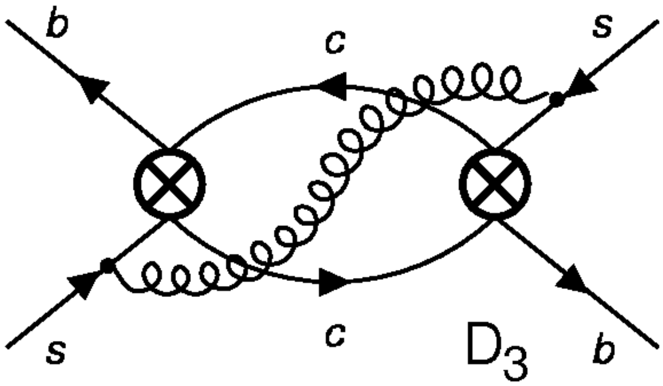
Lifetime differences between b - flavoured mesons beyond leading logarithms



A. Lenz, Regensburg

B-physics at Tevatron, Fermilab

24-Feb-2000

in Collaboration with:

M. Beneke, G. Buchalla, CERN;
C. Greub, Univ. Bern;U. Nierste, FERMILAB

Phys.Lett.**B 459**, (1999), 631; hep-ph/9906317

!! HOT TOPIC !!

Several people spoke in the 1st workshop about $(\Delta\Gamma/\Gamma)_{B_s}$, τ_B...

- Theory:
  Buchalla, Dunietz, Grossman, Hewett, Juge, MacKenzie, Nierste, Rosner, Voloshin

- Experiment: (BTeV, CDF, D0)
  Cheung, Kutschke, Lima, Paus, Taylor, Tseng, Webster

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.34^{+0.31}_{-0.34} \quad \text{or} \quad \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} < 0.83 \;\; \text{at} \;\; 95\% C.L.$$

<u>Questions:</u>

- How relieable is the theoretical prediction $(\Delta\Gamma/\Gamma)_{B_s}(t)$?
- What about violation of quark hadron duality?
- What about nonperturbative contributions??

????????????

Contents:

- Phenomenology of $(\Delta\Gamma/\Gamma)_{B_s}$
- How do we calculate in general
  - Effective Hamiltonian H_{eff}
  - **H**eavy **Q**uark **E**xpansion
  - Quark-Hadron-Duality
  - Why NLO calculations?
- How did we calculate $(\Delta\Gamma/\Gamma)_{B_s}$
- Discussion of the result
- Outlook: Lifetime of B^+ and Λ_b compared to B_d

Phenomenology of $(\Delta\Gamma/\Gamma)_{B_s}$:

- Determination of the unitarity trinagle:
  - theoretical *clean* ratio: $(\Delta M/\Delta\Gamma)_{B_s}$
    Browder, Pakvasa, Beneke, Buchalla, Dunietz, Greub, A.L., Nierste
  - α from inclusive indirect CP-asymmetries
    Beneke, Buchalla, Dunietz
  - novel studies of CP violation without tagging
    Dunietz, Fleischer
- Search for duality violation
- Search for a new CP-violating phase Φ in the B_s mixing amplitude:

$$\Delta\Gamma = \Delta\Gamma_{SM} \cos\Phi \leq \Delta\Gamma_{SM}$$

Grossman

see talks of the first workshop

Effective Hamiltonian I:

Why don't we calculate in the full standard model?

- Look only at the relevant degrees of freedom
  e.g. H-Atom: no quarks, gluons, gravitation,....

  It will make hard calculations easy and impossible calculations doable (Georgi)

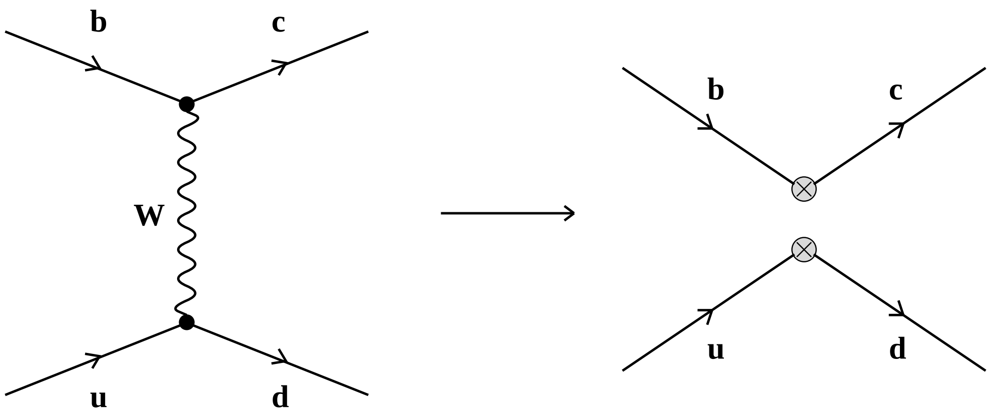
- Big logarithms arise $\alpha_s \ln\left(\frac{m_b^2}{M_W^2}\right) \approx 6\alpha_s \to$ perturbation theory valid?

- Quarks are bound in mesons: nonperturbative effects

B-decays: typical energy 5 GeV

- Integrate out heavy particles (e.g. W-boson)

$$\frac{1}{k^2 - M_W^2} \to -\frac{1}{M_W^2} + \mathcal{O}\left(\frac{k^2}{M_W^4}\right)$$



- Sum up logs with renormalization group

- Separate SD and LD with OPE

Effective Hamiltonian II:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3}^{6} C_j Q_j \right]$$

Current-current-operators

$$Q_2^q = (\bar{q}b)_{V-A} \cdot (\bar{s}q)_{V-A}$$

QCD-penguin operators

$$\begin{aligned} Q_3 &= (\bar{s}b)_{V-A} \cdot \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A} \\ Q_5 &= (\bar{s}b)_{V-A} \cdot \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A} \end{aligned}$$

$C_i(\mu)$: Wilson coefficients

C_1	C_2	C_3	C_4	C_5	C_6
-0.25	1.11	0.01	-0.03	0.01	-0.03

known up to NLO-QCD Buras, Jamin, Lautenbacher, Weisz

Heavy Quark Expansion I:

The standard expression for the decay rate is

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X | \mathcal{H}_{eff} | B \rangle|^2$$

The optical theorem gives

$$\Gamma(B \to X) = \frac{1}{m_B} \mathrm{Im} \langle B | \mathcal{T} | B \rangle$$

with the transition operator

$$\mathcal{T} = i \int d^4x T \left[\mathcal{H}_{eff}(x), \mathcal{H}_{eff}(0) \right]$$

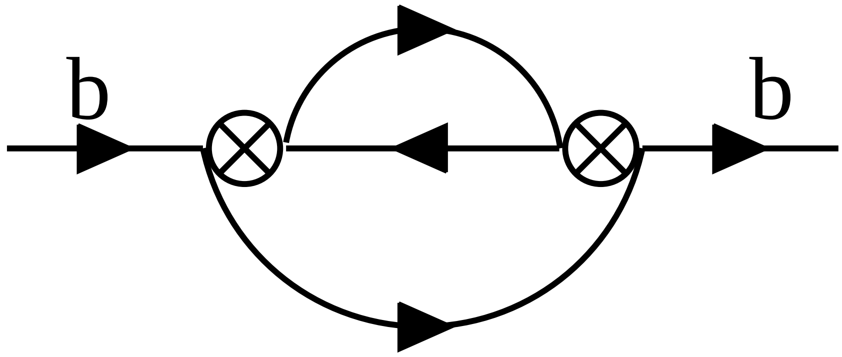
An operator-product-expansion for $\mathcal{T}$ yields

$$\mathcal{T} = c_3 \bar{b} b + \frac{c_5}{m_b^2} \bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b + \frac{\tilde{c}_6}{m_b^3} (\bar{b} q)_\Gamma (\bar{q} b)_\Gamma + \ldots$$

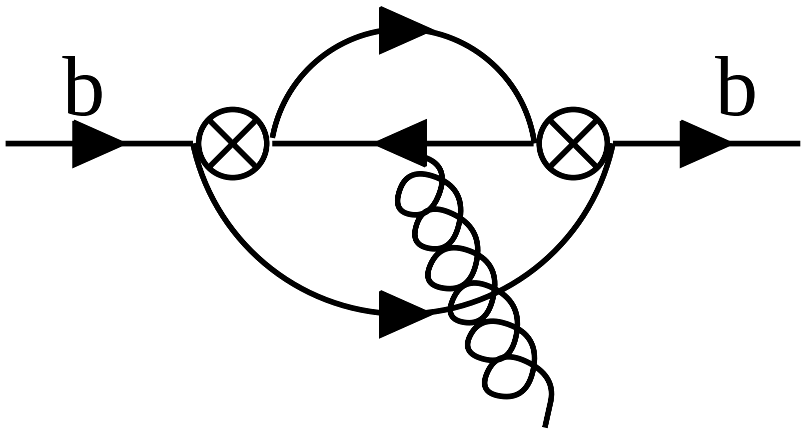
Voloshin, Uraltsev, Khoze, Shifman

Heavy Quark Expansion II:

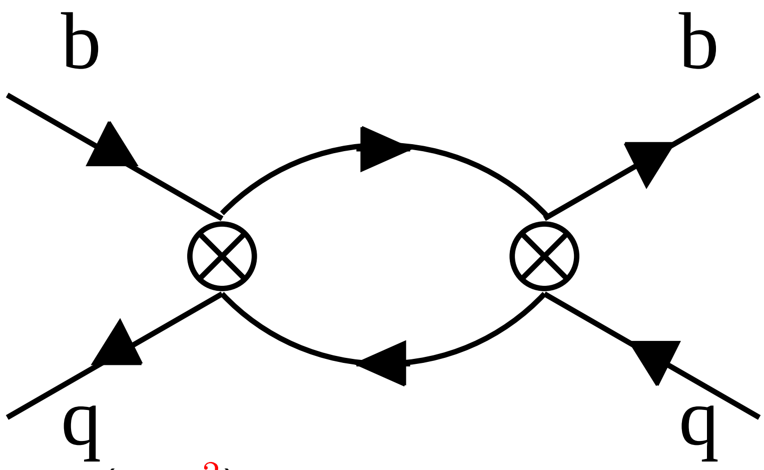
- $\bar{b}b$: Spectator decay



- $\bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b$: QCD corrections



- $(\bar{b}q)_\Gamma (\bar{q}b)_\Gamma$: Weak annihilation



Phase space enhanced ($16\pi^2$)

Spectator quark included

Heavy Quark Expansion III:

With a nonrelativistic expansion

$$\langle \bar{b} b \rangle_B = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2 m_b^2} + \mathcal{O}(m_b^{-3})$$

and the definitions

$$\langle \bar{b} (i\vec{D})^2 b \rangle_B =: \quad \mu_\pi^2 \quad + \mathcal{O}\left(m_b^{-1}\right) \approx 0.35 \pm 0.20 \text{ GeV}^2$$

Ball, Braun; Neubert

$$\langle \bar{b} \frac{g_s}{2} \sigma_{\mu\nu} G^{\mu\nu} b \rangle_B \quad =: \quad \mu_G^2 + \mathcal{O}\left(m_b^{-1}\right) \approx 0.36 \text{ GeV}^2$$

$$\langle (\bar{b} q)_\Gamma (\bar{q} b)_\Gamma \rangle_B \quad =: \quad f_B^2 m_B \sum_\Gamma c_\Gamma B_\Gamma$$

Hashimoto et al; Di Pierro, Sachrajda, Michael

one gets

$$\Gamma = \Gamma_0 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2 m_b^2}\right) \left[c_3 + 2 \frac{c_5}{m_b^2} \mu_G^2 + \frac{16 \pi^2 c_6}{m_b^3} f_B^2 m_B \sum_\Gamma c_\Gamma B_\Gamma + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right]$$

with $\Gamma_0 = (G_F^2 m_b^5)/(192 \pi^3)$

Quark-Hadron-Duality I:

The definition of what duality violation is varies from publication to publication

It is not deviation from parton picture!

—– Deviation from OPE based prediction —–

Bigi, Shifman, Uraltsev, Vainshtein

??What could go wrong??

- Terms in OPE do not decrease. Actual expansion parameter is Λ/Q.

- OPE does not account for exponential terms (e.g. from truncation of the asymptotic OPE, smearing of energy...)

$$X = \sum_n x_n \left(\frac{\Lambda}{Q}\right)^n + \tilde{x} e^{-\frac{Q}{\Lambda}}$$

After analytic continuation: oscillations

Nethertheless: This is QCD based, not like a hadronic model!!!

Quark-Hadron-Duality II:

Why should we believe in duality?

- Poggio-Quinn-Weinberg (1976)
  $\sigma^{tot}(e^- e^+ \to h) \equiv \sigma^{tot}(e^- e^+ \to quarks)$

- Determination of V_{cb} from inclusive and exclusive semileptonic B-decays agree
  e.g. Neubert; Shifman, Uraltsev, Vainsthein

- Experiment and Theory agree in hadronic τ-decays
  Pich et al.; Altarelli et al.; Ball, Beneke, Braun; Neubert; Shifman

- t'Hooft model: QCD in $1+1$ dimensions and $N_c \to \infty$
  Bigi, Shifman, Uraltsev, Vainshtein

- $(\Delta\Gamma/\Gamma)_{B_s} : \sum$ exklusive $=$ inclusive
  in the limit $N_c \to \infty$ and $\Lambda_{QCD} \ll m_b - 2m_c \ll m_b$
  Aleksan et al.

Why do we calculate up to NLO?

To extract parameters and to check the theory a precise theoretical result is needed

- Reduction of μ-dependence
  (theoretical error)

- Renormalization scheme dependence
  (matching to lattice calculations)

- Proper use of $\Lambda_{\overline{MS}}$

- Test of perturbative expansion

- Definition of quark masses
  $m_{pole} \leftrightarrow m_{\overline{MS}}$

- 30 % correction expected
  (comparable to expected QHD violation)

- QHD violation $\leftrightarrow$ non-logarithmic constants

How did we calculate $(\Delta\Gamma/\Gamma)_{B_s}$?

$$\Delta\Gamma = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

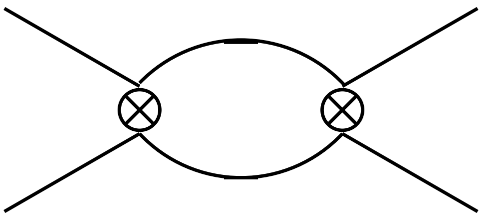
$\Gamma_3^{(0)}$: Hagelin; Buras, Slominski, Steger; Datta, Paschos, Türke, Wu; Voloshin, Uraltsev, Khoze, Shifman; Chau; Franco, Lusignoli, Pugliese

$\Gamma_4^{(0)}$: Beneke, Buchalla, Dunietz

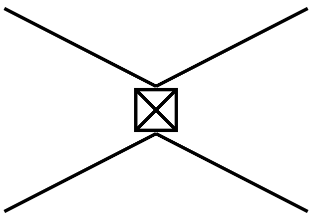
$\Gamma_3^{(1)}$: Beneke, Buchalla, Greub, A.L., Nierste

$$\Delta\Gamma = \frac{1}{2m_{B_s}}\langle \overline{B}_s|\mathcal{T}|B_s\rangle$$

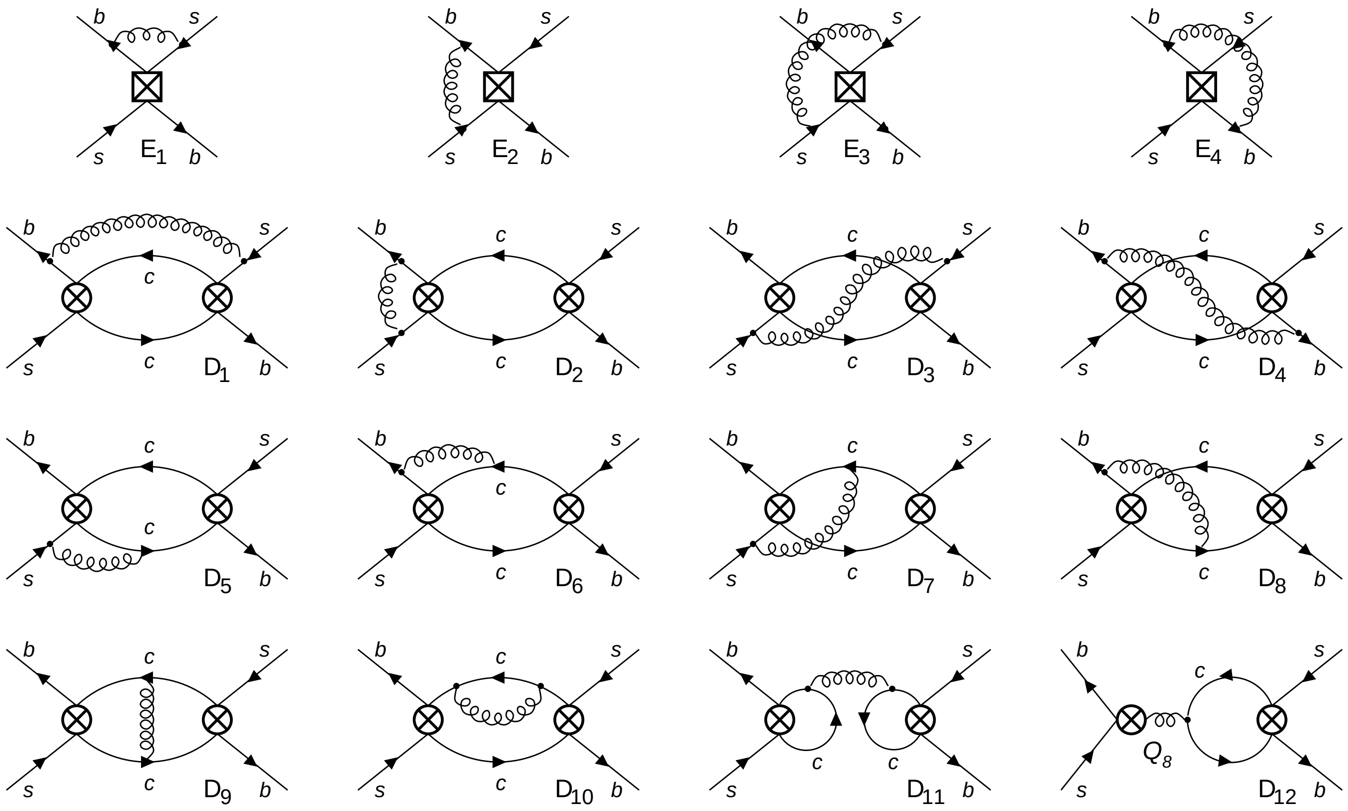
Matching of $\Delta B = 1$ double insertion to $\Delta B = 2$ insertion



$= \quad c_6^{\mathrm{LO}}$



How did we calculate $(\Delta\Gamma/\Gamma)_{B_s}$?



$$
\begin{aligned}
Q &= (\bar{b}_i s_i)_{V-A} \cdot (\bar{b}_i s_i)_{V-A} \\
Q_s &= (\bar{b}_i s_i)_{S-P} \cdot (\bar{b}_i s_i)_{S-P} \\
\langle \bar{B}_S | Q | B \rangle &= \frac{8}{3} f_{B_S}^2 M_{B_S}^2 B \\
\langle \bar{B}_S | Q_S | B \rangle &= -\frac{5}{3} f_{B_S}^2 M_{B_S}^2 \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} B_S
\end{aligned}
$$

Discussion of the result I:

$$\mathcal{T} = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}^* V_{cs})^2 \left[F(z) Q(\mu_2) + F_S(z) Q_S(\mu_2)\right]$$

with

$$F(z) = F_{11}(z) C_1^2(\mu_1) + F_{12}(z) C_1(\mu_1) C_2(\mu_1) + F_{22}(z) C_2^2(\mu_1),$$

$$F_{ij}(z) = F_{ij}^{(0)}(z) + \frac{\alpha_s(\mu_1)}{4\pi} F_{ij}^{(1)}(z)$$

LO:

$$F_{11}^{(0)}(z) = 3\sqrt{1-4z}\,(1-z) \qquad F_{S,11}^{(0)}(z) = 3\sqrt{1-4z}\,(1+2z),$$

$$F_{12}^{(0)}(z) = 2\sqrt{1-4z}\,(1-z) \qquad F_{S,12}^{(0)}(z) = 2\sqrt{1-4z}\,(1+2z),$$

$$F_{22}^{(0)}(z) = \frac{1}{2}(1-4z)^{3/2} \qquad F_{S,22}^{(0)}(z) = -\sqrt{1-4z}\,(1+2z).$$

NLO:

$$\begin{aligned}
F_{11}^{(1)}(z) \quad = \quad & 32(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
+ \quad & 64(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
- \quad & 4(13-26z-4z^2+14z^3)\ln\sigma \\
+ \quad & \sqrt{1-4z}\Big[4(13-10z)\ln z - 12(3-2z)\ln(1-4z) \\
& + \frac{1}{6}(109-226z+168z^2)\Big] + 2\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b},
\end{aligned}$$

$$
\begin{aligned}
F^{(1)}_{S,11}(z) &= 32(1-4z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ 64(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&- 16(4-2z-7z^2+14z^3)\ln\sigma\\
&+ \sqrt{1-4z}\Big[64(1+2z)\ln z-48(1+2z)\ln(1-4z)\\
&\quad -\frac{8}{3}(1-6z)(5+7z)\Big]-32\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b},
\end{aligned}
$$

$$
\begin{aligned}
F^{(1)}_{12}(z) &= \frac{64}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{128}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&+ (2-259z+662z^2-76z^3-200z^4)\frac{\ln\sigma}{6z}\\
&- \sqrt{1-4z}\left[(2-255z+316z^2)\frac{\ln z}{6z}+8(3-2z)\ln(1-4z)\right.\\
&\quad \left.+\frac{2}{9}(127-199z-75z^2)\right]\\
&- 2\sqrt{1-4z}(17-26z)\ln\frac{\mu_1}{m_b}+\frac{4}{3}\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b},
\end{aligned}
$$

$$
\begin{aligned}
F^{(1)}_{S,12}(z) &= \frac{64}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{128}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&+ (1-35z+4z^2+76z^3-100z^4)\frac{4\ln\sigma}{3z}\\
&- \sqrt{1-4z}\left[(1-33z-76z^2)\frac{4\ln z}{3z}+32(1+2z)\ln(1-4z)\right.\\
&\quad \left.+\frac{4}{9}(68+49z-150z^2)\right]\\
&- 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b}-\frac{64}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b},
\end{aligned}
$$

$$
\begin{aligned}
F_{22}^{(1)}(z) &= \frac{4}{3}(4-21z+2z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{4}{3}(1-2z)(5-2z)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&- (7+13z-194z^2+304z^3-64z^4)\frac{\ln\sigma}{6z}-\frac{\pi^2}{3}(1-10z)\\
&+ \sqrt{1-4z}\left[(7+27z-250z^2)\frac{\ln z}{6z}-4(1-6z)\ln(1-4z)\right.\\
&\left.-\frac{1}{18}(115+632z+96z^2)\right]\\
&- 2\sqrt{1-4z}(5-2z)\ln\frac{\mu_1}{m_b}+\frac{4}{3}\sqrt{1-4z}(2-5z)\ln\frac{\mu_2}{m_b},\\
F_{S,22}^{(1)}(z) &= -\frac{32}{3}(1+z)(1+2z)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{32}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&+ (1+7z+10z^2-68z^3+32z^4)\frac{4\ln\sigma}{3z}+\frac{8\pi^2}{3}(1+2z)\\
&- \sqrt{1-4z}\left[(1+9z+26z^2)\frac{4\ln z}{3z}-16(1+2z)\ln(1-4z)\right.\\
&\left.+\frac{8}{9}(19+53z+24z^2)\right]\\
&- 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b}+\frac{32}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b}.
\end{aligned}
$$

We have used:

$$
\mathrm{Li}_2(x) = -\int_0^x dt\frac{\ln(1-t)}{t} \qquad \sigma = \frac{1-\sqrt{1-4z}}{1+\sqrt{1-4z}} \qquad z = \frac{m_c^2}{m_b^2}
$$

Cancellation of IR-singularities in the F's due to

- Bloch-Nordsieck cancellation among different cuts of the same diagram
- Factorization of IR-singularities, which end up in the matrix elements

Discussion of the result II:

LO-result:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\text{MeV}}\right)^2 [0.009 B(m_b) + 0.211 B_S(m_b) - 0.063]$$

$1/m_b$ corrections: Beneke, Buchalla, Dunietz

NLO-result:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\text{MeV}}\right)^2 [0.006 B(m_b) + 0.150 B_S(m_b) - 0.063]$$

Beneke, Buchalla, Greub, A.L., Nierste

- no NLO: $B = B_S = 1 \quad f_{B_s} = 210 MeV$
- Till 1999: $B(m_b) = 0.9 \quad B_S(m_b) = 0.75 \quad f_{B_s} = 210 MeV$
- Summer conferences 1999:
  $B(m_b) = 0.76 \quad B_S(m_b) = 1.19 \quad f_{B_s} = 245 MeV$
  Yamata et al., Hashimoto

$\Delta\Gamma/\Gamma$	LO	LO + $1/m_b$	NLO
B=1	22 %	16 %	-
old	17%	10 %	6 %
new	35 %	27 %	16 %

Outlook: Lifetime of B^+ and Λ_b compared to B_d

	Theory	Experiment
$\tau(B^-)/\tau(B_d^0)$	$1.0 - 1.1$	1.07 ± 0.02
$\tau(B_s^0)/\tau(B_d^0)$	$0.99 - 1.01$	0.94 ± 0.04
$\tau(\Lambda_b)/\tau(B_d^0)$	$0.9 - 1.0$	0.79 ± 0.05

Voloshin, Uraltsev, Khoze, Shifman; Neubert, Sachrajda;
LEP B Lifetime Working Group 7/99

- Voloshin, Uraltsev, Khoze, Shifman; Neubert, Sachrajda:
  LO QCD corrections to the lifetime differences and fit of bag parameters

- Lattice: Di Pierro, Sachrajda, Michael:
  (exploratory) lattice study of spectator effects

Outlook: Lifetime of B^+ and Λ_b compared to B_d

Diagrams

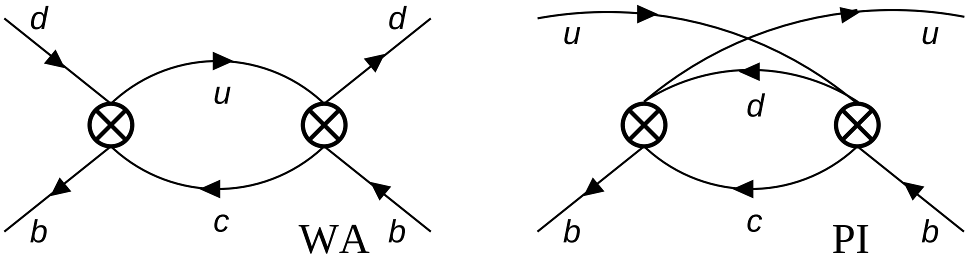


Figure 1: Leading-order *Weak annihilation* (WA) contribution to $\Gamma^{\text{non-spec}}(B_d^0)$ and *Pauli interference* (PI) contribution to $\Gamma^{\text{non-spec}}(B^+)$. The crosses represent $|\Delta B| = 1$ operators.

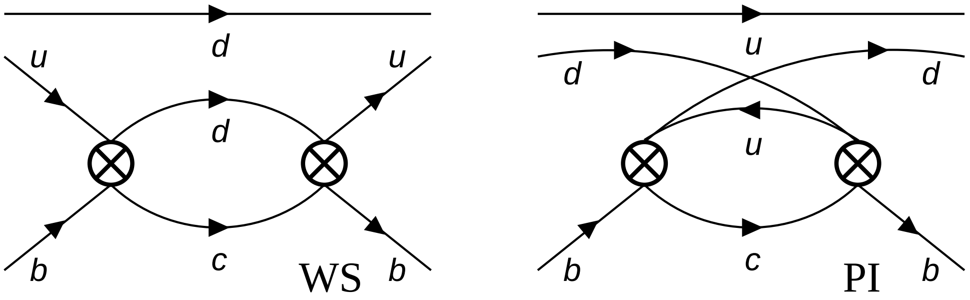


Figure 2: Leading-order *Weak scattering* (WS) and *Pauli interference* contribution to $\Gamma^{\text{non-spec}}(\Lambda_b^0)$.

Work in progress