Facts about Final State Phases

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1. Introduction and motivation
2. Warm-up: final state phases in D decays
3. Final state phases in B decays
    a. perturbative mechanisms
    b. soft phases
4. Conclusions and outlook

Introduction

◇ Why do we care about Final State Interactions?

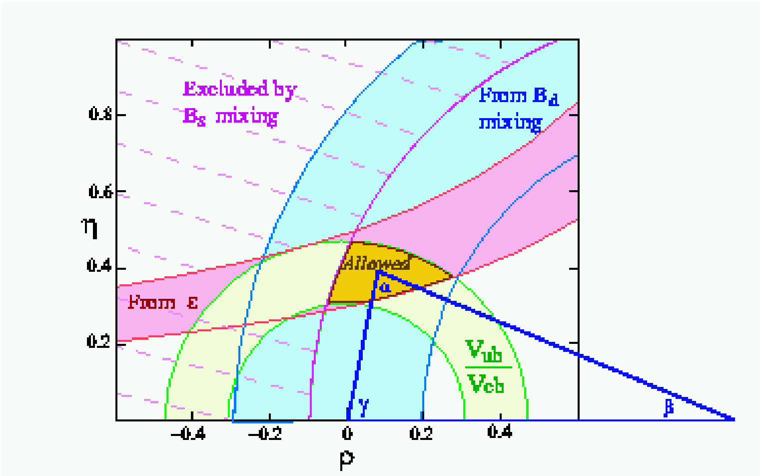
- Branching ratios for rare decays
  Coupled channels

- Extraction of fundamental parameters
  T-violation vs. QED FSI

- Direct CP violation asymmetries
  Explicit dependence on FSI phase

- Studies of CKM unitarity triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



S.Stone, hep-ph/0002025

General features of final state rescatering

◇ Different scale $\Leftrightarrow$ different physics

- K-meson: $m_K \ll 1$ GeV
  only a few channels open
  chiral theory realm

- D-meson: $m_D \sim 1$ GeV
  limited number of channels
  resonance-populated region

- B-meson: $m_B \gg 1$ GeV
  many channels open
  heavy quark limit?

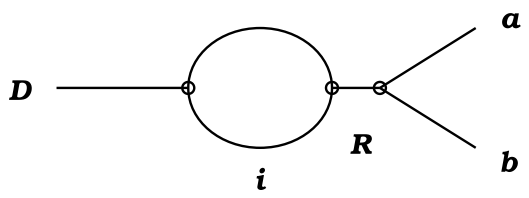
◇ *Different scales suggest different approaches*

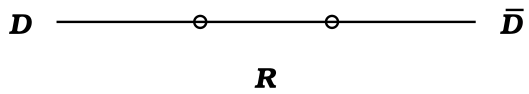
Warm-up: Final State Phases in D decays

$\diamond$ Relevant energy scale $\mu \sim 1$ GeV

$\Rightarrow$ FSI are *large* $\Leftarrow$ nearby resonances $\Rightarrow$ model FSI by assuming resonance dominance

Buccella et. al, Z.Phys.C48, 47 (1990)
El aaoud and Kamal, hep-ph/9910327





- pQCD phases are small, but resonance-generated phases can be large $\Rightarrow$ possibility of large CPV asymmetry?

Buccella et. al, Z.Phys.C48, 47 (1990)
Rosner, PRD60, 114026 (1999)

- Weak phases are tiny $\Rightarrow$ SM $\mathcal{A}_{\rm CP}^{\rm dir} \sim 1$ %...

- Important effects in $D\bar{D}$ mixing

Golowich and A.P., PLB427, 172 (1998)
Falk, Nir, A.P., JHEP 9912, 019 (1999)

Final State Phases in B decays

$\diamond$ Relevant energy scale $\mu \gg 1$ GeV

$\Rightarrow$ FSI are *small* $\Leftarrow$ no resonances nearby...

$\Rightarrow$ FSI are *large* $\Leftarrow$ many resonances contribute...

$$\boxed{\text{Lifeline: } m_b \to \infty \text{ limit}}$$

$\diamond$ Inclusive decays:

$\heartsuit$ Duality $\Leftrightarrow$ perturbative description

CP-violating asymmetries

$\diamond$ Exclusive decays:

$\times$ No duality $\Leftrightarrow$???

branching ratios

CP-violating asymmetries

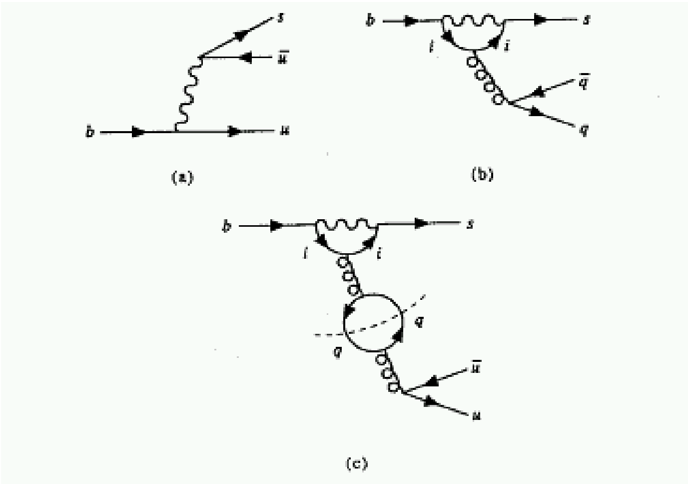
extraction of CKM angle γ

Fleischer and Mannel, PRD57, 2752 (1998)
Neubert and Rosner, PLB441, 403 (1998)

Perturbative QCD: inclusive decays

◇ Example: direct CP violation in $b \to s$

1. "Tree-level" $b \to u\bar{u}s$ diagram
2. "Penguin" $b \to q\bar{q}s$ diagram



Bander, Silverman, Soni, PRL 43, 242 (1979)
Gerard and Hou, PRD43, 2909 (1991)

$$\mathcal{M}_{b\to s} = \xi_u A_u + \xi_c A_c + \xi_t A_t \quad = \quad \xi_u \Delta_{u-t} + \xi_c \Delta_{c-t}$$
$$\xi_u + \xi_c + \xi_t = 0$$

- Direct CP-asymmetry is possible

$$\mathcal{A}^{\rm dir}_{{\rm CP}\, inc} = \frac{\Gamma_{b\to s} - \Gamma_{\bar{b}\to\bar{s}}}{\Gamma_{b\to s} + \Gamma_{\bar{b}\to\bar{s}}} \sim -2Im(\xi_u^* \xi_c)\ Im(\Delta^*_{u-t}\Delta_{u-c})$$

Need two pathways to reach a final state ⇒ FSI phase

Direct CP Violation in Exclusive B decays

- Occurs in both charged and neutral decays

$$
\begin{aligned}
A(B^+ \to \pi^+ K^0) &= A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta^+}, \\
A(B^- \to \pi^- \bar{K}^0) &= A_{cs}^+ - A_{us}^+ e^{-i\gamma} e^{i\delta^+}, \\
A(B^0 \to \pi^- K^+) &= A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta^0}, \\
A(\bar{B}^0 \to \pi^+ K^-) &= A_{cs}^0 - A_{us}^0 e^{-i\gamma} e^{i\delta^0}.
\end{aligned}
$$

$$
\boxed{\mathcal{A}_{\rm CP}^{\rm dir}(B \to f) = \frac{\Gamma_{B\to f} - \Gamma_{\bar{B}\to\bar{f}}}{\Gamma_{B\to f} + \Gamma_{\bar{B}\to\bar{f}}} \sim \sin\gamma \sin\delta}
$$

Need two different pathways to reach a final state

- Non-zero $\mathcal{A}_{\rm CP}^{\rm dir}$ indicates CP violation from CKM
- How to estimate CP conserving phase δ?

Perturbative QCD: exclusive decays

◇ How to calculate ME of 4-fermion operators?

factorization ansatz

$$\langle \pi\pi|O|B\rangle = \langle \pi|j_{1\ \mu}|0\rangle\langle \pi|j_2^{\mu}|B\rangle \ + \ \mathcal{O}(?)$$

Bjorken, SLAC-PUB-5389, Dugan, Grinstein, PLB255, 583 (1991)
Blok, Shifman, NPB389, 534 (1993), Donoghue, A.P. PLB393, 149 (1997)
Buras, Gerard, Ruckl, NPB268, 16 (1986)...

• FSI are perturbative [BSS mechanism] $\Rightarrow \mathcal{O}(\alpha_s(m_b))$

◇ Factorization formula *a'la* pQCD:

1. infrared finitenes
2. scale independence

Beneke, Buchalla, Neubert, Sachrajda PRL 83, 1914 (1999)

$$\langle \pi(p')\pi(q)|O_i|B(p)\rangle = f^{B\to\pi}\int_0^1 dx T_i^I(x)\Phi_\pi(x)$$

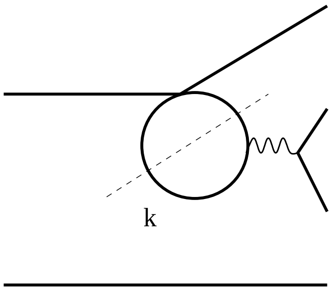
$$+\int_0^1 d\xi dx dy T_i^{II}(\xi,x,y)\Phi_B(\xi)\Phi_\pi(x)\Phi_\pi(y)$$

- Leading amplitude is given by factorization ansatz

  $\Rightarrow$ it is real

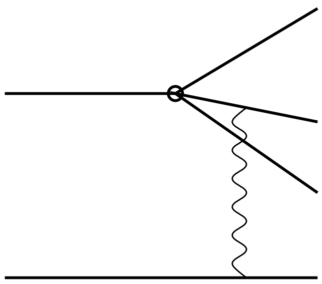
- FSI phases are perturbative, i.e. scale as α_s

$\Rightarrow$ "BSS"-type phases



No ambiguity in k!

$\Rightarrow$ "BBNS"-type phases



- FSI phases are generated by 1-gluon exchange

  $\Rightarrow$ strong phases dissappear as $m_b \to \infty$

Soft phases: are they there?

• Hard FSI – high t, pQCD
• Soft FSI – low t, ???
leading behavior in $m_B \to \infty$ limit?

• Unitarity of $\mathcal{S} = 1 + i\mathcal{T}$ matrix implies

$$\begin{aligned} \mathcal{D}isc\ \mathcal{T}_{B\to f} &\equiv \frac{1}{2i}\left[\langle f|\mathcal{T}|B\rangle - \langle f|\mathcal{T}^\dagger|B\rangle\right] \\ &= \frac{1}{2}\sum_I \langle f|\mathcal{T}^\dagger|I\rangle\langle I|\mathcal{T}|B\rangle \end{aligned}$$

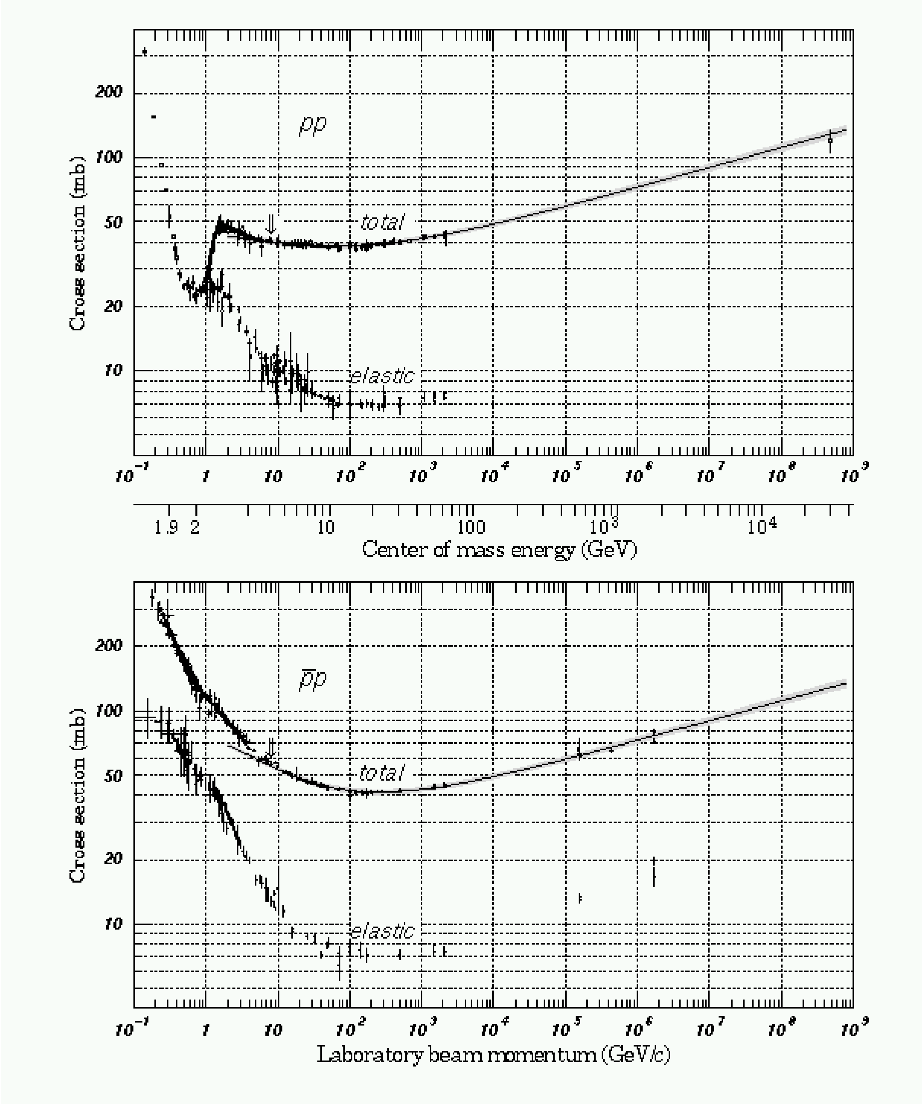
◇ Concentrate on elastic channel (for now)

$\Rightarrow$ relate it to total cross-section

• Optical theorem

$$\mathcal{I}m\ \mathcal{M}_{f\to f}(s,\ t=0) \sim s\sigma_{f\to \mathrm{all}}$$

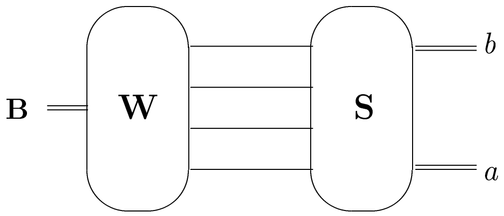
The asymptotic total cross sections are known experimentally to *rise* slowly with energy



RPP, EPJ C3 (1998) 1

$$\sigma(s) = X \left(\frac{s}{s_0}\right)^{0.08} + Y \left(\frac{s}{s_0}\right)^{-0.56}$$

$s_0 = \mathcal{O}(1)$ GeV - *typical hadronic scale*

$Disc\ M_{B\to ab} =$ 

- This implies

$$i\mathcal{I}m\ \mathcal{M}_{f\to f}(s,t) \simeq i\beta_0 \left(\frac{s}{s_0}\right)^{1.08} e^{bt}$$

- $\mathcal{I}m\ \mathcal{M}_{f\to f}$ contribution to discontinuity

$$\begin{aligned}
\mathcal{D}isc\ \mathcal{M}_{B\to f} &= \frac{1}{2}\int d\Phi\ (2\pi)^4\delta^{(4)}(p_B - p_1 - p_2) \\
&\times \mathcal{M}_{B\to f}\left[\,(-i)\ \mathcal{I}m\ \mathcal{M}_{f\to f}(s,t)\,\right] \\
= &- \frac{i\beta_0}{32\pi}\int dx e^{-\frac{bs}{2}(1-x)}\left(\frac{s}{s_0}\right)^{1.08}\mathcal{M}_{B\to f} \\
&= -\frac{1}{16\pi}\frac{i\beta_0}{s_0 b}\left(\frac{m_B^2}{s_0}\right)^{0.08}\mathcal{M}_{B\to f}
\end{aligned}$$

Elastic rescattering *does not* disappear at large m_B!

- Use Regge phenomenology (still elastic channel)

$$\mathcal{M}_{f\to f} = \xi\beta(t)\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{i\pi\alpha(t)/2}, \quad \alpha(t) = \alpha_0 + \alpha' t$$

- Pomeron: $\xi = 1, \alpha_0 \simeq 1.08, \alpha' \simeq 0.25\ \mathrm{GeV}^{-2}$

$$\mathcal{D}isc\ \mathcal{M}_{B\to\pi\pi}|_{\mathrm{Pom}} = -i\epsilon\mathcal{M}_{B\to\pi\pi}, \ \epsilon \sim \left(\frac{m_B^2}{s_0}\right)^0 \simeq 0.21$$

- Elastic channel, $\mathcal{T}$-matrix element
  $\mathcal{T}_{ab\to ab} = 2i\epsilon \Rightarrow \mathcal{S}_{ab\to ab} = 1 - 2\epsilon$
- $\mathcal{S}$-matrix is unitary $\Rightarrow$
  off-diagonal elements are $\mathcal{O}(\sqrt{\epsilon})$
- Since ϵ is $\mathcal{O}(m_B^0)$ and $\epsilon < 1 \Rightarrow$
  inelastic amplitude is $\mathcal{O}(m_B^0)$ and $\sqrt{\epsilon} > \epsilon$!

Donoghue, Golowich, A.P., Soares, PRL 77, 2178 (1996)

- Subleading trajectories are suppressed by $1/m_B$

Donoghue, Golowich, A.P., PRD55, 2657 (1997)
Falk, Kagan, Nir, A.P., PRD57, 4290 (1998)

Example: two-channel model

- Two-channel model with $B \to f_1$ and $B \to f_2$. The S-matrix

$$S = \begin{pmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- We identify $\sin^2\theta \equiv \epsilon$. The unitarity relations

$$\begin{aligned} \mathcal{D}isc\ \mathcal{M}_{B\to f_1} &= -i \sin^2\theta \mathcal{M}_{B\to f_1} + \frac{1}{2} \sin 2\theta \mathcal{M}_{B\to f_2} \\ \mathcal{D}isc\ \mathcal{M}_{B\to f_2} &= \frac{1}{2} \sin 2\theta \mathcal{M}_{B\to f_1} - i \sin^2\theta \mathcal{M}_{B\to f_2} \end{aligned}$$

- These equations are solved by

$$\begin{aligned} \mathcal{M}_{B\to f_1} &= \cos\theta \mathcal{M}_1^0 + i \sin\theta \mathcal{M}_2^0 \\ \mathcal{M}_{B\to f_2} &= \cos\theta \mathcal{M}_2^0 + i \sin\theta \mathcal{M}_1^0 \end{aligned}$$

- Check: insert these solutions back

$$\begin{aligned} \mathcal{D}isc\ \mathcal{M}_{B\to f_1} &= \frac{1}{2}\Big[-2i\epsilon \mathcal{M}_{B\to f_1}^0 + \mathcal{O}(\epsilon^{3/2}) \\ &+ 2\sqrt{\epsilon} \mathcal{M}_{B\to f_2}^0 + 2i\epsilon \mathcal{M}_{B\to f_1}^0 \Big] \end{aligned}$$

- The phase is given by the inelastic scattering

$$\frac{\mathcal{I}m\ \mathcal{M}_{B\to f}}{\mathcal{R}e\ \mathcal{M}_{B\to f}} \sim \sqrt{\epsilon}\ \frac{\mathcal{M}_2^0}{\mathcal{M}_1^0}$$

Direct CP-violating asymmetries

- Inelastic FSI connect *$B \to$ multibody* and *$B \to 2$ body* amplitudes $\Rightarrow$ novel direct CPV effects.

$$\begin{aligned} \mathcal{M}(B^- \to K^- \pi) &= |A_1| e^{i\phi_1} = A_1^w e^{i\phi_w} + A_1^p e^{i\phi_p} \\ \mathcal{M}(B^- \to K^- n\pi) &= |A_n| e^{i\phi_n} = A_n^w e^{i\phi_w} + A_n^p e^{i\phi_p} \end{aligned}$$

with $\phi_1 \neq \phi_n$. In the two channel model

$$\begin{aligned} \mathcal{M}(B^- \to K^- \pi) &= |A_1| e^{i\phi_1} + i\sqrt{\epsilon}\, |A_n| e^{i\phi_n} \\ \mathcal{M}(B^+ \to K^+ \pi) &= |A_1| e^{-i\phi_1} + i\sqrt{\epsilon}\, |A_n| e^{-i\phi_n} \end{aligned}$$

- This leads to a CP-violating decay rate asymmetry

$$\Gamma(B^- \to K^- \pi^0) - \Gamma(B^+ \to K^+ \pi^0) \sim \sqrt{\epsilon} |A_1||A_n| \sin(\phi_n - \phi_1)$$

There is no reliable estimate of this asymmetry

- There is a contribution of inelastic FSI to CP-violating asymmetries at leading order in m_B!!!
- Inelastic FSI modify amplitudes as well.

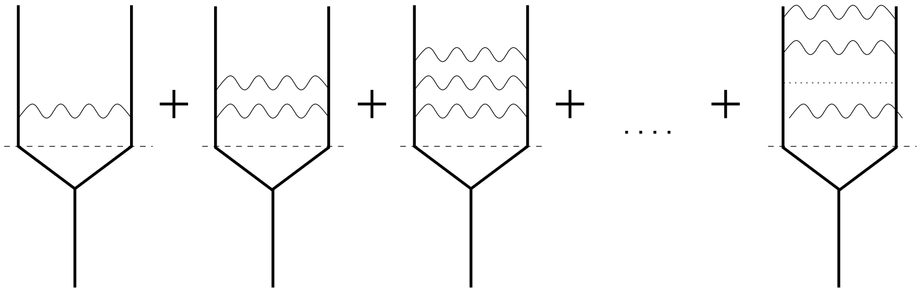
Conflict resolution?

◇ Cancellations among many intermediate hadronic states

Two-channel model is a special case
$\Rightarrow$ Requires conspiracy of **hadronic** states

◇ Different physics cases

One-gluon factorization is a special case
$\Rightarrow$ Pomeron exchange involves 2 and more gluons



◇ $n - th$ order ladder: A_n, interested in $t \to 0$ part

$$\begin{aligned} Reggeon \quad &= \quad \sum_n A_n(s, t \to 0) \sim \sum_n \frac{g^2}{s} \frac{(\log sK(t))^{n-1}}{(n-1)!} \\ &\sim \quad g^2 s^{\alpha(t)}, \quad \alpha(t) = -1 + K(t) \end{aligned}$$

Subleading soft effects

$\diamond$ Do they affect phenomenology?

- calculation of "penguin pollution"
- calculation of direct CPV asymmetries
- FSI "pollution" of Fleischer-Mannel bound
- ...

QCD factorization formalism does not apply beyond leading order in $1/m_b$

- $B^\pm \to \pi^\pm K$ amplitude has no "tree" component...
- ... but it can be induced by FSR from e.g. $B^\pm \to \pi^0 K^\pm$

$$\begin{aligned} A(B^+ \to \pi^+ K^0) &= A'_P - \epsilon_2 A^{n\prime}_T e^{i\gamma} e^{i\delta'_n}, \\ A(B^0 \to \pi^- K^+) &= A'_P - A'_T e^{i\gamma} e^{i\delta'}, \end{aligned}$$

- Regge model for FSI gives $\epsilon_R \simeq 0.06 - 0.09$

$$\epsilon_R = \bar{\gamma} \frac{\sqrt{s_0}}{m_B} \frac{1}{16\pi s_0 \alpha'} \frac{1}{\cos(\pi\alpha_0/2)} \frac{e^{-i\pi\alpha_0/2}}{(\log(s/s_0) - i\pi/2)}$$

This contribution is suppressed by $1/m_B$

Flavor off-diagonal FSI

Example: Fleischer-Mannel ratio

- Construct the ratio R (with $r^0 \equiv A^0_{us}/A^0_{cs}$)

$$R = \frac{BR(B^0 \to \pi^- K^+) + BR(\overline{B^0} \to \pi^+ K^-)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)}$$

$$= \left(\frac{A^0_{cs}}{A^+_{cs}}\right)^2 \frac{1 - 2r^0 \cos\gamma \cos\delta^0 + r^{02}}{1 - 2r^+ \cos\gamma \cos\delta^+ + r^{+2}}.$$

- Use

$$A^+_{us} = 0 \Rightarrow \left\{ \begin{array}{c} A^0_{us} = A_T \\ A^0_{cs} = A^+_{cs} = A_P \end{array} \right\} \Rightarrow r \equiv r_0 = A_T/A_P$$

$$R = 1 - 2r\cos\gamma\cos\delta^0 + r^2$$

$\diamond$ Get a bound, $R_{exp} = 1.00 \pm 0.40$ $\diamond$

$$\boxed{\sin^2\gamma \leq R_{exp}}$$

- EWP: $R \ \Rightarrow \ R(A^+_{cs}/A^0_{cs})^2 \ \Rightarrow \ \pm 10\%$
- FSR might be sizable (10%?) $\Rightarrow$ tree-level parts of the coupled channel amplitudes bring CPV weak phases!

- Decay amplitudes are redefined

$$\boxed{A_i(B \to f) = A_i^{dir}(B \to f) + \epsilon_{I \to f} A_i^{res}(B \to I)}$$

- New expressions for CPV asymmetry and R

$$\mathcal{A}_{\rm CP}^{\rm dir}(B^+ \to \pi^+ K^0) = \frac{2\epsilon \sin\gamma \sin\delta_n}{1 - 2\epsilon \cos\gamma \cos\delta_n},$$

$$R = \frac{1 + r^2 - 2r\cos\gamma\cos\delta}{1 - 2\epsilon\cos\gamma\cos\delta_n}$$

- Now $\epsilon \simeq 0.1 \Longrightarrow \mathcal{A}_{\rm CP}^{\rm dir}(B^+ \to \pi^+ K^0) \simeq 20\%$ is sizable !!!
- New bound on CKM angle γ

$$\boxed{\left\{ \begin{array}{l} \cos\gamma \leq \epsilon R_{exp} - \sqrt{1 - R_{exp}} \\ \\ \cos\gamma \geq -\epsilon R_{exp} + \sqrt{1 - R_{exp}} \end{array} \right.}$$

- Induced error $\Delta = \epsilon R_{exp}/\sqrt{1 - R_{exp}}$

$$\begin{array}{lll} \Delta \simeq 10\% & \text{for} & R_{exp} = 0.65 \\ \Delta \simeq 20 - 25\% & \text{for} & R_{exp} = 0.8 \end{array}$$

- Bound seems to deteriorate as $R_{exp} \to 1$!

What I did last night:

Strong phase in $B_s \to D_s K$

$\diamond$ Extraction of γ via time-dependent studies of $B_s \to D_s K$: enough information to get both γ and δ

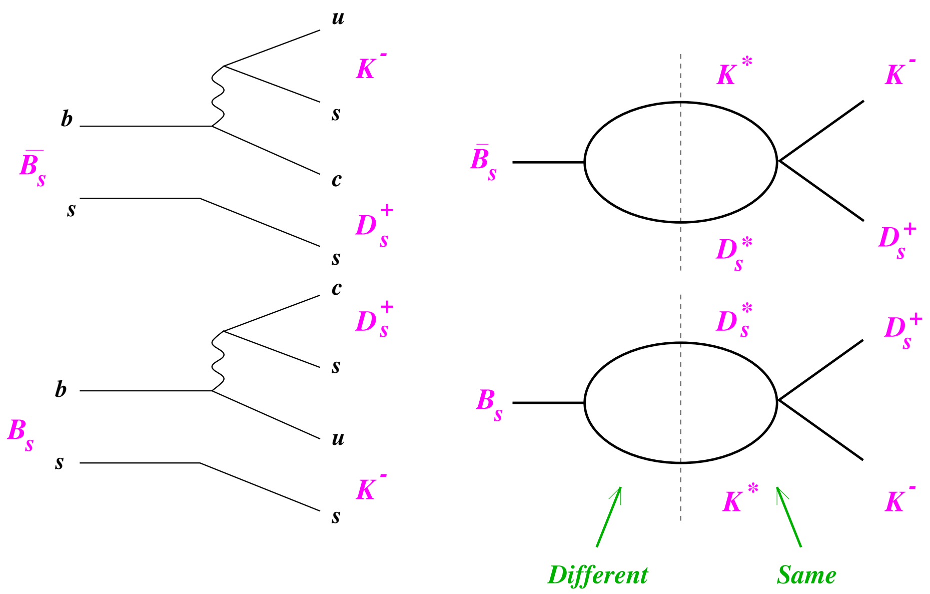
$$\Gamma(B_s^0(t) \to D_s K) = \left|\langle D_s K | B_s^0 \rangle\right|^2 e^{-\Gamma t}\{\cos^2\frac{\Delta m t}{2} + \rho^2 \sin^2\frac{\Delta m t}{2} - \rho \sin(\gamma+\delta)\sin \Delta m t\}$$

and similar for other B_s decays. Strong phase

$$re^{i\delta} = \frac{A(\overline{B_s} \to D_s^+ K^-)}{A(B_s \to D_s^+ K^-)}$$

What is δ?

- Interactions in the final state are the same



Initial states are different (formfactors, etc.)

$\Rightarrow$ do a simple estimate of δ from $B_s \to D_s^* K^* \to D_s K$

D-decays: Falk, Nir, A.P., JHEP 9912, 019 (1999)

- this is just one possible contribution

$$\frac{A(\overline{B_s} \to D_s^+ K^-)}{A(B_s \to D_s^+ K^-)} = \frac{\overline{A}_{direct} + \epsilon_2 \overline{A}_2 + ...}{A_{direct} + \epsilon_2 A_2 + ...}$$

$$\simeq \frac{\overline{A}_{direct}}{A_{direct}} \frac{1 + \epsilon_2 \frac{\overline{A}_2}{\overline{A}_{direct}}}{1 + \epsilon_2 \frac{A_2}{A_{direct}}}$$

- Quick estimate: take $\epsilon_2 = \epsilon_R$

1. $\frac{\overline{A}_2}{\overline{A}_{direct}} \neq \frac{A_2}{A_{direct}}$
2. varying formfactors, masses, etc. get

$$\boxed{\delta < 5^o}$$

- multiparticle states might not be important
- hard to estimate in pQCD factorization formalism
- more robust estimate is needed

Conclusions and outlook

◇ *Perturbative FSI phases are calculable in*
$m_B \to \infty$ limit (inclusive case)

◇ *Soft FSI phases can explain*
large values of CP-violating asymmetries

◇ $1/m_B$ *effects might be important for*
weak decay phenomenology

◇ *Multiparticle FSI are (still) compatible with the*
color transparency argument

◇ *Rare/radiative/OZI-suppressed decays can be used*
to study multiparticle FSI

- *Two body rescattering contribution is*
insufficient for the calculation of the FSI phase