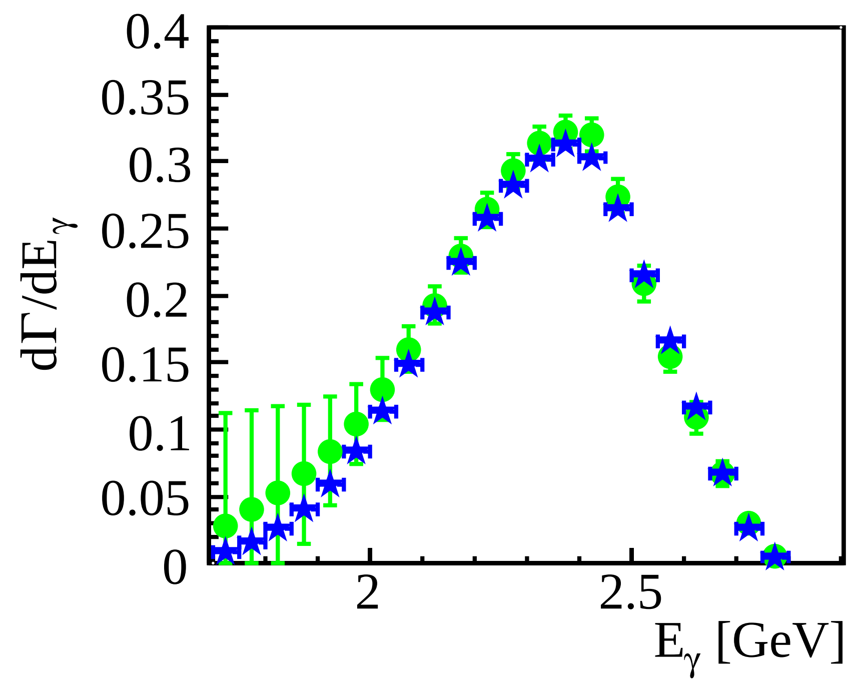
- Belle $B \to X_s\gamma$ spectrum in $\Upsilon(4S)$ restframe

  For demonstration purposes only — there are very strong correlations

  Fit with 4 basis functions in the expansion of the shape function

  Shows that fit works (not as trivial as this plot might indicate); still issues to resolve

[Belle, 0804.1580 + Preliminary; thanks to Antonio Limosani]




- Next step: include various $B \to X_u\ell\bar{\nu}$ measurements

# Conclusions

- Improving accuracy of $|V_{ub}|$ will remain important to constrain new physics (Current situation unsettled, PDG in 2008 inflated error for the first time)

- Qualitatively better inclusive analyses possible than those that exist so far
  - Modeling $F(k)$ instead of $S(\omega, \mu)$
  - Designer orthonormal functions
  - Fully consistent combination of $3$ phase space regions
  - Decouple SF shape variation from $m_b$ variation

- Developments will allow combining all pieces of data with tractable uncertainties
  - Consistently combine $B \to X_u \ell \bar{\nu}$, $B \to X_s \gamma$, $B \to X_c \ell \bar{\nu}$ data to constrain SF's
  - Reduce role of shape function modeling

- To draw conclusions about new physics comparing sides and angles, we'll want $\geq 2$ extractions of $|V_{ub}|$ with different uncertainties (inclusive, exclusive, leptonic)