Weak transitions $\Xi_Q \to \Lambda_Q$

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$\Lambda_c \sim cud \qquad \Xi_c^+ \sim cus \qquad \Xi_c^0 \sim cds$

$\Lambda_b \sim bud \qquad \Xi_c^0 \sim bus \qquad \Xi_c^0 \sim bds$

$M(\Xi_c) - M(\Lambda_c) \approx 180\, MeV$

$M(\Xi_b) - M(\Lambda_b) = M(\Xi_c) - M(\Lambda_c) + O(m_c^{-2} - m_b^{-2})$

Decays $\Xi_Q \to \Lambda_Q\, \pi$ are kinematically possible.

Similar to $\Xi \to (\Sigma, \Lambda)\, \pi$ and $(\Sigma, \Lambda) \to (p, n)\, \pi$ for 'light' baryons.

$(\Gamma \sim 10^{10}\, ps^{-1})$

Also possible 'minor' transitions: $\Xi_Q \to \Lambda_Q\, e\, \nu$, $\Xi_Q \to \Lambda_Q\, \gamma$.

Baryons in the heavy quark limit, $m_Q \to \infty$:

- light 'diquark' is $J^P = 0^+$ state,
- spin of Q = spin of hyperon, and decouples in soft processes $q \ll m_Q$,

i.e. does not change in the transitions.

$\Xi_Q \to \Lambda_Q\, X$ decays are $0^+ \to 0^+$ transitions of the light diquark.

Consequences for the 'minor' processes:

- $\Xi_Q \to \Lambda_Q\, e\, \nu$ are of purely vector (Fermi) type vith $g_V = 1$.

$$\Gamma(\Xi_Q \to \Lambda_Q\, e\, \nu) = G_F^2\, \sin^2\theta_c\, (\Delta M)^5/(60\pi^3) \approx 1.0 \times 10^6\, s^{-1}$$

- $\Xi_Q \to \Lambda_Q\, \gamma$ are suppressed by m_Q^{-2} in the rate.

Consequence for $\Xi_Q \to \Lambda_Q\, \pi$: S wave dominant (P wave amplitude suppressed by m_Q^{-1}: suppressed polarization effects.)

$\boxed{\Xi_Q \to \Lambda_Q\, \pi}$

S wave amplitude $A_S(p_\pi) \approx A_S(0) \to$ current algebra (PCAC):

$$\langle \Lambda_Q\, \pi_i(p=0)\, | H_W |\, \Xi_Q \rangle = \frac{\sqrt{2}}{f_\pi} \langle \Lambda_Q\, | \left[Q_i^5,\, H_W \right] |\, \Xi_Q \rangle$$

Q_i^5: axial charges, $f_\pi \approx 130\, MeV$: charged pion decay constant.

Isotopic spin properties:

$H_W = H_{\Delta I=1/2} + H_{\Delta I=3/2}$,
$[Q_i^5,\, H_{\Delta I=1/2}] = \mathcal{O}_{\Delta I=1/2}$, $[Q_i^5,\, H_{\Delta I=3/2}] = \mathcal{O}_{\Delta I=3/2}$.
But $\langle \Lambda_Q\, | \mathcal{O}_{\Delta I=3/2} |\, \Xi_Q \rangle = 0$.

• $\Xi_Q \to \Lambda_Q\, \pi$ decays should obey $\Delta I = 1/2$ rule.
$\Gamma(\Xi_Q \to \Lambda_Q\, \pi^-) = 2\, \Gamma(\Xi_Q \to \Lambda_Q\, \pi^0)$.

$\boxed{\textit{Two mechanisms of } \Xi_Q \to \Lambda_Q\, \pi \textit{ decay}}$

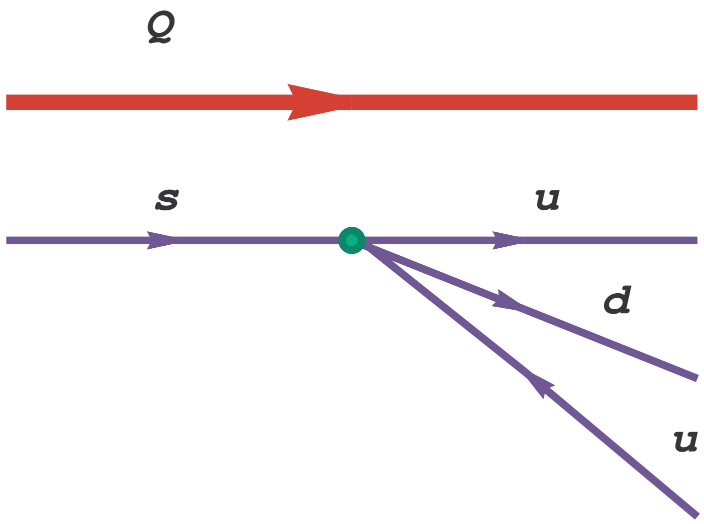

Figure 1: Spectator mechanism for $\Xi_Q \to \Lambda_Q\, \pi$ decays.

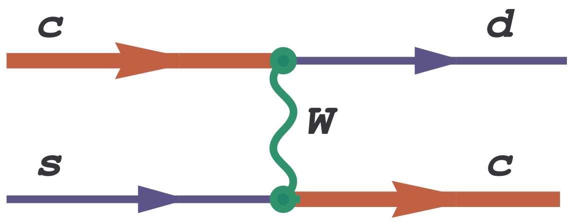

Figure 2: Non-spectator mechanism for $\Xi_c \to \Lambda_c\, \pi$.

Hamiltonian (normalized at $\mu = m_c$):

$$H_W = \sqrt{2}\, G_F\, \cos\theta_c\, \sin\theta_c \times$$
$$\Big\{ (C_+ + C_-) \left[(\overline{u}_L\, \gamma_\mu\, s_L)\, (\overline{d}_L\, \gamma_\mu\, u_L) - (\overline{c}_L\, \gamma_\mu\, s_L)\, (\overline{d}_L\, \gamma_\mu\, c_L) \right] +$$
$$(C_+ - C_-) \left[(\overline{d}_L\, \gamma_\mu\, s_L)\, (\overline{u}_L\, \gamma_\mu\, u_L) - (\overline{d}_L\, \gamma_\mu\, s_L)\, (\overline{c}_L\, \gamma_\mu\, c_L) \right] \Big\} \ .$$

$C_- = C_+^{-2} = (\alpha_s(m_c)/\alpha_s(m_W))^{12/25}$

Spectator contribution:

$$
\begin{aligned}
&\langle \Lambda_b\, \pi^-(p=0)\,|H_W|\,\Xi_b^-\rangle = \frac{\sqrt{2}}{f_\pi} G_F\, \cos\theta_c\, \sin\theta_c \times \\
&\langle \Lambda_b\,|C_- \left[(\overline{u}_L\,\gamma_\mu\, s_L)\,(\overline{d}_L\,\gamma_\mu\, d_L) - (\overline{d}_L\,\gamma_\mu\, s_L)\,(\overline{u}_L\,\gamma_\mu\, d_L) \right] - \\
&\frac{C_+}{3} \left[(\overline{u}_L\,\gamma_\mu\, s_L)\,(\overline{d}_L\,\gamma_\mu\, d_L) + (\overline{d}_L\,\gamma_\mu\, s_L)\,(\overline{u}_L\,\gamma_\mu\, d_L) + \right. \\
&\left. 2\,(\overline{u}_L\,\gamma_\mu\, s_L)\,(\overline{u}_L\,\gamma_\mu\, u_L) \right] |\,\Xi_b^-\rangle
\end{aligned}
$$

Including the non-spectator contribution:

$$
\begin{aligned}
&\langle \Lambda_c\, \pi^-(p=0)\,|H_W|\,\Xi_c^0\rangle = \langle \Lambda_b\, \pi^-(p=0)\,|H_W|\,\Xi_b^-\rangle + \\
&\frac{\sqrt{2}}{f_\pi} G_F\, \cos\theta_c\, \sin\theta_c\, \langle \Lambda_c\,|\,(C_+ + C_-)\,\,(\overline{c}_L\,\gamma_\mu\, s_L)\,(\overline{u}_L\,\gamma_\mu\, c_L) + \\
&(C_+ - C_-)\,\,(\overline{u}_L\,\gamma_\mu\, s_L)\,(\overline{c}_L\,\gamma_\mu\, c_L)|\,\Xi_c^0\rangle
\end{aligned}
$$

Matrix elements: spectator - unknown (model-dependent), non-spectator - related (by flavor SU(3)) to lifetime differences among Ξ_c^+, Ξ_c^0, Λ_c charmed hyperons.

Define

$\langle \Lambda_c \,|(\overline{c}\, \gamma_\mu \, c)(\overline{u}\, \gamma_\mu \, s)|\, \Xi_c^0 \rangle = -x \,, \qquad \langle \Lambda_c \,|(\overline{c}_i\, \gamma_\mu \, c_k)(\overline{u}_k\, \gamma_\mu \, s_i)|\, \Xi_c^0 \rangle = -y$

Then

$$\Delta A_S \equiv \langle \Lambda_c \, \pi^- (p=0)\,|H_W|\, \Xi_c^0 \rangle - \langle \Lambda_b \, \pi^- (p=0)\,|H_W|\, \Xi_b^- \rangle = \frac{G_F \cos\theta_c \, \sin\theta_c}{2\,\sqrt{2}\, f_\pi} \left[(C_- - C_+)\; x - (C_+ + C_-)\; y \right]$$

By the flavor SU(3) symmetry x and y are the same as describing differences of lifetimes within the Ξ_c^+, Ξ_c^0, Λ_c triplet of charmed baryons:

$x = \left\langle \frac{1}{2}\, (\overline{c}\, \gamma_\mu \, c)\, [(\overline{u}\, \gamma_\mu u) - (\overline{s}\, \gamma_\mu s)] \right\rangle_{\Xi_c^0 - \Lambda_c} =$
$\left\langle \frac{1}{2}\, (\overline{c}\, \gamma_\mu \, c)\, \left[(\overline{s}\, \gamma_\mu s) - (\overline{d}\, \gamma_\mu d)\right] \right\rangle_{\Lambda_c - \Xi_c^+}$

$y = \left\langle \frac{1}{2}\, (\overline{c}_i\, \gamma_\mu \, c_k)\, [(\overline{u}_k\, \gamma_\mu u_i) - (\overline{s}_k\, \gamma_\mu s_i)] \right\rangle_{\Xi_c^0 - \Lambda_c} =$
$\left\langle \frac{1}{2}\, (\overline{c}_i\, \gamma_\mu \, c_k)\, \left[(\overline{s}_k\, \gamma_\mu s_i) - (\overline{d}_k\, \gamma_\mu d_i)\right] \right\rangle_{\Lambda_c - \Xi_c^+}$

OPE description of inclusive decay rates of heavy hadrons :
$\Gamma(\Xi_c^0) - \Gamma(\Lambda_c) \approx \frac{G_F^2\, m_c^2}{4\pi}\, (-1.39\, x + 5.56\, y)$
$\Gamma(\Lambda_c) - \Gamma(\Xi_c^+) \approx -\frac{G_F^2\, m_c^2}{4\pi}\, (2.88\, x + 3.16\, y)$
From the data ($\Gamma(\Lambda_c) = 4.85 \pm 0.28\, ps^{-1}$, $\Gamma(\Xi_c^0) = 10.2 \pm 2\, ps^{-1}$, $\Gamma(\Xi_c^+) = 3.0 \pm 0.45\, ps^{-1}$) one extracts $x = -(0.04 \pm 0.01)\, GeV^3$, $y = 0.019 \pm 0.009\, GeV^3$ (at $\mu = m_c$, and $m_c = 1.4\, GeV$). Or directly relate ΔA_S to $\Delta\Gamma$'s:

$$\Delta A_S \approx -\frac{\sqrt{2}\,\pi\, \cos\theta_c\, \sin\theta_c}{G_F\, m_c^2\, f_\pi} \left[0.45\; (\Gamma(\Xi_c^0) - \Gamma(\Lambda_c)) + 0.04\, (\Gamma(\Lambda_c) - \Gamma(\Xi_c^+)) \right] = -10^{-7} \left[0.97\, (\Gamma(\Xi_c^0) - \Gamma(\Lambda_c)) + 0.09\, (\Gamma(\Lambda_c) - \Gamma(\Xi_c^+)) \right] \left(\frac{1.4\, GeV}{m_c} \right)^2 ps$$

$$\Delta A_S = -(5.4 \pm 2) \times 10^{-7}$$

Corresponding rate $\Gamma = |\Delta A_S|^2 \, p_\pi/(2\pi) \approx 0.9 \times 10^{10} \, s^{-1}$

Triangle inequality :

$$\sqrt{\Gamma(\Xi_b^- \to \Lambda_b \, \pi^-)} + \sqrt{\Gamma(\Xi_c^0 \to \Lambda_c \, \pi^-)} \geq \sqrt{0.9 \times 10^{10} \, s^{-1}}$$

We thought from analysis of lifetime differences that the matrix elements x and y are greatly enhanced with respect to a simplistic estimate $y = -x = f_D^2 \, m_D/12 \approx 0.006 \, GeV^3$. From the standpoint of the $\Xi_Q \to \Lambda_Q \, \pi$ decays they look to be of a 'normal' magnitude.

$\boxed{\text{Conclusions}}$

- Decays $\Xi_Q \to \Lambda_Q \, \pi$ are strongly dominated by S wave with $\Delta I = 1/2$.
- The rates should be at the level of 0.01 ps^{-1}, corresponding to branching fractions of about 1% for Ξ_b and $(few) \times 10^{-3}$ for Ξ_c.
- The rates are related to the matrix elements entering the problem of lifetime differences.
- The decays $\Xi_Q \to \Lambda_Q \, e \, \nu$ are well understood, but rare $(Br \sim 10^-6)$.