

Angular distributions in

$B_s \rightarrow J/\psi \phi$ and related decays

Amol Digne
CERN

- $B_s \rightarrow J/\psi \phi$ in SM
- Transversity angle distribution
 - ↳ validity for non-resonant channels
 - ↳ utility for $\Delta\Gamma$, $|A_{\perp}|/|A_0|$
("information content")
- Three angle distribution
 - ↳ accessible parameters
 - ↳ untagged decay samples
- The method of angular moments
 - ↳ weighting functions
 - ↳ efficiency compared to MLE
- The reach of LHC (preliminary) for $\delta\phi$
- Comparison between $B_s \rightarrow J/\psi \phi$ and $B_d \rightarrow J/\psi K^*$
 - removal of discrete ambiguity in β

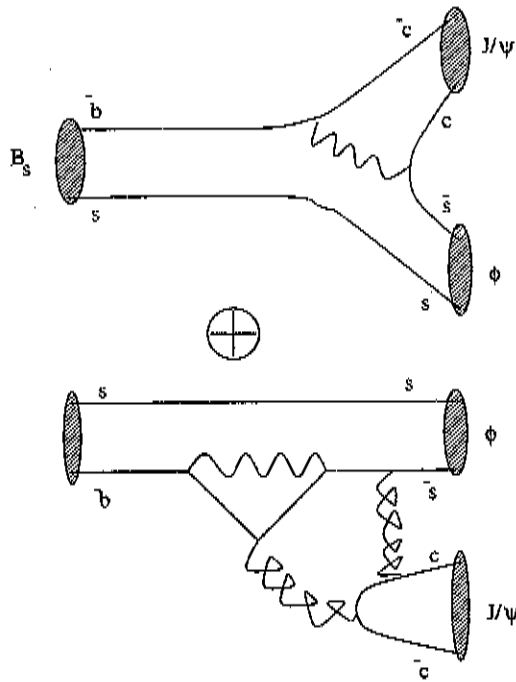
$$B_s \rightarrow J/\psi(\rightarrow \ell^+\ell^-) \phi(\rightarrow K^+K^-)$$

The most general amplitude :

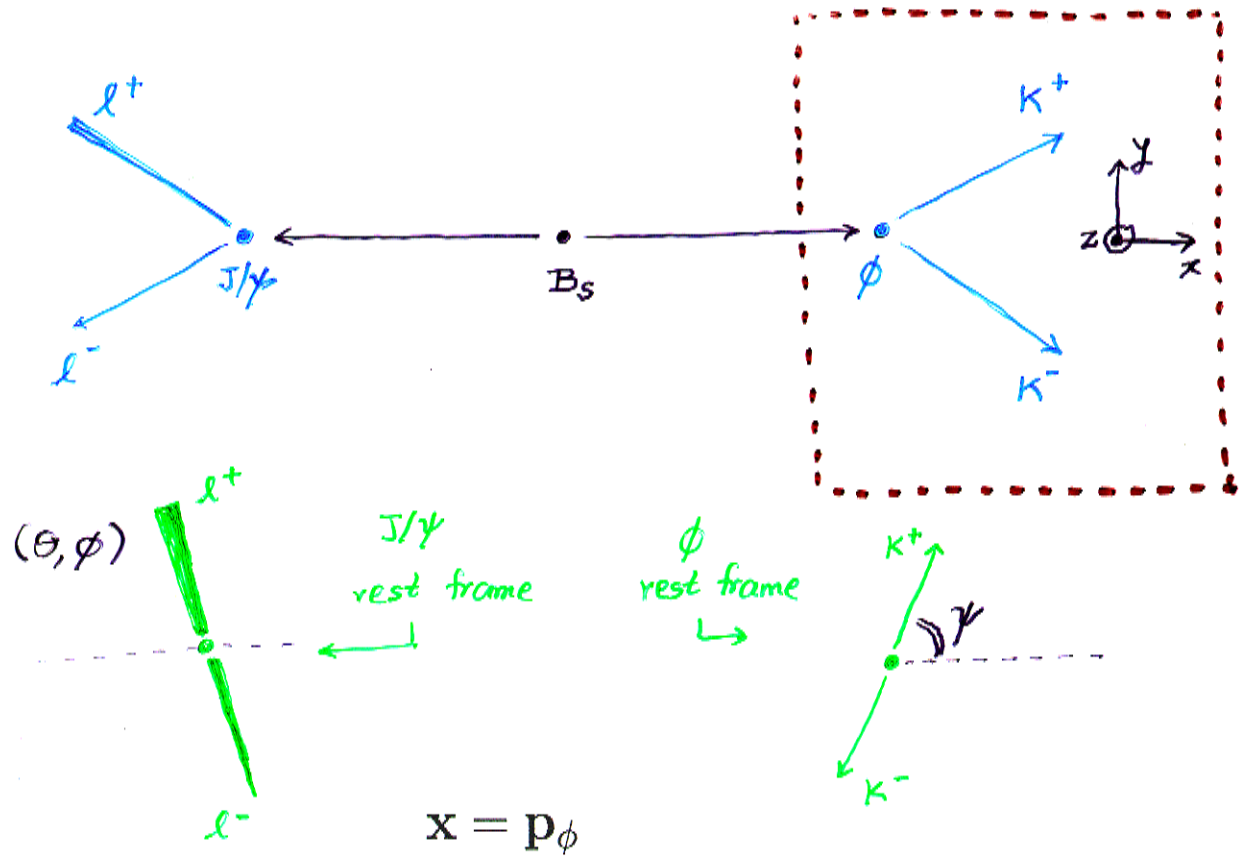
$$\begin{aligned} A(B_s \rightarrow J/\psi\phi) &= A_0 (m_\phi/E_\phi) \epsilon_{J/\psi}^{*L} \epsilon_\phi^{*L} \\ &- A_{\parallel} \epsilon_{J/\psi}^{*T} \cdot \epsilon_\phi^{*T} / \sqrt{2} \\ &- i A_{\perp} \epsilon_{J/\psi}^* \times \epsilon_\phi^* \cdot \hat{\mathbf{p}} / \sqrt{2} \end{aligned}$$

CP-conjugate amplitudes in $\bar{B}_s \rightarrow J/\psi\phi$:

$$\bar{A}_0(0) = A_0(0) \quad , \quad \bar{A}_{\parallel}(0) = A_{\parallel}(0) \quad , \quad \bar{A}_{\perp}(0) = -A_{\perp}(0)$$



Definitions of Angles :



$$y = \frac{\mathbf{p}_{K^+} - \mathbf{p}_\phi(\mathbf{p}_\phi \cdot \mathbf{p}_{K^+})}{|\mathbf{p}_{K^+} - \mathbf{p}_\phi(\mathbf{p}_\phi \cdot \mathbf{p}_{K^+})|}$$

$$\mathbf{z} = \mathbf{x} \times \mathbf{y}$$

$$\sin \theta \cos \varphi = \mathbf{p}_{l^+} \cdot \mathbf{x}$$

$$\sin \theta \sin \varphi = \mathbf{p}_{l^+} \cdot \mathbf{y}$$

$$\cos \theta = \mathbf{p}_{l^+} \cdot \mathbf{z}$$

$$\cos \psi = -\mathbf{p}'_{K^+} \cdot \mathbf{p}'_{J/\psi}$$

$\theta \rightarrow$ transversity angle

Transversity

Single angle distribution :

$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}(|A_0(t)|^2 + |A_{\parallel}(t)|^2)(1 + \cos^2\theta) + \frac{3}{4}|A_{\perp}(t)|^2 \sin^2\theta$$

$$|A_0(t)|^2 = |A_0(0)|^2 [e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi]$$

$$|A_{\parallel}(t)|^2 = |A_{\parallel}(0)|^2 [e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi]$$

$$|A_{\perp}(t)|^2 = |A_{\perp}(0)|^2 [e^{-\Gamma_H t} + e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi]$$



Parameters :

$$\Gamma_L, \Gamma_H, \frac{|A_{\perp}(0)|^2}{|A_0(0)|^2 + |A_{\parallel}(0)|^2 + |A_{\perp}(0)|^2}$$

$$\Delta m, \delta\phi = 2\delta\gamma = \text{Arg} \left(\frac{V_{ts} V_{tb}^* V_{cs}^* V_{cb}}{V_{ts}^* V_{tb} V_{cs} V_{cb}^*} \right)$$

- Angular distribution essential if $\Gamma_H \approx \Gamma_L$

$$p(u, t | \beta, \Gamma_H, \Gamma_L) = \underbrace{\frac{3}{8} \beta \Gamma_L (1+u^2)}_{\cos\theta} e^{-\Gamma_L t} + \frac{3}{4} (1-\beta) \Gamma_H (1-u^2) e^{-\Gamma_H t}$$

$\beta \rightarrow$ "fraction of ~~total CP-even~~ decays that are CP-even"
 ~~$1-\beta$~~
 $1-\beta \rightarrow$ " CP-odd"

Transversity angle distribution

$$(B \rightarrow J/\psi X)$$

↑
two particles

Separates CP even and odd amplitudes
as long as

Ⓐ The particles in X are self-conjugate

OR

Ⓑ The particles in X are CP-conjugates

AND scalars

AND X is a fixed-spin object

No need for X to be a resonance!

Non-resonant channels may be included.

I. Dunietz

H. Quinn

A. Snyder

W. Toki

H. Liphin '90

A.D.
S. Sen
hep-ph/9810381
PRD '99

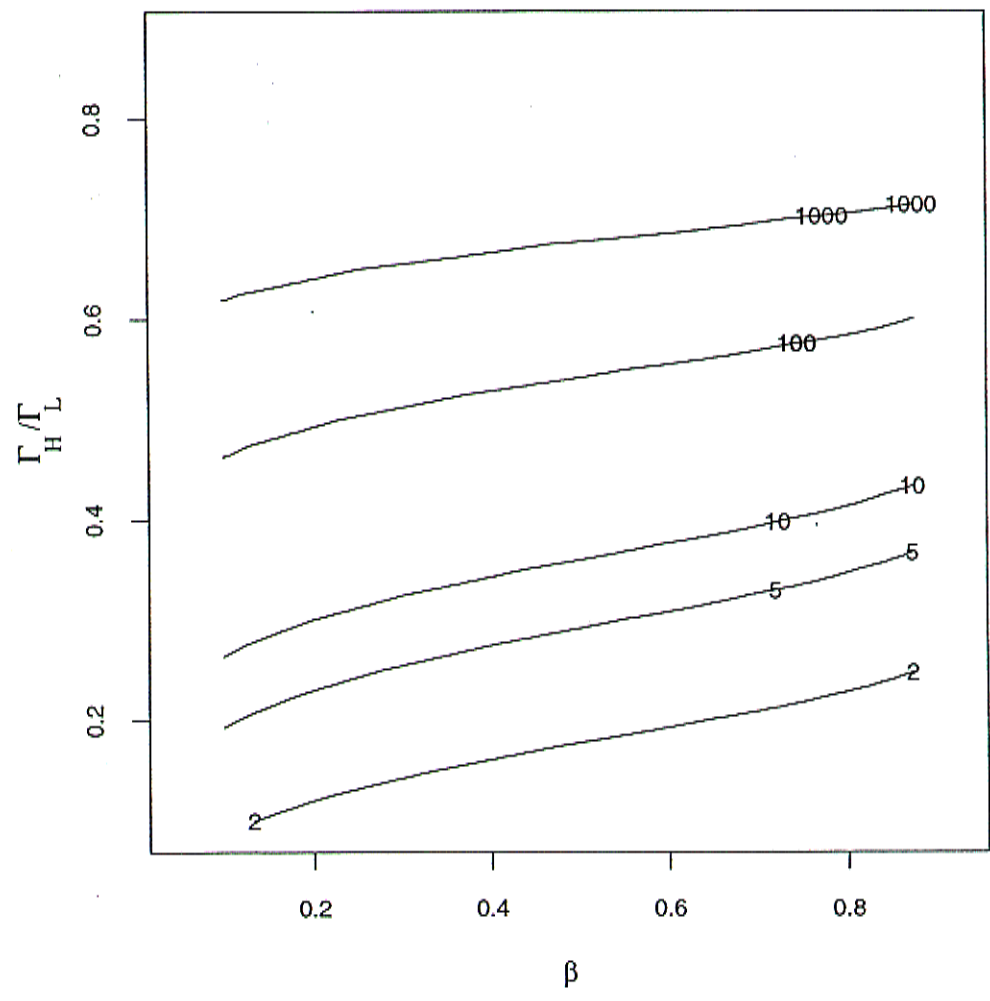


Figure 4: The ratio of the variances, $V(\hat{\theta}_i(t))/V(\hat{\theta}_i(u,t))$ of the estimates of $\theta_3 = \beta$.

$$\beta = \left[1 + \frac{1}{2} \frac{\Gamma_L}{\Gamma_H} \frac{|A_{\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2} \right]^{-1}$$

In $J/4 K^* \rightarrow \beta \sim 0.93 \pm 0.03$

A.S.
S. Sen
hep-ph/9810381
PR9 '99

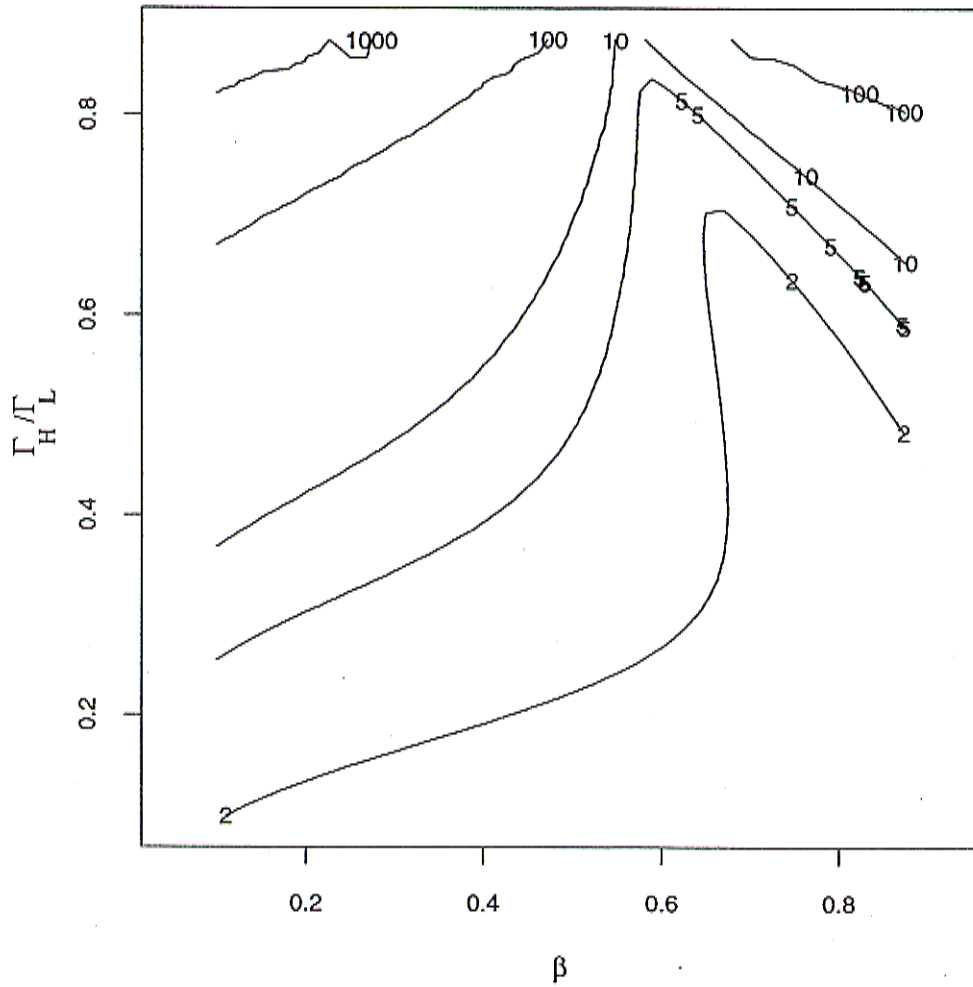


Figure 3: The ratio of the variances, $V(\hat{\theta}_i(t))/V(\hat{\theta}_i(u, t))$ of the estimates of $\theta_2 = \Gamma_L - \Gamma_H$.

A. B.
S. Sen
hep-ph/9810381
PR8'99

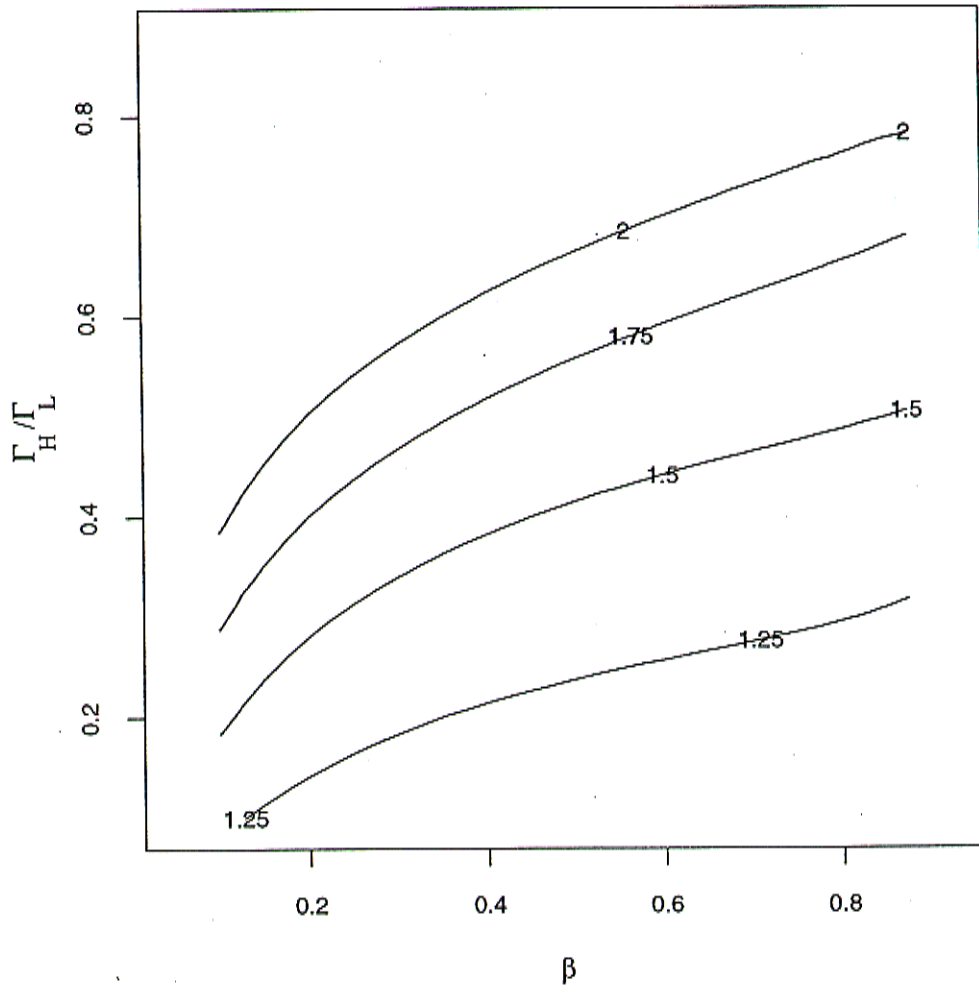


Figure 2: The ratio of the variances, $V(\hat{\theta}_i(t))/V(\hat{\theta}_i(u, t))$ of the estimates of $\theta_1 = \beta\Gamma_L + (1 - \beta)\Gamma_H$.

Three Angle Distribution

$$\frac{d^3\Gamma[B_s(t) \rightarrow J/\psi(\rightarrow l^+l^-)\phi(\rightarrow K^+K^-)]}{d\cos\theta d\varphi d\cos\psi} \propto \frac{9}{32\pi} \times$$

$$\left[\begin{aligned} & 2|A_0(t)|^2 (1 - \sin^2\theta \cos^2\varphi) \cos^2\psi \\ & + |A_{\parallel}(t)|^2 (1 - \sin^2\theta \sin^2\varphi) \sin^2\psi \\ & + |A_{\perp}(t)|^2 \sin^2\theta \sin^2\psi \\ & - \operatorname{Im}(A_{\parallel}^*(t)A_{\perp}(t)) \sin 2\theta \sin\varphi \sin^2\psi \\ & + \operatorname{Re}(A_0^*(t)A_{\parallel}(t)) \sin^2\theta \sin 2\varphi \sin 2\psi / \sqrt{2} \\ & + \operatorname{Im}(A_0^*(t)A_{\perp}(t)) \sin 2\theta \cos\varphi \sin 2\psi / \sqrt{2} \end{aligned} \right]$$

Time Evolutions

Observables	Time evolutions
$ A_0(t) ^2$	$ A_0(0) ^2 \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$ A_{\parallel}(t) ^2$	$ A_{\parallel}(0) ^2 \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$ A_{\perp}(t) ^2$	$ A_{\perp}(0) ^2 \left[e^{-\Gamma_H t} + e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$\operatorname{Re}(A_0^*(t)A_{\parallel}(t))$	$ A_0(0) A_{\parallel}(0) \cos(\delta_2 - \delta_1) \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$\operatorname{Im}(A_{\parallel}^*(t)A_{\perp}(t))$	$ A_{\parallel}(0) A_{\perp}(0) \left[e^{-\bar{\Gamma} t} \sin(\delta_1 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_1) \delta\phi \right]$
$\operatorname{Im}(A_0^*(t)A_{\perp}(t))$	$ A_0(0) A_{\perp}(0) \left[e^{-\bar{\Gamma} t} \sin(\delta_2 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_2) \delta\phi \right]$

Parameters :

$$|A_{\parallel}/A_0| , |A_{\perp}/A_0| , \delta_1 , \delta_2 ,$$

$$\Gamma_H , \Gamma_L , \Delta m , \delta\phi$$

Significance of the accessible parameters :

- Δm

$\Delta m_s/\bar{\Gamma} = \tau_s$ through $J/\psi \phi$

P. Galumian

– SM prediction of $\frac{\Delta m_s}{\Delta m_d} = \frac{\eta_{B_s} M_{B_s} (f_{B_s}^2 B_{B_s})}{\eta_{B_d} M_{B_d} (f_{B_d}^2 B_{B_d})} |V_{ts}|^2$

- Γ_H, Γ_L

T. Browder, S. Pakvasa
PRD 52, '95

– $\frac{\Delta m}{\Delta \Gamma}$ independent of CKM elements (to lowest order) :

M. Beneke
G. Buchalla
I. Dunietz
PRD 54, '96

$$\frac{\Delta m}{\Delta \Gamma} \simeq -\frac{2 m_t^2 h(m_t^2/M_W^2)}{3\pi m_b^2} \left(1 - \frac{8m_c^2}{3m_b^2}\right)^{-1} \simeq -200$$

(+ QCD corrections)

M. Beneke, G. Buchalla, C. Greub, A. Lenz,
U. Nierste hep-ph/4808385

– $(\Delta \Gamma/\Gamma)_{B_s} \sim 0.1$??

– $\Delta \Gamma$ can only decrease with new physics effects

Y. Grossman, PLB 380, '96

– Untagged CP asymmetries proportional to $\Delta \Gamma$

⇒

I. Dunietz, R. Fleischer
PLB 387, '96
PRD 55, '97

- $\delta \phi$

– “squashed” unitarity triangle

R. Aleksan, B. Kayser, D. London, PRL 73, '94

– $\approx 2\lambda^2 \eta \approx 2\lambda^2 R_b \sin \gamma$ means useful for determining η or γ

$$(R_b \equiv \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|})$$

– small SM value (≈ 0.03) and possibilities of significant new physics contributions

- $|A_{\perp}/A_0|, |A_{\parallel}/A_0|$

M. Bauer, B. Stech, M. Wirbel, Z-C29, '85
Z-C34, '87

– Specific model predictions

J. Soares, PRD 53, '96
H.-Y. Cheng, Z-C69, '96

- δ_1, δ_2

– Predicted to be zero in models with factorization

– Useful for getting rid of a discrete ambiguity in β

⇒

A. D.
I. Dunietz
R. Fleischer
PLB 433, '98

*A guidance for the expected values of some parameters:
from $B^+ \rightarrow J/\psi K^*$ results by CLEO*

CLEO conf 96-24

B DecayMode	Parameter	Value
$B^+ \rightarrow J/\psi K^{*+}$	$10^3 \times \mathcal{B}(B \rightarrow J/\psi K^{*+})$	1.41 ± 0.20
	$ A_0 ^2 = \Gamma_L / \Gamma$	0.53 ± 0.11
	$ A_\perp ^2 = P ^2$	$0.11^{+0.15}_{-0.06}$
	$\phi(A_\perp)$	1.57 ± 2.44
	$\phi(A_\parallel)$	3.14 ± 0.67
$B^+ \rightarrow J/\psi K^{*0}$	$10^3 \times \mathcal{B}(B \rightarrow J/\psi K^{*0})$	1.32 ± 0.14
	$ A_0 ^2 = \Gamma_L / \Gamma$	0.52 ± 0.08
	$ A_\perp ^2 = P ^2$	0.18 ± 0.09
	$\phi(A_\perp)$	$-0.53^{+0.53}_{-0.61}$
	$\phi(A_\parallel)$	3.00 ± 0.46
$B^+ \rightarrow J/\psi K^*$ combined	$10^3 \times \mathcal{B}(B \rightarrow J/\psi K^*)$	1.35 ± 0.12
	$ A_0 ^2 = \Gamma_L / \Gamma$	0.52 ± 0.07
	$ A_\perp ^2 = P ^2$	0.16 ± 0.08
	$\phi(A_\perp)$	-0.11 ± 0.46
	$\phi(A_\parallel)$	3.00 ± 0.37

TABLE III. Resulting decay amplitudes from the full fit to the transversity angles. The correlation coefficients of the combined fit are given in Table IV.

CP-conjugate Decay :

For $d^3\Gamma[\overline{B}_s(t) \rightarrow J/\psi(\rightarrow l^+l^-)\phi(\rightarrow K^+K^-)]$,

- $A(0) \leftrightarrow \overline{A}(0)$
- $\Delta m \leftrightarrow -\Delta m$

Untagged Data :

Observables	Time evolutions
$ f_0(t) ^2$	$ A_0(0) ^2 e^{-\Gamma_L t}$
$ f_{\parallel}(t) ^2$	$ A_{\parallel}(0) ^2 e^{-\Gamma_L t}$
$ f_{\perp}(t) ^2$	$ A_{\perp}(0) ^2 e^{-\Gamma_H t}$
$\text{Re}(f_0^*(t)f_{\parallel}(t))$	$ A_0(0) A_{\parallel}(0) \cos(\delta_2 - \delta_1) e^{-\Gamma_L t}$
$\text{Im}(f_{\parallel}^*(t)f_{\perp}(t))$	$ A_{\parallel}(0) A_{\perp}(0) \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_1) \delta\phi$
$\text{Im}(f_0^*(t)f_{\perp}(t))$	$ A_0(0) A_{\perp}(0) \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_2) \delta\phi$

- Assumes no B_s vs. \overline{B}_s production asymmetry

CP Asymmetry :

$$\cos \delta_1 (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \delta\phi \approx \Delta\Gamma t e^{-\Gamma t} \delta\phi \cos \delta_1$$

- Production asymmetry terms would appear as $\Delta m t$ oscillations

The method of angular moments

I. Dunietz
H. Gunn
A. Snyder
W. Toki
H. Lipkin
PRD 43, '91

Angular distribution

$$f(\Theta, \mathcal{P}; t) = \sum b^{(k)}(\mathcal{P}; t) g^{(k)}(\Theta)$$

Find *weighting functions* $w^{(i)}$ such that

$$\int [\mathcal{D}\Theta] w^{(i)}(\Theta) g^{(k)}(\Theta) = \delta_{ik}$$

A.D.
I. Dunietz
R. Fleischer
hep-ph/9804253
EPJC '99

Then

$$b^{(i)} \approx \sum_{events} w^{(i)}(\Theta)$$

- Essential idea : finding 6 vectors, each of which is perpendicular to 5 of the angular terms.
- The task of one fit to a large number of parameters gets split into fits to a smaller number of parameters. (Here, from an 8 parameter-fit to 6 fits with as small as 4 parameters).
- More transparent and easier to implement than the maximal likelihood fit (the *complexity* of *mle* grows exponentially with number of parameters to be fitted).
- Statistically "efficient" - the information loss is small (checked for the single angle distribution).
- Is definitely useful for a 'first guess' with low statistics (to provide the *seed* to *mle*), and maybe even to substitute for *mle* where it may fail (??)

A.D., S.Sen
hep-ph/9810381
PRD '99

Weighting functions

- Exist for all angular distributions
- Can be determined a priori, without any knowledge of $b^{(i)}$
- There are a lot of equivalent weighting functions, the most efficient ones would depend on the detector.
- For $B_s \rightarrow J/\psi\phi$:

Observables: $b^{(i)}(t)$	$w^{(i)}(\theta, \varphi, \psi)$
$ A_0(t) ^2$	$\frac{1}{2}[5(\cos^2 \theta - \sin^2 \theta \cos 2\varphi) - 1]$
$ A_{\parallel}(t) ^2$	$\frac{1}{2}[5(\cos^2 \theta + \sin^2 \theta \cos 2\varphi) - 1]$
$ A_{\perp}(t) ^2$	$2 - 5 \cos^2 \theta$
$Re(A_0^*(t)A_{\parallel}(t))$	$\frac{5}{\sqrt{2}} \sin(2\psi) \sin(2\varphi)$
$Im(A_{\parallel}^*(t)A_{\perp}(t))$	$-\frac{5}{2} \sin(2\theta) \sin \varphi$
$Im(A_0^*(t)A_{\perp}(t))$	$\frac{25}{4\sqrt{2}} \sin(2\psi) \sin(2\theta) \cos \varphi$

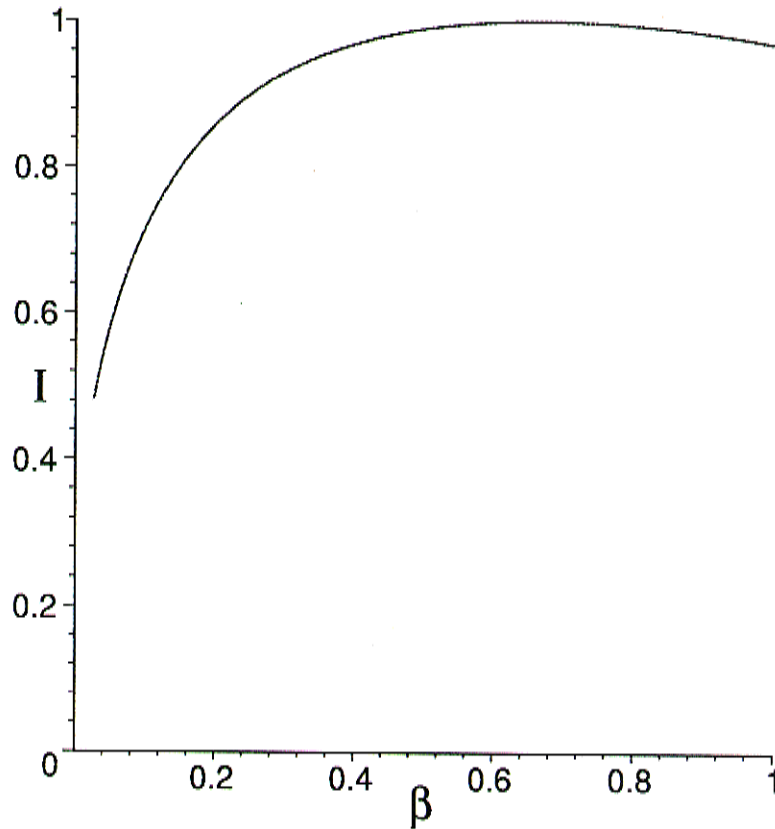


FIG. 5. The ratio of information content about β extracted through the angular moments method and the maximal likelihood method. The X-axis is the actual value of β .

$$\frac{\text{Info (Angular moments)}}{\text{Info (maximal likelihood)}}$$

$$\beta = \left(1 + \frac{1}{2} \frac{\Gamma_L}{\Gamma_H} \frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2} \right)^{-1}$$

$$(\text{in } J/\psi K^* \rightarrow 0.93 \pm 0.03)$$

A. D.
S. Sen
hep-ph/9810381
PRD '99

ang. term	coefficient	moment	value from fit
$\cos^2 \theta_{D^*} \cos^2 \theta_\rho - \frac{8}{25}$	H_0^2	0.751 ± 0.073	0.859
$\sin^2 \theta_{D^*} \sin^2 \theta_\rho - \frac{12}{25}$	$H_+^2 + H_-^2$	0.159 ± 0.034	0.140 ± 0.040
$\sin \chi \sin 2\theta_{D^*} \sin 2\theta_\rho$	$\Im(H_- H_0^* - H_+ H_0^*)$	0.042 ± 0.103	0.110 ± 0.074
$\cos \chi \sin 2\theta_{D^*} \sin 2\theta_\rho$	$\Re(H_+ H_0^* + H_- H_0^*)$	0.352 ± 0.1044	0.341 ± 0.088
$\sin 2\chi \sin^2 \theta_{D^*} \sin^2 \theta_\rho$	$\Im(H_+ H_-^*)$	0.057 ± 0.024	0.053 ± 0.021
$\cos 2\chi \sin^2 \theta_{D^*} \sin^2 \theta_\rho$	$\Re(H_+ H_-^*)$	0.018 ± 0.023	0.023 ± 0.024

TABLE III. Moments of different angular terms for $B^0 \rightarrow D^{*-} \rho^+$ events compared with values expected from fit results

ang. term	coefficient	moment	value from fit
$\cos^2 \theta_{D^*} \cos^2 \theta_\rho - \frac{8}{25}$	H_0^2	0.626 ± 0.074	0.856
$\sin^2 \theta_{D^*} \sin^2 \theta_\rho - \frac{12}{25}$	$H_+^2 + H_-^2$	0.168 ± 0.036	0.143 ± 0.060
$\sin \chi \sin 2\theta_{D^*} \sin 2\theta_\rho$	$\Im(H_- H_0^* - H_+ H_0^*)$	-0.145 ± 0.101	-0.071 ± 0.109
$\cos \chi \sin 2\theta_{D^*} \sin 2\theta_\rho$	$\Re(H_+ H_0^* + H_- H_0^*)$	0.193 ± 0.109	0.2504 ± 0.105
$\sin 2\chi \sin^2 \theta_{D^*} \sin^2 \theta_\rho$	$\Im(H_+ H_-^*)$	0.002 ± 0.027	-0.011 ± 0.032
$\cos 2\chi \sin^2 \theta_{D^*} \sin^2 \theta_\rho$	$\Re(H_+ H_-^*)$	0.043 ± 0.025	0.068 ± 0.029

TABLE IV. Moments of different angular terms for $B^+ \rightarrow \bar{D}^{*0} \rho^+$ events compared with values expected from fit results

CLEO conf 98-23

Comparison of the analysis of actual data through

- *method of angular moments*
- *maximal likelihood fit*

for

- $B^0 \rightarrow D^{*-} \rho^+$
- $B^+ \rightarrow \bar{D}^{*0} \rho^+$

Parameter determination and estimation of precision

Precision of experimental determination of $B_s^0 \rightarrow J/\psi\phi \rightarrow \mu^+\mu^-K^+K^-$ parameters estimated using maximum-likelihood fit to Monte Carlo simulated data.

Maximum likelihood function:

$$L = \prod_{i=1}^{i=N} \frac{\int_0^\infty (\epsilon_1 W^+(t_i, \Omega_i) + \epsilon_2 W^-(t_i, \Omega_i) + b e^{-\Gamma_0 t_i}) Res(t, t_i)}{\int_{t_{min}}^\infty \int_0^\infty (\epsilon_1 W^+(t, \Omega) + \epsilon_2 W^-(t, \Omega) + b e^{-\Gamma_0 t}) Res(t', t_i)} \quad (8)$$

$\Rightarrow \epsilon_1 = \epsilon_2 = 0.5$ for untagged events,

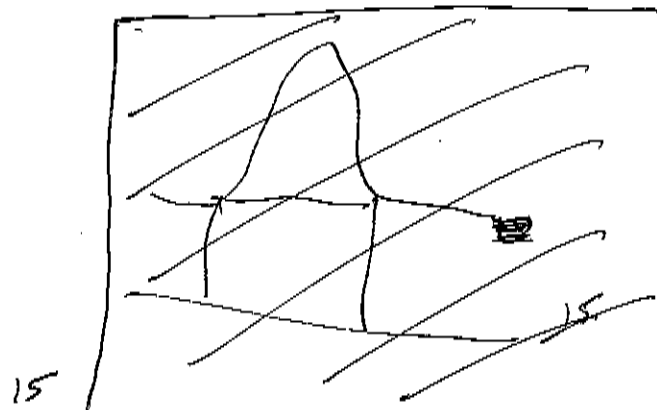
$\epsilon_1 = 1 - w$ $\epsilon_2 = w$ for B_s^0 tagged as particle

$\epsilon_1 = w$ $\epsilon_2 = 1 - w$ for B_s^0 tagged as anti particle

Single maximum likelihood fit done for all events (tagged and untagged).

Free parameters: $\Delta\Gamma_s$, Γ , $|A_{||}(t=0)|$, $|A_{\perp}(t=0)|$, ξ

Fixed parameters: δ_1 , δ_2 , x_s



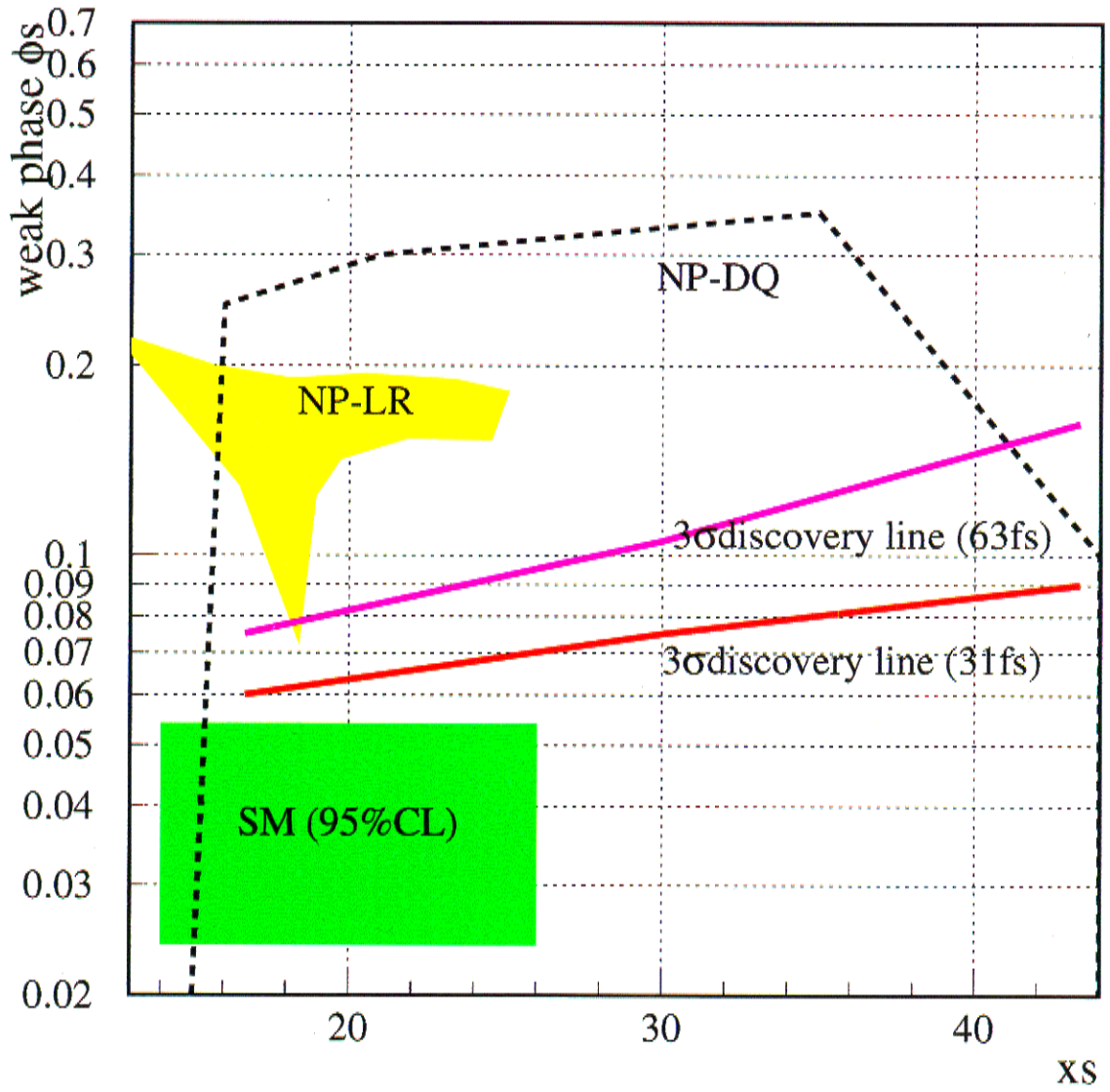
~~CONCLUSIONS~~

The decay $B_s^0 \rightarrow J/\psi\phi$ is a potential source of information on the quantities $\Delta\Gamma_s$ and ϕ_s . The dependence of measurement errors on signal statistics, background and time resolution has been evaluated. The main results of this study are summarised in the Table 3.

height	LHCb	ATLAS / CMS
$\delta\Delta\Gamma_s / \Delta\Gamma_s$	9%	12% / 11%
$\delta\Gamma_s / \Gamma_s$	0.6%	0.7%
$\delta A_{ } / A_{ }$	0.7%	0.8%
$\delta A_{\perp} / A_{\perp}$	2%	3%
$\delta\phi_s (x_s = 20)$	0.02	0.03
$\delta\phi_s (x_s = 40)$	0.03	0.05

Table 3: Summary of the analyses: expected statistical errors of $B_s^0 \rightarrow J/\psi\phi$ parameters for the conditions expected by three LHC experiments. The results correspond to ATLAS and CMS running three years at luminosity $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, for LHCb five years at luminosity $2 \cdot 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$. The channel $B_s^0 \rightarrow J/\psi(\mu^+\mu^-)\phi(K^+K^-)$ has been used for analyses.

The measurement precision for $\delta\Delta\Gamma_s / \Delta\Gamma_s$ typically varies between 9% and 12%, for the conditions expected by the three LHC experiments. The weak phase error ϕ_s is sensitive to the time resolution. The results indicate that, while LHC experiments may not be sensitive to the ϕ_s values expected by the Standard Model, it would be possible to measure the larger ϕ_s values predicted by some models containing new physics. The full potential of measuring this process at LHC has not been exhausted in the present study and will be further investigated.



The same parameters across the board :

$$B \rightarrow V(\rightarrow ab)V(\rightarrow cd)$$

color suppressed

$$B_s \rightarrow \begin{matrix} J/\psi & \phi \\ (\ell^+\ell^-) & (K^+K^-) \end{matrix} \xleftrightarrow[\delta\phi \leftrightarrow 2\beta]{\text{SU}(3)} B^0 \rightarrow \begin{matrix} J/\psi & K^* \\ (\ell^+\ell^-) & (K\pi) \end{matrix}$$

color allowed

$$B_s \rightarrow \begin{matrix} D_s^{*+} & D_s^{*-} \\ (D_s^+\gamma) & (D_s^-\gamma) \end{matrix} \xleftrightarrow[\delta\phi]{\text{SU}(3)} B^0 \rightarrow \begin{matrix} D_s^{*+} & \bar{D}^* \\ (D_s^+\gamma) & (\bar{D}\pi) \end{matrix}$$

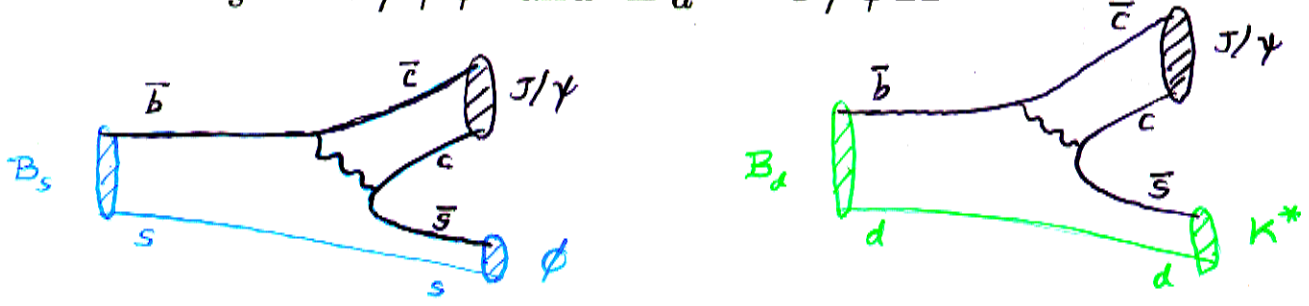
• Factorization tests :

$$\text{Im}[A_0^*(0)A_\perp(0)] = 0$$

$$\text{Im}[A_\parallel^*(0)A_\perp(0)] = 0$$

• Checking predictions of specific models

$$B_s \rightarrow J/\psi\phi \quad \text{and} \quad B_d \rightarrow J/\psi K^*$$



Observables	Time evolutions
$ A_0(t) ^2$	$ A_0(0) ^2 \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$ A_{\parallel}(t) ^2$	$ A_{\parallel}(0) ^2 \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$ A_{\perp}(t) ^2$	$ A_{\perp}(0) ^2 \left[e^{-\Gamma_H t} + e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$\text{Re}(A_0^*(t)A_{\parallel}(t))$	$ A_0(0) A_{\parallel}(0) \cos(\delta_2 - \delta_1) \left[e^{-\Gamma_L t} - e^{-\bar{\Gamma} t} \sin(\Delta m t) \delta\phi \right]$
$\text{Im}(A_{\parallel}^*(t)A_{\perp}(t))$	$ A_{\parallel}(0) A_{\perp}(0) \left[e^{-\bar{\Gamma} t} \sin(\delta_1 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_1) \delta\phi \right]$
$\text{Im}(A_0^*(t)A_{\perp}(t))$	$ A_0(0) A_{\perp}(0) \left[e^{-\bar{\Gamma} t} \sin(\delta_2 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_2) \delta\phi \right]$

Table 1: Time evolution of the decay $B_s \rightarrow J/\psi(\rightarrow l^+l^-)\phi(\rightarrow K^+K^-)$ of an initially (i.e. at $t=0$) pure B_s meson.

Observable	Time evolution
$ A_0(t) ^2$	$ A_0(0) ^2 e^{-\Gamma t} [1 + \sin(2\beta) \sin(\Delta m t)]$
$ A_{\parallel}(t) ^2$	$ A_{\parallel}(0) ^2 e^{-\Gamma t} [1 + \sin(2\beta) \sin(\Delta m t)]$
$ A_{\perp}(t) ^2$	$ A_{\perp}(0) ^2 e^{-\Gamma t} [1 - \sin(2\beta) \sin(\Delta m t)]$
$\text{Re}(A_0^*(t)A_{\parallel}(t))$	$ A_0(0) A_{\parallel}(0) \cos(\delta_2 - \delta_1) e^{-\Gamma t} [1 + \sin(2\beta) \sin(\Delta m t)]$
$\text{Im}(A_{\parallel}^*(t)A_{\perp}(t))$	$ A_{\parallel}(0) A_{\perp}(0) e^{-\Gamma t} [\sin(\delta_1) \cos(\Delta m t) - \cos(2\beta) \cos(\delta_1) \sin(\Delta m t)]$
$\text{Im}(A_0^*(t)A_{\perp}(t))$	$ A_0(0) A_{\perp}(0) e^{-\Gamma t} [\sin(\delta_2) \cos(\Delta m t) - \cos(2\beta) \cos(\delta_2) \sin(\Delta m t)]$

Table 2: Time evolution of the decay $B_d \rightarrow J/\psi(\rightarrow l^+l^-)K^{*0}(\rightarrow \pi^0 K_S)$ of an initially (i.e. at $t=0$) pure B_d meson.

\Rightarrow Removal of the discrete ambiguity $\beta \leftrightarrow \pi/2 - \beta$ by using $\text{sign}[\cos(\delta)]$ from $J/\psi\phi$.

A.P.
I. Dunietz
R. Fleischer
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Another way: using $B_d \rightarrow \Lambda_c^+ \bar{\Lambda}_c^+$ angular distributions

J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal
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