NRQCD Analysis of Bottomonium Production at the Tevatron

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NRQCD factorization formalism for Production in $P\bar{P}$ Collisions

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$$\sigma(p\bar{p} \to H+X) = \int dx_1\, dx_2\, f_{i/p}(x_1)\, f_{j/p}(x_2)\, \hat{\sigma}(ij \to Q\bar{Q}(n)+\tilde{X}) \times \langle 0|O^H(n)|0\rangle + \ldots$$

... power suppressed corrections

$\hat{\sigma}(ij \to Q\bar{Q}(n)+\tilde{X})$ expansion in α_s

$\langle 0|O^H(n)|0\rangle$ expansion in v

$v^2 \approx \frac{1}{10}$ in bottomonium

P_T distribution for bottomonium production

$$\frac{d\sigma}{dP_T^2} = \int_{-0.4}^{+0.4} dy \int dx_1\, dx_2\, f_{i/p}(x_1)\, f_{j/\bar{p}}(x_2)\, \frac{d\hat{\sigma}}{dt}(ij \to b\bar{b}(n)+k)\ J \cdot \delta$$

$$\times \langle 0 |\, O^H(n) \,| 0 \rangle$$

$$\frac{d\hat{\sigma}}{dt}(ij \to b\bar{b}(n)+k) \sim \alpha_s^3$$

$\langle 0 |\, O^H(n) \,| 0 \rangle$:

$$\langle 0 |\, O^{\Upsilon}(1, {}^3S_1) \,| 0 \rangle \sim v^0$$

$$\langle 0 |\, O^{\Upsilon}(8, {}^3S_1) \,| 0 \rangle \sim v^4$$

$$\langle 0 |\, O^{\Upsilon}(8, {}^1S_0) \,| 0 \rangle \sim v^4$$

$$\langle 0 |\, O^{\Upsilon}(8, {}^3P_J) \,| 0 \rangle \sim v^4$$

$$\langle 0 |\, O^{\chi_b}(1, {}^3P_J) \,| 0 \rangle \sim v^2$$

$$\langle 0 |\, O^{\chi_b}(8, {}^3S_1) \,| 0 \rangle \sim v^2$$

why keep v^4 terms?

for $n = 1, {}^3S_1$ $\qquad \frac{d\sigma}{dP_T^2} \sim \frac{m_b^4}{P_T^8}$

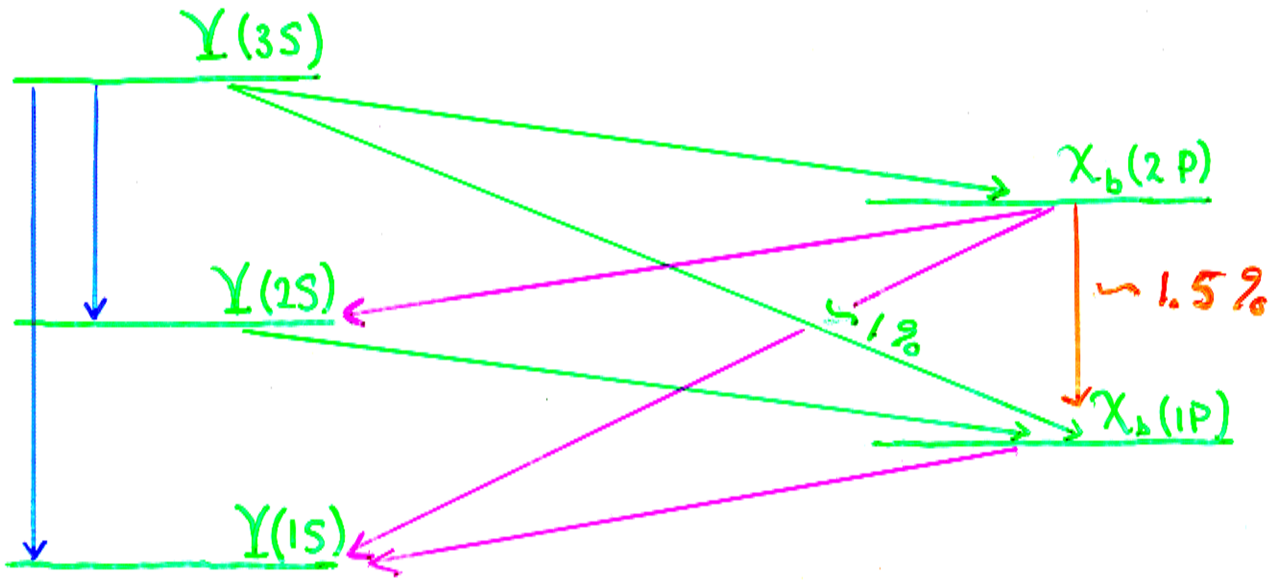
$8, {}^3S_1$ $\qquad \frac{d\sigma}{dP_T^2} \sim \frac{1}{P_T^4}$

$8, {}^1S_0 / 8, {}^3P_J$ $\qquad \frac{d\sigma}{dP_T^2} \sim \frac{m_b^2}{P_T^6}$

will fit data in the range

$8\ \text{GeV} < P_T < 20\ \text{GeV}$

Feeddown



- neglect $\chi_b(3P)$ — not observed
- expect additional cont. to be small

"Total" NRQCD matrix elements

$$\langle 0|O(n)|0\rangle^{\Upsilon(mS)}_{total} = \sum_H B_{H\to\Upsilon(mS)} \langle 0|O^H(n)|0\rangle$$

Consider $\Upsilon(1S)$ production—
the $8, {}^3S_1$ total matrix element

$$\langle 0|O_8({}^3S_1)|0\rangle^{\Upsilon(1S)}_{total} = \langle 0|O_8^{\Upsilon(1S)}({}^3S_1)|0\rangle + B_{\Upsilon(2S)\to\Upsilon(1S)}\langle 0|O_8^{\Upsilon(2S)}({}^3S_1)|0\rangle$$

$$+ B_{\Upsilon(3S)\to\Upsilon(1S)}\langle 0|O_8^{\Upsilon(3S)}({}^3S_1)|0\rangle + B_{\chi(2P)\to\Upsilon(1S)}\langle 0|O_8^{\chi_b(2P)}({}^3S_1)|0\rangle$$

$$+ B_{\chi(1P)\to\Upsilon(1S)}\langle 0|O_8^{\chi_b(1P)}({}^3S_1)|0\rangle$$

- the total cross-section

$$\hat{\sigma}(\Upsilon(1S)) = \hat{\sigma}(b\bar{b}_1(^3S_1))\langle 0|O_1(^3S_1)|0\rangle^{\Upsilon(1S)}_{total}$$

$$+ \hat{\sigma}(b\bar{b}_8(^3S_1))\langle 0|O_8(^3S_1)|0\rangle^{\Upsilon(1S)}_{total} + \hat{\sigma}(b\bar{b}_8(^1S_0))\langle 0|O_8(^1S_0)|0\rangle^{\Upsilon(1S)}_{total}$$

$$+\left(\sum_J (2J+1)\,\hat{\sigma}(b\bar{b}_8(^3P_J))\right)\langle 0|O_8(^3P_0)|0\rangle^{\Upsilon(1S)}_{total}$$

$$+ \sum_J \hat{\sigma}(b\bar{b}_1(^3P_J))\langle 0|O_1(^3P_J)|0\rangle^{\Upsilon(1S)}_{total}$$

Fitting the matrix elements

least χ^2 fit of

$$\frac{d\hat{\sigma}}{dP_T}(\Upsilon(1S)) \qquad \frac{d\hat{\sigma}}{dP_T}(\Upsilon(2S)) \qquad \frac{d\hat{\sigma}}{dP_T}(\Upsilon(3S))$$

$$\hat{\sigma}(\chi_{bJ}(1P)) \qquad \hat{\sigma}(\chi_{bJ}(2P))$$

to CDF data to obtain

$$\left.\begin{aligned} &\langle 0|O_8(^3S_1)|0\rangle^{\Upsilon(nS)}_{\text{total}} \\ &\langle 0|O_8(^1S_0)|0\rangle^{\Upsilon(nS)}_{\text{total}} \\ &\langle 0|O_8(^3P_0)|0\rangle^{\Upsilon(nS)}_{\text{total}} \end{aligned}\right\} \qquad n = 1, 2, 3$$

$$\langle 0|O_8(^3S_1)|0\rangle^{\chi_b(nS)}_{\text{total}} \qquad n = 1, 2$$

- use vacuum saturation to relate color-singlet production to color-singlet decay m.e.s

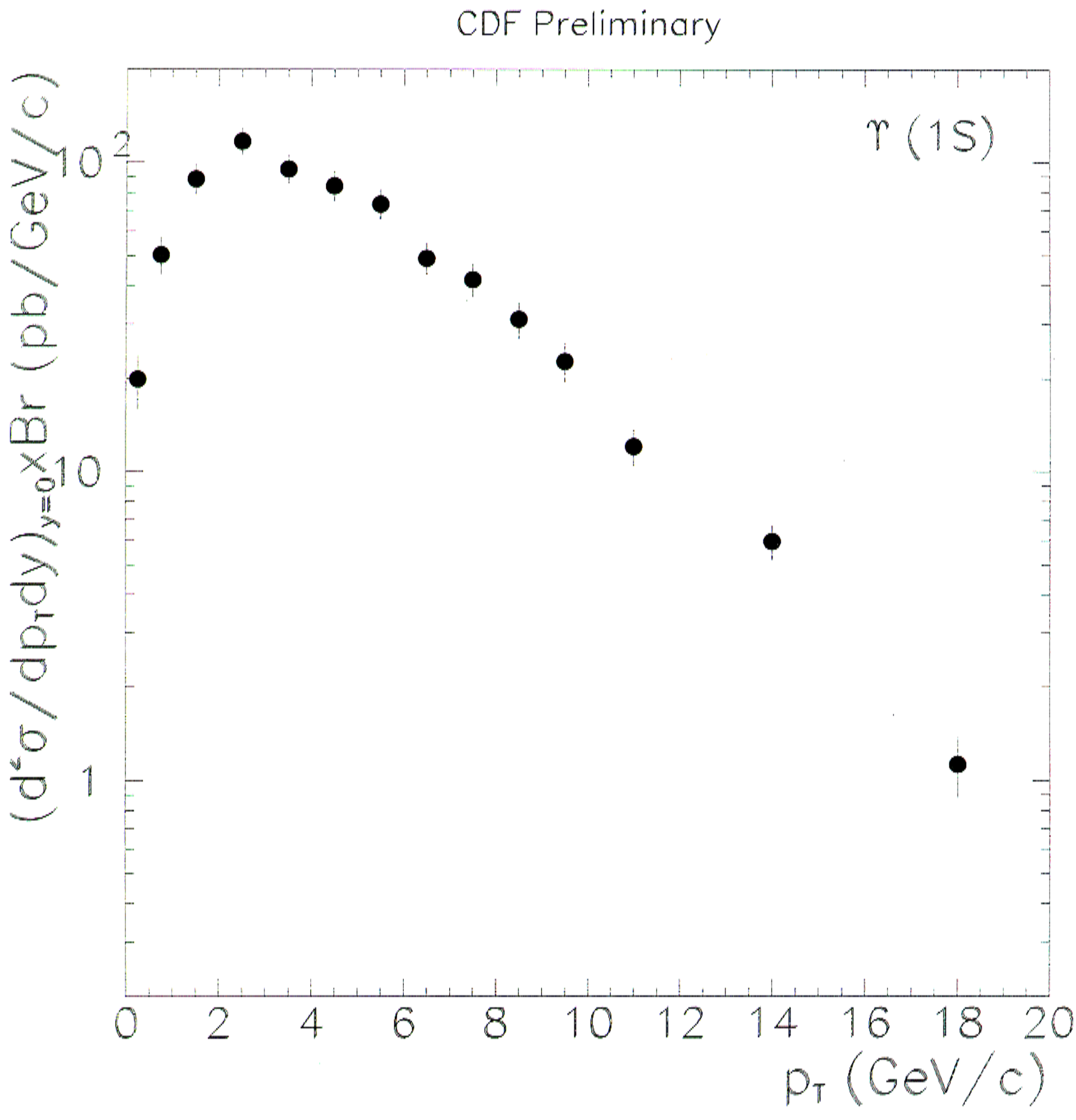
	$\langle O_1^{\Upsilon(nS)}(^3S_1)\rangle$			$\langle O_1^{\chi_{b0}(nP)}(^3P_0)\rangle$
	phenomenology	potential models		potential models
$\Upsilon(3S)$	4.3 ± 0.9	3.7 ± 1.5	$\chi_b(3P)$	2.7 ± 0.7
$\Upsilon(2S)$	4.5 ± 0.7	5.0 ± 1.8	$\chi_b(2P)$	2.6 ± 0.5
$\Upsilon(1S)$	10.9 ± 1.6	10.8 ± 5.5	$\chi_b(1P)$	2.4 ± 0.4

- use cteq5l & mrs98lo pdfs

$$\mu = \mu_R = \mu_F$$

$$\mu_0 = \sqrt{m_b^2 + P_T^2}$$

$$\mu = \mu_0,\ \tfrac{1}{2}\mu_0,\ 2\mu_0$$



CDF Preliminary
Υ (1S)
$(d^2\sigma/dp_T dy)_{y=0} \times Br$ (pb/GeV/c)
10^2
10
1
0
2
4
6
8
10
12
14
16
18
20
p_T (GeV/c)

- $m_b = 4.77 \pm 0.11$ GeV   1S mass

- fit only for $P_T > 8$ GeV
  avoid soft gluon effects

- once all the "total" m.ε.'s
  are fit extract NRQCD mε's using

	$\Upsilon(3S)$	$\chi_{b2}(2P)$	$\chi_{b1}(2P)$	$\chi_{b0}(2P)$	$\Upsilon(2S)$	$\chi_{b2}(1P)$	$\chi_{b1}(1P)$	$\chi_{b0}(1P)$	$\Upsilon(1S)$
$\chi_{bJ}(3P)$	0 ?	0 ?	0 ?	0 ?	0 ?	0 ?	0 ?	0 ?	0 ?
$\Upsilon(3S)$	1	11.4 ± 0.8	11.3 ± 0.6	5.4 ± 0.6	10.6 ± 0.8	0.6 ± 0.1	0.6 ± 0.1	0.4 ± 0.1	11.2 ± 0.5
$\chi_{b2}(2P)$		1			16.2 ± 2.4	1.1 ± 0.2	1.1 ± 0.2	0.7 ± 0.2	12.1 ± 1.3
$\chi_{b1}(2P)$			1		21 ± 4	1.4 ± 0.3	1.4 ± 0.3	0.9 ± 0.3	15.0 ± 1.8
$\chi_{b0}(2P)$				1	4.6 ± 2.1	0.3 ± 0.1	0.3 ± 0.1	0.2 ± 0.1	2.3 ± 0.9
$\Upsilon(2S)$					1	6.6 ± 0.9	6.7 ± 0.9	4.3 ± 1.0	31.1 ± 1.6
$\chi_{b2}(1P)$						1			22 ± 4
$\chi_{b1}(1P)$							1		35 ± 8
$\chi_{b0}(1P)$								1	< 6

Results:

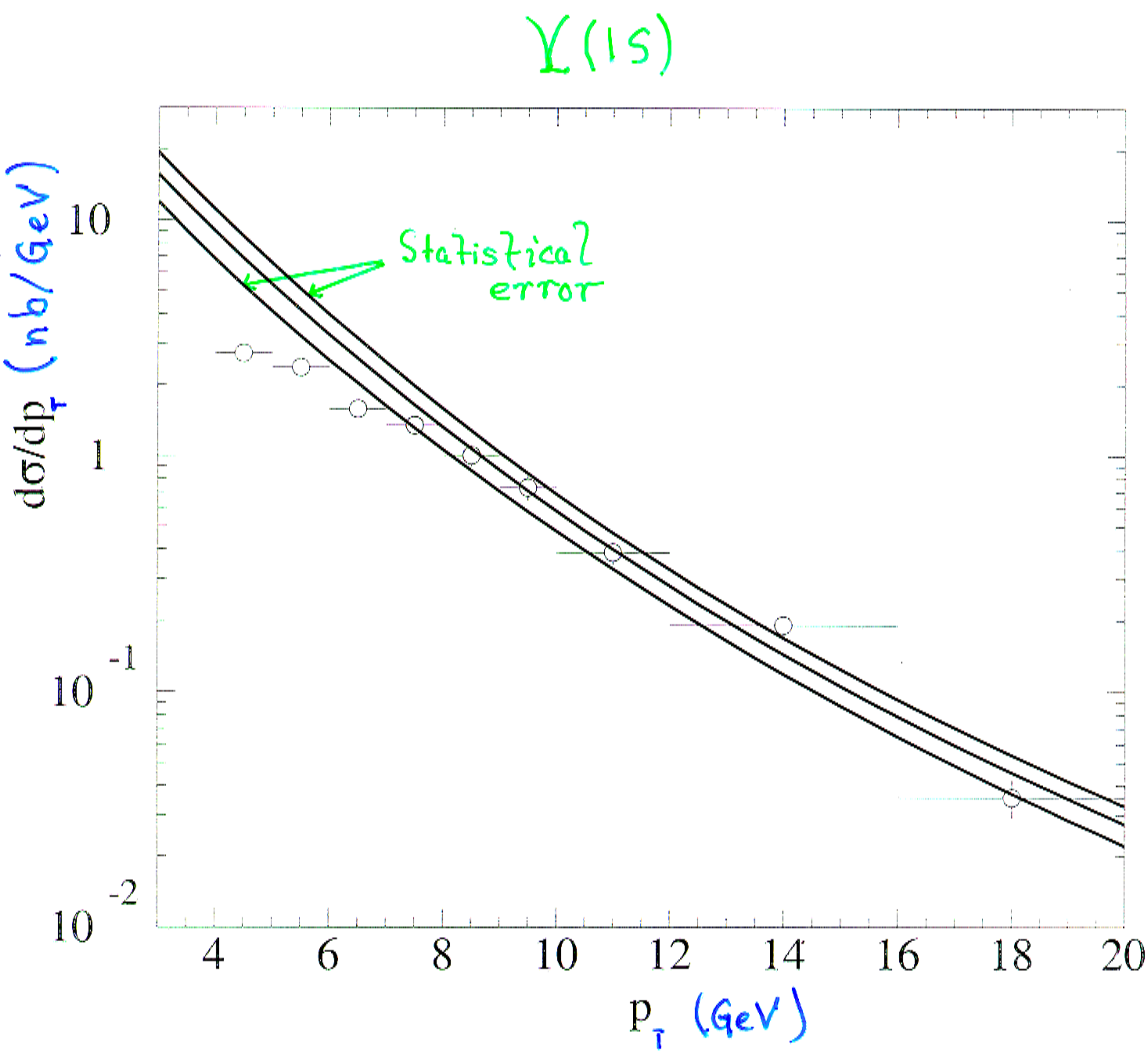
$$\text{fit} \pm \text{Stat.}{}^{+2\mu_0}_{-\frac{\mu_0}{2}}\ {}^{+m_b=4.88}_{-m_b=4.66}$$

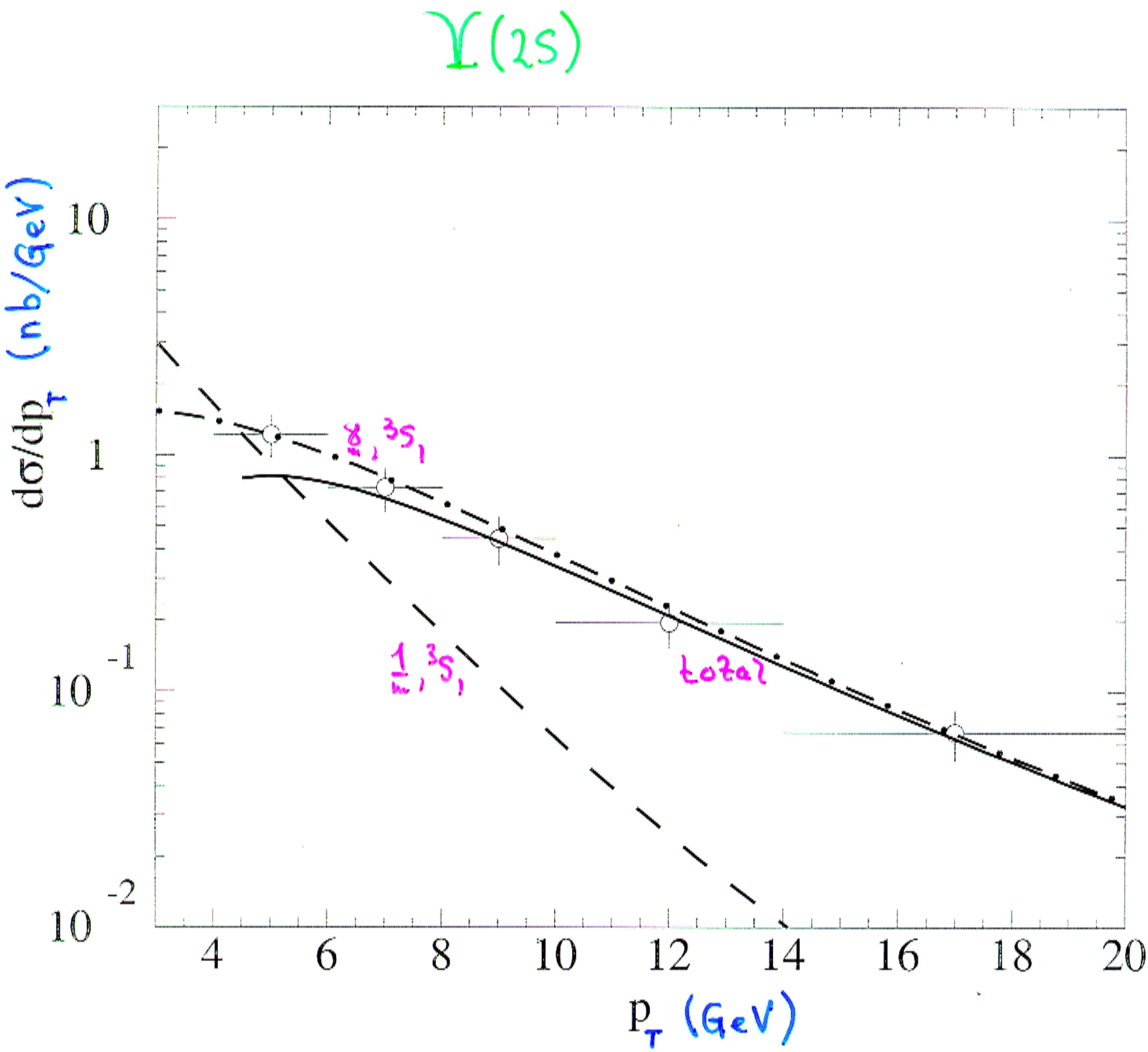
$m_b = 4.77$ GeV

$\mu_0 = \sqrt{p_T^2 + m_b^2}$

	CTEQ5L		MRSTLO	
$\langle O_8^{\chi_{b0}(2P)}(^3S_1)\rangle$	$0.8 \pm 1.1^{+1.1+0.3}_{-0.8-0.2}$		$1.2 \pm 1.3^{+1.1+0.3}_{-0.8-0.3}$	
$\langle O_8^{\chi_{b0}(1P)}(^3S_1)\rangle$	$1.5 \pm 1.0^{+1.3+0.4}_{-1.0-0.3}$		$1.9 \pm 1.2^{+1.4+0.4}_{-1.0-.04}$	
$\langle O_8^{\Upsilon(2S)}(^3S_1)\rangle$	$16.4 \pm 5.6^{+7.1+1.3}_{-5.1-1.1}$	$15.6 \pm 5.0^{+6.9+1.3}_{-4.9-1.1}$	$17.4 \pm 6.3^{+7.0+1.3}_{-5.1-1.2}$	$16.8 \pm 5.7^{+6.8+1.3}_{-5.0-1.2}$
$\langle O_8^{\Upsilon(2S)}(^1S_0)\rangle$	$-10.8 \pm 9.7^{-3.4+0.6}_{+2.0-0.5}$	0	$-9.5 \pm 11.1^{-2.8+0.9}_{+2.1-0.8}$	0
$\frac{5}{m_b^2}\langle O_8^{\Upsilon(2S)}(^3P_0)\rangle$	0	$-11.2 \pm 10.2^{-3.3+0.7}_{+2.4-0.7}$	0	$-9.7 \pm 11.6^{-2.9+1.0}_{+2.1-0.9}$
$\langle O_8^{\Upsilon(1S)}(^3S_1)\rangle$	$2.0 \pm 4.0^{-0.6-0.8}_{+0.5+0.6}$	$3.0 \pm 3.8^{+0.2-0.6}_{-0.1+0.4}$	$0.4 \pm 4.6^{-1.0-1.1}_{+0.7+0.9}$	$1.8 \pm 4.3^{-0.2-0.8}_{+0.1+0.6}$
$\langle O_8^{\Upsilon(1S)}(^1S_0)\rangle$	$13.6 \pm 6.0^{-9.5+3.1}_{+6.7-2.9}$	0	$20.2 \pm 6.9^{-10.9+3.7}_{+7.7-3.3}$	0
$\frac{5}{m_b^2}\langle O_8^{\Upsilon(1S)}(^3P_0)\rangle$	0	$13.9 \pm 6.4^{+11.4+3.1}_{-8.0-2.9}$	0	$20.7 \pm 7.3^{-11.4+3.6}_{+8.0-3.4}$

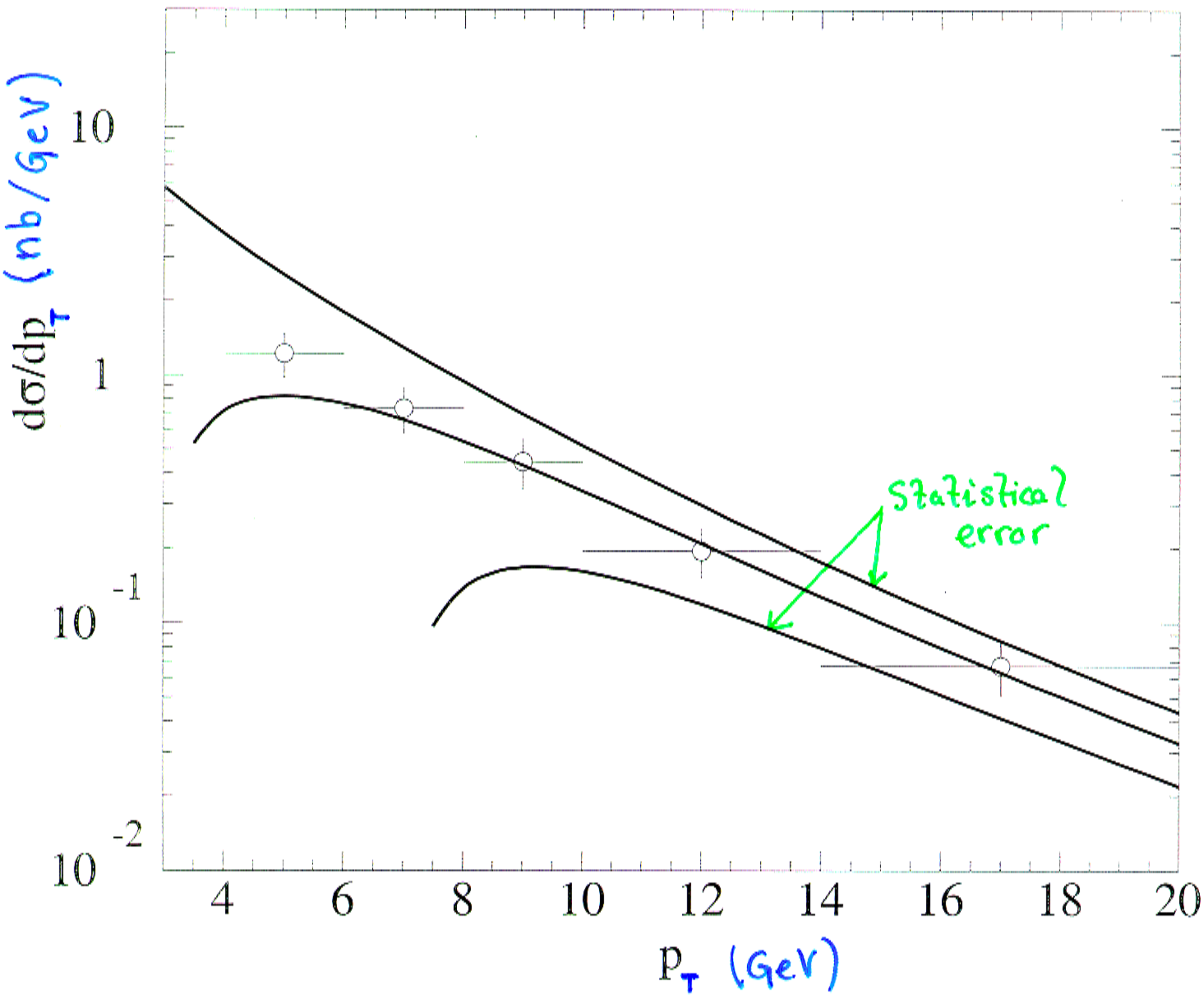
$\langle 0|O_8^{\Upsilon(nS)}(^1S_0)|0\rangle$, $\langle 0|O_8^{\Upsilon(nS)}(^3P_0)|0\rangle$ fit by data at lower P_T.


Υ(1S)
Statistical error
10
1
10 -1
10 -2
dσ/dp_T (nb/GeV)
4
6
8
10
12
14
16
18
20
p_T (GeV)

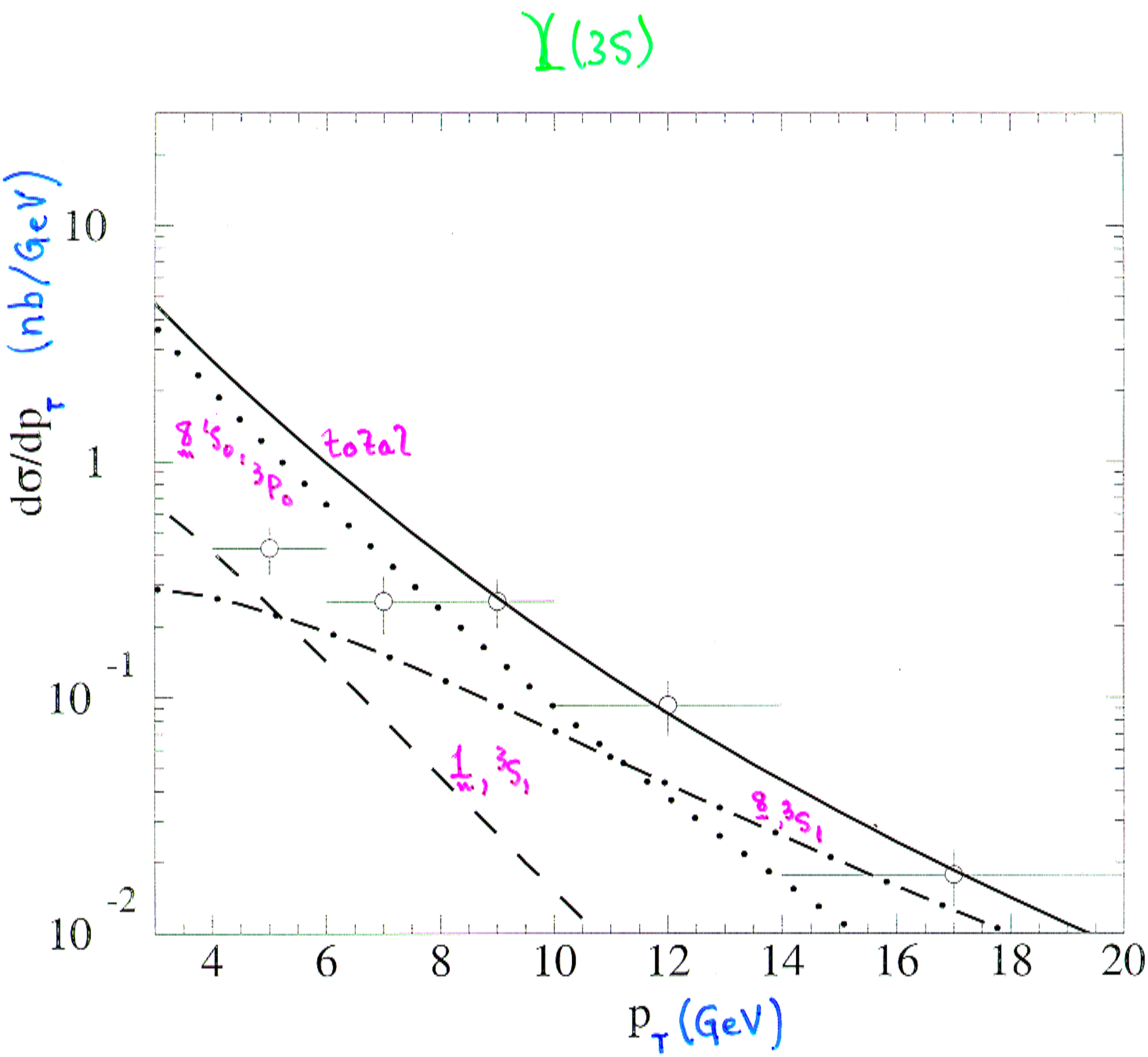


Υ(2S)
10
1
10^{-1}
10^{-2}
$d\sigma/dp_T$ (nb/GeV)
4
6
8
10
12
14
16
18
20
p_T (GeV)
$\underline{8}, {}^3S_1$
$\underline{1}, {}^3S_1$
total

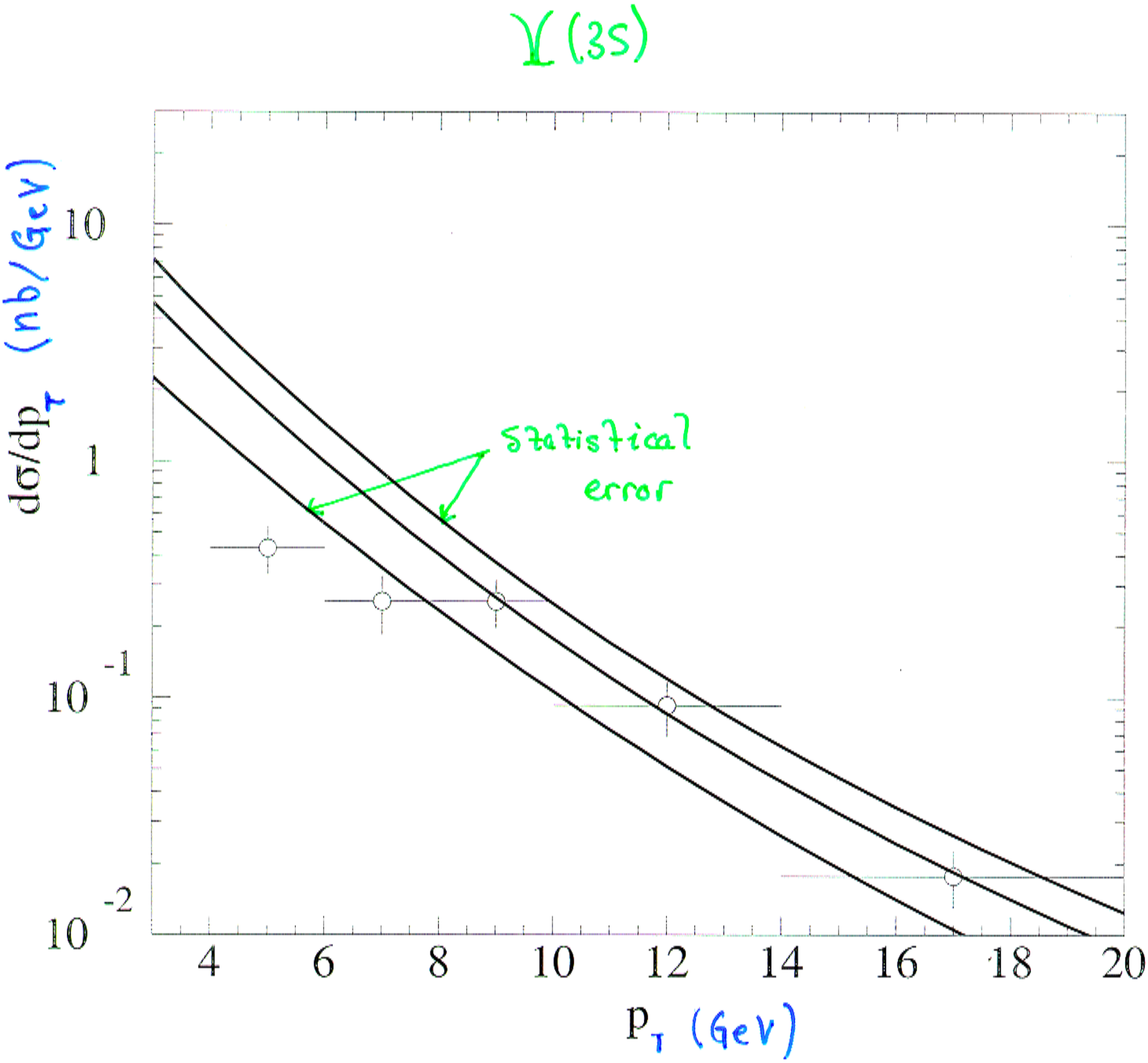
$\Upsilon(2S)$



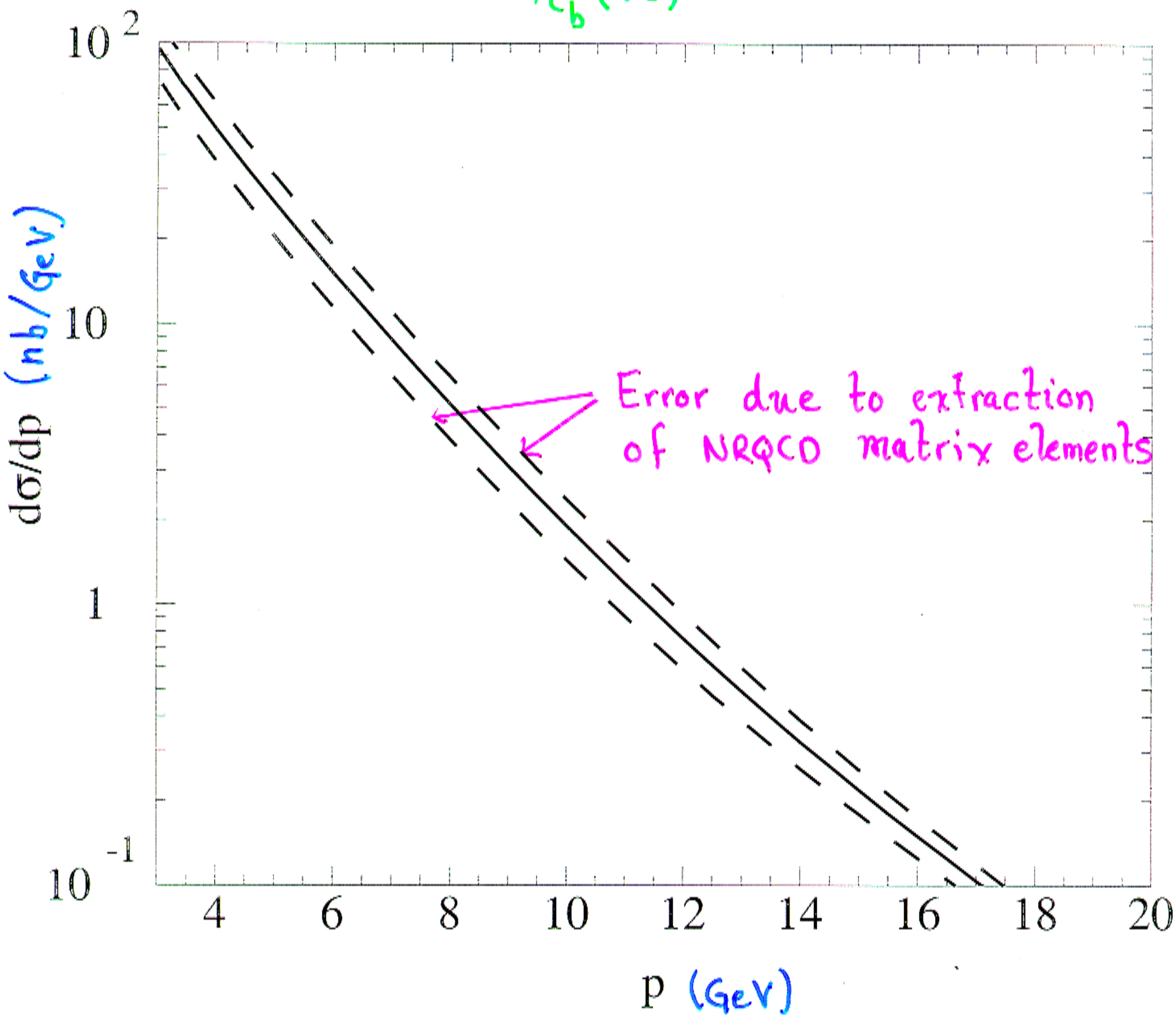
10
1
10^{-1}
10^{-2}
$d\sigma/dp_T$ (nb/GeV)
4
6
8
10
12
14
16
18
20
p_T (GeV)
statistical error



Υ(3S)
dσ/dp_T (nb/GeV)
10
1
10^-1
10^-2
total
8 ¹S₀, ³P₀
1, ³S₁
8, ³S₁
4
6
8
10
12
14
16
18
20
p_T (GeV)

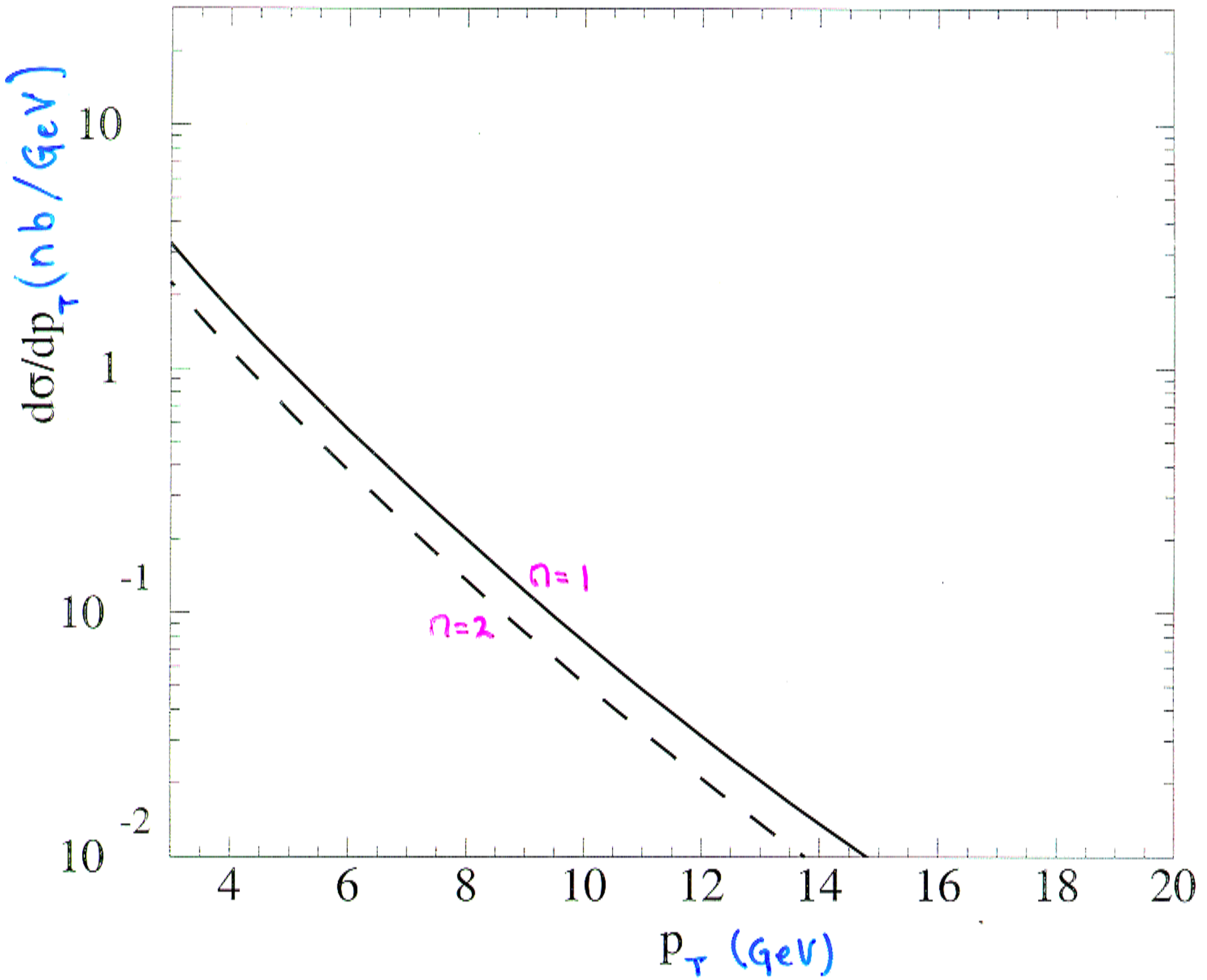

Υ(3S)
10
1
10 -1
10 -2
dσ/dp_T (nb/GeV)
4
6
8
10
12
14
16
18
20
p_T (GeV)
statistical error

$\eta_b(1S)$



$\sigma(p_T > 8\ \mathrm{GeV}) = 10.8\ \mathrm{nb}$

$h_b(1,2)$



$$\sigma_{h_b(1S)}(p_T > 8\ \text{GeV}) = 0.43\ \text{nb}$$