Introduction to Heavy Meson Decays and CP Asymmetries

Zoltan Ligeti

Lawrence Berkeley Laboratory

Thanks to: Adam Falk, Sandrine Laplace, Mike Luke, Yossi Nir

References

An incomplete list:

- Y. Nir, hep-ph/0109090
- A. Falk, hep-ph/0007339
- The Babar Physics Book, SLAC-R-0504
- B Physics at the Tevatron: Run II and Beyond, hep-ph/0201071
- Manohar and Wise, *Heavy Quark Physics,* Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 10 (2000)
- Bigi and Sanda, CP *Violation,* Cambridge University Press, New York, 2000
- Branco, Lavoura and Silva, CP *Violation,* Clarendon Press, Oxford, 1999

Disclaimers

In these lectures:

... I will not talk about the strong CP problem

see: H. Quinn, hep-ph/0110050
M. Dine, hep-ph/0011376

$$\mathcal{L} = \frac{\theta_{QCD}}{16\pi^2} F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

... I will not talk about lattice QCD

see: lectures of P. Lepage

... I will not talk about beyond SM physics

see: lectures of A. Kagan

Dictionary

- SM $=$ standard model
- NP $=$ new physics
- CPV $=$ CP violation/violating
- CPC $=$ CP conserving
- UT $=$ unitarity triangle

Central questions about SM

1. Origin of electroweak symmetry breaking:

   $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}}$

   spontaneous breaking of a gauge symmetry by $v \sim 250$ GeV VEV

   $W_L W_L \rightarrow W_L W_L$ breaks unitarity ~ 1 TeV ... determines scale of Higgs / NP

2. Origin of flavor symmetry breaking:

   $U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_{\text{Baryon}}$ (for leptons don't even know yet!)

   global symmetries (e.g, d_R, s_R, b_R identical if massless) broken by dimensionless Yukawa couplings ... we do not know what scale to look

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It would be nice if flavor and electroweak symmetry breaking were connected

Flavor physics depends on both — Yukawa couplings determine quark masses, mixing, and CP violation

Central questions of flavor physics

1. Does the SM (i.e., only virtual quarks, W, and Z interacting through CKM matrix in tree and loop diagrams) explain all flavor changing interactions?
2. If it does not, then at what level and where can we see deviations?

To answer these questions, we need: Experimental precision
Theoretical precision — cleanliness

corollary:

The point is not simply to measure CKM elements, but to overconstrain the SM description of flavor by many “redundant” measurements

The key processes are those which can teach us about high energy phsyics without hadronic uncertainties

Interplay between weak and strong interactions

- Can we learn about high energy physics from low energy hadronic processes?

QCD coupling is scale dependent:

$$\alpha_s(\mu) = \frac{\alpha_s(M)}{1 + \frac{\alpha_s}{2\pi}\beta_0 \ln \frac{\mu}{M}}$$

High energy (short distance): perturbation theory is useful

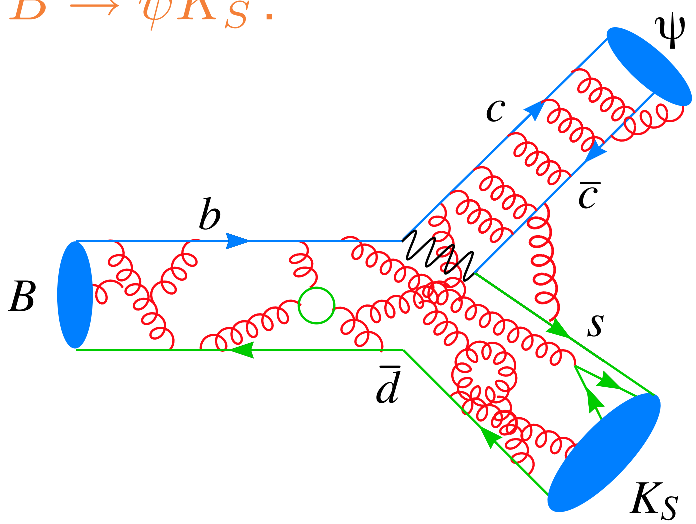
Low energy (long distance): QCD becomes nonperturbative $\Rightarrow$ It is usually very hard, if not impossible, to make precise calculations

- Solutions: – Use symmetries of QCD (exact or approximate)
  – Certain processes are determined by short-distance physics

Sometimes it is possible to use data and symmetries together to eliminate uncalculable hadronic mess

(1) Want to learn about CP violation

- $\sin(2\beta)$ from $B \to \psi K_S$:



energy release:

$m_B - m_\psi - m_K \simeq 1.7\,$GeV

Contributions of diagrams with many soft gluons are not suppressed

Theoretically clean measurement of $\sin(2\beta)$ possible (at $< 1\%$ level), because amplitudes with one weak phase dominate

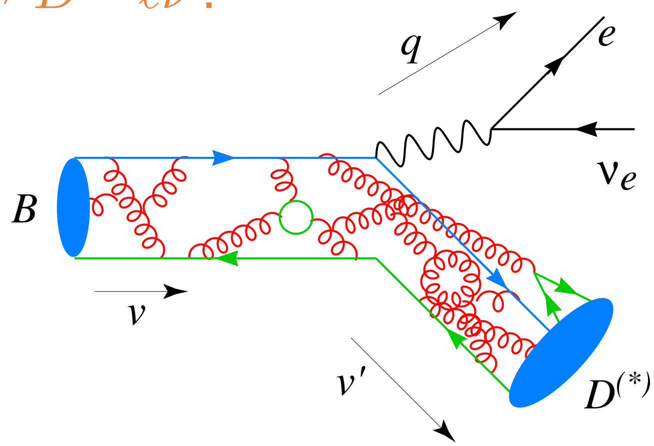
- Solution: CP symmetry of strong interactions (exact symmetry)

The magnitude of the amplitude does not matter, only need the relation:

$$\langle \psi K_S | \mathcal{H} | B^0 \rangle = -\langle \psi K_S | \mathcal{H} | \overline{B}^0 \rangle \times [1 + \mathcal{O}(\alpha_s \lambda^2)]$$

(2) Want to learn about CKM elements

- $|V_{cb}|$ from $B \to D^{(*)}\ell\bar{\nu}$:



Contributions of diagrams with many soft gluons are not suppressed

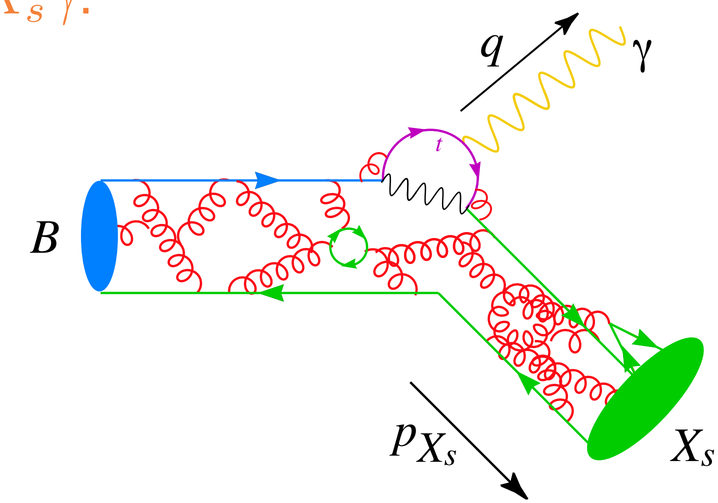
Theoretically clean measurement of $|V_{cb}|$ possible (at 5% level), because hadronic matrix element is known in the $m_{c,b} \to \infty$ limit at "zero recoil" $v \cdot v' = 1$

- Solution: Heavy quark symmetry in heavy mesons (approximate symmetry)

determines rate at zero recoil: $\langle D^*(v)|\, J\, |B(v)\rangle = 1 + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(2m_c)^2}\right)$

(3) Want to learn about physics beyond the SM

- E.g.: $B \to X_s\gamma$:



Inclusive decay:
$X_s = K^*,\ K^{(*)}\pi,\ K^{(*)}\pi\pi$, etc.

Diagrams with many gluons are crucial, resumming certain subset of them affects rate at factor-of-two level

Rate calculable at 10% level, using several effective theories, renormalization group, operator product expansion... one of the most involved SM analyses

- Solution: Short distance dominated; unknown corrections suppressed by

$$\Gamma(B \to X_s\gamma) = [\text{known}] \times \left\{1 + \mathcal{O}\left(\alpha_s^3 \ln \frac{m_W}{m_b},\, \frac{\Lambda^2_{\rm QCD}}{m^2_{b,c}},\, \frac{\alpha_s \Delta m_c}{m_b}\right)\right\}$$

Outline (1)

1. Introduction to flavor physics and CPV

   ... Brief SM review

   ... How CKM matrix arises from Yukawa couplings

   ... Present status

   Mixing and CPV in neutral meson systems (K, D, B, B_s)

   ... Ways to obtain clean information about short distance physics

   ... Mixing: Δm_{B_d} and Δm_{B_s}

   ... CPV: $B \to \psi K_s$, $B \to \phi K_s$

Outline (2–3)

2. The heavy quark limit

   ... Heavy quark symmetry: spectroscopy, strong decays

   ... Exclusive $B \to D^{(*)}\ell\nu$ decays and $|V_{cb}|$ (HQET)

   ... Inclusive semileptonic decays, $|V_{cb}|$ (OPE)

   ... Inclusive $|V_{ub}|$ measurements and rare decays

3. Some clean CP measurements

   ... $B_s \to D_s K$, $B \to \pi\pi$ isospin analysis, $B \to DK$

   Nonleptonic decays, factorization

   ... Factorization in $B \to D^{(*)}X$ decays; tests of factorization

   ... Factorization in charmless decays

   ... Tests / applications in decays to pseudoscalars (α, γ)

   Final thoughts

Introduction

The Standard Model (SM)

Gauge symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y$ — parameters

8 gluons $\quad W^\pm$, Z^0, γ — 3

Particle content: 3 generations of quarks and leptons

$Q_L(3,2)_{1/6},\ u_R(3,1)_{2/3},\ d_R(3,1)_{-1/3}$ — 10

$L_L(1,2)_{-1/2},\ \ell_R(1,1)_{-1}$ — $3(+9)$

quarks: $\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}$ leptons: $\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}$

Symmetry breaking: $SU(2)_L \times U(1)_Y \ \to\ U(1)_{\rm EM}$

$\phi(1,2)_{1/2}$ Higgs scalar, $\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ — 2

- The SM agrees (too well...) with all observed particle physics phenomena

Why is CPV interesting?

"CPV is a mystery"

... the SM with 3-generations "predicts" it

"CPV is one of the least understood parts of the SM"

... $\sin 2\beta$, ϵ_K, ϵ' are all in the right ballpark

BUT:

- Almost all extensions of the SM contain new sources of CP and flavor violation
- Major constraint for model building, may distinguish between NP models
- The observed baryon asymmetry of the Universe requires CPV beyond the SM (not necessarily in flavor changing processes)

If $\Lambda_{\rm CPV} \gg \Lambda_{\rm EW}$: no observable effects in B decays $\Rightarrow$ precise SM measurements

If $\Lambda_{\rm CPV} \sim \Lambda_{\rm EW}$: sizable effects possible $\Rightarrow$ could get detailed information on NP

The track record

Bits of history: $K\bar{K}$ mixing $\Rightarrow$ GIM & charm

CP violation $\Rightarrow$ three generations, CKM

$B\bar{B}$ mixing $\Rightarrow$ heavy top

Best sensitivity to some particles predicted in the MSSM comes from (crudely...)

experiment	energy scale	best sensitivity to
Tevatron	~ 2 TeV	squarks, gluinos
LEP	~ 200 GeV	sleptons, charginos
$B \to X_s\gamma$	~ 5 GeV	charged Higgs

SM: where can CP violation occur?

- Kinetic terms: $\mathcal{L}_{\rm kin} = -\frac{1}{4}\sum_{\rm groups} (F^a_{\mu\nu})^2 + \sum_{\rm rep's} \overline{\psi}\, i \not{D}\, \psi$

  always CPC (ignoring $F\widetilde{F}$)

- Higgs terms: $\mathcal{L}_{\rm Higgs} = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger\phi)^2 \qquad (v^2 = \mu^2/\lambda)$

  CPC if $\exists$ only one Higgs doublet; CPV possible with extended Higgs sector

- Yukawa couplings in interaction basis:

$$\mathcal{L}_Y = -Y^d_{ij}\, \overline{Q^I_{Li}}\, \phi\, d^I_{Rj} - Y^u_{ij}\, \overline{Q^I_{Li}}\, \widetilde{\phi}\, u^I_{Rj} - Y^\ell_{ij}\, \overline{L^I_{Li}}\, \phi\, \ell^I_{Rj} + \text{h.c.}$$

cannot write mass term for ν's!

$i, j \sim$ generations

$$\widetilde{\phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \phi^*$$

CPV is related to unremovable phases of Yukawa couplings:

$$Y_{ij}\, \overline{\psi_{Li}}\, \phi\, \psi_{Rj} + Y^*_{ij}\, \overline{\psi_{Rj}}\, \phi^\dagger\, \psi_{Li}$$

$\Downarrow$ CP exchanges fermion bilinears

$$Y_{ij}\, \overline{\psi_{Rj}}\, \phi^\dagger\, \psi_{Li} + Y^*_{ij}\, \overline{\psi_{Li}}\, \phi\, \psi_{Rj}$$

Quark mixing

- Replacing ϕ with its VEV in Yukawa couplings:

$$\mathcal{L}_{\text{mass}} = -(M_d)_{ij}\,\overline{d^I_{Li}}\,d^I_{Rj} - (M_u)_{ij}\,\overline{u^I_{Li}}\,u^I_{Rj} - (M_\ell)_{ij}\,\overline{\ell^I_{Li}}\,\ell^I_{Rj} + \text{h.c.}$$

$M_f = \frac{v}{\sqrt{2}} Y^f$ — want to diagonalize these ($f = u, d, \ell$)

$M_f^{\text{diag}} \equiv V_{fL}\, M_f\, V_{fR}^\dagger$ — $V_{L,R}$ unitary matrices

Define mass eigenstates:
$$f_{Li} \equiv (V_{fL})_{ij}\, f^I_{Lj}$$
$$f_{Ri} \equiv (V_{fR})_{ij}\, f^I_{Rj}$$

- The quark mass matrices are diagonalized by different transformations for u_{Li} and d_{Li}, which are part of the same $SU(2)_L$ doublet Q_L

$$\begin{pmatrix} u^I_{Li} \\ d^I_{Li} \end{pmatrix} = (V^\dagger_{uL})_{ij} \begin{pmatrix} u_{Lj} \\ (V_{uL}V^\dagger_{dL})_{jk}\, d_{Lk} \end{pmatrix}, \qquad V_{\text{CKM}} \equiv V_{uL} V^\dagger_{dL}$$

Which terms in the Lagrangian get modified by this transformation?

SM: where can flavor violation occur?

- In mass basis, charged current ($W^\pm$) weak interactions become complicated:

$$-\frac{g}{2}\,\overline{Q^I_{Li}}\,\gamma^\mu\,W^a_\mu\,\tau^a\,Q^I_{Li} + \mathrm{h.c.} \;\Rightarrow\; -\frac{g}{\sqrt{2}}\left(\overline{u_L},\ \overline{c_L},\ \overline{t_L}\right)\gamma^\mu\,W^+_\mu\,\underset{\Uparrow}{(V_{uL}V^\dagger_{dL})}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathrm{h.c.}$$

Cabibbo-Kobayashi-Maskawa matrix: $V_{\rm CKM}$

Only source of CPV in flavor changing processes in the SM

- The neutral current (Z^0) interactions remain flavor conserving in the mass basis (True in all models with only left handed doublet and right handed singlet quarks)

$\Rightarrow$ In the SM, only charged current interactions change flavor

How do we know that CP is violated?

- Prior to 1964, the explanation of the large lifetime ratio of the two neutral kaons was CP symmetry (before 1956, it was C alone...)

$$|K^0\rangle = \overline{s}\, d\,, \qquad |\overline{K}^0\rangle = \overline{d}\, s\,, \qquad CP|K^0\rangle = +|\overline{K}^0\rangle$$ (convention dependent)

states of definite CP: $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\overline{K}^0\rangle\right)$

$$CP|K_1\rangle = |K_1\rangle\,, \qquad CP|K_2\rangle = -|K_2\rangle$$

If CP were an exact symmetry: $\left.\begin{array}{l}\text{only } K_1 \to \pi\pi \\ \text{both } K_{1,2} \to \pi\pi\pi\end{array}\right\} \Rightarrow \tau(K_1) \ll \tau(K_2)$

- But $K_L \to \pi\pi$ decay was also observed (1964) at the 10^{-3} level!

$$\eta_{00} = \frac{\langle\pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle\pi^0\pi^0|\mathcal{H}|K_S\rangle} \quad \eta_{+-} = \frac{\langle\pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle\pi^+\pi^-|\mathcal{H}|K_S\rangle} \quad \epsilon_K \equiv \frac{1}{3}\left(\eta_{00} + 2\eta_{+-}\right) \quad \epsilon'_K \equiv \frac{1}{3}\left(\eta_{+-} - \eta_{00}\right)$$

Was <1 yr to propose superweak, but 9 till KM (before 2nd generation complete!)

Baryogenesis

$$\frac{\#\ \text{baryons}}{\#\ \text{photons}} \sim 10^{-9}\ \text{now} \quad \Longleftrightarrow \quad \frac{n_q - n_{\bar q}}{n_q + n_{\bar q}} \sim 10^{-9}\ \text{at}\ t < 10^{-6}\,\text{sec}\ (T > 1\,\text{GeV})$$

- To produce such an asymmetry from symmetric initial conditions, need

1. baryon number violating interactions
2. C and CP violation
3. deviation from thermal equilibrium

- SM contains 1–3, but

A. CP violation is too small

B. deviation from thermal equilibrium too small with just one Higgs doublet

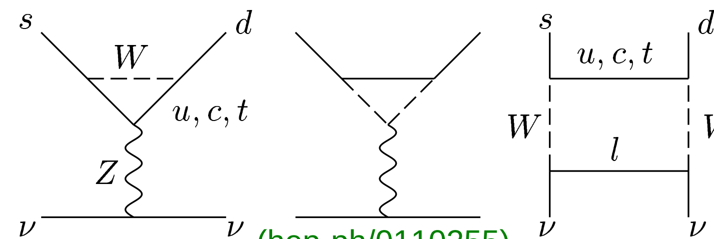
NP models can solve A–B near the weak scale, and may have observable effects (possibly only in flavor diagonal processes, such as electric dipole moments)

Why B physics?

- Observed CPV in K system is at the right level (ϵ_K can be described with $\mathcal{O}(1)$ CKM phase), but hadronic uncertainties preclude precision tests (ϵ'_K is notoriously hard to calculate)

  Plan to measure $K \to \pi\nu\overline{\nu}$ — theoretically clean, but $\mathcal{B} \sim 10^{-10}(K^\pm),\ 10^{-11}(K_L)$

  $$\mathcal{A} \propto \begin{cases} (\lambda^5\, m_t^2) + i(\lambda^5\, m_t^2) & t: \text{CKM suppressed} \\ (\lambda\, m_c^2) + i(\lambda^5\, m_c^2) & c: \text{GIM suppressed} \\ (\lambda\, \Lambda_{\text{QCD}}^2) & u: \text{GIM suppressed} \end{cases}$$

  (hep-ph/0110255)

- In D decays the SM predicts small CPV, interesting for NP (few words later)

- In the B meson system, large variety of interesting processes:
  - top quark loops neither GIM nor CKM suppressed (large mixing, rare decays)
  - large CP violating effects possible, some of which have clean interpretation
  - some of the hadronic physics understood model independently ($m_b \gg \Lambda_{\text{QCD}}$)

CKM matrix and the unitarity triangle

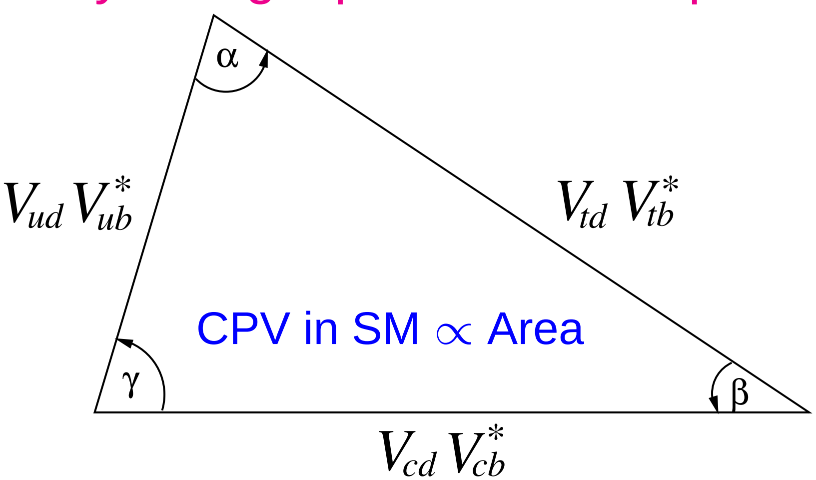
- CKM matrix is hierarchical

$$(u,\, c,\, t) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \begin{matrix} \sim 1 \\ \sim \lambda \\ \sim \lambda^2 \\ \sim \lambda^3 \end{matrix} \qquad \lambda \sim 0.22$$

Elements depend on 4 real parameters (3 angles $+$ 1 CPV phase)

$\mathbf{V}_{\mathrm{CKM}}$ is the only source of CPV in the SM

- The unitarity triangle provides a simple way to visualize the SM constraints



$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td}\, V_{tb}^* = 0$$

The angles and sides are directly measurable — want to overconstrain this picture

Wolfenstein parameterization

- It is convenient to exhibit the hierarchical structure by expanding in $\lambda = \sin\theta_C$

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Present uncertainties: $\lambda \sim 1\%$, $A \sim 5\%$, $\eta/\rho \sim 7\%$, $\sqrt{\rho^2+\eta^2} \sim 20\%$,

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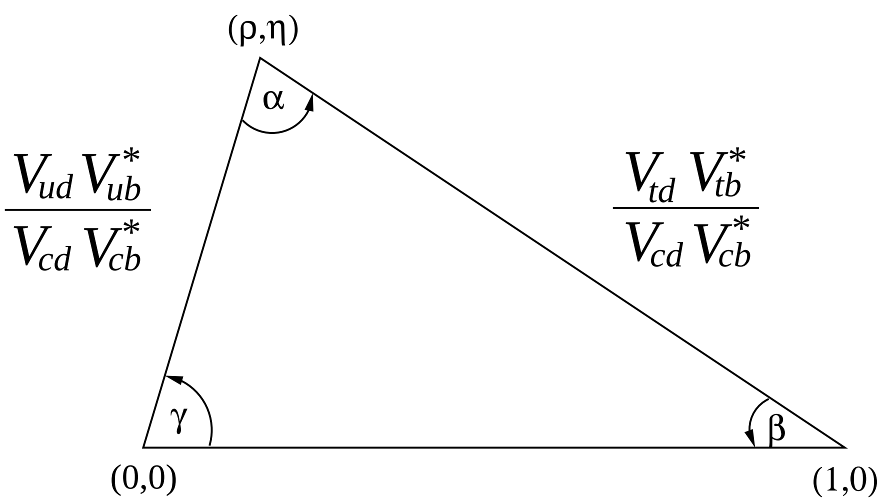
- Constraints on CKM usually plotted on the $(\bar\rho, \bar\eta)$ plane, $\bar\rho + i\bar\eta \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$



$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Experimental program

- Goal: precision tests of the flavor sector via redundant measurements, which in the SM determine CKM elements, but sensitive to different short distance physics

New physics could easily modify:

– SM loop processes: mixing
rare decays

– CP violation

So we want to measure:

– mixing & rare decays

– CPV asymmetries

– compare tree and loop processes

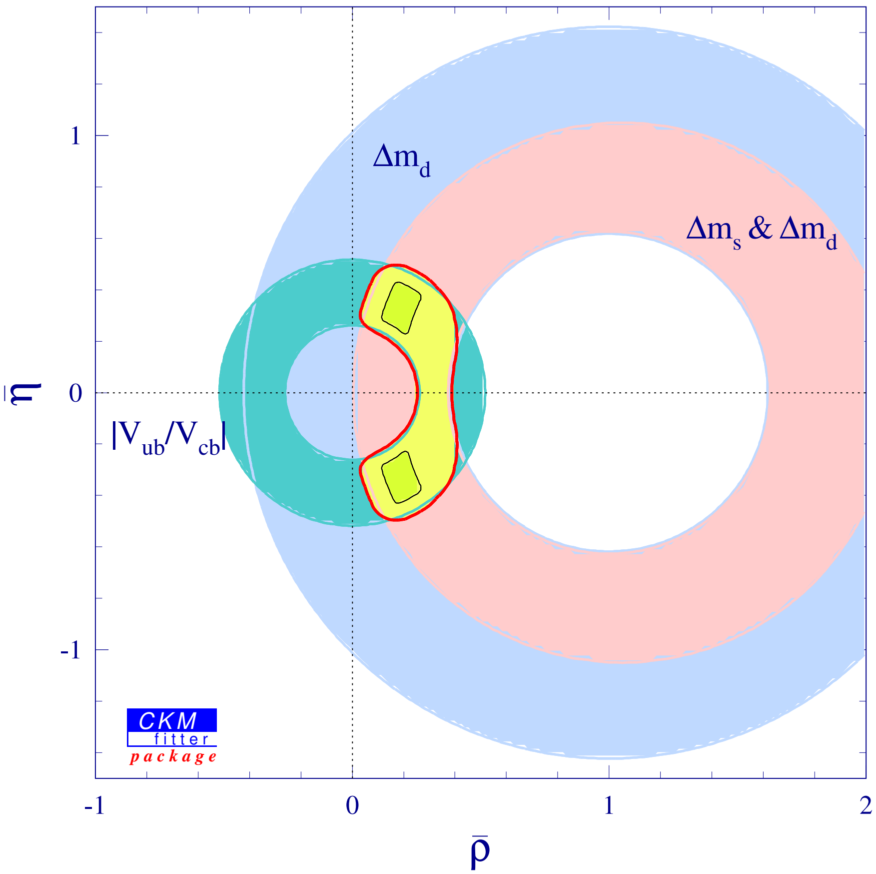
- In the presence of NP, many independent and large CPV phases are possible; Then "α, β, γ" is only a language and two "would-be" γ measurements can be sensitive to different NP contributions (similarly for $|V_{td}|, |V_{ts}|$)

Do all possible measurements which have clean interpretation; correlations may be crucial to narrow down type of NP

$\Rightarrow$ Very broad program — independent measurements are searching for NP!

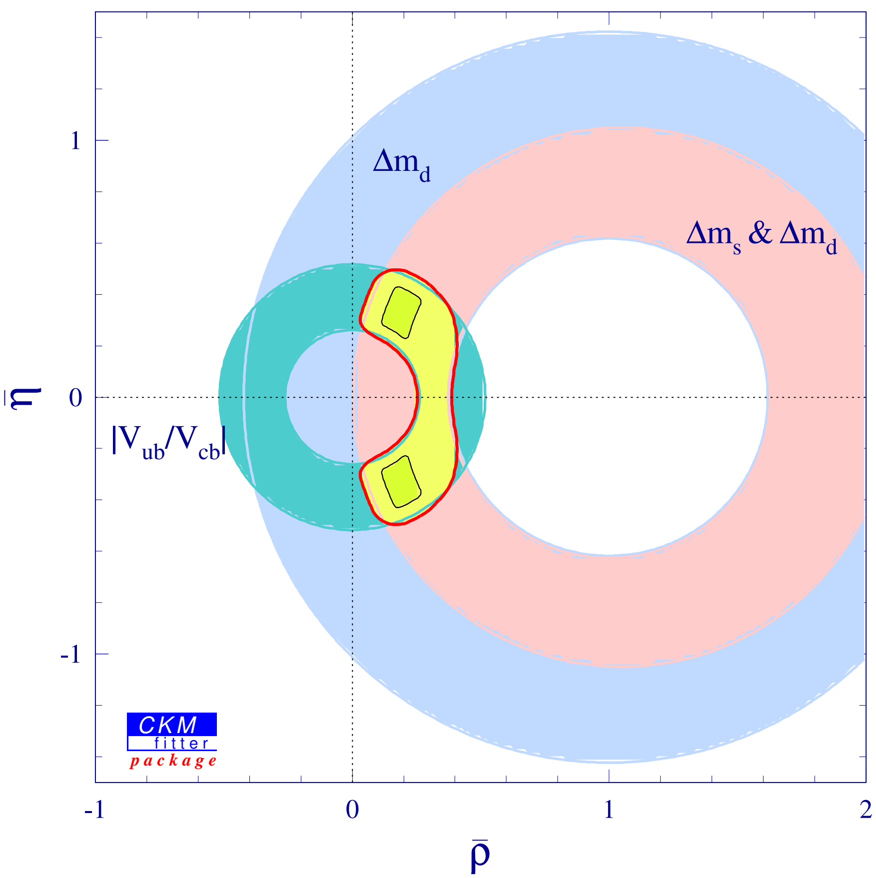
Present knowledge of $(\bar{\rho}, \bar{\eta})$

Tree level $+$ CP conserving only

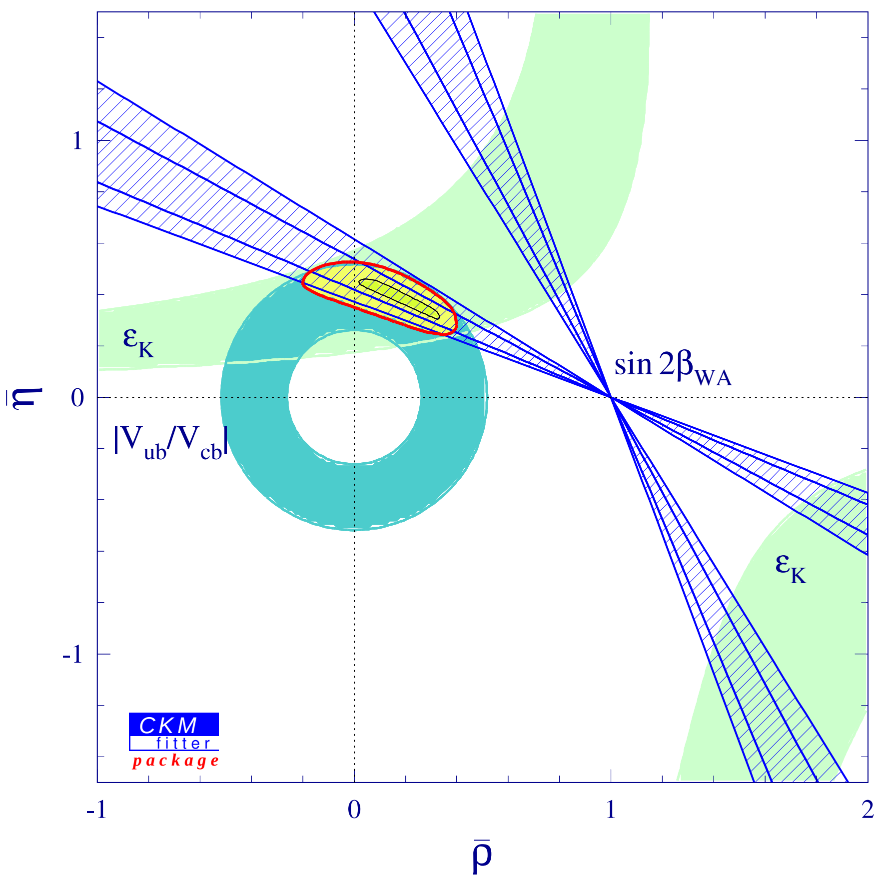


Present knowledge of $(\bar{\rho}, \bar{\eta})$

Tree level $+ \; CP$ conserving



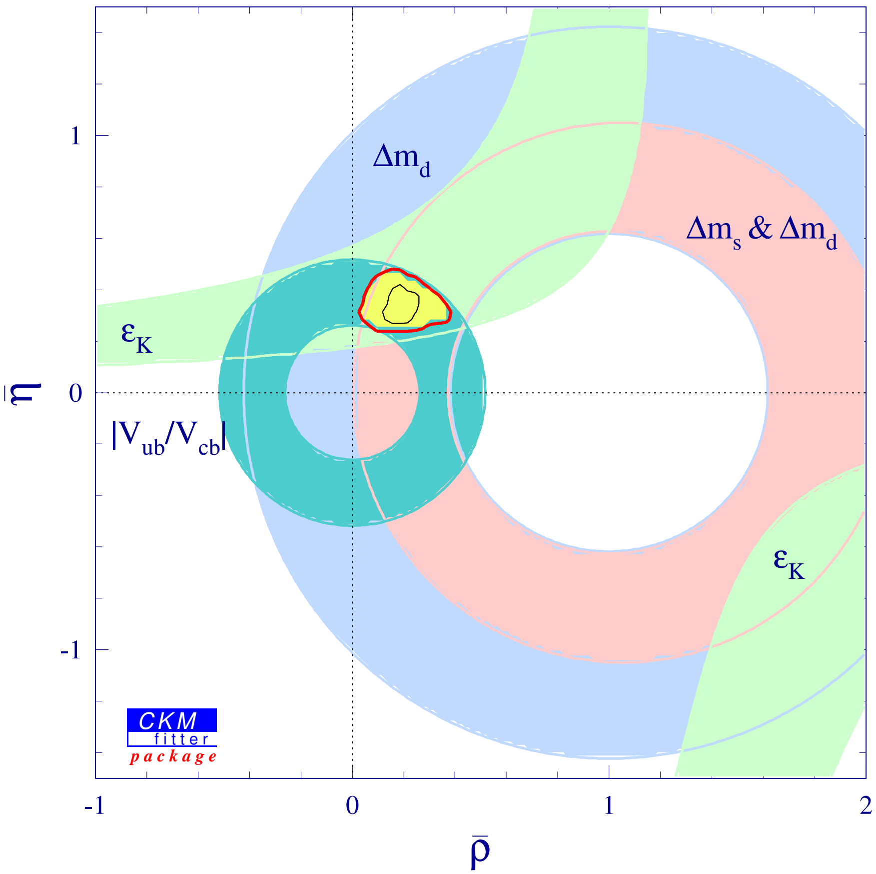
Tree level $+ \; CP$ violating

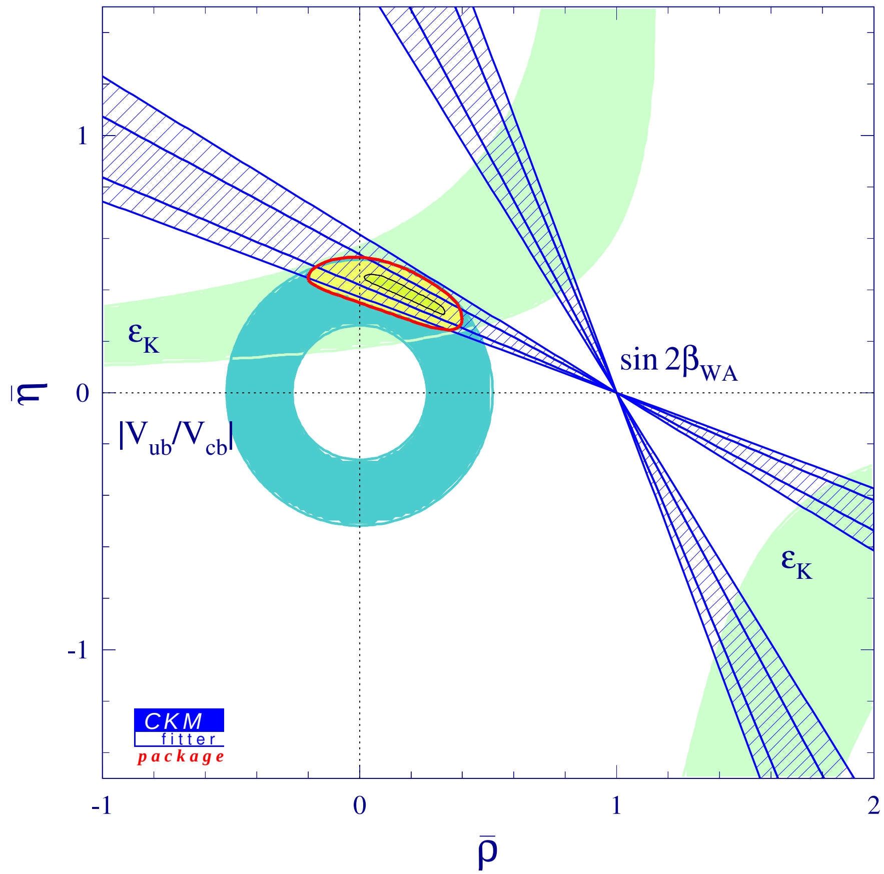


Present knowledge of $(\bar{\rho}, \bar{\eta})$

Tree level $+$ CP conserving $+$ ϵ_K

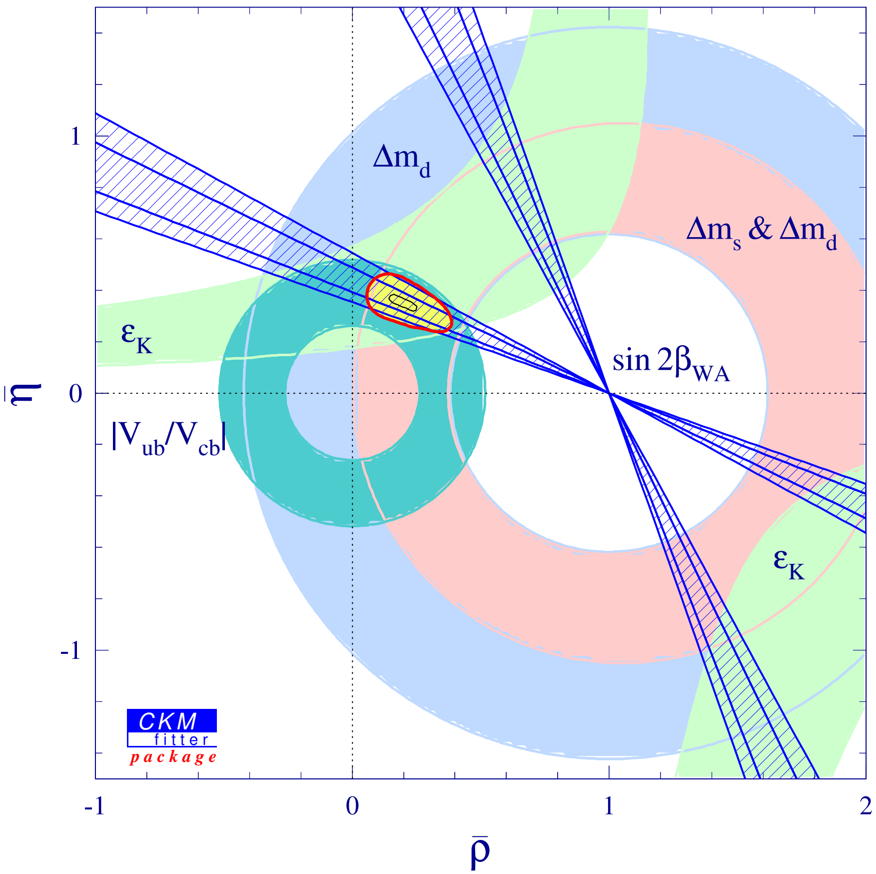
Tree level $+$ CP violating



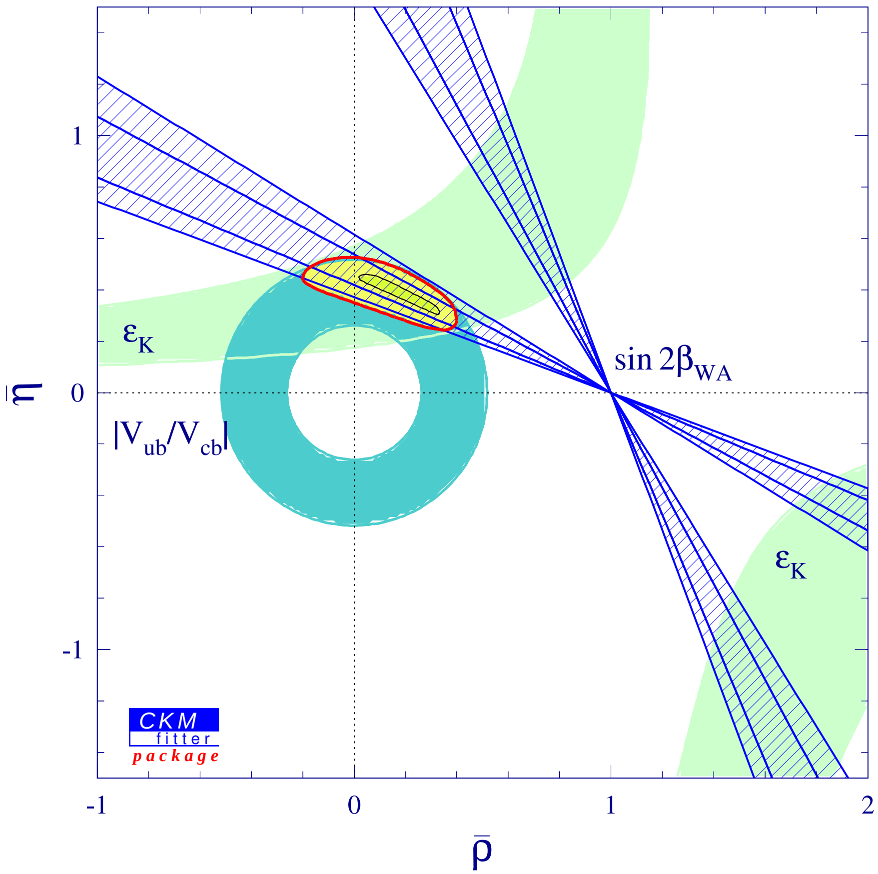


Present knowledge of $(\bar{\rho}, \bar{\eta})$

Full standard model fit



Tree level $+$ CP violating



Summary — so far

- The CKM picture of CPV passed its first non-trivial test; $\sin 2\beta$ has become the best known ingredient of the unitarity triangle
  Paradigm change: look for corrections to – rather than alternatives to CKM picture

  Questions: Is the SM the *only* source of CPV?
  Does the SM *fully* explain flavor physics?

  Key measurements: ones that are theoretically clean and experimentally doable

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- Heading towards $\leq 10\%$ test of CKM: Our ability to test CKM in B decays depends on precision of measurements besides $\sin 2\beta$ and $|V_{td}/V_{ts}|$ (today)

  Central themes: 1) How to determine $|V_{ub}|$ model independently (2nd lecture)
  2) Utility of factorization & $SU(3)$ to determine $\alpha\,/\,\gamma$ from rates or "simple" time dependent asymmetries (3rd lecture)
  3) "Zero prediction" observables: $a_{CP}(B_s \to \psi\phi)$, $a_{dir}(B \to s\gamma)$

Mixing and CPV in neutral mesons

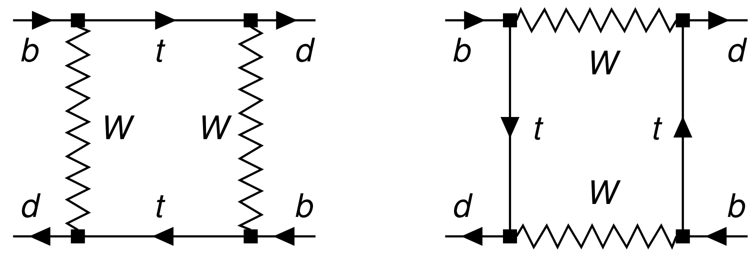
Neutral meson mixing

- Two flavor eigenstates, e.g.: $|B^0\rangle = |\bar{b}\, d\rangle,\ |\overline{B}{}^0\rangle = |b\, \bar{d}\rangle$; time evolution satisfies

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|B^0(t)\rangle\\ |\overline{B}{}^0(t)\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|B^0(t)\rangle\\ |\overline{B}{}^0(t)\rangle\end{pmatrix}$$

  M, Γ are 2×2 Hermitian matrices; CPT implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

  Off-diagonal elements due to box diagrams dominated by top quarks $\Rightarrow$ sensitive to high scales



  Mass eigenstates are eigenvectors of $\mathcal{H}$:

  $|B_L\rangle = p|B^0\rangle + q|\overline{B}{}^0\rangle\,, \qquad |B_H\rangle = p|B^0\rangle - q|\overline{B}{}^0\rangle$

  $|B_{H,L}(t)\rangle = e^{-(iM_{H,L}+\Gamma_{H,L}/2)t}|B_{H,L}\rangle$ time dependence involves mixing and decay

- In the $|\Gamma_{12}| \ll |M_{12}|$ limit, which holds for both $B_{d,s}$ within and beyond the SM

  $$\Delta m = 2|M_{12}|\,,\quad \Delta\Gamma = 2|\Gamma_{12}|\cos\phi_{12}\,,\quad \phi_{12} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

  $\Rightarrow$ NP cannot enhance B_s width difference

Aside: importance of $|\Gamma_{12}| \ll |M_{12}|$

- New physics in mixing modifies M_{12}; new CPV phases may alter $\phi \equiv \arg(q/p)$[1]
  Observing ϕ different from the SM prediction may be the best hope to find NP

  $B_{d,s}$: $\Gamma_{12} \ll M_{12}$, $\qquad$ K: $M_{12} \sim \Gamma_{12}$, $\qquad$ D: $\Gamma_{12} \sim$ or $> M_{12}$

  Solving the eigenvalue equation:

  – If $\Delta m \gg \Delta\Gamma$, the CPV phase can be **LARGE**: $\phi = \arg(M_{12}) + \mathcal{O}(\Gamma_{12}^2/M_{12}^2)$

  – If $\Delta\Gamma \gg \Delta m$, the CPV phase is SMALL: $\phi = \mathcal{O}(M_{12}^2/\Gamma_{12}^2) \times \sin(2\,\phi_{12})$

- If $\Delta\Gamma \gg \Delta m$ then even if new physics dominates M_{12}, the sensitivity of any physical observable to it is suppressed by $\Delta m/\Delta\Gamma$

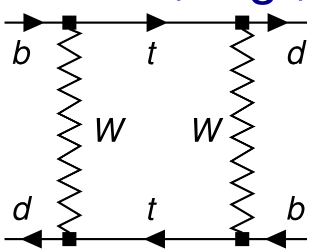
  In the D system it is possible that long distance contributions and $SU(3)$ breaking enhance $\Delta\Gamma$ compared to Δm, this would make looking for NP hard

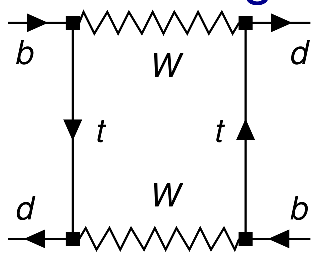
[1] Note: $\arg(q/p)$ is convention dependent; think of it in D decay as the relative phase between q/p and the phase of a tree level decay assumed to be real.

Aside: effective Hamiltonians

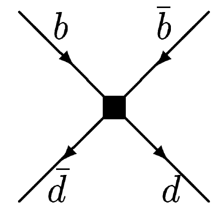
- Interactions at high scale (weak or new physics) produce local operators at lower scales (hadron masses)

Consider, e.g., $B^0 - \overline{B}^0$ mixing:





$\Rightarrow$



$$Q(\mu) = (\bar{b}_L\, \gamma_\nu\, d_L)\, (\bar{b}_L\, \gamma^\nu\, d_L)$$

New physics can modify coefficients and/or induce new operators

Going from operators to observables is equally important

In SM:
$$M_{12} = (V_{tb} V_{td}^*)^2\, \frac{G_F^2}{8\pi^2}\, \frac{M_W^2}{m_B}\, S\!\left(\frac{m_t^2}{M_W^2}\right) \eta_B\, b_B(\mu)\, \langle B^0 | Q(\mu) | \overline{B}^0 \rangle$$

what we are after calculable perturbatively nonperturbative

$\eta_B\, b_B(\mu)$: Resumming $\alpha_s^n \ln^n(m_W/\mu)$, where $\mu \sim m_b$, is often very important

$\langle B^0 | Q(\mu) | \overline{B}^0 \rangle = \frac{2}{3}\, m_B^2\, f_B^2 \frac{\widehat{B}_B}{b_B(\mu)}$: Hadronic uncertainties enter here

$B_{d,s}$ mixing: $|V_{td}|$ and $|V_{ts}|$

$$\Delta m_q = 2|M_{12}| = |V_{tb}V_{tq}^*|^2 \underbrace{f_{B_q}^2 B_{B_q}} \times [\text{known factors}]$$

Need from lattice QCD — ratio of $q = d, s$ is easier:

$$\xi^2 \equiv \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} = 1 \text{ in } SU(3) \text{ limit}$$

Lattice QCD: $\sim [1.15(6)]^2$ “typical lattice average”

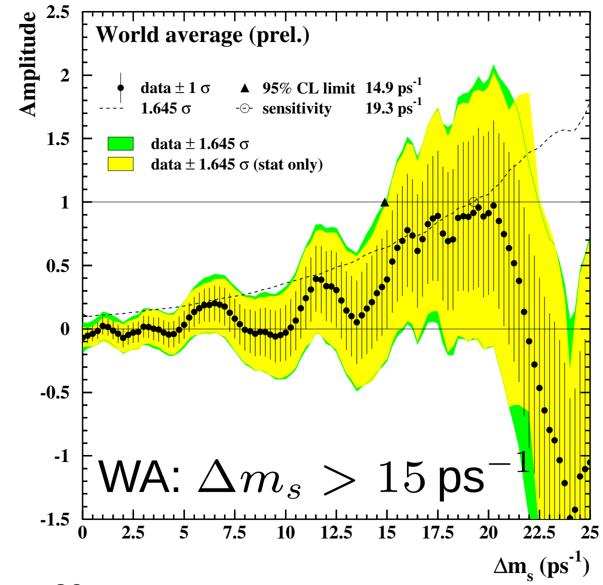
Chiral logs: ~ 1.3 (Grinstein *et al.*, ’92)

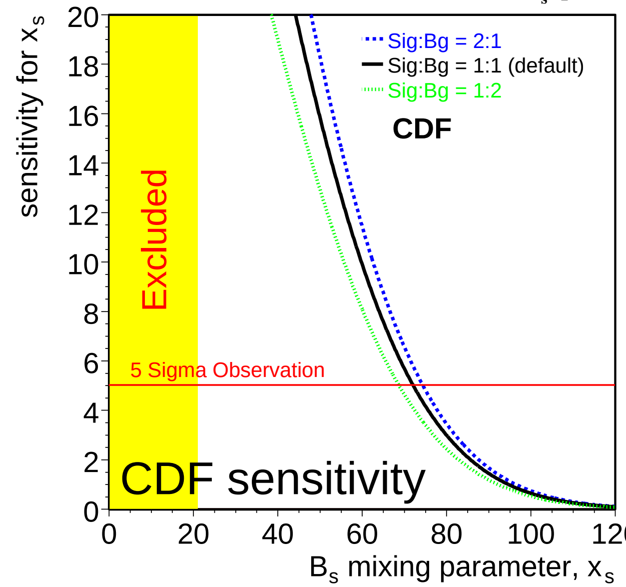
Recent lattice calculation: $\xi = 1.32 \pm 0.1$ (Kronfeld&Ryan)

A conservative error of ξ is probably sizable at present

This will soon be the main limitation to extract $|V_{td}/V_{ts}|$

Effects of light quarks need to be reliably controlled





CPV in mixing

- If CP is conserved then physical states are $\frac{1}{\sqrt{2}}(|B^0\rangle \pm |\overline{B}^0\rangle)$, corresponding to $|q/p| = 1$ and $\arg M_{12} = \arg \Gamma_{12}$

$$\left|\frac{p}{q}\right| \neq 1 \ \Rightarrow \text{CPV in mixing} \qquad \text{occurs iff } \langle B_H|B_L\rangle = |p|^2 - |q|^2 \neq 0$$

- Simplest example is decay to “wrong sign” lepton

$$A_{\rm SL} = \frac{\Gamma[\overline{B}^0(t)\to\ell^+X]-\Gamma[B^0(t)\to\ell^-X]}{\Gamma[\overline{B}^0(t)\to\ell^+X]+\Gamma[B^0(t)\to\ell^-X]} = \frac{1-|q/p|^4}{1+|q/p|^4} = \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}$$

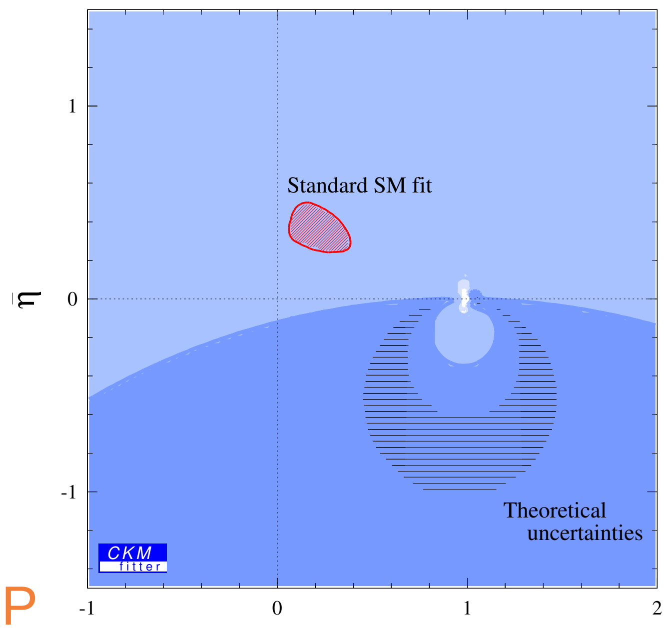
Has been observed in K decay, not yet in B decay

Calculation of Γ_{12} has large hadronic uncertainties

Nevertheless interesting to look for new physics:

$|\Gamma_{12}/M_{12}| = \mathcal{O}(m_b^2/m_W^2)$ model independently

$\arg(\Gamma_{12}/M_{12}) = \mathcal{O}(m_c^2/m_b^2)$ in SM, maybe $\mathcal{O}(1)$ with NP



(hep-ph/0202010) $\bar\rho$

CPV in decay

- Decay amplitudes can, in general, receive many contributions:

$$A_f = \langle f|\mathcal{H}|B\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{i\phi_k} \qquad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{-i\phi_k}$$

"weak phases" ϕ_k — complex parameters in Lagrangian (in $V_{\rm CKM}$ in the SM)

"strong phases" δ_k — on-shell intermediate states rescattering, absorptive parts

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \neq 1 \ \Rightarrow \text{CPV in decay}$$

Can also occur in charged meson and baryon decays

Requires at least two decay amplitudes with different strong and weak phases:

$$|A|^2 - |\overline{A}|^2 = 4 A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$$

Calculations of A_k and δ_k have large model dependence

Can be interesting for looking for NP, when SM prediction is small (e.g., in $b \to s\gamma$)

CPV in interference between decay and mixing

- If both B^0 and $\overline{B}^0$ can decay to same final state, there's another possibility; e.g., if $|f\rangle$ is a CP eigenstate:

$$\begin{tikzcd} B^0 \arrow[rr, "\text{decay}"] \arrow[dr, "\text{mixing}"'] & & f_{CP} \\ & \bar{B}^0 \arrow[ur, "\text{decay}"'] & \end{tikzcd}$$

$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\frac{q}{p}\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$$

$$a_{f_{CP}} = \frac{\Gamma[\overline{B}^0(t)\to f] - \Gamma[B^0(t)\to f]}{\Gamma[\overline{B}^0(t)\to f] + \Gamma[B^0(t)\to f]} = -\frac{(1-|\lambda_f|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\lambda_f \sin(\Delta m t)}{1+|\lambda_f|^2}$$

CP is violated either if $|\lambda| \neq 1$ due to CPV in mixing and/or decay, or if

$|\lambda_f| = 1$, but $\mathrm{Im}\,\lambda_f \neq 0$ $\Rightarrow$ CPV in interference

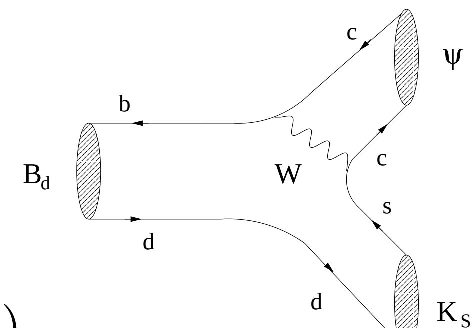
- In such cases ($|\lambda_f| = 1$), CP asymmetry measures phase difference in a theoretically clean way

$$a_{f_{CP}} = \mathrm{Im}\,\lambda_f \sin(\Delta m t)$$

In the $B_{d,s}$ systems $|q/p| - 1 < \mathcal{O}(10^{-2})$, so the question is usually $|\overline{A}/A| \stackrel{?}{=} 1$

$B \to \psi K_{S,L}$ — a decay everyone loves

- There are many amplitudes, nevertheless $|\overline{A}/A| - 1 < 10^{-2}$



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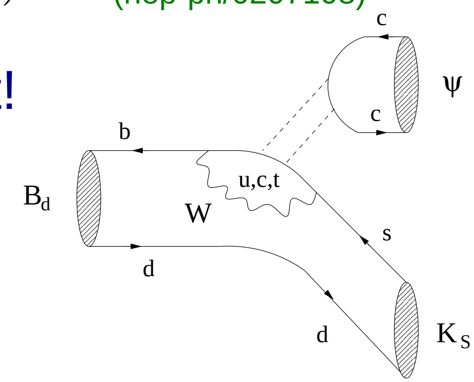


"Tree" ($b \to c\bar{c}s$): $\overline{A}_T \sim \overset{[\lambda^2]}{V_{cb}V_{cs}^*}$

"Penguin": $\overline{A}_P \sim \overset{[\lambda^2]}{V_{tb}V_{ts}^*}\, f(m_t) + \overset{[\lambda^2]}{V_{cb}V_{cs}^*}\, f(m_c) + \overset{[\lambda^4]}{V_{ub}V_{us}^*}\, f(m_u)$

Separation between T and P is scheme and scale dependent!

Rewrite P using unitarity, $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$

$$\overline{A}_P \sim \underbrace{\overset{[\lambda^2]}{V_{cb}V_{cs}^*}}_{\text{same as Tree phase}} [f(m_c) - f(m_t)] + \underbrace{\overset{[\lambda^4]}{V_{ub}V_{us}^*}}_{\text{suppressed by } \lambda^2} [f(m_u) - f(m_t)]$$

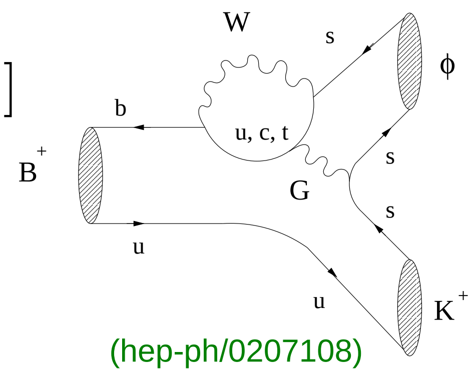
- $|\overline{A}/A| - 1 = \mathcal{O}[\lambda^2 \times (\text{loop})]$ $\Rightarrow$ theoretically very clean

$$\lambda_{\psi K_{S,L}} = \mp \left(\frac{V_{tb}^* V_{td}}{V_{tb}V_{td}^*}\right) \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^* V_{cs}}\right) \left(\frac{V_{cs}V_{cd}^*}{V_{cs}^* V_{cd}}\right) = \mp e^{-2i\beta} \quad \Rightarrow \quad \mathrm{Im}\lambda_{\psi K_{S,L}} = \pm \sin 2\beta$$

$B \to \phi K_{S,L}$ — window to NP?

- “Naively” no tree contribution to $b \to s\bar{s}s$, use unitarity to write penguins:

Penguin: $\overline{A}_P \sim \underbrace{\overset{[\lambda^2]}{V_{cb}V_{cs}^*}}_{\text{dominant contribution}}[f(m_c)-f(m_t)]+\underbrace{\overset{[\lambda^4]}{V_{ub}V_{us}^*}}_{\text{suppressed by } \lambda^2}[f(m_u)-f(m_t)]$



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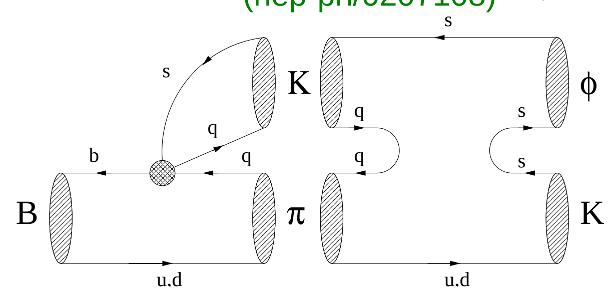
Tree: $b \to u\bar{u}s$ followed by $u\bar{u} \to s\bar{s}$ rescattering

Constrain rescattering by measuring $B^+ \to \phi\pi^+, K^*K^+$

(Grossman, Isidori, Worah)



ψK_S: NP expected to enter $\lambda_{\psi K}$ mainly through q/p

ϕK_S: NP could enter $\lambda_{\phi K}$ through both q/p and $\overline{A}/A$

Expect $\sin 2\beta_{\phi K} = \sin 2\beta_{\psi K}$ to hold in the SM at $\sim 5\%$ level

- Measuring same angle in decays sensitive to different short distance physics is important! [See also the data for $\eta' K_S$ and $K^+K^-K_S$]

Summary

- Seeking experimentally precise and theoretically reliable measurements that in the SM relate to CKM elements but can probe different short distance physics
- The CKM picture passed its first nontrivial test; we can no longer claim to be looking for alternatives of CKM, but to seek corrections due to new physics (Except maybe B_s system, $\mathrm{Im}\lambda_{s\bar{s}s}$, ...)
- Very broad program — a lot more interesting as a whole than any single measurement alone; redundancy / correlations may be the key to new physics
- $B_{d,s}$ mixing ($|V_{td}/V_{ts}|$) and $B \to \psi K$ ($\sin 2\beta$) are "easy" (i.e., both theory and experiment under control)
- Tomorrow we'll start looking at harder things...

Second Lecture

- Heavy quark symmetry

  ... Spectroscopy with HQS

- Exclusive semileptonic decays

  ... $B \to D^{(*)}\ell\nu$ decays and $|V_{cb}|$

  ... Heavy to light decays

- Inclusive semileptonic decays

  ... $B \to X_c \ell \bar{\nu}$ and $|V_{cb}|$

  ... Inclusive $|V_{ub}|$ measurements and rare decays

- Summary

- Additional topics

  ... B decays to excited D mesons; exclusive & inclusive rare decays

Preliminaries

- Theoretical tools to analyze semileptonic and rare decays are similar

  Allow measurements of CKM elements and are sensitive to new physics

  Improved understanding of hadronic physics and accuracy of theoretical predictions affects sensitivity to new physics

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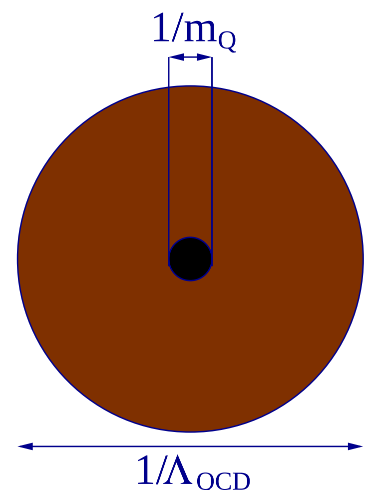
- For the purposes of this and tomorrow's talks, [strong interaction] model independent $\equiv$ theoretical uncertainty suppressed by small parameters

  Most of the recent progress comes from expanding in powers of Λ/m_Q, $\alpha_s(m_Q)$
  ... a priori not known whether $\Lambda \sim 200\mathrm{MeV}$ or $\sim 2\mathrm{GeV}$ $(f_\pi, m_\rho, m_K^2/m_s)$
  ... need experimental guidance to see which cases work how well

Heavy quark symmetry

$Q\,\overline{Q}$: positronium-type bound state, perturbative in $m_Q \gg \Lambda_{\rm QCD}$ limit

$Q\,\bar{q}$: wave function of the light degrees of freedom
("brown muck") insensitive to spin and flavor of Q



B meson is a lot more complicated than just a $b\,\bar{q}$ pair

In the $m_Q \to \infty$ limit, the heavy quark acts as a static color source with fixed four-velocity v^μ

$\Rightarrow SU(2n)$ heavy quark spin-flavor symmetry at fixed v^μ

Similar to atomic physics ($m_e \ll m_N$):

1. Flavor symmetry $\sim$ isotopes have similar chemistry [Ψ_e independent of m_N]
2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

Spectroscopy of heavy-light mesons

- In $m_Q \to \infty$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

$$\left.\begin{array}{r} \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \\ \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \end{array}\right\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$$

For a given s_l, two degenerate states:

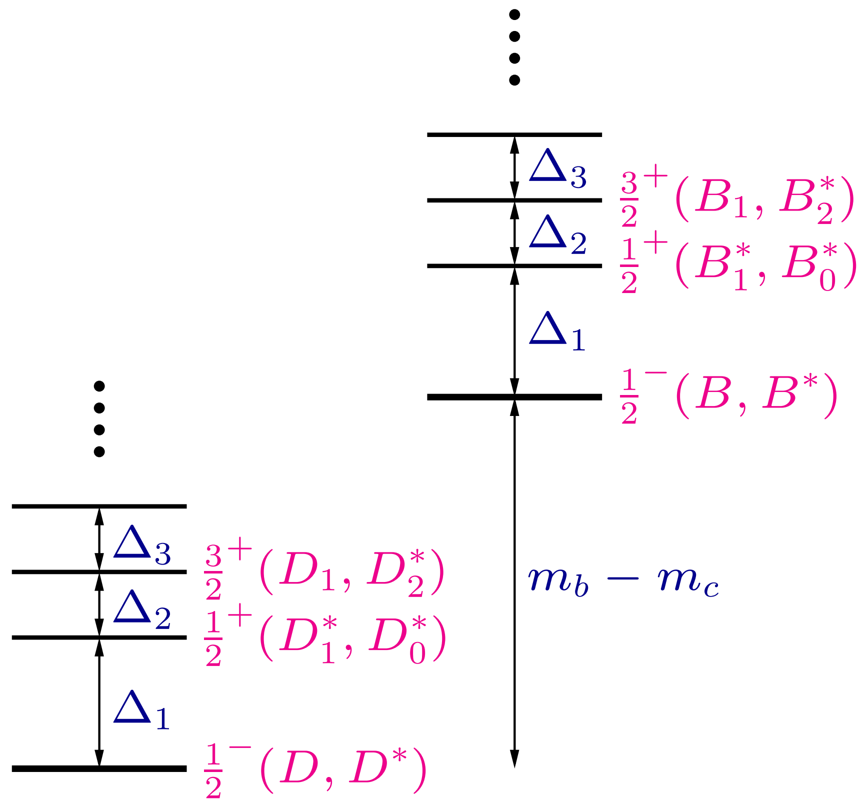
$$J_\pm = s_l \pm \tfrac{1}{2}$$

$\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda^2_{\rm QCD}/m_Q$, e.g.:

$m_{D^*} - m_D \simeq 140$ MeV

$m_{B^*} - m_B \simeq 45$ MeV



Aside: a puzzle

Since vector–pseudoscalar mass splitting $\propto 1/m_Q$, expect: $m_V^2 - m_P^2 = \text{const.}$

This argument relies on $m_Q \gg \Lambda_{\text{QCD}}$

Experimentally:

$$m_{B^*}^2 - m_B^2 = 0.49\,\text{GeV}^2$$

$$m_{B_s^*}^2 - m_{B_s}^2 = 0.50\,\text{GeV}^2$$

$$m_{D^*}^2 - m_D^2 = 0.54\,\text{GeV}^2$$

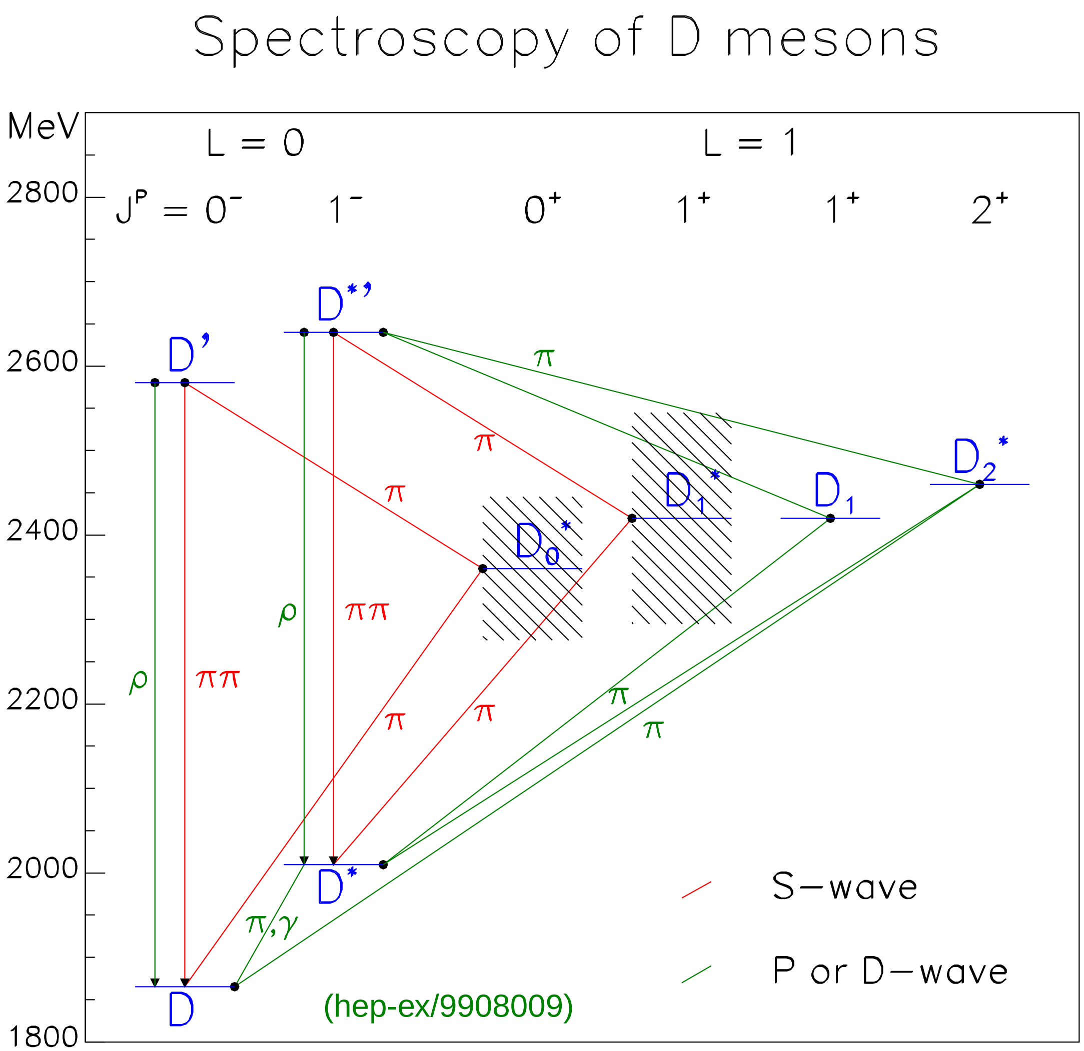
$$m_{D_s^*}^2 - m_{D_s}^2 = 0.58\,\text{GeV}^2$$

$$m_\rho^2 - m_\pi^2 = 0.57\,\text{GeV}^2$$

$$m_{K^*}^2 - m_K^2 = 0.55\,\text{GeV}^2$$

Not understood... there is something more going on than just HQS!

Charmed meson spectrum



"Successes:"

D_1 is narrow: S-wave $D_1 \to D^*\pi$ amplitude allowed by angular momentum conservation, but forbidden in the $m_Q \to \infty$ limit by heavy quark spin symmetry

Mass splittings of orbitally excited states is small:

$m_{D_2^*} - m_{D_1} = 37\,\text{MeV} \ll m_{D^*} - m_D$

vanishes in the quark model, since it arise from $\langle \vec{s}_Q \cdot \vec{s}_{\bar{q}}\, \delta^3(\vec{r}\,)\rangle$

Aside: strong decays of D_1 and D_2^*

- The strong interaction Hamiltonian conserves the spin of the heavy quark and the light degrees of freedom separately

$(D_1, D_2^*) \to (D, D^*)\pi$ — four amplitudes related by heavy quark spin symmetry

$$\Gamma(j \to j'\,\pi) \propto (2s_l + 1)(2j' + 1) \left| \begin{Bmatrix} L & s_l' & s_l \\ \frac{1}{2} & j & j' \end{Bmatrix} \right|^2$$

Multiplets have opposite parity $\Rightarrow$ π must be in $L = 2$ partial wave

$\Gamma(D_1 \to D\pi)$	:	$\Gamma(D_1 \to D^*\pi)$	:	$\Gamma(D_2^* \to D\pi)$	:	$\Gamma(D_2^* \to D^*\pi)$
0	:	1	:	$\frac{2}{5}$	:	$\frac{3}{5}$
0	:	1	:	2.3	:	0.92

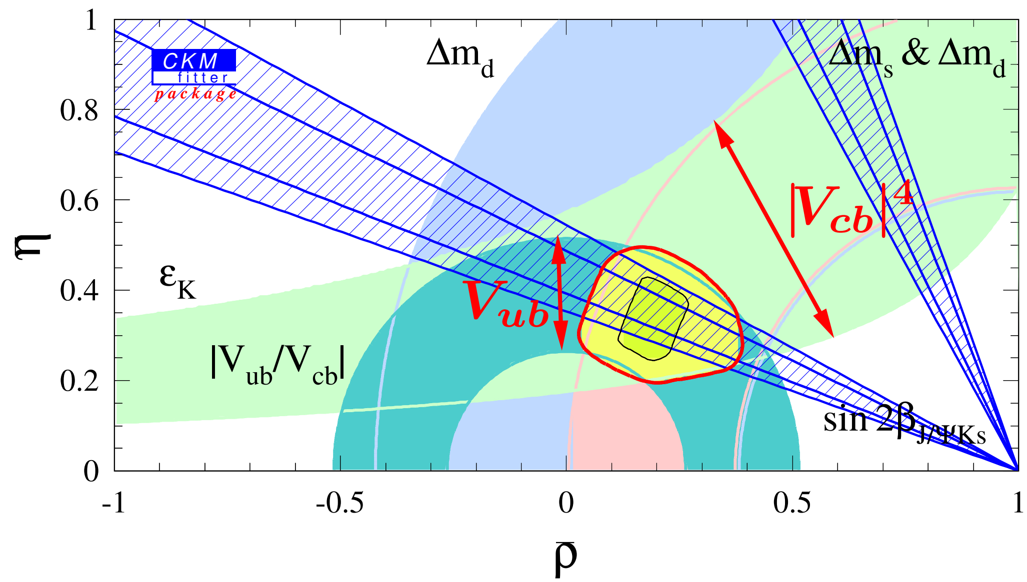
- Last line includes large $|p_\pi|^5$ HQS violation from phase space, which changes $\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi)$ from $2/3$ to 2.5 (data: 2.3 ± 0.6)

[Note: prediction for ratio of D_1 and D_2^* total widths works less well (Falk & Mehen)]

Semileptonic and rare B decays

$|V_{ub}|$ is the dominant uncertainty of the side of the UT opposite to $\beta = \phi_1$

Error of $|V_{cb}|$ is a large part of the uncertainty in the ϵ_K constraint, and in $K \to \pi\nu\bar{\nu}$ when it's measured



Rare decays mediated by $b \to s\gamma$, $b \to s\,\ell^+\ell^-$, and $b \to s\,\nu\bar{\nu}$ transitions are sensitive probes of the Standard Model

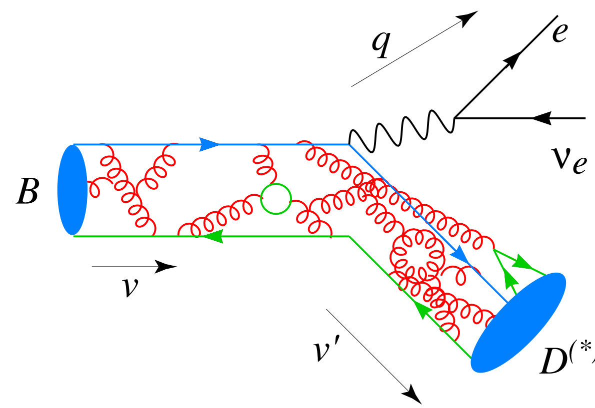
Exclusive $B \to D^{(*)}\ell\bar{\nu}$ decay

- In the $m_{b,c} \to \infty$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin

  Weak current changes $b \to c$, i.e.:

  $\vec{p}_b \to \vec{p}_c$ and possibly flips $\vec{s}_Q$, on a time scale $\ll \Lambda_{\rm QCD}^{-1}$

  In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit brown muck only feels $v_b \to v_c$



  Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$

  $\Uparrow$

  Contains all nonperturbative low-energy hadronic physics

- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

$B \to D^{(*)}\ell\bar{\nu}$ form factors

- Lorentz invariance $\Rightarrow$ 6 form factors

$$\langle D(v')|V_\nu|B(v)\rangle = \sqrt{m_B m_D}\left[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\right]$$

$$\langle D^*(v')|V_\nu|B(v)\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma$$

$$\langle D(v')|A_\nu|B(v)\rangle = 0$$

$$\langle D^*(v')|A_\nu|B(v)\rangle = \sqrt{m_B m_{D^*}}\left[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\right]$$

$V_\nu = \bar{c}\gamma_\nu b,\quad A_\nu = \bar{c}\gamma_\nu\gamma_5 b,\quad w \equiv v\cdot v' = \dfrac{m_B^2+m_D^2-q^2}{2m_B m_D},$ and $h_i = h_i(w,\mu)$

- In $m_Q \to \infty$ limit, up to corrections suppressed by α_s and $\Lambda_{\rm QCD}/m_{c,b}$

$$h_- = h_{A_2} = 0\,, \qquad h_+ = h_V = h_{A_1} = h_{A_3} = \xi(w)$$

α_s corrections calculable

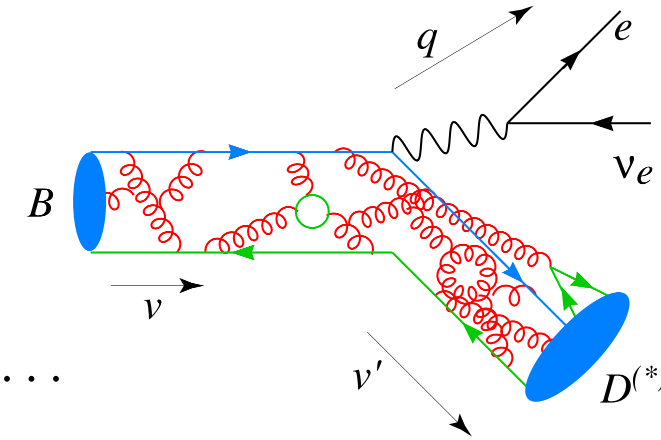
$\Lambda_{\rm QCD}/m_{c,b}$ corrections is where model dependence enters

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar\nu$

- Extract $|V_{cb}|$ from $w \equiv v \cdot v' = \frac{m_B^2+m_D^2-q^2}{2m_B m_D} = 1$ limit of $B \to D^{(*)}\ell\bar\nu$ rate

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = \text{(known factors)}\ |V_{cb}|^2\, \mathcal{F}^2_{(*)}(w)$$

$\mathcal{F}_{(*)}(w) =$ Isgur-Wise function $+\, \mathcal{O}(\alpha_s, \Lambda_{\mathrm{QCD}}/m_{c,b})$



$$\mathcal{F}(1) = 1_{\mathrm{Isgur-Wise}} + 0.02_{\alpha_s, \alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \ldots$$

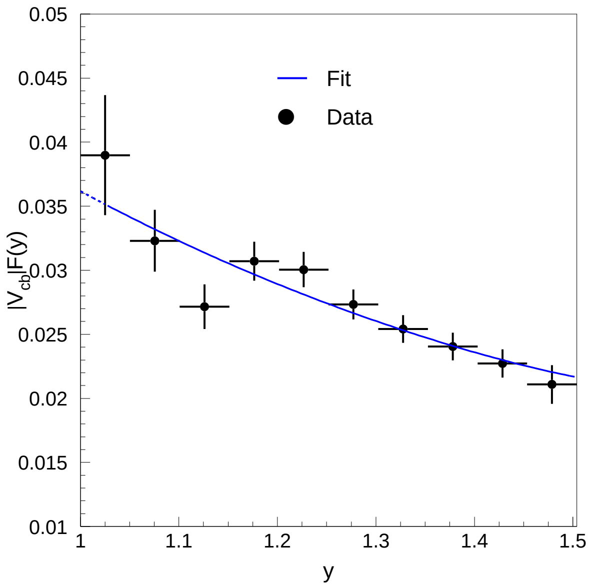
$$\mathcal{F}_*(1) = 1_{\mathrm{Isgur-Wise}} - 0.04_{\alpha_s, \alpha_s^2} + \frac{0_{\mathrm{Luke}}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \ldots$$

$\Rightarrow$ theorists argue about small corrections

Near zero recoil: $\mathrm{d}\Gamma/\mathrm{d}w \propto \begin{cases} \sqrt{w^2-1} & \text{for } B \to D^* \\ (w^2-1)^{3/2} & \text{for } B \to D \ \text{(helicity!)} \end{cases}$

$B \to D^*$ preferred both experimentally and theoretically (except lattice QCD)

Experimental status of $|V_{cb}|_{\text{exclusive}}$



Functional form used to extrapolate to zero recoil is very important — shape related to $B \to D^{**}\ell\bar{\nu}$ decay

Experiments measure: $|V_{cb}|\,\mathcal{F}_*(1)$

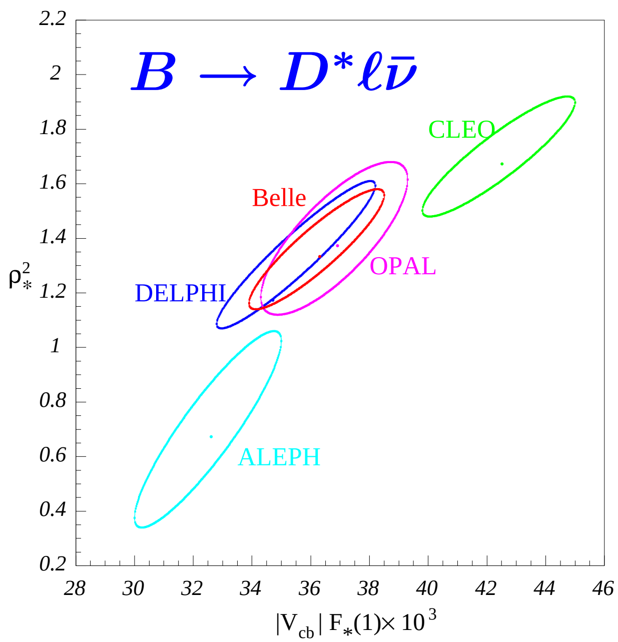
Theory predicts: $\mathcal{F}_*(1) = 0.91 \pm 0.04$

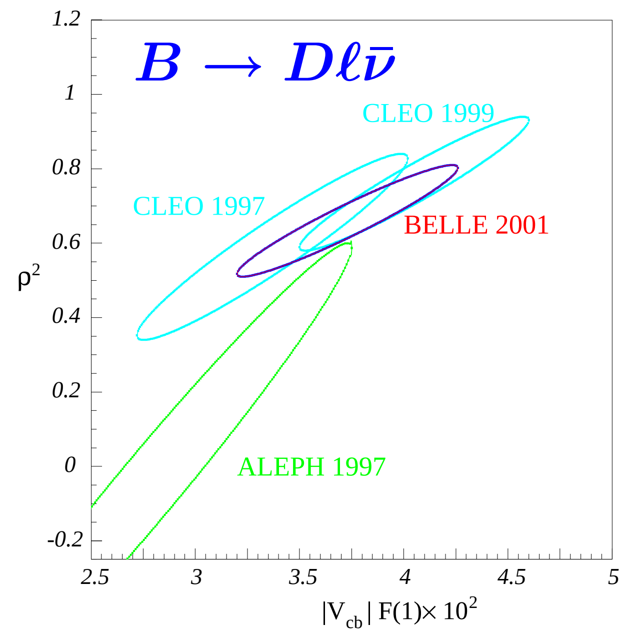
$\Rightarrow |V_{cb}| = (41.9 \pm 1.1 \pm 1.9) \times 10^{-3}$ (Battaglia @ ICHEP)

$B \to D\ell\bar{\nu}$ may be important:

Difference of slopes is an order $\Lambda_{\rm QCD}/m_{c,b}$ effect...

Corellation between slope and $|V_{cb}|$ very large





Uncertainties in $|V_{cb}|_{\rm exclusive}$

- Nonperturbative correction at zero recoil
  - Bounds from sum rules or models[2]
  - Lattice QCD: Calculate $\mathcal{F}_{(*)} - 1$ from a double ratio of correlation functions

    $\mathcal{F}(1) = 1.06 \pm 0.02\,,\ \mathcal{F}_*(1) = 0.91 \pm 0.03$, D not harder than D^* (FNAL, quenched)

  Checks: consistency between $B \to D^*$ and D, and the form factor ratios ($R_{1,2}$)

- Extrapolation to zero recoil
  - Unitarity constraints: strong correlation between slope & curvature of $\mathcal{F}_{(*)}(w)$

    (Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert)
  - Constrain slopes by studying decays to excited D^{**}, $B \to D^{**}\ell\bar\nu$, near $w = 1$

[2] "When you have to descend into the brown muck, you abandon all pretense of doing elegant, pristine, first-principles calculations. You have to get your hands dirty with uncontrolled approximations and models. When you are finished with the brown muck you should wash your hands." (H. Georgi, TASI' 1991)

$B \longrightarrow$ light form factors

- Limited use of HQS: relate $B \to \rho\ell\bar\nu,\ K^*\ell^+\ell^-,\ K^*\gamma$ form factors in large q^2 region, but HQS neither reduces number of form factors, nor determines their normalization at any value of q^2

$$\begin{array}{cccccc} & \bar B & \xrightarrow{\bar u \Gamma b\, V_{ub}} & \rho\,\ell\bar\nu & & \\ \begin{array}{c}\text{flavor}\\ SU(2)\end{array} & \updownarrow & & \updownarrow & \begin{array}{c}\text{chiral}\\ SU(3)\end{array} & \Rightarrow\ \text{relations at same } v\cdot v' \\ & D & \xrightarrow{\bar d \Gamma c\, V_{cs}} & K^*\ell\bar\nu & & \end{array}$$

Can predict $B \to \rho\ell\bar\nu$ rate from measured $D \to K^*\ell\bar\nu$ form factors

---

- Corrections to heavy quark symmetry and chiral symmetry could be $\sim 20\%$ each (First order corrections can be eliminated — complicated)

Large q^2 region is also what's most accessible to lattice QCD

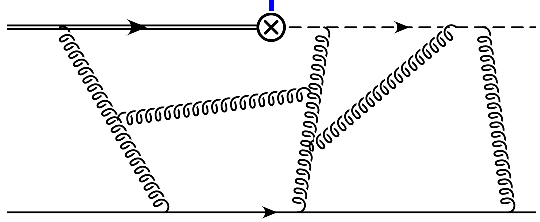
Soft-collinear effective theory

- Recently proposed: for $q^2 \ll m_B^2$, 7 vector meson form factors (V, A, T currents) related to 2 functions; 3 pseudoscalar form factors related to just 1 (Charles *et al.*)

  SCET: a new effective field theory for energetic particles (simplify power counting, helps to make all-order proofs, etc.) (Bauer, Fleming, Luke, Pirjol, Stewart)

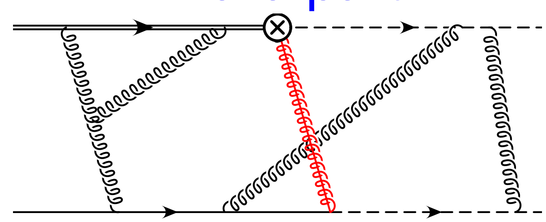
  Systematic framework to describe form factors when light hadron is very energetic

  soft part

  extra symmetries

  hard part

  calculable corrections

  Consistency of separation only proven to 1-loop yet (Beneke & Feldman)

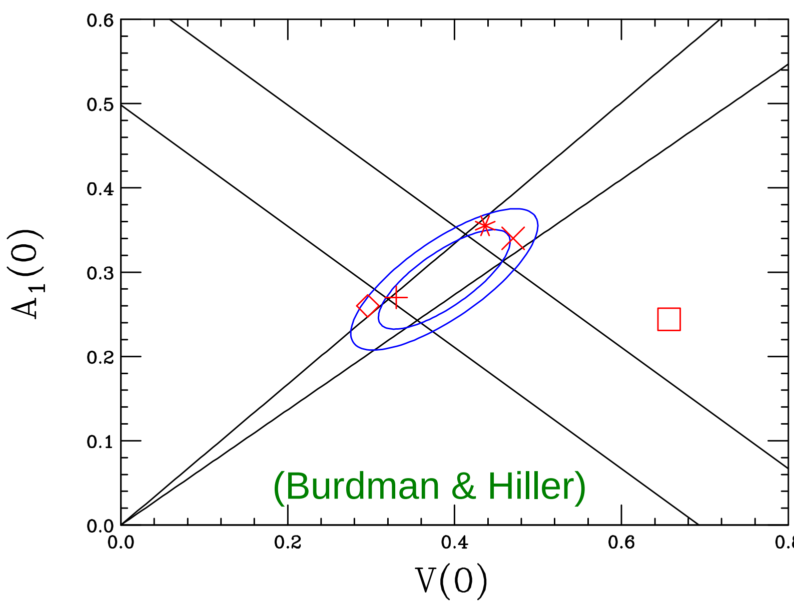
  (In $B \to D^{(*)}\ell\bar{\nu}$, nonperturbative part is in Isgur-Wise function to all orders)

  ... Expect progress!

Aside: an application

- The hope is to use some measurements in a theoretically controlled way to predict other decay rates; e.g., use $B \to K^* \gamma$ data to reduce uncertainty of $B \to K^* \ell^+ \ell^-$ and $B \to \rho \ell \bar{\nu}$ predictions, and also constrain models

Perturbative order α_s corrections have been computed (Beneke, Feldman, Seidel)



Crucial questions: all orders proof and understand power suppressed corrections

Exclusive decays — Summary

- Heavy quark symmetry provides many model independent predictions, similar to chiral symmetry

  Spectroscopy, strong and weak decays much better understood

- $B \to D^{(*)} \ell \bar{\nu}$: six semileptonic form factors depend on a single Isgur-Wise function in the $m_Q \to \infty$ limit; at zero recoil $\xi(1) = 1$, sometimes no $\Lambda_{\rm QCD}/m_Q$ corrections

  $|V_{cb}|$ known at $\sim 5\%$ level from exclusive decays (improvements will rely on lattice)

- Progress to understand exclusive heavy $\to$ light semileptonic and rare decays for small q^2; SCET might lead to rigorously proving

  Form factor relations between $B \to \pi \ell \bar{\nu}$, $B \to \rho \ell \bar{\nu}$, $B \to K^* \gamma$, $B \to K^{(*)} \ell^+ \ell^-$

  – increase sensitivity to new physics

  – tests some assumptions for factorization in charmless decays (more tomorrow)

Inclusive decays

Operator product expansion

- Consider semileptonic $b \to c$ decay: $O_{bc} = -\frac{4G_F}{\sqrt{2}} V_{cb} \underbrace{(\bar{c}\,\gamma^\mu P_L\, b)}_{J_{bc}^\mu} \underbrace{(\bar{\ell}\,\gamma_\mu P_L\, \nu)}_{J_{\ell\mu}}$

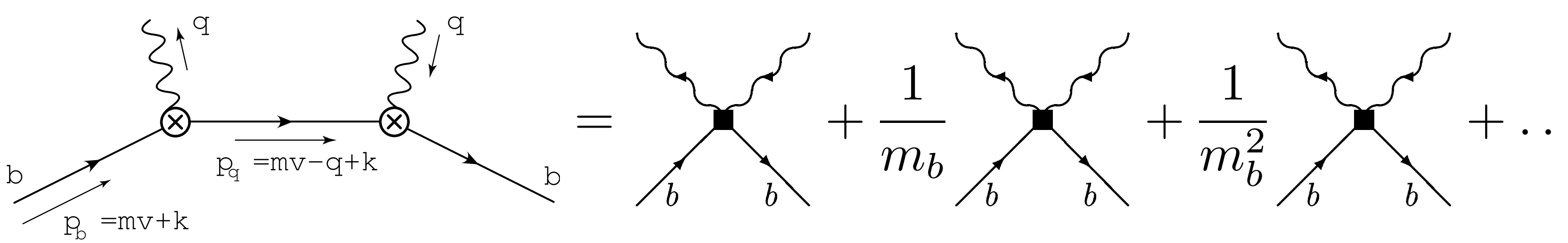
Decay rate: $\Gamma(B \to X_c \ell\bar{\nu}) \sim \sum_{X_c} \int \mathrm{d[PS]}\, \left| \langle X_c \ell \bar{\nu} | O_{bc} | B \rangle \right|^2$

Factor to: $B \to X_c W^*$ and $W^* \to \ell\bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X) \left| \langle B | J_{bc}^{\mu\dagger} | X_c \rangle \, \langle X_c | J_{bc}^\nu | B \rangle \right|^2$$

(optical theorem) $\qquad \sim \mathrm{Im} \int \mathrm{d}x\, e^{-iq\cdot x} \langle B |\, T\{ J_{bc}^{\mu\dagger}(x)\, J_{bc}^\nu(0) \} \,| B \rangle$

In $m_b \gg \Lambda_{\mathrm{QCD}}$ limit, time ordered product dominated by $x \ll \Lambda_{\mathrm{QCD}}^{-1}$



OPE (cont.)

- The $m_b \to \infty$ limit is given by free quark decay

  No $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections

  Order $\Lambda_{\rm QCD}^2/m_b^2$ corrections depend on two hadronic matrix elements

$$\lambda_1 = \frac{1}{2m_B}\,\langle B|\,\bar{b}\,(iD)^2\,b\,|B\rangle \qquad \lambda_2 = \frac{1}{6m_B}\,\langle B|\,\bar{b}\,\frac{g}{2}\,\sigma_{\mu\nu}\,G^{\mu\nu}\,b\,|B\rangle$$

not well-known $\qquad \lambda_2 = (m_{B^*}^2 - m_B^2)/4$

- OPE predicts decay rates in an expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$

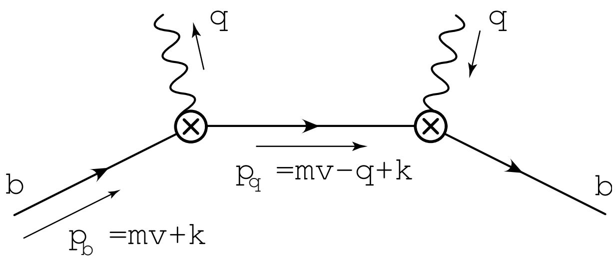
$$\mathrm{d}\Gamma = \begin{pmatrix} b\text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1,\lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

  Interesting quantities computed to order α_s, $\alpha_s^2\beta_0$, and $1/m^3$

  When can we trust the result?

Inclusive decay rates

- In which regions of phase space can we expect the OPE to converge?



Can think of the OPE as an expansion in $k \sim \Lambda_{\rm QCD}$

$$[(m_b v + k - q)^2 - m_q^2]^{-1}$$
$$= [(m_b v - q)^2 - m_q^2 + 2k\cdot(m_b v - q) + k^2]^{-1}$$

Need to allow: $m_X^2 - m_q^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$

Implicit assumption: "quark-hadron duality" valid once $m_X \gg m_q$ allowed

---

- Good news: Total rates calculable at few ($\lesssim 5$) percent level (duality...) $\Rightarrow$ $|V_{cb}|$

Need to know m_b (or $\bar\Lambda = m_B - m_b$) and λ_1

$$|V_{cb}| \sim \Big[42 \pm (\text{error mostly in } m_b \ \&\lambda_1)\Big] \times 10^{-3} \left(\frac{\mathcal{B}(B\to X_c \ell\bar\nu)}{0.105}\,\frac{1.6\,\text{ps}}{\tau_B}\right)^{1/2}$$

- Bad news: In certain restricted regions of phase space the OPE breaks down

To determine $|V_{ub}|$, cuts required to eliminate ~ 100 times larger $b\to c$ background

Determination of m_b & $\lambda_1 \Rightarrow |V_{cb}|$

- Progress likely to come from determining m_b and λ_1 from "shape variables" in inclusive B decays $\sim\ \langle E_\gamma^n \rangle$ in $B \to X_s\gamma$, $\langle E_\ell^n \rangle$ and $\langle m_{X_c}^n \rangle$ in $B \to X_c \ell\bar{\nu}$

These have been computed to $\alpha_s^2\beta_0$ and $(\Lambda_{\rm QCD}/m_Q)^3$

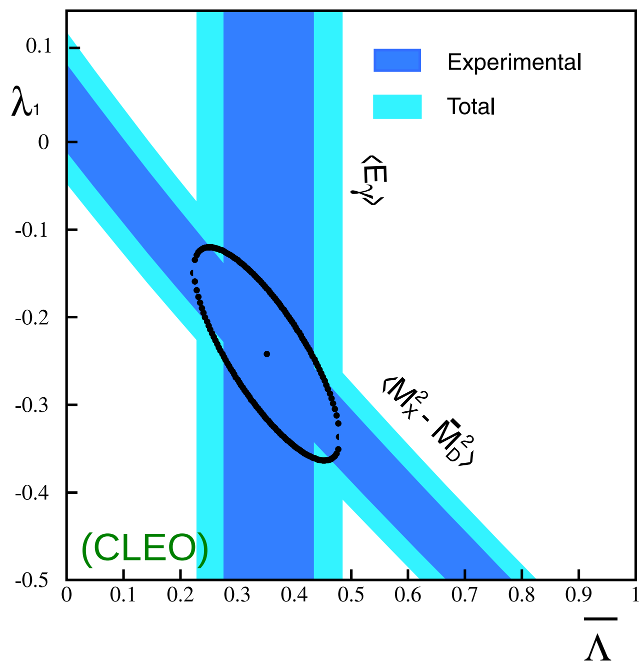
CLEO:

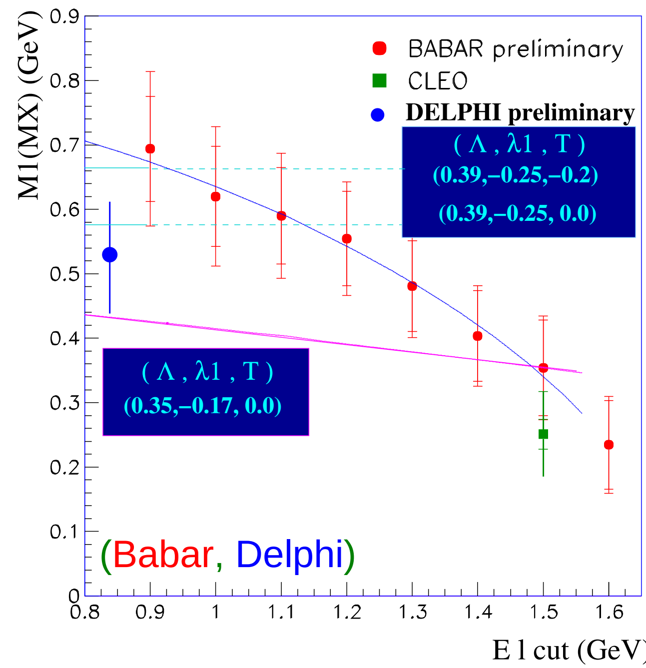
$$\bar{\Lambda} = 0.35 \pm 0.13\,\text{GeV}$$

$$\lambda_1 = -0.24 \pm 0.11\,\text{GeV}^2$$

$$\Downarrow$$

$$|V_{cb}| \sim (40.4 \pm 1.6) \times 10^{-3}$$

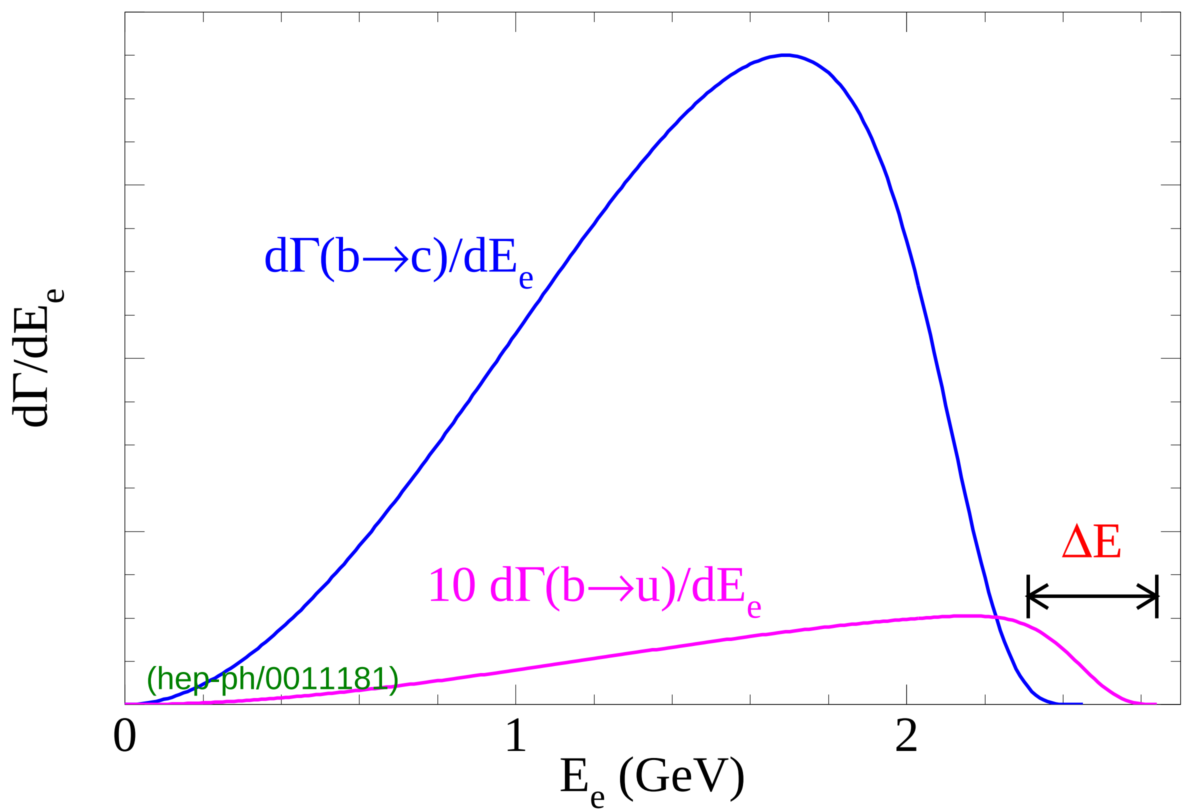




Level of (in)consistency will test accuracy of OPE and quark-hadron duality

$\Rightarrow$ May lead to $\sigma(V_{cb}) \sim 2-3\%$ if all works out

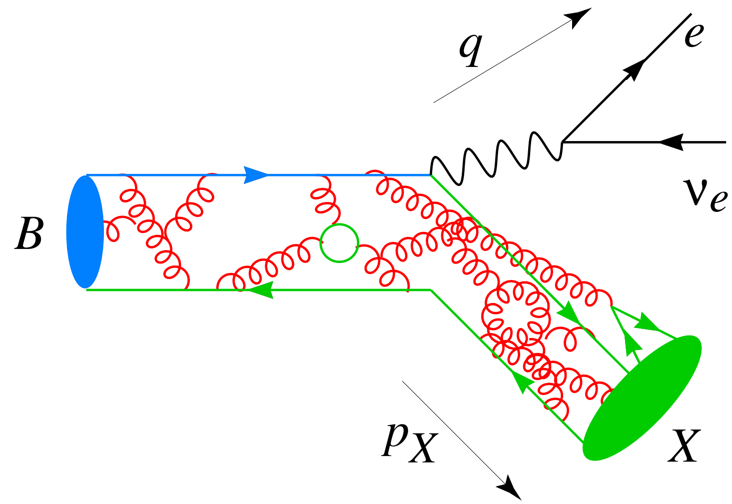
Inclusive $b \to u$: the problem



$$|V_{ub}| \sim \frac{1}{10} |V_{cb}| \quad \Rightarrow \text{cuts}$$

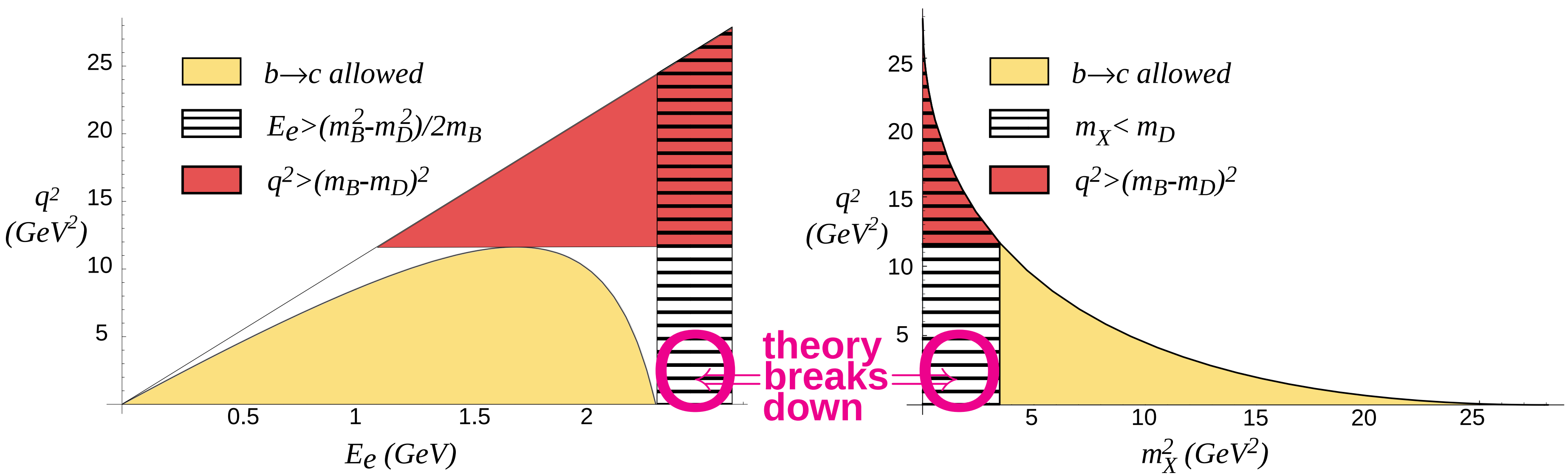
... and the troubles begin

Inclusive $B \to X_u \ell \bar{\nu}$ decay and $|V_{ub}|$



Proposals to measure $|V_{ub}|$:

- Lepton spectrum: $E_\ell > (m_B^2 - m_D^2)/2m_B$
- Hadronic mass spectrum: $m_X < m_D$
- Dilepton mass spectrum: $q^2 > (m_B - m_D)^2$



$B \to X_u \ell \bar{\nu}$ spectra

- Three qualitatively different regions of phase space:

  1) $m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: the OPE converges, first few terms can be trusted

  2) $m_X^2 \sim E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: infinite set of terms in the OPE equally important

  3) $m_X \sim \Lambda_{\rm QCD}$: resonance region — cannot compute reliably

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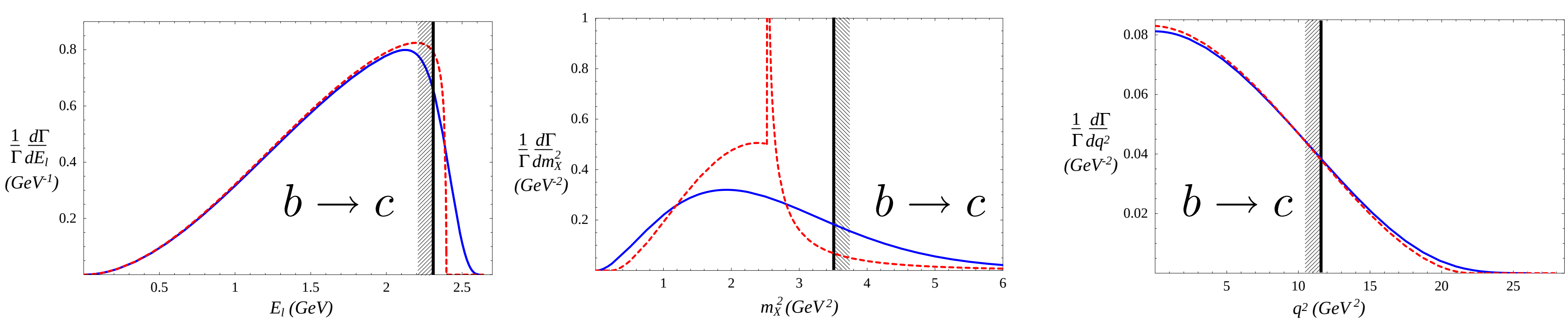
- Problem: $E_\ell > (m_B^2 - m_D^2)/2m_B$ and $m_X < m_D$ are in (2) since $m_B \Lambda_{\rm QCD} \sim m_D^2$



— b quark decay to $O(\alpha_s)$
— incl. “Fermi-motion” (model)

Theory happy

Experiment happy

V_{ub}: lepton endpoint region

- Bad: an infinite set of terms in the OPE are equally important

  Good: it is related to $B \to X_s\gamma$ photon spectrum (Neubert; Bigi, Shifman, Uraltsev, Vainshtein)

  Recently: Perturbative corrections worked out to higher order (Leibovich, Low, Rothstein)

  Terms in the OPE not related to $B \to X_s\gamma$ are also significant

  (Leibovich, ZL, Wise; Bauer, Luke, Mannel)

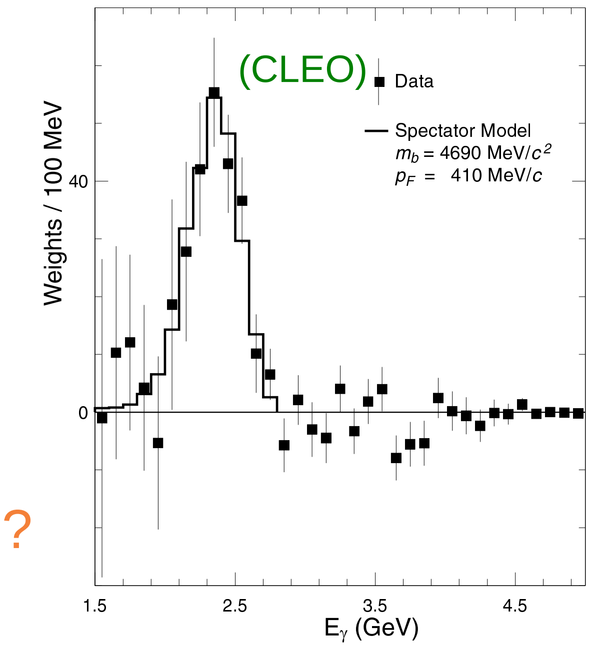
CLEO used the $B \to X_s\gamma$ photon spectrum as an input to determine $|V_{ub}|$

... measures the "Fermi-motion" of the b quark

$$|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$$

Limiting uncertainties: subleading corrections

quark-hadron duality applicable?



V_{ub}: q^2 spectrum

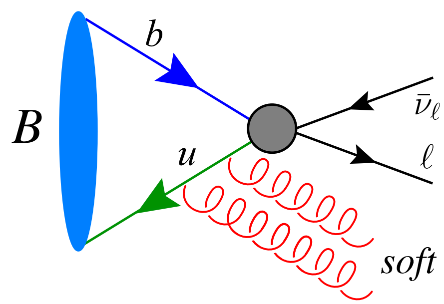
- In large q^2 region, first few terms in OPE can be trusted (Bauer, ZL, Luke)

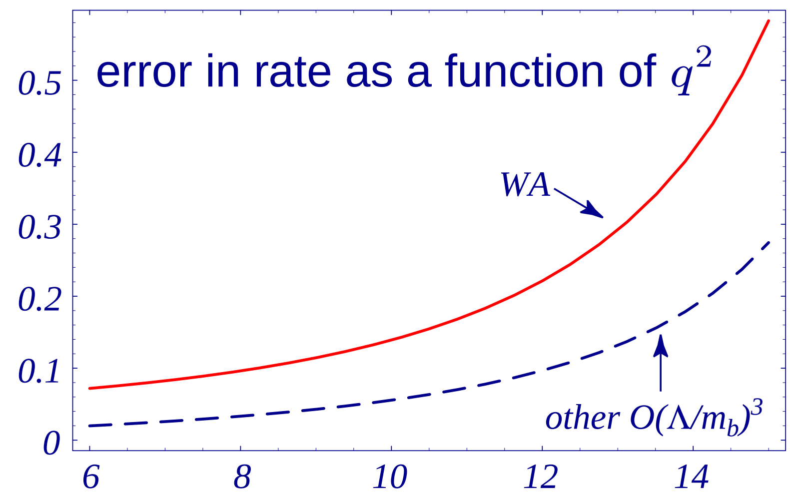
Reason: $q^2 > (m_B - m_D)^2$ cut implies $E_X < m_D$, therefore $m_X^2 \gg E_X \Lambda_{\rm QCD}$

Some nonperturbative corrections are $(\Lambda_{\rm QCD}/m_c)^3$, and not $(\Lambda_{\rm QCD}/m_b)^3$ (Neubert)

Possibly sizable corrections at $\mathcal{O}(\Lambda_{\rm QCD}/m_b)^3$ from weak annihilation (Voloshin)

Guesstimate: $\sim$ 2–3% of $b \to u$ semileptonic rate; delta-function at maximal q^2 and maximal E_ℓ





Comparing D^0 vs. D_s SL widths, or V_{ub} from $B^\pm$ vs. B^0 decay can constrain WA

V_{ub}: combine q^2 & m_X cuts

- Can get $|V_{ub}|$ with theoretical uncertainty at the 5–10% level, from up to $\sim 45\%$ of the events (Bauer, ZL, Luke)

Such precision can be achieved even with cuts away from the $b \to c$ threshold

Cuts on $(q^2,\, m_X)$	included fraction of $b \to u\ell\bar{\nu}$ rate	error of $\lvert V_{ub}\rvert$ $\delta m_b = 80/30\,\mathrm{MeV}$
$6\,\mathrm{GeV}^2,\, m_D$	46%	$8\%/5\%$
$8\,\mathrm{GeV}^2,\, 1.7\,\mathrm{GeV}$	33%	$9\%/6\%$
$(m_B - m_D)^2, m_D$	17%	$15\%/12\%$

Strategy: (i) reconstruct q^2 and m_X; make cut on m_X as large as possible
(ii) for a given m_X cut, reduce q^2 cut to minimize overall uncertainty

... Would significantly reduce the uncertainty of a side of the unitarity triangle

Semileptonic & rare decays — Summary

- $|V_{cb}|$ is known at the $\sim 5\%$ level; error may become half of this in the next few years using both inclusive and exclusive determinations (latter will rely on lattice)
- Situation for $|V_{ub}|$ may become similar to present $|V_{cb}|$; for precise inclusive determination the neutrino reconstruction seems crucial; the exclusive will use lattice
- For both $|V_{cb}|$ and $|V_{ub}|$ it is important to pursue both inclusive and exclusive
- Progress in understanding exclusive rare decays for $q^2 \ll m_B^2$ (expect more!) $B \to K^{(*)}\gamma$ and $B \to K^{(*)}\ell^+\ell^-$ below the $\psi \Rightarrow$ increase sensitivity to new physics

  Related to some issues in factorization in charmless decays (tomorrow)

Additional Topics

- B decays to excited D mesons
- Exclusive rare decays
- Inclusive rare decays

Decays to excited states: $B \to D^{**}\ell\bar\nu$

- HQS $\Rightarrow$ matrix elements of weak currents vanish at zero recoil for excited states
  Become non-zero at $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ — most of the phase space is near zero recoil

$m_Q \to \infty$: for each doublet, all form factors are related to an Isgur-Wise function

$\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$: in $B \to (D_1, D_2^*)\ell\bar\nu$, 8 subleading I-W fn's, but only 2 independent

$$\frac{{\rm d}\Gamma(B \to D_1 \ell\bar\nu)}{{\rm d}w} \propto \sqrt{w^2-1}\,[\tau(1)]^2 \Big\{ 0 + 0\,(w-1) + (\ldots)(w-1)^2 + \ldots$$

$$+\frac{\Lambda_{\rm QCD}}{m_Q}\big[\,0 + (\text{almost calculable})(w-1) + \ldots\big]$$

$$w \equiv \frac{m_B^2+m_{D_1}^2-q^2}{2m_B\, m_{D_1}} \in (1, 1.3)$$

$$+\frac{\Lambda_{\rm QCD}^2}{m_Q^2}\big[\,(\text{calculable!}) + \ldots\big] + \ldots \Big\}$$

In $B \to$ (orbitally excited D) decays, the zero recoil matrix element at $\mathcal{O}(1/m_Q)$ is given by mass splittings and the $m_Q \to \infty$ Isgur-Wise fn. (Leibovich, ZL, Stewart, Wise)

More $B \to D^{**}\ell\bar\nu$

- Bjorken sum rule for the slope of Isgur-Wise function ($\exists$ many more sum rules):

$$\rho^2 = \frac{1}{4} + \sum_m \frac{|\zeta^{(m)}(1)|^2}{4} + 2\sum_p \frac{|\tau^{(p)}(1)|^2}{3} + \text{nonresonant}$$

$\zeta^{(m)}$ and $\tau^{(p)}$ are Isgur-Wise fn's for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states

$B \to D_1\ell\bar\nu$ rate is enhanced at order $\Lambda_{\rm QCD}/m_Q$ by much more than $D_2^*\ell\bar\nu$

The present world average is about 0.4 ± 0.15

Approximation	$\Gamma_{D_2^*}/\Gamma_{D_1}$
$m_Q \to \infty$	1.65
Finite m_Q B_1	0.52
Finite m_Q B_2	0.67

- To compare $B \to (D_1, D_2^*)$ with (D_0^*, D_1^*), need to know the Isgur-Wise functions

Quark models (ISGW, etc.) and QCD sum rules predict that the Isgur-Wise function for the broad doublet is not larger than for the narrow doublet

If you buy these arguments, then the large $B \to (D_0^*, D_1^*)\ell\bar\nu$ rate is a puzzle

$B \to D^{**}\pi$ decays

- Factorization is expected to work as well as in $B \to D^{(*)}\pi$

$$\Gamma_\pi = \frac{3\pi^2\,|V_{ud}|^2\,C^2\,f_\pi^2}{m_B^2\,r} \times \left(\frac{\mathrm{d}\Gamma_{\mathrm{sl}}}{\mathrm{d}w}\right)_{w_{\max}}$$

$$r = m_{D^{**}}/m_B\,, \quad w_{\max} = (1+r^2)/(2r) \simeq 1.3\,, \quad f_\pi \simeq 132\,\text{MeV}\,, \quad C\,|V_{ud}| \simeq 1$$

- An interesting ratio from which Isgur-Wise function cancels out:

$$\frac{\mathcal{B}(B^- \to D_2^{*0}\pi^-)}{\mathcal{B}(B^- \to D_1^0\pi^-)} = 0.89 \pm 0.14$$ (BELLE @ ICHEP)

This looks OK and can teach us about $1/m$ corrections (in '97 ratio was 1.8 ± 0.9, theory could not accommodate such a large central value) (Leibovich, ZL, Stewart, Wise)

Sorting out these semileptonic and nonleptonic decays to excited D's will be important for HQET, factorization, and will impact $|V_{cb}|$ determinations

Exclusive rare decays

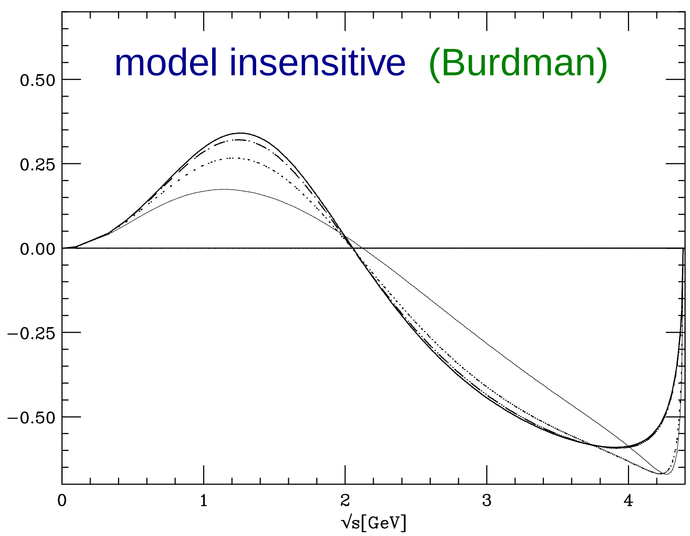
- Important probes of NP — measurements of $|V_{ij}|$

  Exclusive decays are experimentally easier — need to understand form factors

  – $B \to K^*\gamma$ or $B \to X_s\gamma$: best $m_{H^\pm}$ limits in 2HDM — in SUSY many param's

  – $B \to K^{(*)}\ell^+\ell^-$ or $B \to X\,\ell^+\ell^-$: bsZ penguins, SUSY, right handed couplings

- There is an observable insensitive to the precise values of the form factors:



Forward-backward asymmetry in $B \to K^*\ell^+\ell^-$ changes sign:

$$C_9^{\rm eff}(s_0) = -\frac{2m_B m_b}{s_0}\, C_7^{\rm eff} \times \left[\, 1 + O(\alpha_s, \Lambda_{\rm QCD}/m_b)\right]$$

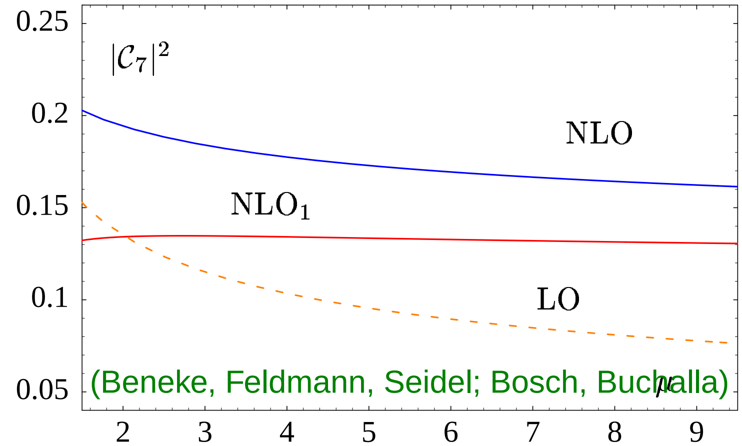
$\mathcal{O}(\alpha_s)$ corrections computed (Beneke, Feldman, Seidel)

May give clean measurement of C_9 (sensitive to NP)

$B \to K^*\gamma$ briefly

Large ($\sim 80\%$) enhancement of $B \to K^*\gamma$ decay rate found at NLO

$\Rightarrow$ $1/m$ correction large or/and form factors significantly different from model predictions



Form factors also enter predictions for isospin splitting — power suppressed correction, but claimed to be calculable

$$\Delta_{0-} = \frac{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) + \Gamma(B^- \to K^{*-}\gamma)} = 0.02 \pm 0.07 \qquad \text{(data)}$$

$$\Delta_{0-} = (0.08^{+2.1}_{-3.2})\% \times \frac{0.3}{T_1^{B\to K^*}} \qquad \text{(Kagan \& Neubert)}$$

Testing these predictions may be important for understanding various approaches to factorization in charmless decays

Inclusive rare B decays

- Important probes of new physics — measurements of CKM elements
  - $B \to K^*\gamma$ or $X_s\gamma$: Best $m_{H^\pm}$ limits in 2HDM — in SUSY many param's
  - $B \to K^{(*)}\ell^+\ell^-$ or $X_s\ell^+\ell^-$: bsZ penguins, SUSY, right handed couplings

A crude guide... ($\ell = e$ or μ)

Decay	$\sim$SM rate	physics examples
$B \to s\gamma$	3×10^{-4}	$\lvert V_{ts}\rvert$, $H^\pm$, SUSY
$B \to s\nu\nu$	4×10^{-5}	new physics
$B \to \tau\nu$	4×10^{-5}	$f_B\lvert V_{ub}\rvert$, $H^\pm$
$B \to s\ell^+\ell^-$	7×10^{-6}	new physics
$B_s \to \tau^+\tau^-$	1×10^{-6}	
$B \to s\tau^+\tau^-$	5×10^{-7}	$\vdots$
$B \to \mu\nu$	3×10^{-7}	
$B_s \to \mu^+\mu^-$	4×10^{-9}	
$B \to \mu^+\mu^-$	1×10^{-10}	

Replacing $b \to s$ by $b \to d$ costs factor ~ 20 (in SM)

In $B \to q\, l_1\, l_2$ decays expect ~ 10–20% K^*/ρ, and ~ 5–10% K/π (model dependent)

So far the $b \to s\ell^+\ell^-$ data agrees with the SM expectation within the still sizable errors

Something to worry about?

$$\mathcal{B}(B \to \psi X_s) \sim 4 \times 10^{-3}$$

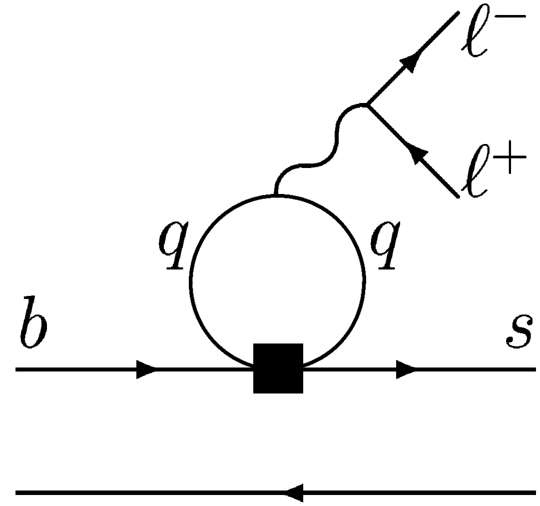
$$\downarrow$$

$$\mathcal{B}(\psi \to \ell^+\ell^-) \sim 6 \times 10^{-2}$$

So this “long distance” contribution is:

$$\mathcal{B}(B \to X_s\ell^+\ell^-) \sim 2 \times 10^{-4}$$

This is ~ 30 times the short distance contribution!



Averaged over a large region of invariant masses (and $0 < q^2 < m_B^2$ should be large enough), the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \ldots$ This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but NOT here

Is it consistent to “cut out” the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)

Third Lecture

- Clean determination of angles

  ... $B_s \to D_s K$ or $B \to D^{(*)}\pi$

  ... $B \to \pi\pi$ with isospin analysis, etc.

- Factorization in $B \to D^{(*)}X$ decay

  ... How / why to test it

- Factorization in charmless decays

  ... different approaches

  ... some predictions / applications

- Summary / Conclusions

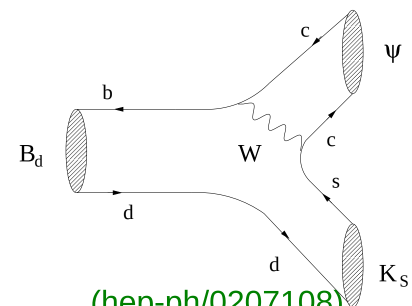
Angles — cleanly

$B \to \psi K_{S,L}$ — saw this before

- Clean measurement possible because $|\lambda_{\psi K_{S,L}}| - 1 \ll 1$

$$a_{f_{CP}} = \frac{\Gamma[\overline{B}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\overline{B}^0(t) \to f] + \Gamma[B^0(t) \to f]} = S_f \sin(\Delta m\, t) - C_f \cos(\Delta m\, t)$$

$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\,\frac{q}{p}\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}\,, \quad S_f = \frac{2\mathrm{Im}\,\lambda_f}{1+|\lambda_f|^2}\,, \quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$$



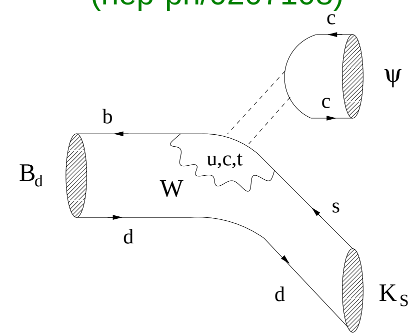
(hep-ph/0207108)

Tree: $\overline{A}_T \sim V_{cb}V_{cs}^*$

Penguin: $\overline{A}_P \sim \overset{[\lambda^2]}{V_{tb}V_{ts}^*}\, f(m_t) + \overset{[\lambda^2]}{V_{cb}V_{cs}^*}\, f(m_c) + \overset{[\lambda^4]}{V_{ub}V_{us}^*}\, f(m_u)$

$$= \underbrace{V_{cb}V_{cs}^*}\,[f(m_c)-f(m_t)] + \underbrace{V_{ub}V_{us}^*}\,[f(m_u)-f(m_t)]$$

same as Tree phase — suppressed by λ^2

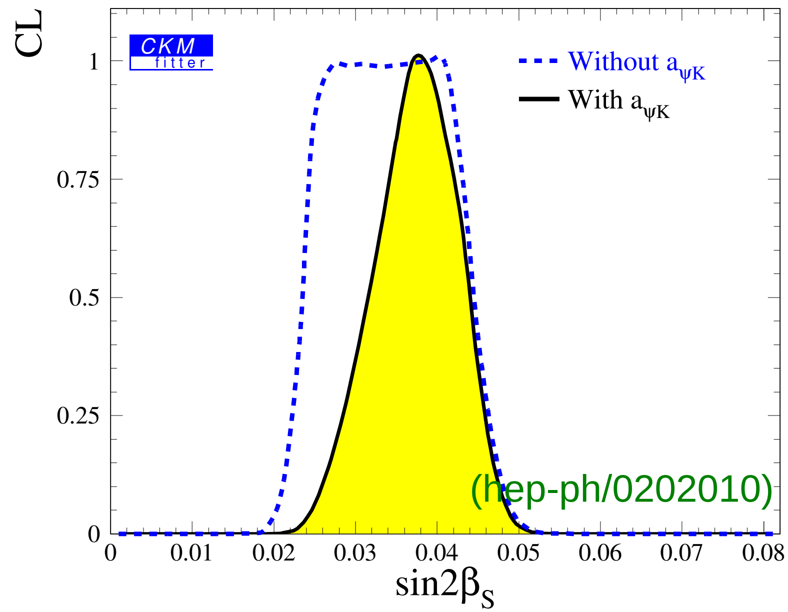


$$\lambda_{\psi K_{S,L}} = \mp\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right) = \mp e^{-2i\beta} \quad \Rightarrow \quad \mathrm{Im}\lambda_{\psi K_{S,L}} = \pm\sin 2\beta$$

$B_s \to \psi\phi$ and $B_s \to \psi\eta^{(\prime)}$

- Analog of $B \to \psi K_S$ in B_s decay — determines the phase between B_s mixing and $b \to c\bar{c}s$ decay, β_s, as cleanly as the determination of β

  β_s is a small angle (of order λ^2) in one of the "squashed" UT's

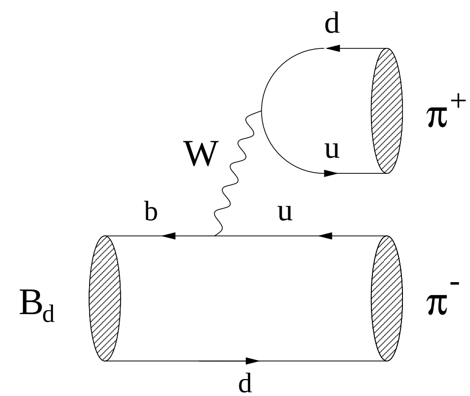


- $\psi\phi$ is a VV final state, so the asymmetry will be diluted by the CP-odd component
  $\Rightarrow$ A large asymmetry would clearly signal NP

  $\psi\eta^{(\prime)}$, on the other hand, is pure CP-even

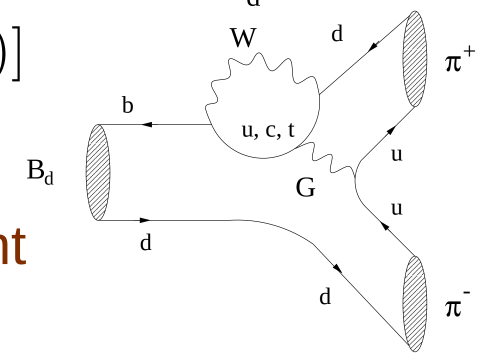
$B \to \pi\pi$ — the problem

- There are tree and penguin amplitudes, just like for ψK_S



"Tree" ($b \to u\bar{u}d$): $\overline{A}_T \sim \overset{[\lambda^3]}{V_{ub}V_{ud}^*}$

"Penguin": $\overline{A}_P \sim \overset{[\lambda^3]}{V_{tb}V_{td}^*}\, f(m_t) + \overset{[\lambda^3]}{V_{cb}V_{cd}^*}\, f(m_c) + \overset{[\lambda^3]}{V_{ub}V_{ud}^*}\, f(m_u)$



(unitarity) $\sim \underbrace{\overset{[\lambda^3]}{V_{ub}V_{ud}^*}}_{\text{same as Tree phase}} [f(m_u) - f(m_t)] + \underbrace{\overset{[\lambda^3]}{V_{cb}V_{cd}^*}}_{\text{not suppressed}} [f(m_c) - f(m_t)]$

Two amplitudes with different weak- and possibly different strong phases; their values not known model independently

Define P and T by: $A_{\pi^+\pi^-} = T(V_{ub}V_{ud}^*) + P(V_{cb}V_{cd}^*)$

Ratio of $K\pi$ and $\pi\pi$ rates indicates $|P/T| \sim 0.2 - 0.4$, i.e., $|P/T| \not\ll 1$

- Possible solutions: (1) eliminate P; or (2) attempt to calculate P

$B \to \pi\pi$ — isospin analysis

$(u,\, d)$: I-spin doublet

other quarks and gluons: $I = 0$

$\gamma,\, Z$: mixtures of $I = 0, 1$

$$(\pi\pi)_{\ell=0} \quad \to \quad I_f = 0 \quad \text{or} \quad I_f = 2$$

$$(1 \times 1) \qquad (\Delta I = \tfrac{1}{2}) \qquad (\Delta I = \tfrac{3}{2})$$

$I = 0$ final state forbidden by Bose symmetry

Hamiltonian has two parts: $\Delta I = \frac{1}{2} \;\Rightarrow\; I_f = 0$

$\Delta I = \frac{3}{2} \;\Rightarrow\; I_f = 2$ ... only two amplitudes

3 rates: $\overline{B}^0 \to \pi^+\pi^-$, $\overline{B}^0 \to \pi^0\pi^0$, and $B^- \to \pi^0\pi^-$ determine magnitudes and relative phase of two amplitudes ... similarly for B^0 and B^+ decay

In practice, need all (tagged) rates $+$ time dependent asymmetry in $B \to \pi^+\pi^-$

Note: γ and Z penguins violate isospin and yield some (small) uncertainty

Isospin analysis (cont.)

$$
\begin{aligned}
A^{+-} &\equiv A(B^0 \to \pi^+\pi^-), & \bar{A}^{+-} &\equiv A(\bar{B}^0 \to \pi^+\pi^-),\\
A^{00} &\equiv A(B^0 \to \pi^0\pi^0), & \bar{A}^{00} &\equiv A(\bar{B}^0 \to \pi^0\pi^0),\\
A^{+0} &\equiv A(B^+ \to \pi^+\pi^0), & \bar{A}^{-0} &\equiv A(B^- \to \pi^-\pi^0).
\end{aligned}
$$

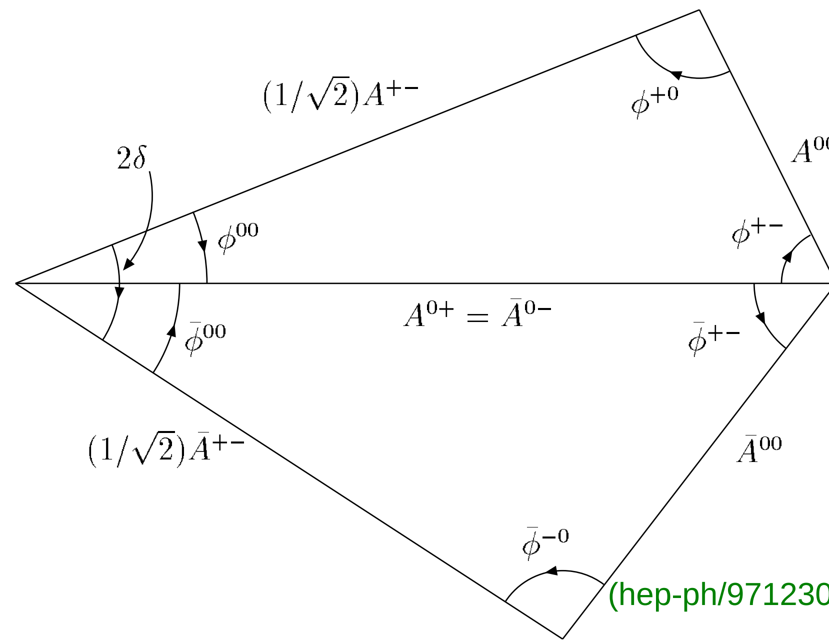
Isospin symmetry implies that 6 amplitudes form two triangles with a common base

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}, \quad \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{-0}$$

$$|A^{+0}| = |\bar{A}^{-0}|$$

$2\delta =$ difference between $\arg\lambda_{\pi^+\pi^-}$ and 2α

(constrained by any limit on $\pi^0\pi^0$ rate – later)



(hep-ph/9712306)

$B \to \rho\pi$: 4 isospin amplitudes $\Rightarrow$ pentagon relations

Dalitz plot analysis would allow considering $\pi^+\pi^-\pi^0$ final state only

Implications of current data

- Babar and Belle measured:

$$S_{\pi^+\pi^-} = \frac{2\,\mathrm{Im}\,\lambda_{\pi\pi}}{1+|\lambda_{\pi\pi}|^2} \equiv \sin 2\alpha_{\rm eff}\,, \qquad C_{\pi^+\pi^-} = \frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2}$$

$$\lambda_{\pi\pi} = e^{-2i\beta}\,\frac{e^{-i\gamma}+P/T}{e^{i\gamma}+P/T}$$

If P/T were small, then $|\lambda_{\pi\pi}| \simeq 1$ and $S_{\pi^+\pi^-} = \mathrm{Im}\lambda_{\pi\pi} \simeq -\sin 2(\beta+\gamma) = \sin 2\alpha$

$C_{\pi^+\pi^-}$ measures: $|\lambda_{\pi\pi}|^2 = \dfrac{1-C_{\pi^+\pi^-}}{1+C_{\pi^+\pi^-}}$ (note: $S^2+C^2 \le 1$, and $=1$ iff $\mathrm{Re}\lambda = 0$)

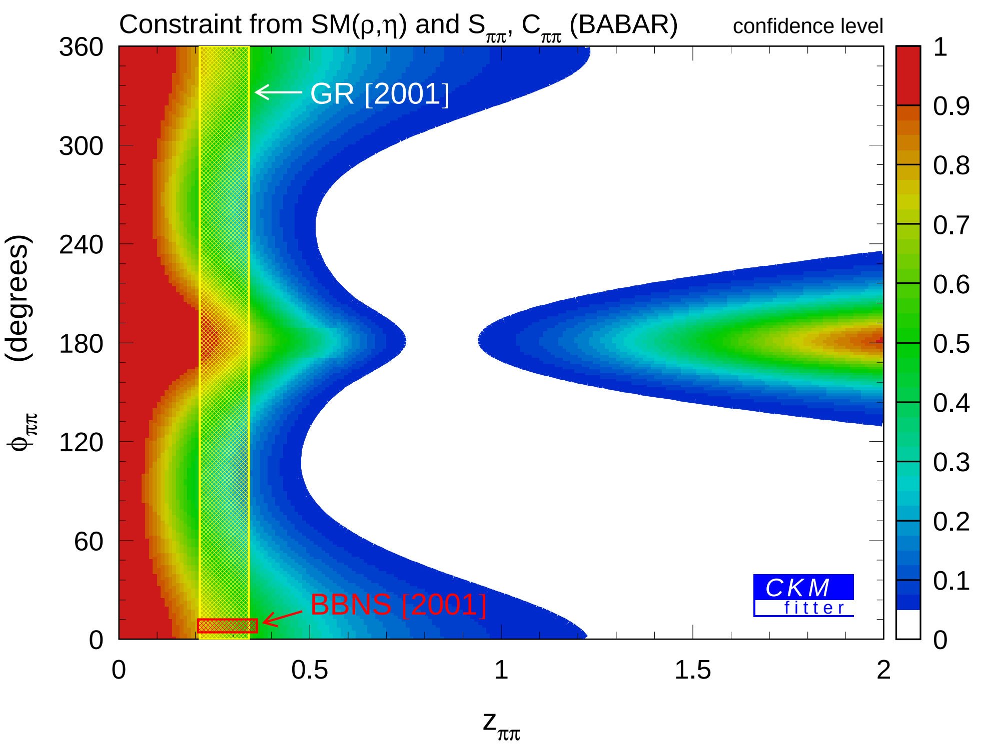
Central values of $C_{\pi^+\pi^-}$ imply Babar: $-0.30 \pm 0.25 \pm 0.04$ $\Rightarrow$ modest P/T

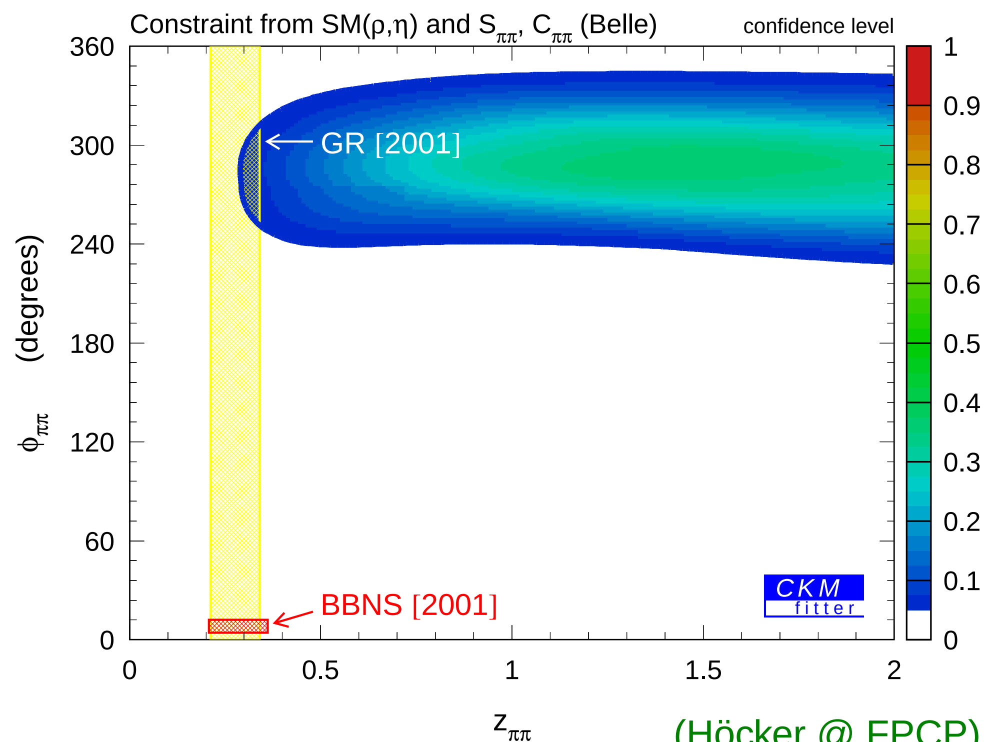
Belle: $-0.94^{+0.31}_{-0.25} \pm 0.09$ $\Rightarrow$ **large P/T**

- To extract α from $S_{\pi^+\pi^-}$ alone, need to know magnitude and phase of P/T

Implications for P/T — another way

Assume the SM is correct, use $S_{\pi\pi}$ and $C_{\pi\pi}$ measurements to constrain magnitude and phase of $P/T (\equiv z_{\pi\pi}\, e^{i\phi_{\pi\pi}})$

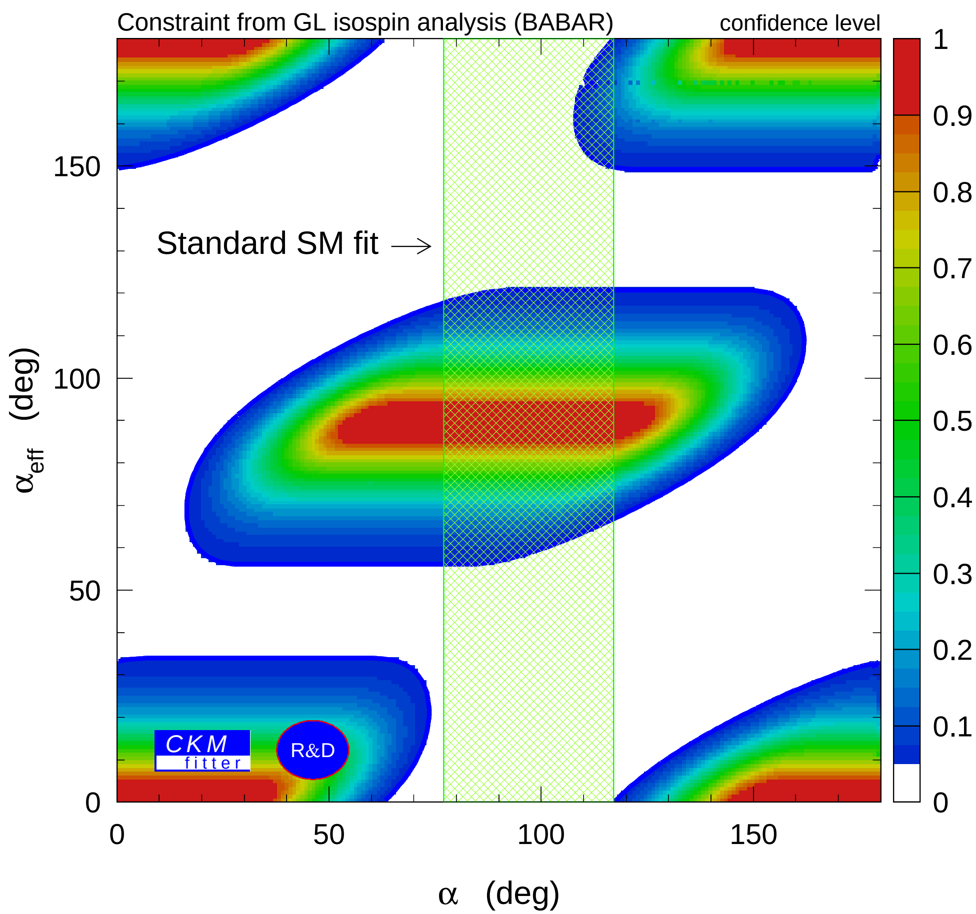




(Höcker @ FPCP)

need more data...

Bounding $\alpha - \alpha_{\text{eff}}$



Isospin relations $+$ branching ratios $+$ limit on $B \to \pi^0\pi^0 \;\Rightarrow\;$ bound on $\alpha - \alpha_{\text{eff}}$

No strong constraint from present bound on $B \to \pi^0\pi^0$

$B_s \to D_s^\pm K^\mp$ — a different story

- Interference between B_s and $\overline{B}_s$ decay — only one tree amplitude in each case

Four amplitudes:

$$\overline{B}_s \stackrel{A_1}{\to} D_s^+ K^- \quad (b \to c\bar{u}s)\,, \qquad \overline{B}_s \stackrel{A_2}{\to} K^+ D_s^- \quad (b \to u\bar{c}s)$$

$$B_s \stackrel{A_1}{\to} D_s^- K^+ \quad (\bar{b} \to \bar{c}u\bar{s})\,, \qquad B_s \stackrel{A_2}{\to} K^- D_s^+ \quad (\bar{b} \to \bar{u}c\bar{s})$$

$$\frac{\overline{A}_{D_s^+K^-}}{A_{D_s^+K^-}} = \frac{A_1}{A_2}\left(\frac{V_{cb}V_{us}^*}{V_{ub}^*V_{cs}}\right), \qquad \frac{\overline{A}_{D_s^-K^+}}{A_{D_s^-K^+}} = \frac{A_2}{A_1}\left(\frac{V_{ub}V_{cs}^*}{V_{cb}^*V_{us}}\right)$$

Relative strong phase and magnitudes of A_1 and A_2 are unknown, still theory error is eliminated if four time dependent rates are measured:

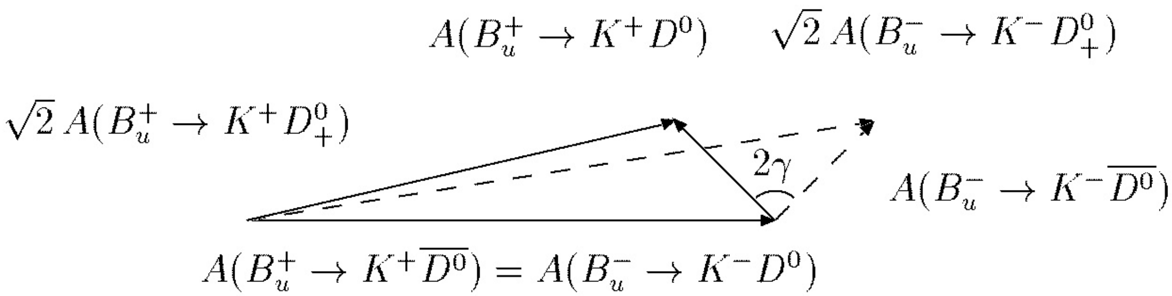
$$\lambda_{D_s^+K^-}\,\lambda_{D_s^-K^+} = \left(\frac{V_{tb}^*V_{ts}}{V_{tb}V_{ts}^*}\right)^2 \left(\frac{V_{cb}V_{us}^*}{V_{ub}^*V_{cs}}\right)\left(\frac{V_{ub}V_{cs}^*}{V_{cb}^*V_{us}}\right) = e^{-2i(\gamma-2\beta_s-\beta_K)}$$

- Similarly, $B_d \to D^{(*)\pm}\pi^\mp$ determines $\gamma + 2\beta$: $\quad \lambda_{D^+\pi^-}\,\lambda_{D^-\pi^+} = e^{-2i(\gamma+2\beta)}$

... ratio of amplitudes $\mathcal{O}(\lambda^2) \Rightarrow$ expected asymmetries are small

$B^\pm \to (D^0, \overline{D}^0) K^\pm \to f_i\, K^\pm$

- $B^\pm \to K^\pm D$: theoretically clean, experimentally very hard (Gronau-Wyler)



$$\frac{|A(B^+ \to K^+ D^0)|}{|A(B^+ \to K^+ \overline{D}^0|} \sim \frac{\lambda}{N_c}$$

- $B^\pm \to K^\pm (D^0, \bar{D}^0) \to K^\pm f_i \qquad (i = 1, 2)$ (Atwood, Dunietz, Soni)

  make use of large final state interaction in D decay

  Idea: $B^+ \to K^+ \overline{D}^0 \to K^+ f_i$ in doubly Cabibbo suppressed $\overline{D}^0$ decay

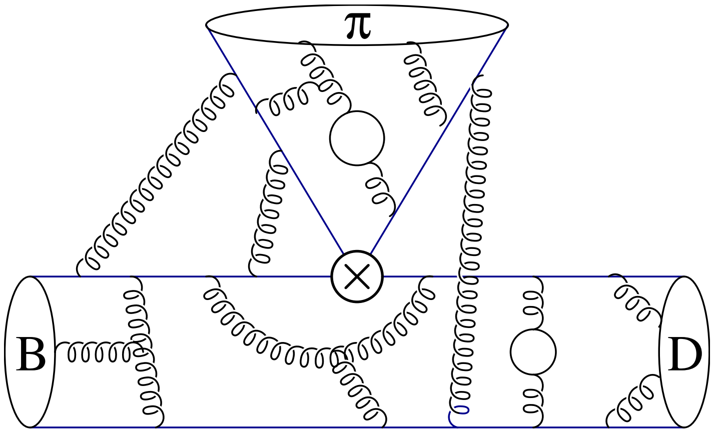
  $B^+ \to K^+ D^0 \to K^+ f_i$ in Cabibbo allowed D^0 decay (e.g.: $f_i = K^- \pi^+ / \rho^+$)

  Need at least 2 final states   Total Br's $\sim 10^{-7}$ — statistics?

- Many ideas on the market would become a lot simpler if some of the hadronic decay amplitudes were understood

Factorization in $b \to c$

Factorization in $b \to c$ exclusive decays



Start from OPE; estimate matrix elements of four-quark operators by grouping the quark fields into two that mediate $B \to D$, and two that can describe vacuum $\to \pi$ — Are gluons connecting B & D to π either calculable or power suppressed?

- "Naive" factorization: $\langle D\pi|\bar{c}b\bar{u}d|B\rangle \sim F_{B\to D}\, f_\pi$

  Since B and D are "soft" and π is "collinear", "color transparency" provides a physical picture for factorization (early 90's: Bjorken; Dugan, Grinstein)

  Configuration of brown muck in D changes only slightly, π is a fast color dipole

  This picture expected to hold for $B \to D^{(*)}X$, as long as $E_X/m_X \gg 1$

  Cannot be the full story: Wilson coefficients (of $\bar{c}b\bar{u}d$ operators) scale dependent

Factorization in $b \to c$ (cont.)

- "Generalized" factorization: $\langle D\pi|\bar{c}b\bar{u}d|B\rangle \sim F_{B\to D}\int_0^1 \mathrm{d}x\, T(x,\mu)\,\phi_\pi(x,\mu)\, f_\pi$

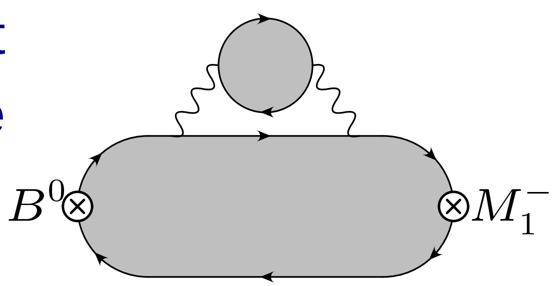
  (proposed: Politzer, Wise; 2-loop proof: Beneke, Buchalla, Neubert, Sachrajda; all orders proof: Bauer, Pirjol, Stewart)

  Fully consistent formulation, scale and scheme dependence cancels order-by-order between Wilson coefficients $C_i(\mu)$ and matrix elements $\langle O_i(\mu)\rangle$

  No OPE — corrections presumed to be $\mathcal{O}(\Lambda/m_b)^n$ but this is not firmly established (Depends on details of B, D, π wave-functions)

  Proof applies when meson that inherits the spectator quark from the B is heavy and the other is light

- Factorization also holds in the large number of colors limit ($N_c \to \infty$ with $\alpha_s N_c =$ const.) in all $B^0 \to M_1^- M_2^+$ type decays, corrections $\propto 1/N_c^2$



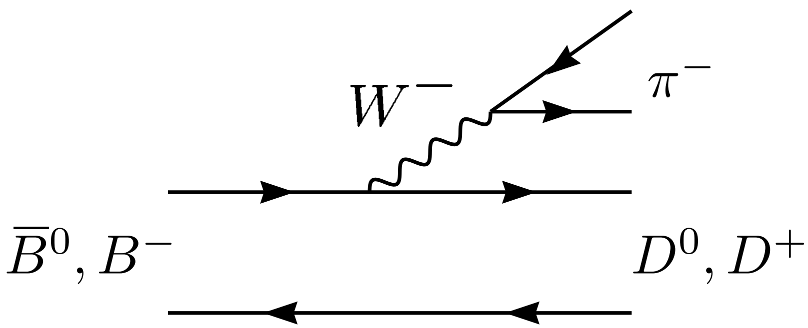
Factorization tests

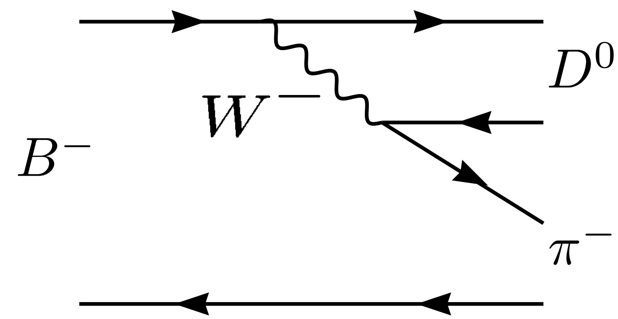
- Factorization has been observed to work in $\overline{B}^0 \to D^{(*)+}\pi^-$ decays at the $\lesssim 10\%$ level (in amplitudes) ...it gets really interesting just below this ($\sim 1/N_c^2$)

  Want to understand quantitatively accuracy of factorization in different processes

- $\sim 35\%$ corrections for B^- matrix elements have been observed

  Spectator in B going into π expected to be power suppressed





$$\frac{\mathcal{B}(B^- \to D^0\pi^-)}{\mathcal{B}(\overline{B}^0 \to D^+\pi^-)} = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b})\,, \qquad \text{data: } \sim 1.8 \pm 0.2$$

Ratio appears universal across channels (D/D^*, π/ρ)

Isospin again: $B \to D\pi$

- Classify amplitudes in terms of isospin (conserved by strong interaction) instead of “tree” and “color suppressed tree”, etc.

  Two isospin amplitudes for B decay to $(D\pi)$ in $I_f = \frac{1}{2}$ or $I_f = \frac{3}{2}$
  Three measurable rates $\Rightarrow$ 1 relation:

$$A(B^- \to D^0\pi^-) = A(\overline{B}^0 \to D^+\pi^-) + \sqrt{2}\, A(\overline{B}^0 \to D^0\pi^0)$$

- Three rates determine $|A_{1/2}|$, $|A_{3/2}|$ and their relative strong phase

$$\delta \sim 30^\circ$$

(CLEO, Belle, Babar)

  QCD factorization predicts $\delta \sim \mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b})$

  Not clear yet what sets the scale for the size of corrections

Origin of factorization?

- The color transparency argument relies on M_2 being fast ($m/E \ll 1$); the large-N_c argument is independent of this

  Would be nice to observe deviations that clearly distinguish between expectations

---

- At the level of existing data, factorization also works in $B \to D_s^{(*)} D^{(*)}$ when both particles are heavy
- Check if factorization is worse in $\overline{B}^0 \to D_s^{(*)-}\pi^+$ than in $\overline{B}^0 \to D^{(*)+}\pi^-$?

  Need $B \to \pi$ form factor ↖ should be $|V_{ub}/V_{cb}|^2 \times$ power suppressed
- "Designer mesons": Study factorization breaking in decays that vanish in naive factorization (so α_s & $1/m$ corrections important), e.g, $B^0 \to D^{(*)+} a_0/b_1/\pi_2$

  Rates at 10^{-6} level — soon accessible? (Diehl & Hiller)

Secrets of the B Meson XXX SLAC Summer Institute ssi2002, August 5–16, 2002

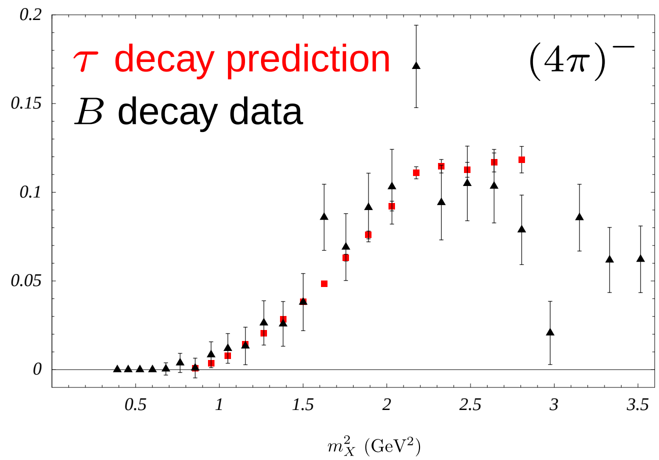
Berkeley Lab

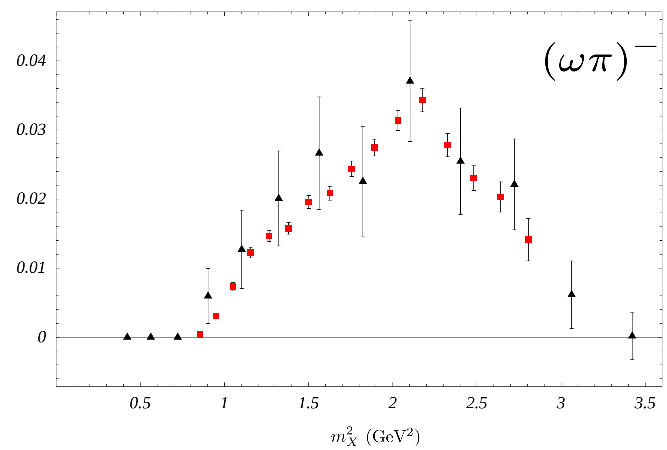
Factorization in $B \to D^{(*)}X$

- Study accuracy as a function of the kinematics, with fixed multi-body final states

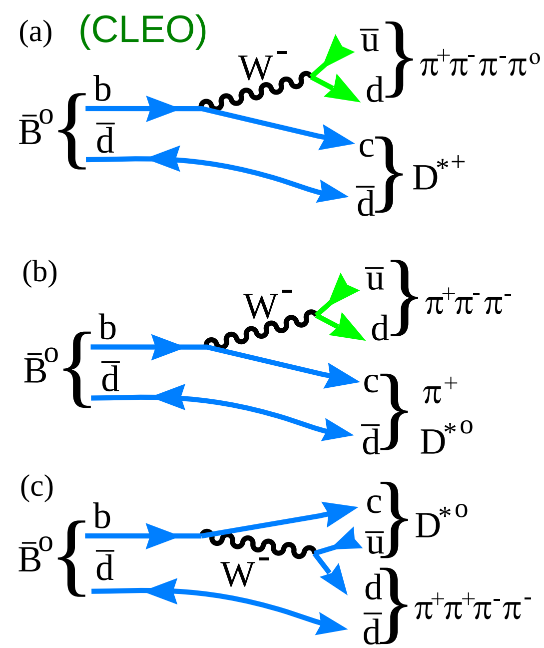
Expect some nonperturbative corrections to grow as m_X increases (ZL, Luke, Wise)

Compare $B \to D^* 4\pi$ with $\tau \to 4\pi$ (allows $0.4 \lesssim m_X/E_X \lesssim 0.7$)





Different charge modes can disentangle backgrounds from D^{**}, etc.



Observing deviations that grow with m_X would provide evidence that perturbative QCD is an important part of the success of factorization in $B \to D^* X$

Factorization in charmless decays

Factorization in charmless B decays

- Two contributions:

Two proposals:

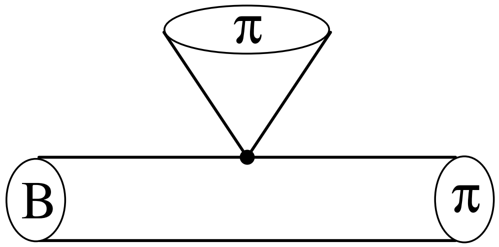
(1) "QCD factorization:" (Beneke, Buchalla, Neubert, Sachrajda)

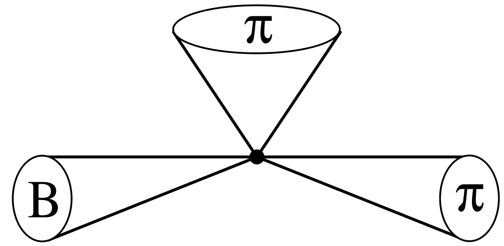
$$\langle \pi\pi | O_i | B \rangle \sim \; F_{B\to\pi} \, T(x) \otimes \phi_\pi(x) \; + \; T(\xi, x, y) \otimes \phi_B(\xi) \otimes \phi_\pi(x) \otimes \phi_\pi(y)$$

- Sudakov suppression at the b mass scale is not effective in the endpoint regions of quark distributions
- Two terms have same size in $\Lambda_{\rm QCD}/m_b$ power counting
- Second term suppressed by $\alpha_s(m_b)$
- Small strong phases

Factorization in charmless B decays

- Two contributions:





Two proposals:

(2) "Perturbative QCD:" (Keum, Li, Sanda)

$$\langle \pi\pi | O_i | B \rangle \sim \; 0 \; + \; T(x_i, b_i) \otimes \phi_B(x_3, b_3) \otimes \phi_\pi(x_2, b_2) \otimes \phi_\pi(x_1, b_1)$$

- Sudakov suppression effective in the regions $x_i \sim \Lambda_{\mathrm{QCD}}/m_b$ and $1/b_i \sim \Lambda_{\mathrm{QCD}}$
- k_T factorization, $1/b_i \sim \sqrt{\Lambda_{\mathrm{QCD}} m_b}$ dominates
- Larger strong phases, annihilation & penguin contributions

Main issues

Huge body of literature due to importance for CP violation

– Power counting depends on treatment of Sudakovs

BBNS: form factors are nonperturbative inputs

KLS: Sudakov suppression renders form factors calculable

– Some formally $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$ terms in BBNS are known to be large

Chirally enhanced terms: $2m_K^2/m_b m_s \sim 1$

– Other issues raised:

Charming penguins (Ciuchini *et al.*)

Intrinsic charm (Brodsky & Gardner)

Scale where π form factor becomes asymptotic $\sim x(1-x)$

It very hard to test the assumptions ... need large variety of rates and direct CPV, for some of which the predictions differ

$B \to hh$ rates vs. predictions

- BBNS and KLS predictions vs Experiment (CLEO, BELLE, BABAR):

	BBNS	KLS	World Average
$\frac{\mathcal{B}(\pi^+\pi^-)}{\mathcal{B}(\pi^\mp K^\pm)}$	$0.3-1.6$	$0.30-0.69$	0.28 ± 0.04
$\frac{\mathcal{B}(\pi^\mp K^\pm)}{2\mathcal{B}(\pi^0 K^0)}$	$0.9-1.4$	$0.78-1.05$	1.0 ± 0.3
$\frac{2\mathcal{B}(\pi^0 K^\pm)}{\mathcal{B}(\pi^\pm K^0)}$	$0.9-1.3$	$0.77-1.60$	1.3 ± 0.2
$\frac{\tau_+}{\tau_0}\frac{\mathcal{B}(\pi^\mp K^\pm)}{\mathcal{B}(\pi^\pm K^0)}$	$0.6-1.0$	$0.70-1.45$	1.1 ± 0.1
$\frac{\tau_+}{\tau_0}\frac{\mathrm{B}(\pi^+\pi^-)}{2\mathrm{B}(\pi^\pm\pi^0)}$	$0.6-1.1$		0.56 ± 0.14

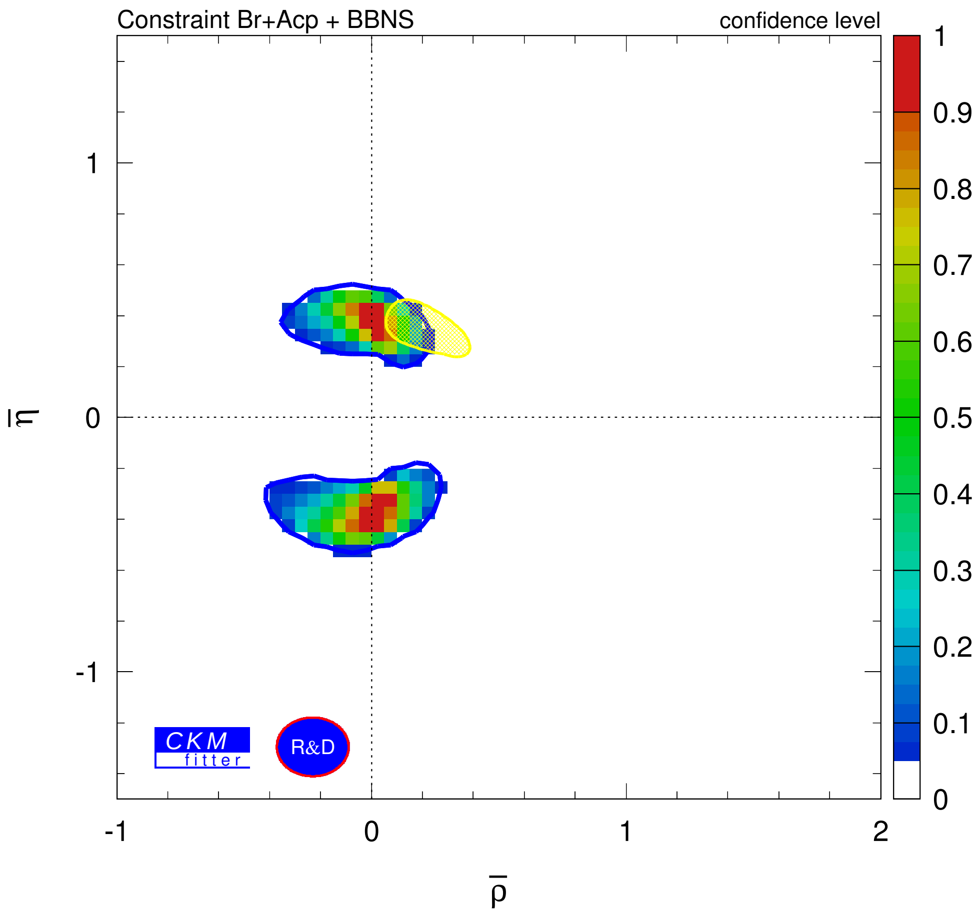
(Nir @ ICHEP)

Although the starting points, and predictions for direct CPV and phases in P/T differ, both BBNS and KLS can reproduce these rates by now

Experimental data is crucial

(ρ, η) using charmless rates

Inputs: $B \to \pi\pi/K\pi$ rates $+$ BBNS



γ from $B_{d,s} \to hh$

- $B \to K\pi/\pi\pi$: Combination of rates — careful

  Need some assumptions on (some of): rescattering effects, penguins, factorization, $SU(3)$

- $B_d \to \pi^+\pi^-$ vs. $B_s \to K^+K^-$: see: F. Würthwein's talk tomorrow (Fleischer)

  Idea: two decays related by u-spin, that exchanges $d \leftrightarrow s$

  Corrections to the u-spin limit are order m_s/Λ, just as for $SU(3)$
  Need to constrain it somehow from other measurements

  My feeling / hope: measure all possible $B_{d,s} \to \pi\pi$, $K\pi$, KK asymmetries and rates, we'll figure out something, build a case...

Summary — factorization

- In decays such as $\overline{B}^0 \to D^{(*)+} \pi^-$ factorization has become well established
- Some power suppressed corrections (formally order $\Lambda_{\rm QCD}/m_c$) appear sizable
  No evidence yet of factorization becoming a worse approximation in $B \to D^{(*)} X$ as m_X increases
- In charmless decays there are two approaches to factorization, BBNS and KLS
- Different assumptions and power counting, and sometimes different predictions
  Testing the assumptions in a conclusive way does not seem easy
- Theoretical progress in understanding semileptonic form factors in the small q^2 region may help to understand importance of Sudakov effects
- New and more precise data will be crucial to test factorization and tell us about significance of unknown power suppressed terms in various processes

Summary

Final remarks

- To overconstrain CKM, all possible clean measurements are very important, both CP violating and conserving, even if redundant in SM (correlations important)
- The key processes are those which give clean information on short distance parameters ...one theoretically clean measurement is worth ten dirty ones
- It changes with time what is theoretically clean — significant recent progress for:
  - Determination of $|V_{ub}|$ from inclusive B decay
  - Exclusive rare & semileptonic form factors at small q^2
  - Factorization in certain nonleptonic decays
- Studying CKM/CPV and hadronic physics is complementary; except for a few very clean cases several measurements needed to minimize theoretical uncertainties — data will help to get rid of nasty things hard to constrain otherwise

A (near future & personal) best buy list

- $|V_{td}/V_{ts}|$: Tevatron should nail this, hopefully very soon (lattice caveats?)
- Rare decays: $B \to X_s\gamma$ near theory limited; q^2 distribution in $B \to X_s\ell^+\ell^-$ will be very interesting
- $|V_{ub}|$: reaching $\lesssim 10\%$ would be very significant (assumes understanding $|V_{cb}|$; a Babar/Belle measurement that may well survive LHCB/BTeV)
- β: reduce error in ϕK_S, $\eta' K_S$ (and $D^{(*)}D^{(*)}$) modes
- β_s: is CPV in $B_s \to \psi\phi$ small?
- α: how small is $B \to \pi^0\pi^0$? How big are other resonances in $\rho - \pi$ Dalitz plot?
- γ: clean modes hard, test $SU(3)$ relations, factorization and other approaches
- try $B \to \ell\nu$, search for "null observables" [$a_{CP}(b \to s\gamma)$, etc.], for enhancement of $B \to \ell^+\ell^-$, etc.

(apologies for omissions!)

Conclusions

- The CKM picture is predictive and testable — it passed its first nontrivial test and is probably the dominant source of CPV in flavor changing processes
- The point is not only to measure the sides and angles of the unitarity triangle, (ρ, η) and (α, β, γ), but to probe CKM by overconstraining it in as many ways as possible (rare decays, correlations!)
- The program as a whole is a lot more interesting than any single measurement; all possible clean measurements are important, both CPV and CPC
- Many processes can give clean information on short distance physics, and there is progress towards being able to model independently interpret many interesting observables

"This is not the end. It is not even the beginning of the end.
But it is, perhaps, the end of the beginning."
— W. Churchill (Nov. 10, 1942)