Factorization of the Heavy-Quarkonium Production Cross Section

- Production in nuclear matter: J. Qiu, J. Vary, and X. Zhang.
- Production in pp collisions: J. Qiu and G. Sterman.

Production in pp collisions

Production of Quarkonium at large p_T

- A plausible general argument exists.
- First step: factorize the cross section into a parton hard-scattering cross section convolved with parton distribution functions and fragmentation functions for $g, q, Q \to$ quarkonium.
  - This step relies on techniques developed for proving factorization of single-particle inclusive cross sections for light quarks.
- Second step: relate the fragmentation functions for single quantum $\to$ quarkonium to fragmentation functions for $Q\bar{Q} \to$ quarkonium.
- Third step: expand the fragmentation functions for $Q\bar{Q} \to$ quarkonium (nonlocal operators) in terms of NRQCD matrix elements (local operators).
- The second and third steps are less well-established than the first step.
  - It is planned to check these steps by working out an explicit example.

- Corrections to the factorized cross section:
  - $1/p_T^2$ for the unpolarized cross section;
  - $1/p_T$ for the polarized cross section.
- Do the errors go as m_Q^2/p_T^2 (m_Q/p_T) or as $\Lambda_{\rm QCD}^2/p_T^2$ $(\Lambda_{\rm QCD}/p_T)$?
  - There are certainly contributions of order m_Q^2/p_T^2 (m_Q/p_T), but they may cancel, leaving only corrections of order $\Lambda_{\rm QCD}^2/p_T^2$ $(\Lambda_{\rm QCD}/p_T)$.
  - It is hoped that calculation in an explicit example will resolve the question.
- Plan: complete work at Aspen this summer.

The Total Production Cross Section (Integrated over p_T)

- The factorization proof involves additional difficulties.
  - In general, there is no large momentum to facilitate the factorization of soft-gluon interactions between the $Q\bar{Q}$ and beam spectators.
  - Must rely on the heavy-quark mass.
  - But, at low relative velocity, $1/v$ interactions complicate the analysis.
- Work on this subject has been set aside until the proof of factorization at large p_T is complete.

Hunches

(GTB)

- Key ingredients in proving the first step are factorization and cancellation of soft singularities, factorization of collinear singularities, and cancellation of final-state spectator-spectator interactions.

- Factorization and cancellation of soft singularities
  - The soft approximation (leads to factorization) is accurate up to terms of order $(k/Q)^2$.
    (k is the gluon momentum, $Q \sim p_T$ is the large scale).
  - Holds even if the gluon exchange involves massive quarks.
  - Cancellation of factored soft gluons (KLN and Bloch-Nordsieck) holds up to terms of order $(k/Q)^2$.
  - Residual terms of order $(k/Q)^2$ give an IR-finite contribution that has a renormalon ambiguity of order $\Lambda^2_{\mathrm{QCD}}/Q^2$.

- Factorization of collinear singularities into the parton distributions
  - The collinear approximation (leads to factorization) is accurate up to terms of order $(k_T/Q)^2$.
    (k_T is defined relative to the massless quantum that emits the gluon and is almost collinear to it.)
  - Holds if the gluon is exchanged between massless quanta or between a massless quantum and a massive quark.
  - For active-spectator interactions, $k_T \sim \Lambda_{\mathrm{QCD}}$, so the residual terms are of order $\Lambda^2_{\mathrm{QCD}}/Q^2$.
  - For active-active interactions, residual terms of order $(k_T/Q)^2$ give collinear-finite contributions that have a renormalon ambiguity of order $\Lambda^2_{\mathrm{QCD}}/Q^2$.

- Cancellation of final-state spectator-spectator interactions
  - The final-state-unitarity argument requires being able to neglect the energy and momentum of soft gluons relative to Q.
  - Holds up to terms of order $(k/Q)^2$.
  - Independent of the presence of massive quarks.
  - For spectator-spectator interactions, $k \sim \Lambda_{\rm QCD}$, so the residual terms are of order $\Lambda_{\rm QCD}^2/Q^2$.
- The second and third steps involve re-writing the fragmentation function in terms of local NRQCD matrix elements.
  - If the NRQCD effective-field-theory formalism is correct, then this must work out.
  - From the first step, we know that the interactions that create the fragmenting quantum are local in terms of the effective theory.
  - However, the derivation of the NRQCD expressions from QCD can be complicated.
- Conclusion: the proof of factorization at large p_T likely goes through with errors of order $\Lambda_{\rm QCD}^2/p_T^2$ in the unpolarized case and errors of order $\Lambda_{\rm QCD}/p_T$ in the polarized case.