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QCD Factorization for Exclusive Non-leptonic B Decays

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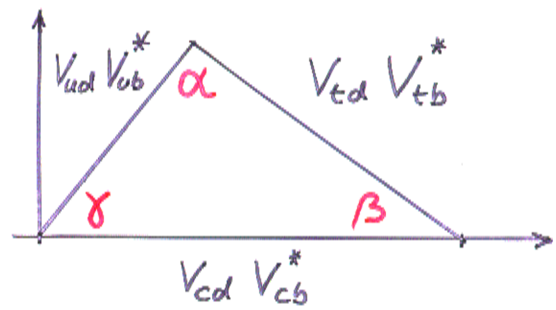
Introduction

two-body nonleptonic B decays $\quad B \to \pi\pi, \pi K, \ldots$

- probe fundamental weak interactions of b-quarks
  $\to$ CKM, CP violation, penguins, ...

$$\mathcal{A}_{CP}(t) = \frac{\Gamma(B(t)\to\pi^+\pi^-) - \Gamma(\bar{B}(t)\to\pi^+\pi^-)}{+}$$



pure direct CPV : $B^\pm \to K^\pm \pi^0, \ldots$

- many channels $\pi, K, \rho, K^*, \phi, \ldots$
- experimentally well accessible

CERN, Cornell, DESY, FNAL, KEK, SLAC

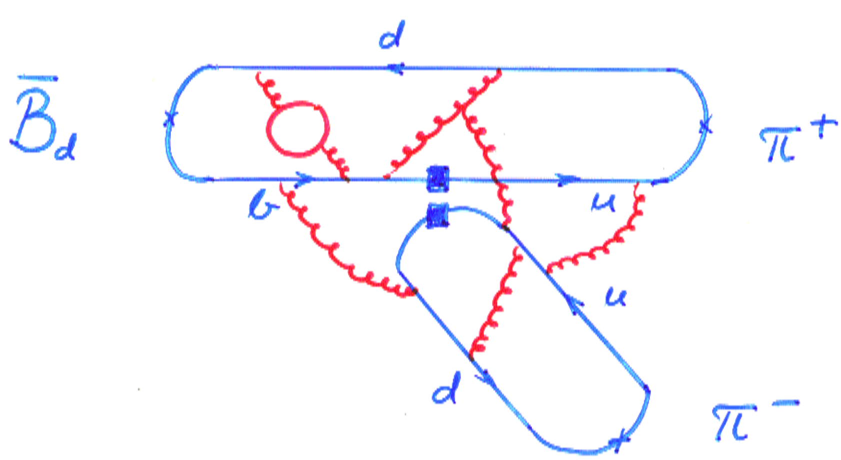
$\Rightarrow$ rich phenomenology to study flavour physics

theory $\qquad \mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \lambda_{CKM} C_i(\mu) Q_i$

problem : matrix elements $\quad \langle \pi\pi | Q_i | B \rangle$

$\hookrightarrow (\bar{u}b)_{V-A} (\bar{d}u)_{V-A}$

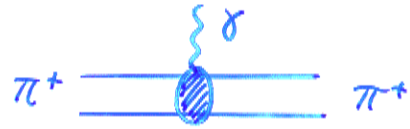
all hadronic final state

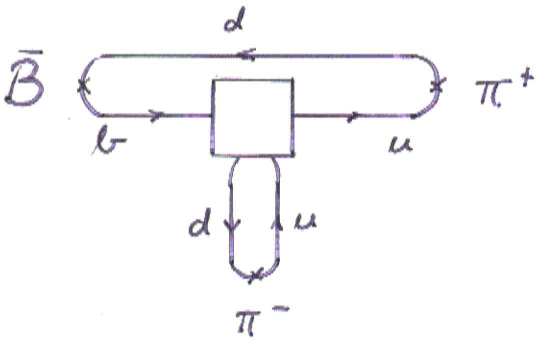


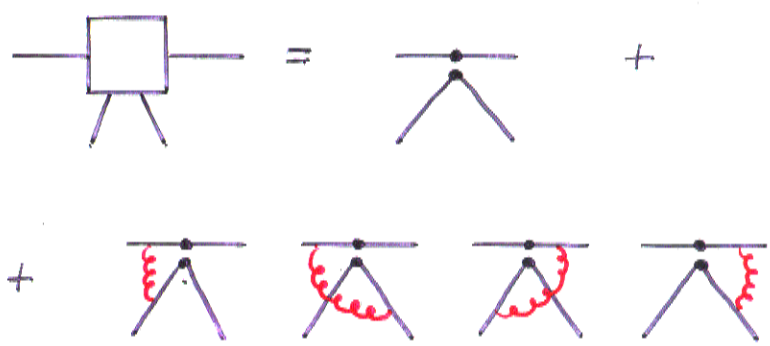
QCD factorization

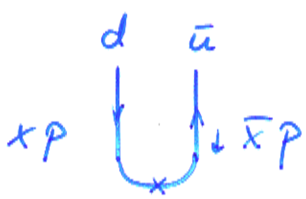
$m_B \gg \Lambda_{QCD}$

$$A(B\to\pi\pi) \sim \underbrace{\langle\pi|j_1|B\rangle\langle\pi|j_2|0\rangle\left[1 + \mathcal{O}(\alpha_s)\right.}_{\text{calculable in terms of universal quantities } f_+(0),\, f_\pi,\, \phi_\pi,\ldots} \left. + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_B}\right)\right]$$

compare: hard exclusive processes as 

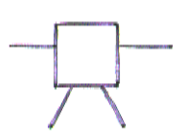






high-energy π^-, momentum p

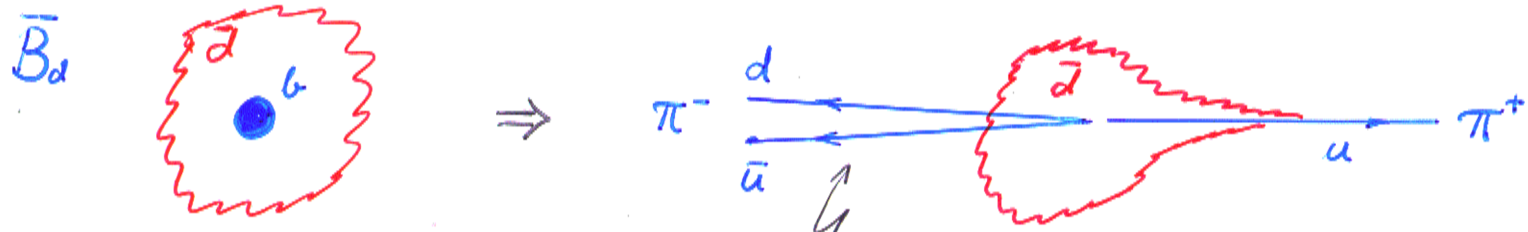
$f_\pi\,\phi_\pi(x) = f_\pi \cdot 6x\bar{x}$ $\qquad\qquad \bar{x} = 1-x$

- $p_{quark} = x\,p$ to leading power
-  free of soft + collinear singularities
  → perturbative
- power suppression: end point region, higher Fock states

→ factorization formula

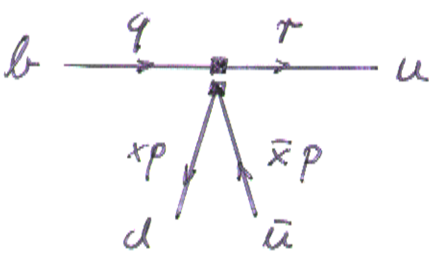
$$\langle\pi\pi|Q|B\rangle = F(B\to\pi)\cdot\int_0^1 dx\; T(x)\,\phi_\pi(x)$$

intuitive picture

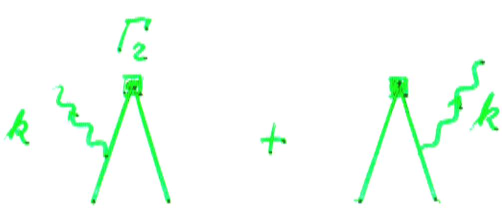


$m_b \gg \Lambda_{QCD}$

collinear, colour-singlet $\bar{u}d$ pair, decoupled from IR interactions



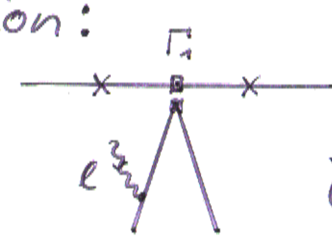
soft cancellation:



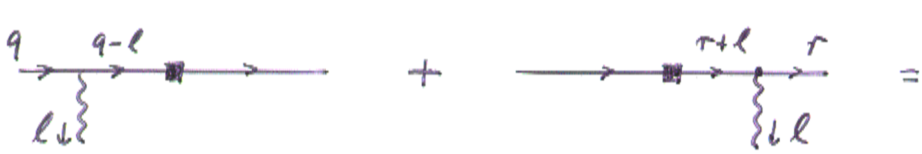
$$= \frac{\gamma^\lambda (x\not{p} + \not{k})\Gamma_2}{(xp+k)^2} + \frac{\Gamma_2(-\bar{x}\not{p} - \not{k})\gamma^\lambda}{(\bar{x}p+k)^2}$$

$$\overset{k\ \text{soft}}{=} \frac{2x\,p^\lambda\,\Gamma_2}{2x\,p\cdot k} + \frac{-2\bar{x}\,p^\lambda\,\Gamma_2}{2\bar{x}\,p\cdot k} = 0$$

collinear cancellation:

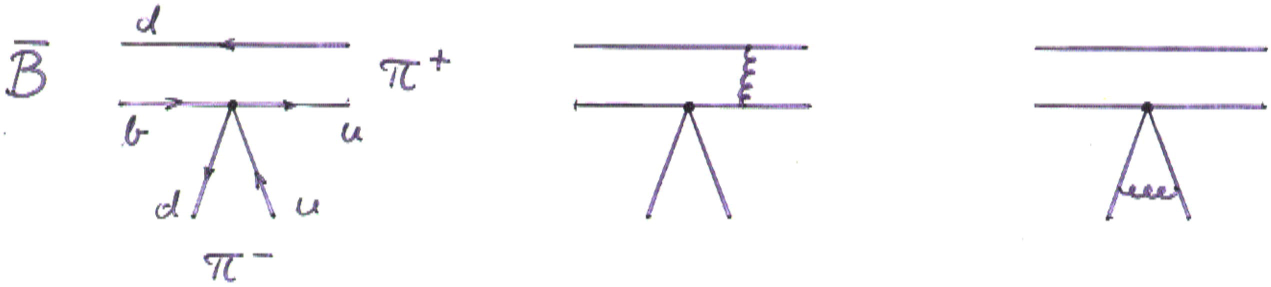


$$\gamma^\lambda (x\not{p} - \not{\ell})\Gamma_2 \overset{\ell \sim p}{\sim} p^\lambda \sim \ell^\lambda$$

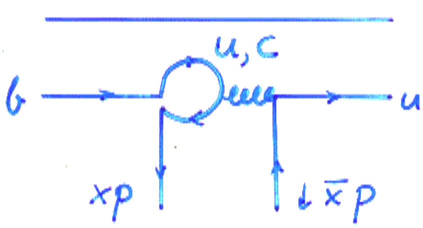


$$= \frac{\Gamma_1(\not{q} - \not{\ell} + m_b)\not{\ell}}{(q-\ell)^2 - m_b^2} + \frac{\not{\ell}(\not{r} + \not{\ell} + m_u)\Gamma_1}{(r+\ell)^2 - m_u^2} = -\Gamma_1 + \Gamma_1 = 0$$

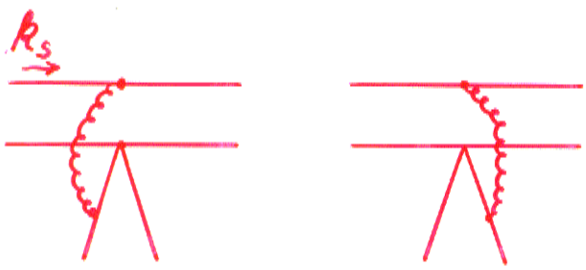
$B \to \pi\pi$ - Overview



FSI $\to$ phases

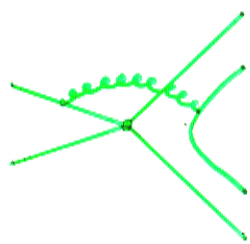


- gluon hard $k^2 = \bar{x}\, m_B^2$
- BSS mechanism
- $\langle G(k^2) \rangle = \int_0^1 dx\, G(\bar{x})\, \phi_\pi(x)$



gluon hard $k^2 \sim \Lambda m_B$

suppressed



$+ \ \ldots$

$$\langle \pi\pi | Q | \bar{B} \rangle = F(B\to\pi) \cdot \int_0^1 dx\, T^{I}(x)\, \phi_\pi(x) \; +$$

$$+ \int_0^1 d\xi\, dx\, dy\, T^{II}(\xi, x, y)\, \phi_B(\xi)\, \phi_\pi(x)\, \phi_\pi(y)$$

$$\xi = k_s^+ / p_B^+$$

$B \to \pi\pi$ amplitude from

$$\hat{\mathcal{H}}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \cdot$$

$$\cdot \Big[a_1^p \, (\bar{u}b)_{V-A} \otimes (\bar{d}u)_{V-A} + a_2^p \, (\bar{d}b)_{V-A} \otimes (\bar{u}u)_{V-A} +$$

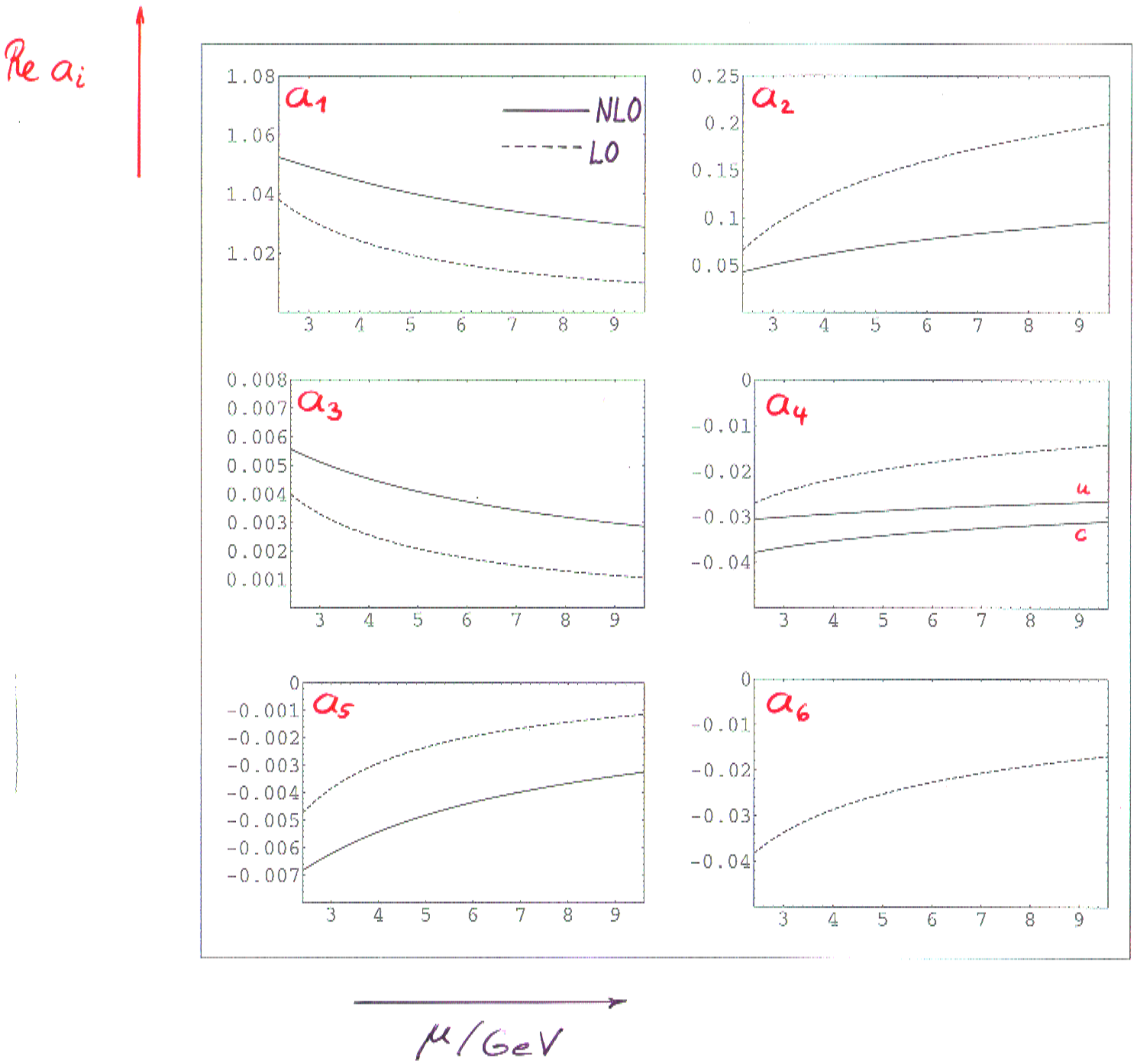
$$+ a_3 \, (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V-A} + a_4^p \, (\bar{q}b)_{V-A} \otimes (\bar{d}q)_{V-A} +$$

$$+ a_5 \, (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V+A} + a_6^p \, (-2)(\bar{q}b)_{S-P} \otimes (\bar{d}q)_{S+P} \Big]$$

$$a_1^u = C_1 + \frac{1}{3} C_2 + \frac{\alpha_s}{9\pi} \left(-12 \ln \frac{\mu}{m_b} - 18 + \int_0^1 g(x) \phi_\pi(x) \, dx \right) C_2 +$$

$$+ \frac{4\pi\alpha_s}{27} \frac{f_\pi f_B}{f_+ m_B^2} \int_0^1 \frac{\phi_\pi(x)}{x} \frac{\phi_\pi(y)}{y} dx\, dy \underbrace{\int_0^1 \frac{\phi_B(\xi)}{\xi} d\xi}_{\equiv m_B/\lambda_B} \cdot C_2$$

$$A(B \to \pi^+\pi^-) \sim e^{i\gamma} + 0.23\, e^{i\delta} \qquad \delta \approx 6^\circ$$



Re a_i
a_1
NLO
LO
a_2
a_3
a_4
u
c
a_5
a_6
μ/GeV

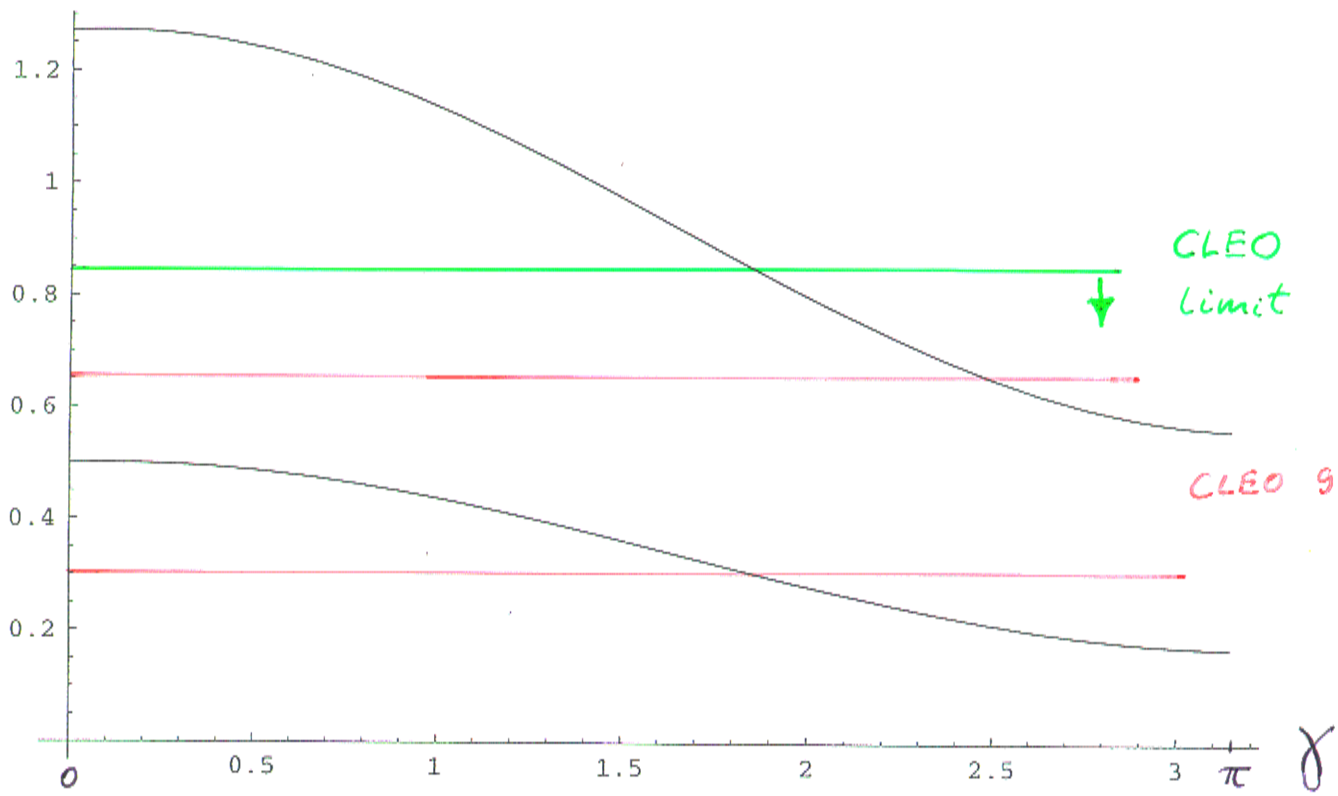
$$0.07 \le \left| \frac{V_{ub}}{V_{cb}} \right| \le 0.09$$

$$0.038 \le |V_{cb}| \le 0.040$$

$$0.25 \le f_+(0) \le 0.30$$ Bagan, Ball, Braun

$$0.1 \le \lambda_B/\mathrm{GeV} \le 0.4$$

$B(B \to \pi^+\pi^-)/10^{-5}$



$$B(B^+ \to \pi^+\pi^0) = \left[\frac{f_+}{0.275} \, \frac{V_{cb}}{0.039} \, \frac{|V_{ub}/V_{cb}|}{0.08} \right]^2 (0.40 \pm 0.03) \cdot 10^{-5}$$

$$< 1.2 \cdot 10^{-5} \quad \text{CLEO}$$

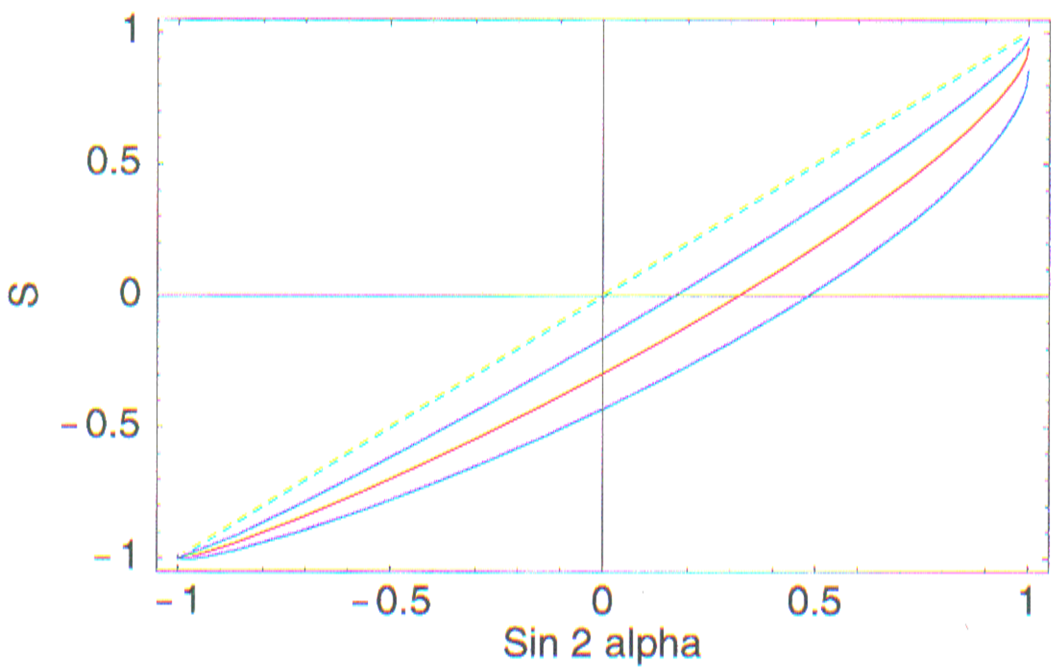
$$B(B \to \pi^0\pi^0) < 1.2 \cdot 10^{-6}$$

$\sin(2\alpha)$ from $B_d \to \pi^+\pi^-$ decays

$$\mathcal{A}(t) = -S \cdot \sin(\Delta M t) + C \cdot \cos(\Delta M t)$$

$$\begin{aligned} S &= \sin(2\alpha) + \mathcal{O}(P/T) \\ C &= \mathcal{O}(P/T) \end{aligned}$$

Fix $\sin(2\beta) = 0.7$.



green: no penguin amplitude (i.e. single weak phase contributes).

red: $a_6(\pi\pi)$ at leading order in α_s.

blue: twice $a_6(\pi\pi)$ and $a_6(\pi\pi) = 0$.

Summary

- factorization holds for $B \to \pi\pi$ in heavy quark limit

- calculated at LO and NLO in QCD

$$A(B\to\pi\pi) \sim f_+(0)\, f_\pi\, m_B^2 \left[1 + c\,\frac{1}{\ln\frac{m_B}{\Lambda}} + \mathcal{O}\left(\frac{\Lambda}{m_B}\right) \right]$$

non-factorizable, but calculable

- rescattering phases calculable

- other applications $B \to h h'$, $h, h' = \pi, \rho, K, K^*, \ldots$

CPV $\overset{(-)}{B} \to \pi^+\pi^- \longrightarrow \sin 2\alpha$

$B \to \pi K \longrightarrow \gamma$ Fleischer, Mannel Neubert, Rosner

$\bar{B} \to D^+\pi^-$, $B^+ \to D^+\pi^0$ Politzer, Wise

however: $\bar{B} \to D^0\pi^0$ no factorization, but rel. power suppr.

- importance of power corrections ?

- QCD predictions for $B \to \pi\pi, \ldots$ in HQ limit

$\downarrow$

to be tested in experiment