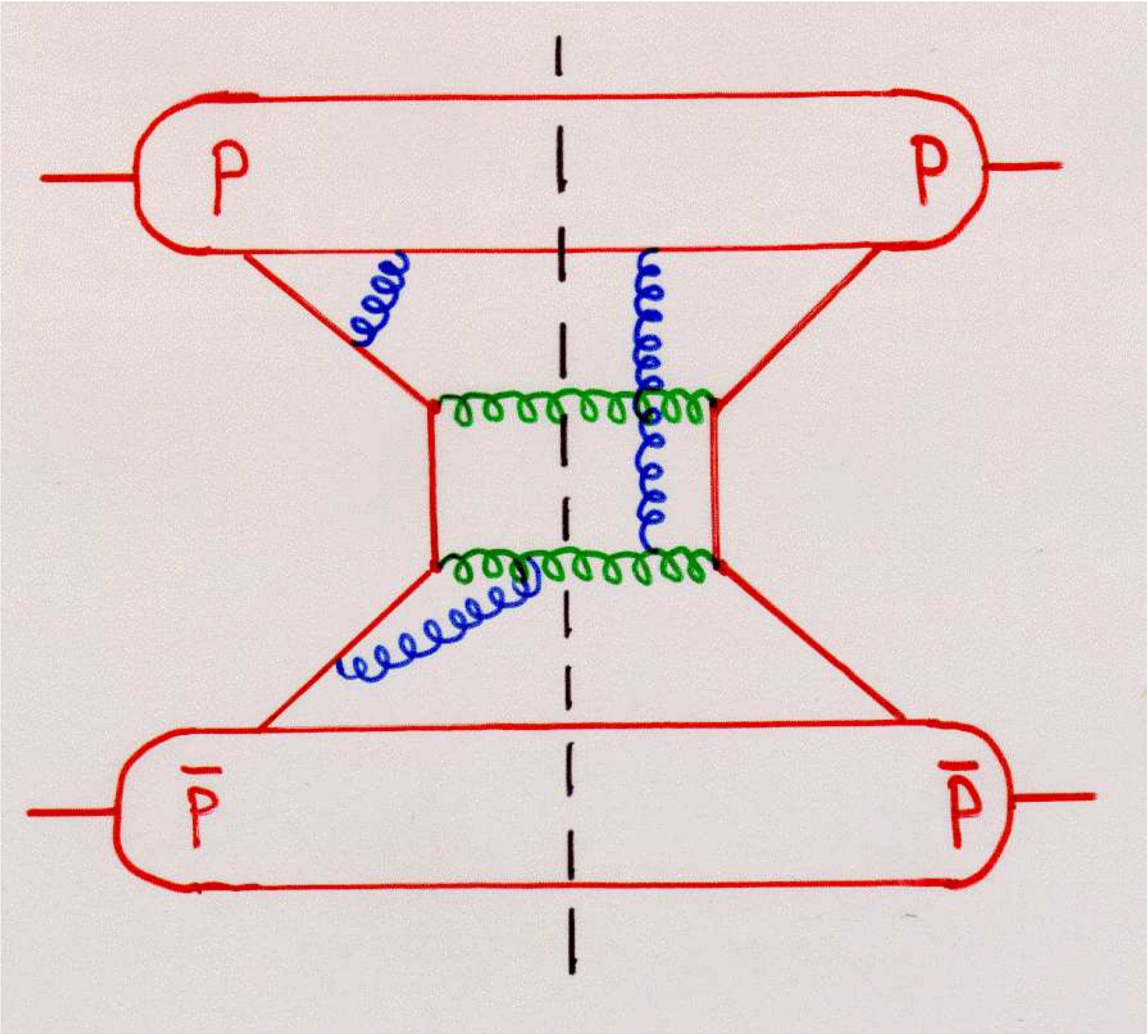
Inclusive Production of Heavy Quarkonium in Hadronic Collisions

(Geoffrey Bodwin, Argonne)

- Factorization of the Inclusive Hadro-Production Cross Section
- Uncertainties in the Theory of Quarkonium Production
- Experimental Tests
- Wish List

Factorization of the Inclusive Hadro-Production Cross Section

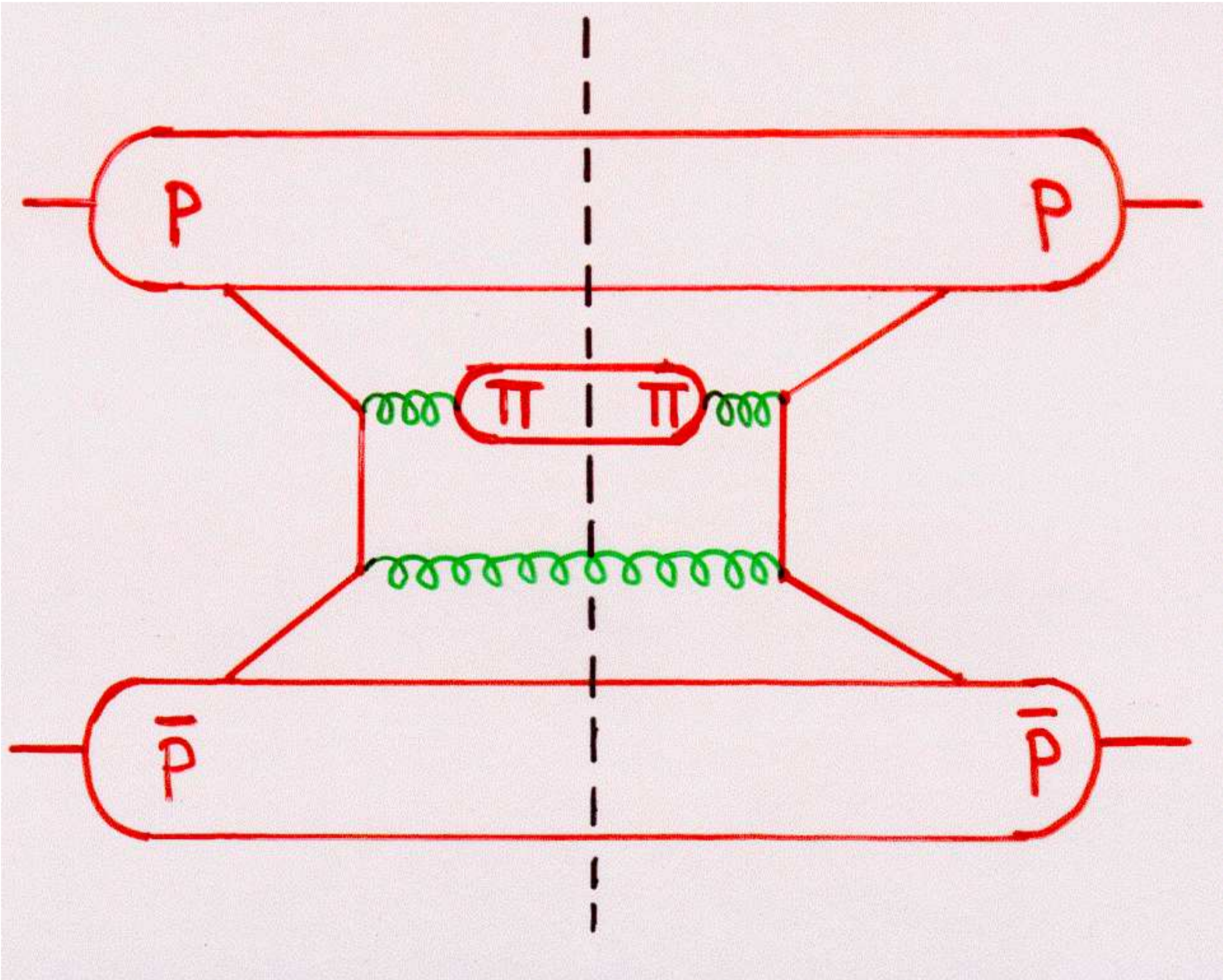
Jet Production at Large p_T



- The simple parton model is complicated in QCD by additional gluon exchanges, possibly involving spectator quarks.
- Many years ago, the technology was developed for dealing with the gluon exchanges to all orders in perturbation theory for sufficiently inclusive processes (Collins, Soper, Sterman; GTB; many others).

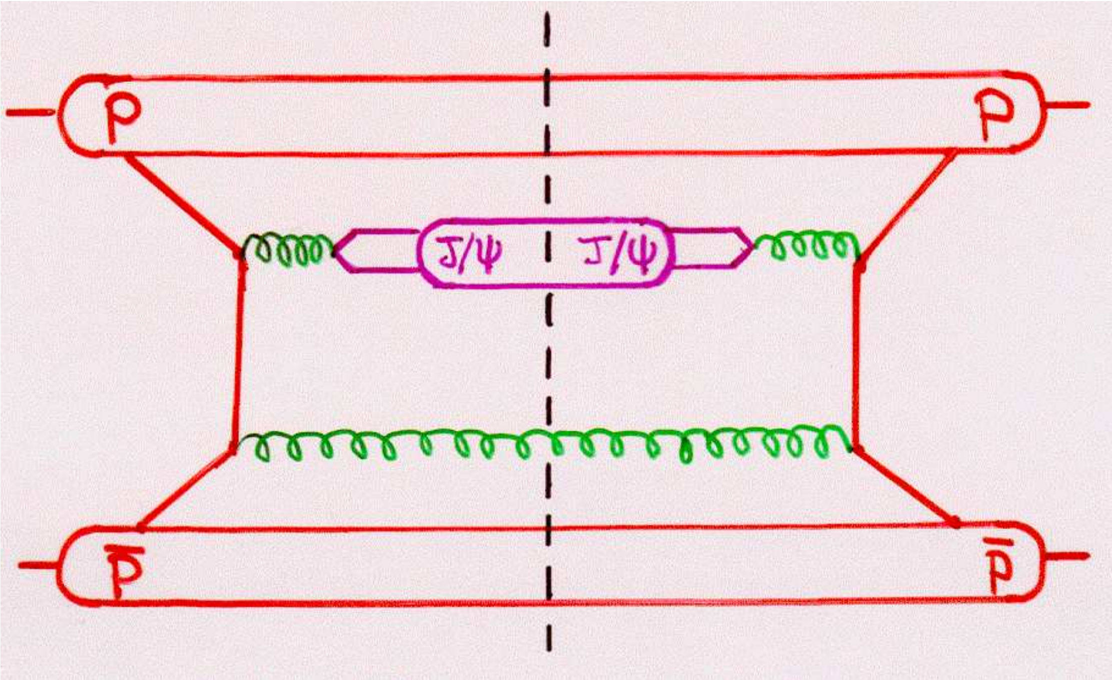
- Result
  - Soft gluons factor completely from the original diagram.
  - Collinear gluons factor into separate contributions for each beam or jet direction.
  - Initial-state collinear contributions factor into PDF's.
  - Factored soft and final-state collinear contributions cancel by unitarity.
  - The remaining hard contributions involve only active partons and can be calculated in QCD perturbation theory.
  - The cross section is a convolution of PDF's with perturbatively calculable partonic cross sections.

Inclusive Single-Particle Production at Large p_T



- Radiated soft gluons and/or light quarks turn the final-state parton into a light hadron.
- The cross section is no longer completely inclusive, so the unitarity cancellation of contributions from gluons collinear to the final hadron fails.
- These collinear contributions are factored into the fragmentation function for the parton to become a hadron.
- The cross section is a convolution of PDF's and the fragmentation function with a perturbatively calculable partonic cross section.

Inclusive Heavy-Quarkonium Production at Large p_T



- The heavy $Q\bar{Q}$ pair turns into a heavy-quarkonium, possibly by radiating gluons and/or light quarks.
- Note that the $Q\bar{Q}$ pair can be produced in a color-singlet state or a color-octet state.
- Again, the cross section is no longer completely inclusive, so the unitarity cancellation of contributions from gluons collinear to the quarkonium fails.
- These collinear contributions are factored into the fragmentation function for the $Q\bar{Q}$ pair to become a quarkonium.
- The cross section is a convolution of PDF's and the fragmentation function with a perturbatively calculable partonic cross section.
- We could stop here, but there is more to be learned from the fact that the quarkonium is a nonrelativistic system. (In the CM frame, the heavy Q velocity $v << 1$.)

The NRQCD Factorization Formalism

(GTB, Braaten, Lepage)

- The dynamics of the quarkonium system is described by an effective field theory, Nonrelativistic QCD (NRQCD).
- Soft gluons ($k << M$) are also accurately described.
  - The gluons collinear to the quarkonium in the lab frame are soft in the CM frame.
- Choose the NRQCD cutoff Λ (NRQCD factorization scale) large enough so that $Mv < \Lambda << M$ but $\alpha_s(\Lambda) << 1$. (Typically take $\Lambda = M$ and hope dynamics is dominated by $k << M$.)
- The fragmentation function for a $Q\bar{Q}$ pair to turn into a quarkonium can be written in terms of NRQCD matrix elements of the $Q\bar{Q}$ operator between the vacuum and the quarkonium state plus anything.

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{M^{d_n-4}} \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle,$$

$$\mathcal{O}_n^H = \chi^\dagger \kappa_n \psi \Big(\sum_X |H+X\rangle\langle H+X| \Big) \psi^\dagger \kappa_n' \chi.$$

- The short-distance coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
  - They have an expansion in powers of α_s.
- The operator matrix elements are universal (process independent).

- NRQCD power-counting rules allow one to organize the sum over operators as an expansion in powers of v.
  - Non-spin-flip gluon emission costs a factor v^2
  - Spin-flip gluon emission costs a factor v^3.
- Through a given order in v only a limited number of operator matrix elements contribute.
- At leading orders in v there are simplifying relations between operator matrix elements.
  - Heavy-quark spin symmetry
  - Vacuum-saturation approximation
- Qiu, Vary, X. Zhang have recently completed a detailed proof of the factorization formula.
  - Their analysis confirms the factorization of the inclusive cross section at large p_T.
  - Their analysis shows that the total inclusive cross section (integrated over p_T) also factorizes, but the operator matrix elements are not universal (process-dependent eikonal-line factors).
  - Work is in progress for the case of production of polarized quarkonium.

Models of Quarkonium Production

- The NRQCD factorization formula is often called the "color-octet model", but it accounts for production of the $Q\bar{Q}$ pair in an octet or a singlet state.
- The NRQCD factorization formula not a model, but a consequence of QCD, at the level of rigor of other hadronic hard-scattering formulas.
- The color-singlet model is incomplete in that it allows the $Q\bar{Q}$ pair to be created only in a color-singlet state.
- In the color-evaporation model, the $Q\bar{Q}$ pair is assumed to evolve into a quarkonium with a probability that is independent of the $Q\bar{Q}$ quantum numbers (Amundson, Éboli, Gregores, Halzen).
  - It ignores the hierarchy of matrix elements in the v expansion.
  - It agrees fairly well with much of the data, but there is no known way to make systematic improvements on this approximation.
  - In contrast with NRQCD factorization,
    * it predicts zero polarization for quarkonium production,
    * it predicts the ratio of χ_{c1} production to χ_{c2} production to be $3/5$ (~ 0.1 in NRQCD).

Uncertainties in the Theory of Quarkonium Production

- Corrections to the factorization of the hard-scattering process
  - In the unpolarized case, errors of order M^2/p_T^2 (Qiu, Vary, Zhang).
    - Getting to high enough p_T to suppress these errors is difficult for bottomonium.
  - Can reduce to order M^4/p_T^4 by including higher-twist partonic processes, but involves unknown higher-twist PDF's.
  - In the case of polarized quarkonium, errors of order M/p_T? (Qiu, Sterman; Qiu, Vary, Zhang).
  - In the total cross section (integrated over p_T), errors of order $\Lambda^2_{\rm QCD}/M^2$, but matrix elements are non-universal.
- Corrections to the NRQCD factorization
  - Typically relative order v^2. ($v^2 \approx .3$ for charmonium; $v^2 \approx .1$ for bottomonium.)
  - For some processes, can be relative order v^1 (spin-flip matrix elements).
  - Systematically improvable to any accuracy, but at the cost of additional matrix elements.
  - In the case of quarkonium decays, some corrections are known to have large coefficients ($\sim 5v^2$).
  - Near the edge of phase space
    - the momentum of accompanying gluons may be important or
    - the difference between $2M$ and the quarkonium mass may be important.
    - Then the corresponding class of corrections must be resummed to all orders in v (Beneke, Rothstein, Wise).

- NRQCD operator matrix elements.
  - Not currently known from first principles, except for the color-singlet matrix elements, which are approximately equal to decay matrix elements.
    * Lattice calculations of NRQCD decay matrix elements exist (GTB, Kim, Sinclair).
    * It is not known how to formulate the calculation of NRQCD production matrix elements on a Euclidean lattice.
  - Must be extracted from the data.
  - Often several matrix elements contribute to a given process.
    * Difficult to disentangle the contributions by using their kinematic dependences.
    * The important linear combinations vary from process to process, making tests of universality difficult.
- Corrections to the perturbative expressions for the short-distance coefficients.
  - Nominally relative order α_s, but coefficients can be large.
    * May need to resum $\log(p_T^2/M^2)$, $\log(z)$, $\log(1-x)$, $\log(x)$.
    * If the p_T spectrum is steeply falling, multiple soft gluon emission can greatly increase the cross section ($\sim$ factor 4 for J/ψ at the Tevatron) (Cano-Coloma, Sanchis-Lozano).
    * May need to resum corrections proportional to β_0 (Beneke, Braun).
    * Some calculated corrections are large ($\sim 5\alpha_s$) for no known reason.

- Uncertainty in M.
  - $\sim 25\%$ for M_c; $\sim 5\%$ for M_b.
  - Can be very significant for charmonium rates that are proportional to a large power of the mass.
- Uncertainties in PDF's
  - Uncertainties in distribution shapes may make it difficult to use kinematic dependences to disentangle contributions from various operator matrix elements.
- Uncertainty in α_s.

Experimental Tests

Quarkonium Production at the Tevatron (Unpolarized)

- Quarkonium production at the Tevatron is more than an order of magnitude larger than the prediction of the color-singlet model.
- Can fit the data for J/ψ, χ_c, ψ', Υ and $\Upsilon(2S)$ production by an appropriate choice of the color-octet NRQCD matrix elements.
- For J/ψ, the relevant linear combinations are $\langle \mathcal{O}_8(^3S_1) \rangle$ and $\Delta_8(3) = (3/M^2)\langle \mathcal{O}_8(^3P_0) \rangle + \langle \mathcal{O}_8(^1S_0) \rangle$.
- For $p_T \gtrsim 3M_c$, J/ψ production is dominated by gluon fragmentation into quarkonium through $\mathcal{O}_8(^3S_1)$.
  - Goes as $1/p_T^4$, while other contributions go as $1/p_T^6$.
- The extracted color-octet matrix elements are compatible with estimates from the NRQCD v-scaling rules.
- However, a much more stringent test of the theory is to check the universality of the extracted matrix elements in another process.

Cho & Leibovich

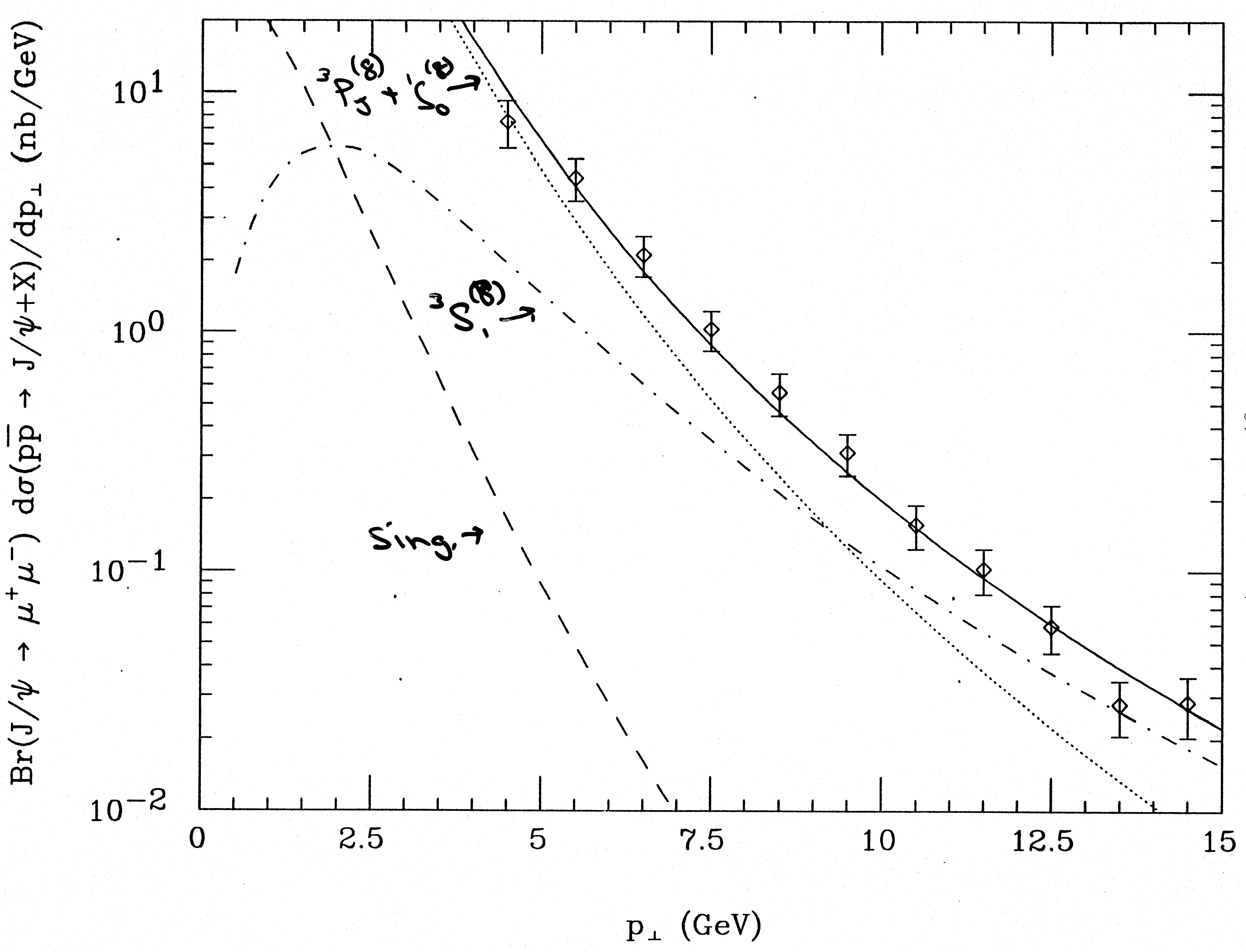


Figure 6

Caveats

- The extracted values of the matrix elements are probably overestimates because multiple soft-gluon emission hasn't been taken into account.
- The relative-order α_s corrections are not complete.
  - Complete for χ_0 and χ_2 (Petrelli, Cacciari, Greco, Maltoni, Mangano).
  - Calculation of real-gluon corrections to color-singlet $^3S_1$ production (Petrelli, Maltoni).
  - Resummation of $\log(p_T^2/M^2)$ for the fragmentation process (Braaten, Doncheski, Fleming, Mangano; Beneke, Rothstein).
  - Order α_s for fragmentation process (Beneke, Rothstein).

πN and pN Fixed-Target Charmonium Total Cross Sections

(Beneke, Rothstein)

- The relevant linear combinations are now $\langle \mathcal{O}_8(^3S_1) \rangle$ and $\Delta_8(7) = (7/M^2)\langle \mathcal{O}_8(^3P_0) \rangle + \langle \mathcal{O}_8(^1S_0) \rangle$.
- Comparison involves total cross sections, so can fit only $\Delta_8(7)$ for J/ψ, ψ'.
  - Use Tevatron octet matrix elements in other cases.
- $\Delta_8(7)$ seems to be in disagreement with with Tevatron results (4–7 times too small).
  - The Tevatron is sensitive to $\Delta_8(3)$, so comparisons are somewhat uncertain.
    - $\overline{\mathrm{MS}}$ parton distributions need not be positive in some factorization schemes.
    - But the factorization-scheme dependence isn't resolved until relative-order α_s.
  - Kinematic corrections from the difference between $2M$ and quarkonium mass may be large.
  - Tevatron values may be an overestimate (soft gluons).
  - Relative-order α_s corrections give a large k-factor (Petrelli, Cacciari, Greco, Maltoni, Mangano).
    - The NLO $\Delta_8(6.4)$ is a factor 3 smaller than the LO $\Delta_8(7)$.
    - NLO uses CTEQ4M PDF's; LO uses CTEQ3L PDF's.

- Discrepancies in polarized production results.
  - $d\sigma/d(\cos\theta) \propto 1 + \alpha \cos^2\theta$.
    - $\alpha = 1$ is completely transverse;
    - $\alpha = -1$ is completely longitudinal.
  - J/ψ theory: $0.31 < \alpha < 0.63$; J/ψ expt.: $0.024 < \alpha < 0.032$.
  - ψ' theory: $0.15 < \alpha < 0.44$; ψ' expt.: $-0.12 < \alpha < 0.16$.
  - J/ψ theory and data include feeddown from χ_c states.
- χ_{c1}/χ_{c2} ratio in πN production. Theory: 0.13. Expt.: 1.4 ± 0.4.
- Beneke and Rothstein suggest that higher-twist (M^2/p_T^2) corrections may be large.
- Evidence for non-universality of matrix elements for total cross section?

Inelastic Quarkonium Photoproduction at HERA

(Cacciari, Krämer; Amundson, Fleming, Maksymyk; Ko, Lee, Song; Kniehl, Krämer)

- Data seem to be incompatible with color-octet matrix elements extracted at the Tevatron.
- But...
  - Large k-factors?
    - Color-octet contribution is calculated only at leading order in α_s for photoproduction.
    - Relative-order-α_s phenomenology has not been completed for Tevatron data.
  - The v expansion breaks down near $z = 1$. Resummation is needed to get the shape.
  - Uncertainties in M_c and α_s result in uncertainties in color-singlet photoproduction, leaving room for color-octet photoproduction.
  - Different linear combinations of color-octet matrix elements appear in Tevatron hadroproduction ($\Delta_8(3)$) and HERA photoproduction ($\Delta_8(7)$).
  - $p_T > 1$GeV cut. Non-universality of matrix elements for small p_T?
  - Color-octet matrix elements were overestimated by $\sim$ factor 4 in the analysis of Tevatron data because soft-gluon effects were omitted (Kniehl, Krämer).

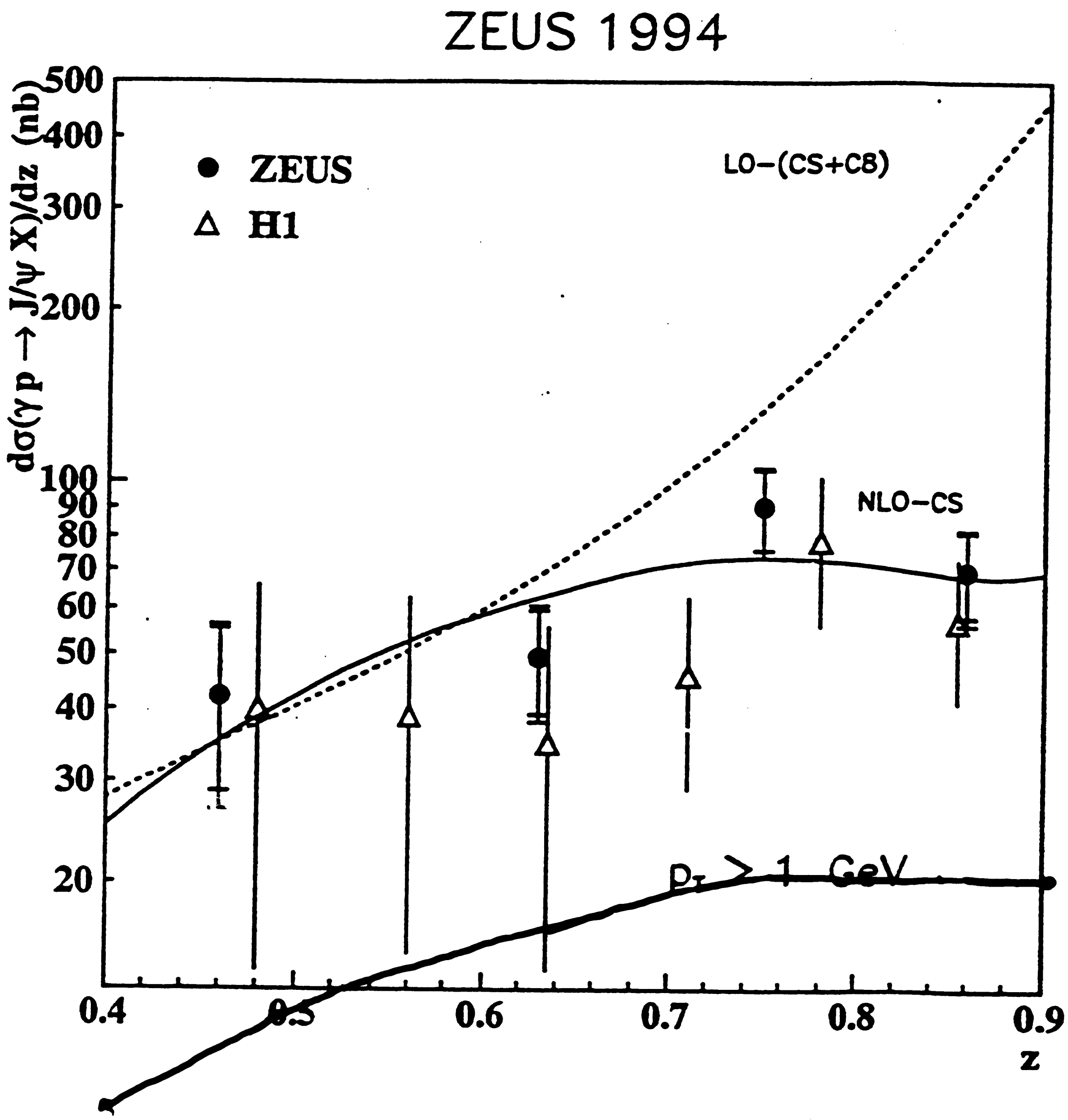


Figure 7: Differential cross section $d\sigma/dz$ for the inelastic J/ψ sample ($J/\psi \to \mu^+\mu^-$), for $p_T > 1$ GeV and $50 \leq W \leq 180$ GeV. Data from ZEUS and H1 [14] are shown. The inner error bars indicate the statistical uncertainties, the outer error bars the quadratic sum of statistical and systematic uncertainties. The prediction of the NLO pQCD computation [1] with the GRV[29] structure function is shown as a solid line. The prediction from the colour-octet calculation [7] is shown as a dashed line. In the theoretical curves the correction for the ψ' is not included.

J/ψ **Production at LEP**

(Boyd, Leibovich, Rothstein)

- Fragmentation of Z into J/ψ.
- Resummed $\log(M_Z^2/M_\psi^2)$, $\log(z^2)$.
- $\langle \mathcal{O}_8(^3S_1)\rangle$ contributions dominate.
- The extracted $\langle \mathcal{O}_8(^3S_1)\rangle$ value is a factor of 2 larger than the Tevatron value and has smaller theory errors, but includes feeddown from χ_c states.
- A bump at small z is a signature for color-octet production, but errors in the data are too large to confirm this feature.

Boyd, Leibovich, Rothstein

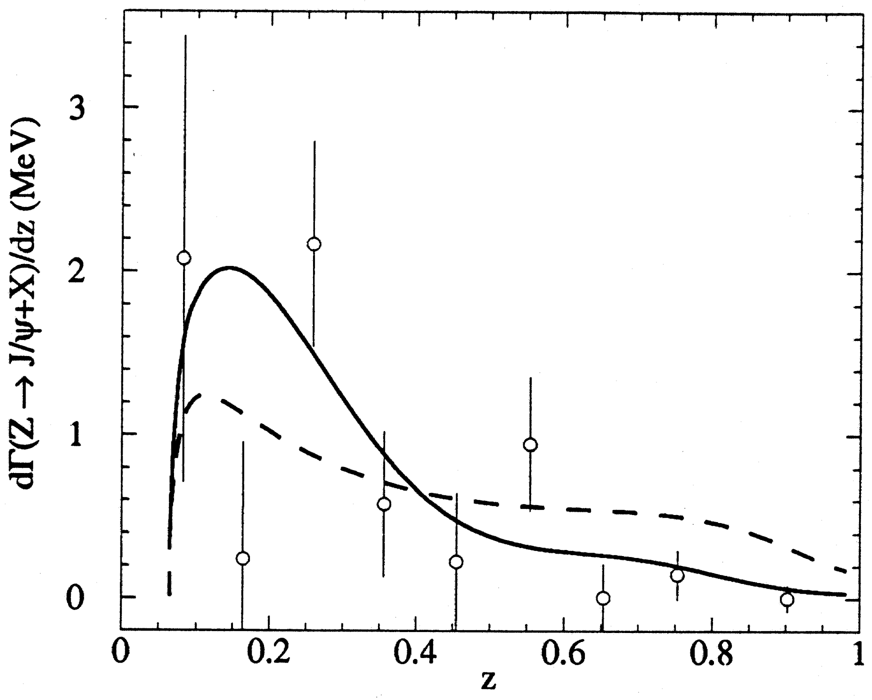


FIG. 3. *Differential rate* $\frac{d\Gamma}{dz}$ *as a function of* $z = 2E_\psi/M_Z$ *vs data. The dashed line is the sum of the tree-level octet and singlet results and the solid line is the interpolation between the large and small z region octet resummation plus the singlet resummation.*

Polarized Quarkonium Production at the Tevatron

- Potentially a "smoking gun" for the Color-Octet Mechanism.
- For large-p_T quarkonium production ($p_T \gtrsim 3M_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle \mathcal{O}_8(^3S_1) \rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, 100% of the gluon's polarization is transferred to the J/ψ (Cho, Wise).
- Radiative corrections, color-singlet production dilute this (Beneke, Rothstein; Beneke, Krämer).
- Reminder: $d\sigma/d(\cos\theta) \propto 1 + \alpha \cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- The observed J/ψ polarization is much smaller than the prediction and seems to decrease with p_T.
- Feeddown from χ_c states is about 30% of the J/ψ sample.
  - Preliminary result $\alpha \approx 0.2$ (Braaten, J. Lee).
  - Reduces the polarization by about a factor 0.7.
- Feeddown from ψ' is about 10% of the J/ψ sample and is expected to be largely transversely polarized.
- Polarization depends on a ratio of matrix elements.
  - It probably is not strongly affected by multiple-soft-gluon emission or k-factors.
- ψ' data has no feeddown. Should be a cleaner test.

Beneke + Krämer

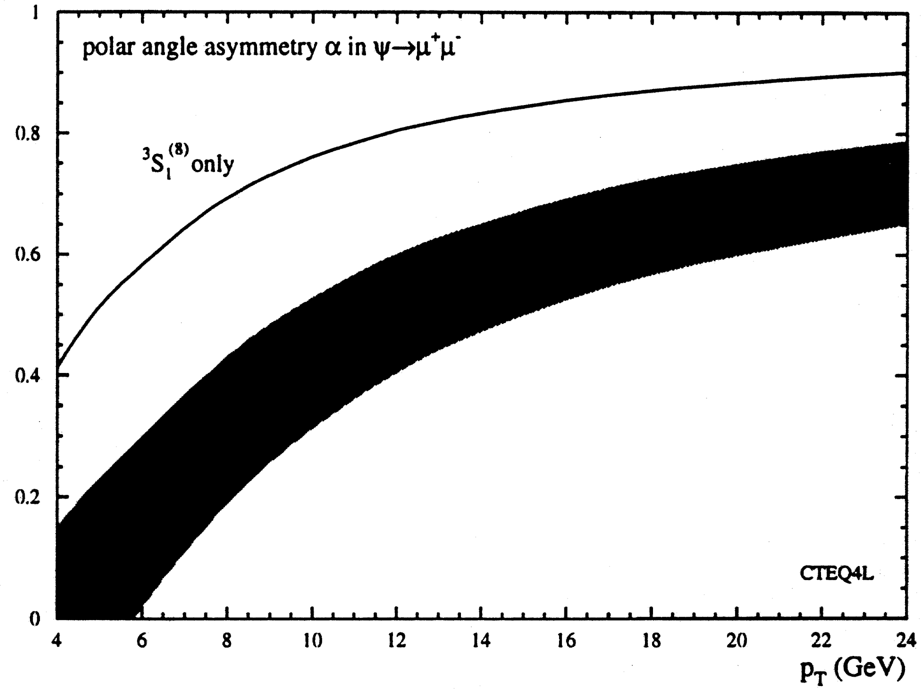
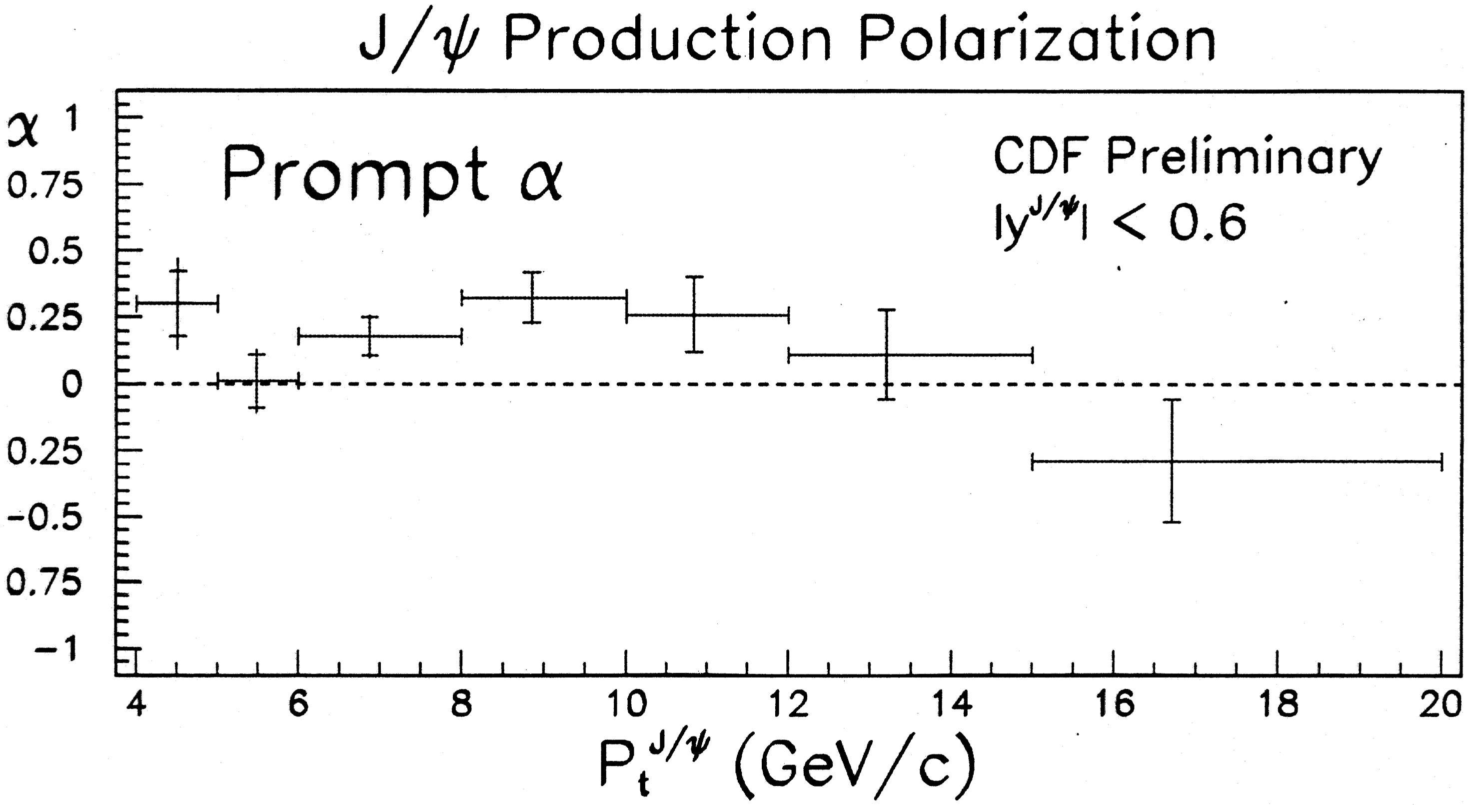


Figure 3: α as a function of p_t. Parameter specifications as for Fig. 1. The band is obtained from Eq. (8) and the $\mathcal{O}(\alpha_s^4)$ fragmentation contributions from [6] as described in the text. The solid curve is obtained with $M_{3.5}^{\psi}({}^1S_0^{(8)}, {}^3P_0^{(8)}) = 0$ and shows the importance of this parameter for polarization. The band does not include theoretical uncertainties in the short-distance coefficients, Tab. 2.



J/ψ Production Polarization
α
Prompt α
CDF Preliminary
|y^J/ψ| < 0.6
1
0.75
0.5
0.25
0
0.25
-0.5
0.75
-1
4
6
8
10
12
14
16
18
20
P_t^J/ψ (GeV/c)

- Theoretical uncertainties from higher orders in α_s and v could conceivably change the rates for the various spin states by a factor of 2.
- It is difficult to see how there could not be substantial polarization in J/ψ or ψ' polarization for $p_T > 3M_c$.
- Υ polarization
  - Same calculations as in the J/ψ case, but the numbers haven't been worked out.
  - Compared to J/ψ polarization, Υ polarization has smaller v-expansion uncertainties.
  - But it may be necessary to go to higher p_T to insure that the fragmentation mechanism dominates and to avoid corrections to the factorization formula.
  - CDF finds $\alpha = -0.06 \pm 0.20$ for $1\text{GeV} < p_T < 20\text{GeV}$.

Wishlist

Theory

- Complete treatment of multiple-soft-gluon emission, including virtual corrections. (Resummation methods?)
- Lattice computation of production matrix elements.
- Complete relative-order-α_s calculations for all production processes.
- Complete resummations of all large logarithms.
- Understanding (resummation?) of other large corrections.

Experiment

- Clean separation of quarkonium states.
  - Distinguish P-wave states with different quantum numbers.
  - Distinguish feeddown from higher states from direct production.
- More statistics at high p_T.