1. A study of heavy quark non-perturbative fragmentation in HERWIG

Stefano Frixione and Michelangelo Mangano

In this Section we present the results of a phenomenological study of the non-perturbative hadronization of b-quarks. According to the standard QCD lore, distributions for an observable hadron H can be computed by convoluting the short-distance cross section $\hat{\sigma}(p)$ with a fragmentation function $D_H^{(h)}(z)$ that describes the way in which the heavy quark h hadronizes into H:

$$\mathrm{d}\sigma_H(p) = \int dz\, D_H^{(h)}(z)\, \mathrm{d}\hat{\sigma}(p/z). \tag{1}$$

The precise definition of $D_H^{(h)}(z)$ depends on how much of the heavy quark evolution after its production is absorbed into the perturbative part $\hat{\sigma}(p)$, and how much is assigned to the non-perturbative component parameterised by $D_H^{(h)}(z)$. Since perturbation theory (PT) is well defined for a massive quark, the standard prescription is to absorb into $\hat{\sigma}(p)$ not only the hard matrix elements, but also the perturbative part of the fragmentation function, defined by the evolution in Q^2 down to a scale equal to the heavy quark mass m_h. $D_H^{(h)}(z)$ will therefore account for the transition of an "on-shell" quark h into the hadron H. The assumptions built into eq. (1) are that $D_H^{(h)}(z)$ depends neither on the type of hard process, nor on the scale at which h was produced. Under these assumptions, $D_H^{(h)}(z)$ can be extracted from data in one given reaction (typically, e^+e^-), and eventually used to predict the cross section in some other reaction ($p\bar{p}$, DIS and so on).

QCD factorization theorems indicate that this universality of $D_H^{(h)}(z)$ holds in the asymptotic limit, and up to corrections of order m_h/Q, Q being the scale of the hard process. The size of these corrections cannot be calculated, today, in any rigorous way. A possible approach to this problem is to turn to the phenomenological models of hadronization implemented in QCD-based parton-shower Monte Carlo (PSMC) codes. In PSMC the full final-state kinematical configuration is available at both the parton and hadron levels. Therefore, it is possible to "measure" $D_H^{(h)}(z)$ using eq. (1), both $\mathrm{d}\sigma_H$ and $\mathrm{d}\hat{\sigma}$ being known. In the present section, we carry out this program using the PSMC HERWIG [1]. HERWIG evolves quarks according to perturbative QCD down to small scales. The quarks are paired up at the end of the evolution into colour singlet clusters, which are then decayed to the physical hadrons using phenomenological distributions. The study of the heavy quark hadronisation process in HERWIG will allow us to test the universality assumption, and to measure the size of possible deviations.

We should stress that, at this moment, our conclusions are only relevant for the hadronization model implemented in HERWIG; other PSMC's, which treat the hadronization process differently (for example, by adopting a string model), may well lead to different conclusions.

In order to precisely define our procedure for extracting $D_H^{(h)}(z)$, we need to consider in more details the way in which HERWIG generates events. Regardless of the type of initial-state particles, we can distinguish the following steps.

- *Hard subprocess:* at this stage, the PSMC generates the kinematics for the basic $2 \to 2$ hard reaction. We denote the momentum of the b-quark (or antiquark) as p_b^{hard}.
- *Parton shower:* the partons resulting from the hard subprocess undergo successive branchings, until their virtuality is smaller than a fixed cutoff value. We denote the momentum of the b-quark at the end of this phase as p_b^{ps}.
- *Gluon splitting and cluster formation:* the gluons present at the end of the shower are decayed into light-quark pairs. Colour-singlet, two-body clusters are formed, according to colour parenthood and closeness in the phase-space. If there exist one or more cluster whose mass is too large (relative to a given threshold), part of the cluster rest energy is transformed into new $q\bar{q}$ pairs, and new clusters are defined. In this process, energy-momentum is redistributed among the cluster

	e^+e^-				$p\bar{p}$			
	μ_1	μ_2	μ_3	μ_4	μ_1	μ_2	μ_3	μ_4
$\lvert\vec{p}_b^{ps}\rvert < 5$ GeV	0.87	0.78	0.71	0.66	0.95	0.94	0.94	0.96
$10 < \lvert\vec{p}_b^{ps}\rvert < 15$ GeV	0.92	0.85	0.80	0.76	0.85	0.74	0.66	0.60
$20 < \lvert\vec{p}_b^{ps}\rvert < 25$ GeV	0.92	0.86	0.81	0.76	0.83	0.71	0.63	0.57
$30 < \lvert\vec{p}_b^{ps}\rvert < 35$ GeV	0.92	0.85	0.80	0.75	0.82	0.70	0.62	0.56

Table 1: Normalized Mellin moments of the b-quark non-perturbative fragmentation function. Results are given for the case of e^+e^- collisions at $\sqrt{S} = 91.2$ GeV and for $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. All numbers have a statistical accuracy of ± 0.01.

elements, and the momentum of the b-quark can therefore be modified with respect to p_b^{ps}. The momentum of the b quark after completion of the clustering process will be denoted by p_b^{glsp}.

- *Cluster decay and hadron formation:* the clusters decay into observable hadrons, according to the flavour and to tabulated mass spectra. We therefore obtain b-flavoured hadrons, whose momentum we denote as p_B.

The hard subprocess and parton shower stages are based on perturbative QCD. Thus, we identify the predictions given by HERWIG at the end of the parton shower with the cross section $\hat{\sigma}$ that appears in eq. (1). On the other hand, the gluon splitting and cluster decay stages do not contain QCD information, as they are performed according to a phenomenological model. The $g \to q\bar{q}$ splitting and the decay kinematics are induced by simple phase-space considerations. We thus identify these stages as the long-distance, non perturbative part of the process, which gives rise to $D_H^{(h)}(z)$. We therefore determine the fragmentation function by comparing the results for p_b^{ps} and p_B, defining, on an event-by-event basis, the following variables:

$$z_1 = \frac{\vec{p}_B \cdot \vec{p}_b^{ps}}{\left|\vec{p}_b^{ps}\right|^2}, \qquad z_2 = \frac{E_B + \vec{p}_B \cdot \hat{p}_b^{ps}}{E_b^{ps} + \left|\vec{p}_b^{ps}\right|}, \tag{2}$$

where $\hat{p}_b^{ps} = \vec{p}_b^{ps} / \left|\vec{p}_b^{ps}\right|$. Our conclusions will apply to both z_1 and z_2; thus, we will collectively denote them by z. In hadronic collisions, the momenta and energies have to be substituted by transverse momenta and transverse energies respectively. Our results are summarized in table 1; we considered e^+e^- collisions at $\sqrt{S} = 91.2$ GeV and $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. In the table, we present four of the (normalized) Mellin moments of the z distribution, defined as follows:

$$\mu_n = \int dz\, z^n\, D_H^{(h)}(z) \Big/ \int dz\, D_H^{(h)}(z). \tag{3}$$

Usually, $0 \le z \le 1$. In the present case, as we will see, we can also have $z > 1$; thus, in eq. (3) the range of integration coincide with the support of $D_H^{(h)}(z)$. The Mellin moments appearing in table 1 have been evaluated by considering bins in $\left|\vec{p}_b^{ps}\right|$ (in the case of hadronic collisions, the momentum is the transverse one). In e^+e^- collisions larger (smaller) values of $\left|\vec{p}_b^{ps}\right|$ correspond to less (more) energy lost to gluons. In hadronic collisions larger (smaller) values of $\left|\vec{p}_b^{ps}\right|$ are more likely to correspond to larger (smaller) values of the hard process momentum before evolution. In either case, dependence of $D_H^{(h)}(z)$ on $\left|\vec{p}_b^{ps}\right|$ signals therefore a departure from universality.

By inspection of the table, we see that $D_H^{(h)}(z)$ is scale-independent to a very good extent (the situation appears to be slightly better in the case of e^+e^- collisions), except for the very low p_b^{ps} region; this is what we should expect, since in that region the factorization theorem on which eq. (1) is based is bound to fail. On the other hand, there seems to emerge a clear difference between the fragmentation functions extracted from e^+e^- and $p\bar{p}$ "data", the latter being substantially softer than the former. The first moment, which is the average value of the fragmentation variable, changes by about 10%. This variation can change the rate of predicted b-hadrons in hadronic collisions by almost 50%.

This suggests that transporting to hadronic collisions the non-perturbative fragmentation functions obtained by fitting e^+e^- data may not be correct. Of course, a much more detailed investigation on the subject is required before reaching a firm conclusion; however, this simple exercise of ours shows that universality better not be taken for granted.

We now concentrate on the separate role played in the fragmentation process by the purely perturbative evolution and by the non-perturbative gluon-splitting phase, before the cluster formation and decay take place. We shall confine ourselves to the case of e^+e^- collisions. The variables relevant to our study are the following:

1. Energy fraction retained *during* the perturbative evolution:

$$z_{ps} = \frac{2\left|\vec{p}_b^{\,ps}\right|}{\sqrt{S}} = \frac{\left|\vec{p}_b^{\,ps}\right|}{\left|\vec{p}_b^{\,hard}\right|}, \tag{4}$$

where $\sqrt{S}$ is the e^+e^- CM energy.

2. Energy fraction retained *during* the gluon-splitting:

$$z_{glsp} = \frac{\left|\vec{p}_b^{\,glsp}\right|}{\left|\vec{p}_b^{\,ps}\right|}. \tag{5}$$

3. Energy fraction left *after* the perturbative evolution and the gluon-splitting:

$$z = z_{ps} \times z_{glsp} = \frac{2\left|\vec{p}_b^{\,glsp}\right|}{\sqrt{S}} = \frac{\left|\vec{p}_b^{\,glsp}\right|}{\left|\vec{p}_b^{\,hard}\right|}, \tag{6}$$

The left panel in fig. 1 shows the three distributions for b quarks at $\sqrt{S} = 91.2$ GeV. The solid histogram represents the distribution of z_{ps}. The distribution has the shape of a Gribov-Lipatov, with no indication of a Sudakov turn-over at large z_{ps}. The dotted line is the distribution in z. A strong deformation of the purely perturbative curve is clearly seen. The dashed line corresponds to the z_{glsp} distribution. This is part of what the MC treats as a non-perturbative component of the fragmentation process. The peak of the dashed histogram at $z_{glsp} = 1$ corresponds to events where the cluster containing the heavy quark does not need to be further split, while the tail corresponds to events where the invariant mass of the heavy-quark cluster mass is too large, and additional light-quark pairs have to be produced by hand. Notice that almost as much energy is lost during this non-perturbative phase, as is lost during the perturbative evolution.

For comparison, we show the same set of curves for the evolution of the charm quark (right panel of fig. 1). Notice that while the effect of the perturbative evolution is to soften the quark spectrum relative to the b-quark case, the amount of energy lost due to gluon splitting is similar ($\langle z^c_{glsp}\rangle = 0.82$, as opposed to $\langle z^b_{glsp}\rangle = 0.85$). This is bizarre, since one expects the non-perturbative part to scale with $1/m_h$. The same result is found for the fragmentation of the s quark (left panel of fig. 2). Here $\langle z^s_{glsp}\rangle$ is 0.81. Again, a violation of the expected $1/m_h$ scaling.

Things improve for the top quark, whose distributions for $\sqrt{S} = 2$ TeV are shown on the right panel of fig. 2. The gluon-splitting part has only a minor impact on the overall spectrum of the top quark.

We are a bit bothered by the dominant role played by the gluon-splitting phase. By comparison, the next step in the evolution, namely the cluster formation and decay, plays only a minor role, as will be shown next. We would have anticipated that the cluster formation and decay should be the place where most of the non-perturbative physics should show up. This suggests that the thresholds for the perturbative evolution in the MC shold be lowered, so that the impact of the non-perturbative gluon splitting phase is reduced, and purely perturbative Sudakov effects can manifest themselves.

We now turn again to the non-perturbative part of the fragmentation function. The most striking feature, that cannot be inferred from the simple study of Mellin moments as done in table 1, is the

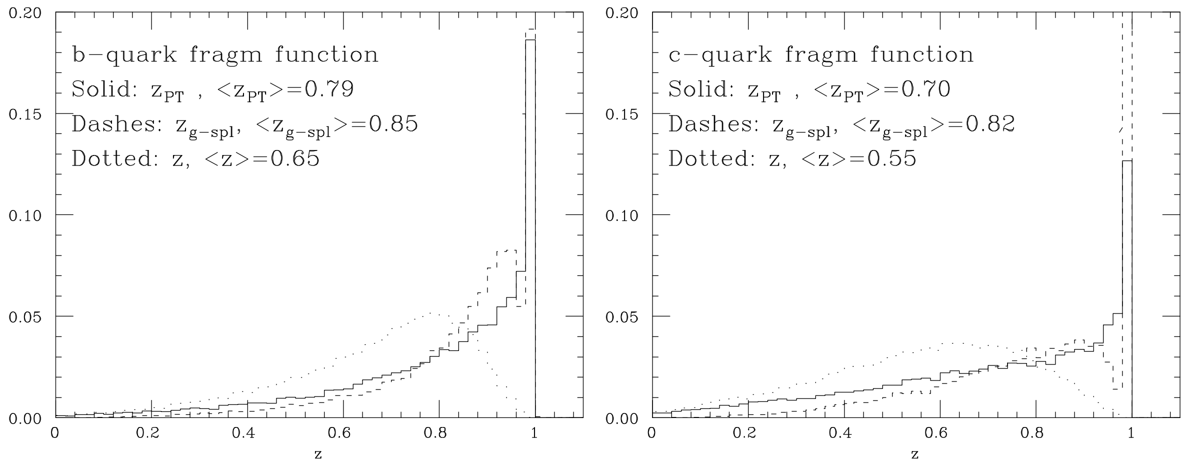

Fig. 1: Fragmentation functions for b (left) and c (right) quarks, produced in e^+e^- collisions at $\sqrt{s} = 91.2$ GeV.

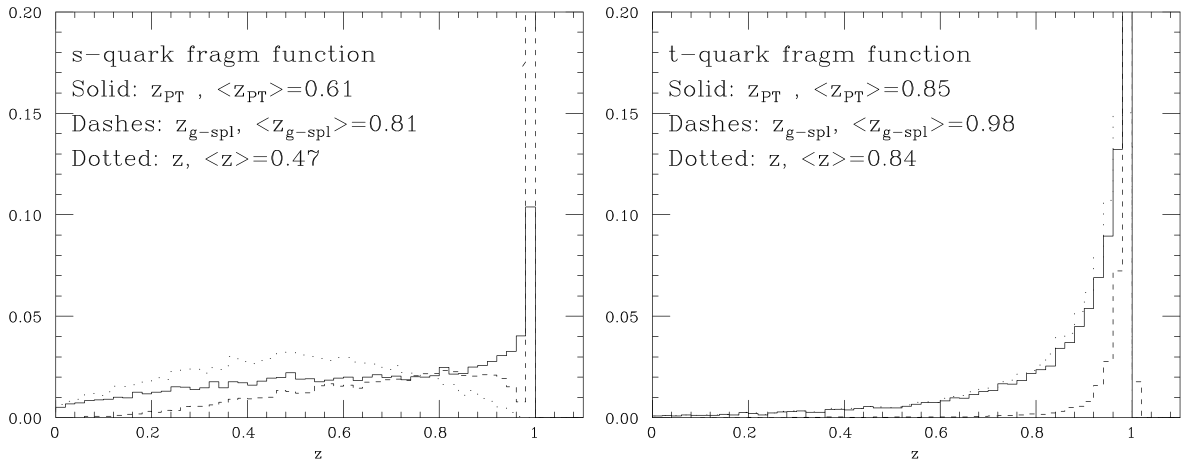

Fig. 2: Fragmentation functions for s (left) and t (right) quarks, produced in e^+e^- collisions at $\sqrt{s} = 91.2$ (left) and 2000 (right) GeV.

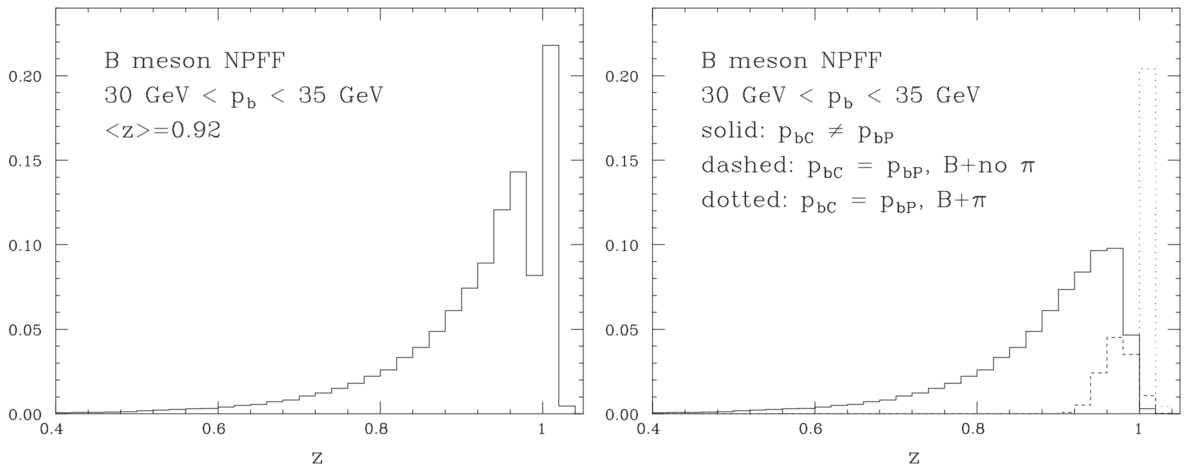

Fig. 3: B-meson non-perturbative fragmentation function in e^+e^- at $\sqrt{S} = 91.2$ GeV, for 30 GeV< $|\vec{p}_b|$ <35 GeV. CLDIR=1, CLSMR=0. See the text for details.

presence of a double peak in the high-z region (see the left panel of fig. 3). A first peak (which we will call peak **A**) is seen at z values around 0.97. A second peak (peak **B**) is at $z = 1.01$ (we have a z-bin size of 0.02. We verified that the events contributing to the peak **B** do not have $z = 1 + \epsilon$, i.e., the peak is not due to a roundoff error). The latter peak is higher than the former.

The origin of this double peak can be traced back to the following facts. First, the momentum of the emitted B meson is very strongly correlated with the momentum of the b quark which enters the cluster. Therefore, the z distribution closely reflects the mass spectrum of the light hadron emitted, together with the B meson, in the cluster decay. Second, the peak **B** is almost entirely due to events where the cluster decay into $B + \pi$: at this peak, $z > 1$ because the mass of the pion is lighter than the mass of the lightest quarks in HERWIG. More in detail, we have observed the following facts.

- The structure of the double peak is strongly influenced by the value taken by the two input parameters **CLDIR** and **CLSMR**. If the default is used (**CLDIR=1**, **CLSMR=0**), the double peak is observed (see the plot on the left of fig.3). On the other hand, by setting **CLSMR≠0**, the z distributions display a single peak (broader that the previous ones) at about $z = 0.97$. For small **CLSMR** values and large b momenta, a second peak at $z = 1.01$ tends to re-appear, although smaller than observed before.
- The double peak disappears also if one chooses **CLDIR=0**, as in older HERWIG versions. In this case, the z distributions peak at about $z = 0.9$, this peak being much broader that those obtained with **CLDIR=1**, regardless of the value of **CLSMR**.
- We then set **CLDIR=1** and **CLSMR=0**. For any given B meson, we looked for the parent cluster $\mathcal{C} = \{b_C q\}$, and for the parent bottom quark, b_P (the parent quark is defined as in the HERWIG routine **HWCHAD**). We observed what follows.
  - Plotting the z distributions for the events with $\vec{p}_{b_C} \neq \vec{p}_{b_P}$ (i.e. events where the original cluster was split), we see a single peak, at the same z value as for the peak **A**(solid line, plot on the right of fig.3).
  - The z distributions for events such that $\vec{p}_{b_C} = \vec{p}_{b_P}$ display again a double peak. The two peaks are at the same z values as peaks **A** and **B**, the latter one being by far dominant.
  - Selecting only events with $\vec{p}_{b_C} = \vec{p}_{b_P}$, we found that the peak at the position of **B** corresponds to those clusters decaying into a B meson and a π, while the peak at the position of **A** is relevant for all the other two-body decays (dotted and dashed lines respectively, plot on the right of fig.3).

Overall, notice also that the amount of energy retained after the gluon-splitting phase is of the same size as that retained at the end of the full hadronization process, indicating that cluster formation and decay have a minor impact on the total amount of energy lost during the non-perturbative part of the evolution.

We were also able to reproduce the previous findings with a very simple model. Given a momentum for a quark b, we generate randomly the momentum for a light quark q, to be combined with b into a cluster, which eventually decays into a B meson and a particle of given mass m_P. The momentum of the quark q is allowed to have a (small) transverse momentum with respect to the direction of the quark b. After evaluating the cluster mass, we performed the decay in the rest frame of the cluster, either in a isotropic manner (thus mimicking the choice **CLDIR=0**), or by letting the momentum of the meson B to be parallel to that of the quark b (which corresponds to **CLDIR=1** and **CLSMR=0**). In the latter case, depending upon the value of m_P, we got a peak for $z < 1$ (if $m_P > m_q$) or $z > 1$ (if $m_P < m_q$).

In conclusion, the z distributions we find when using HERWIG seem not to contain a lot of dynamical information, the most important features being those implemented in the cluster-decay routine. If the decay is not smeared out (**CLDIR=0**), we get a structure which is very difficult to reconcile with the idea of fragmentation we have from QCD. After smearing, the distribution still has a $z > 1$ tail which will be extremely difficult to fit with a function vanishing for $z \to 1$. This problem is related to the fact that the mass of the lightest quarks in the MC is 320 MeV, that is much larger than the pion mass. We performed a test by reducing the light quark masses to 20 MeV, and increasing the shower cutoff **VQCUT** in such a way as to maintain the default value of the effective infrared threshold. The double peak structure, as expected, disappeared. It remains to be seen, however, whether such a small value of the quark masses is, more generally, acceptable.

References

[1] G. Marchesini and B. R. Webber, Nucl. Phys. **B310**, 461 (1988). G. Marchesini, B. R. Webber, G. Abbiendi, I. G. Knowles, M. H. Seymour and L. Stanco, HERWIG 5.1, Comput. Phys. Commun. **67**, 465 (1992). G. Corcella *et al.*, HERWIG 6.1, hep-ph/9912396. For this study the public version 5.8 of HERWIG was used. We checked that the results discussed here have not changed with higher versions. Version features and various parameter tunings can be found from: `http://hepwww.rl.ac.uk/theory/seymour/herwig/`.