B_c meson :

Production and Decays (Run-II @ Tevatron)

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- **General features:**
  - Flavor explicit (weak decays ···)
  - Weak-binding (nonrelativistic ···)
  - "Double heavy" (perterbative ---)
- **Production :**
  - Double heavy quark production
  - mechanisms
    - fragmentation
    - Combination
    - ----
- **Decays :**
  - Three comparable "components" (lifetime ···)
  - Recoil effects
  - Typical decays and signals
  - ----
- **Outlooks**

- **General features:**

Explicit-flavored:

B_c meson : $(c\bar{b})$ $^{2S+1}L_J = {}^1S_0$

$\bar{B}_c$ " : $(\bar{c}b)$ "

B_c^* meson : $(c\bar{b})$ $^{2S+1}L_J = {}^3S_1$

$\bar{B}_c^*$ " " "

B_c^{**} ····· $(c\bar{b})$ excited states

$m_b > m_c \gg \Lambda_{QCD} \Longrightarrow$ "Double heavy"

Weak-binding : $\mu = \frac{m_b m_c}{m_b + m_c} \simeq 1.2\ \text{GeV}$

$\left(\frac{m_c^2}{m_c + m_c} \simeq .75\ \text{GeV},\ \frac{m_b^2}{m_b + m_b} \simeq 5.0\ \text{GeV} \right)$

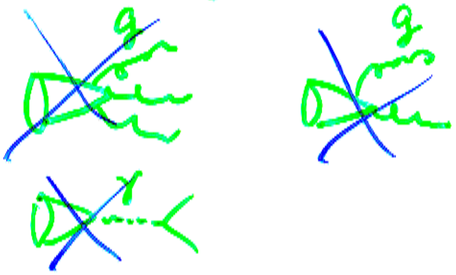
$(c\bar{c})$ $(b\bar{b})$

$m_{B_c} = 6.2 - 6.4\ \text{GeV}$ ←

W. Kwong, J. Rosner
E. Eichten, C. Quigg
UKQCD hep-lat/9902025
⋮

Weak decay only → study b, c flavors

no such decays (at tree level):



(strong)

(electromagnetic)

- **Production :**

The most favorable production way

1st. 4 heavy quark production : $(c\bar{c})$ & $(b\bar{b})$

2d. inclusive combination : $(c\bar{b}) \Rightarrow B_c, B_c^*, \cdots$

$(\bar{c}b) \Rightarrow \bar{B}_c, \bar{B}_c^*, \cdots$

$\Rightarrow$ NRQCD factorization (PQCD + PM applicable)

PQCD : 4 heavy quark production

PM : wave function — possibility of recombination

(the matrix element of a proper operator — NRQCD)

(color-singlet one always dominant over the color-octet — due to flavor being explicit)

At Tevatron & LHC : gluon-gluon fusion is dominant over quark-antiquark fusion

$$g+g \to B_c + \cdots \gg q+\bar{q} \to B_c + \cdots$$

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Mechanisms:

- **Fragmentation:**

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K. Cheung
V. V. Kiselev, A. K. Likhoded
- - - -

A "high energy scale" e.g. $P_T \gg m_{B_c}$

$\bar{b}$-quark (or c-quark) fragmentates the meson

$D^{B_c}_{\bar{b}(c)}(x_3, \mu_F)$

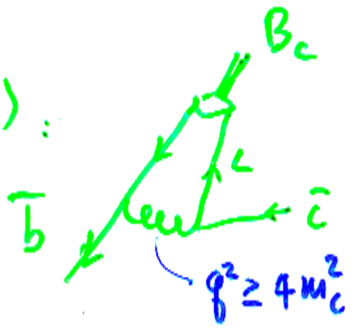
$$d\sigma = \sum_{ijk} \int dx_1 \int dx_2 \int dx_3\, F^i_{h_1}(x_1, \mu_F)\, F^j_{h_2}(x_2, \mu_F) \cdot$$

$$\cdot\, d\sigma_{ij \to \bar{b}(c) \ldots}(x_1 x_2 x_3, \mu_F)\, D^{B_c}_{\bar{b}(c)}(x_3, \mu_F)$$

$F^i_{h_1}(x_1, \mu_F)$
$F^j_{h_2}(x_2, \mu_F)$ $>$ S. F.

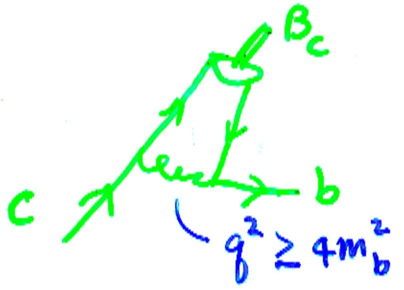
$D^{B_c}_{\bar{b}(c)}(x_3, \mu_F)$: F. F.

$D^{B_c}_{\bar{b}}(x, 4m_c^2)$:



calculable at $q^2 \geq 4m_c^2$
with PQCD & $\psi_{B_c}(0)$

$D_c^{B_c}(x, 4m_b^2)$:



calculable at $q^2 \geq 4m_b^2$
with PQCD & $\psi_{B_c}(0)$

For instance

$$D_{\bar{b}}^{B_c}(x, 2m_c) \propto \alpha_s^2(4m_c^2)\,|\psi_{B_c}(0)|^2 \frac{x(1-x)}{(xa_2-1)^6}\{[(2a_1x-3(a_2-a_1)\cdot$$

$$\cdot(1-a_2x)(2-x)]\cdot(1-a_2x)x+6(1+a_1x)^2(1-a_2x)^2-8a_1a_2x^2(1-x)\}$$

$$D_{\bar{b}}^{B_c^*}(x, 2m_c) \propto \alpha_s^2(4m_c^2)\,|\psi_{B_c}(0)|^2 \frac{x(1-x)^2}{(1-a_2x)^6}\{(2a_2^2x^2-3a_2x+4a_2^2x$$

$$+4a_2^2x+4a_2x-9x-4a_2+6)(1-a_2x)+2[(1+a_1x)^2+2x^2]\cdot$$

$$\cdot(1-a_2x)^2-8a_1a_2x^2(1-x)\}$$

$$D_{\bar{b}}^{B_c^{**}}(x, 2m_c) \propto \cdots$$

$a_1 = \frac{m_c}{m_{B_c}}$ $a_2 = \frac{m_b}{m_{B_c}}$ x : fragmentation variable

- "Combination" (non-fragmentation)

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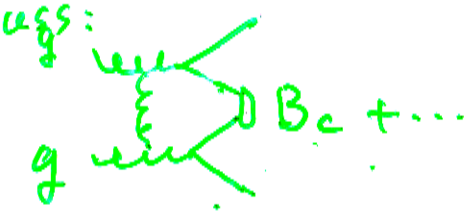
K. Kolodziej, A. Leike, R. Rückl
A.V. Berezhnoi, A.K. Likhoded, M.V. Shevlyagin

Subprocess:



(typical one).

m_b, m_c, m_{B_c} only,
the mechanism may be important at certain kinetics region!

- **Comparison** (through a complete α_s^4-calculation)

:

subprocess:

= 36 Feynman diagrams

(5 independent and gauge-invariant subsets ; interference ; ···)

(See Figures)

Conclusions:

① P_T-distribution: Fortuitous (accidental) consistency — gluon structure function

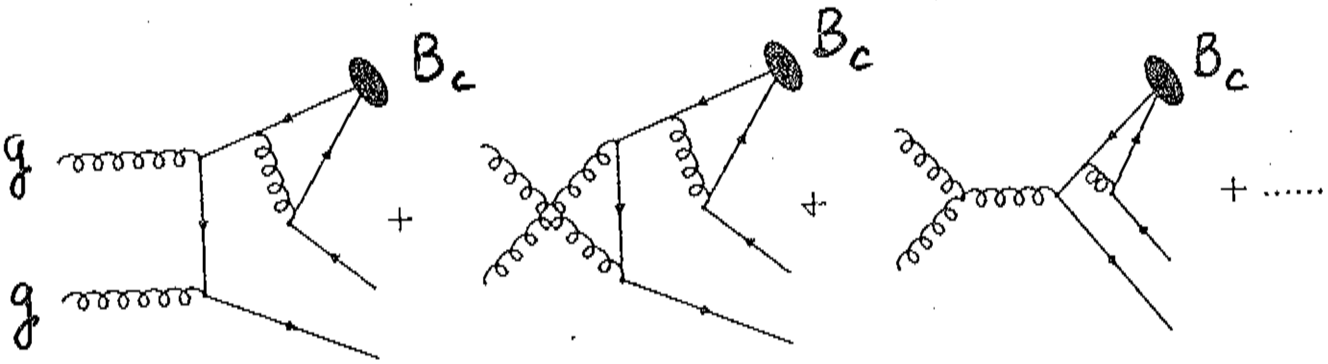
② $P_T \gg m_{B_c}$ ($P_T \gtrsim 30$ GeV when $\sqrt{\hat{s}} \gtrsim 200$ GeV) fragmentation Mechanism is dominant

③ At $P_T \sim m_{B_c}$ there are "extra" contributions besides F.M. indecates ⇒ combination dominant

36 Feynman diagrams:

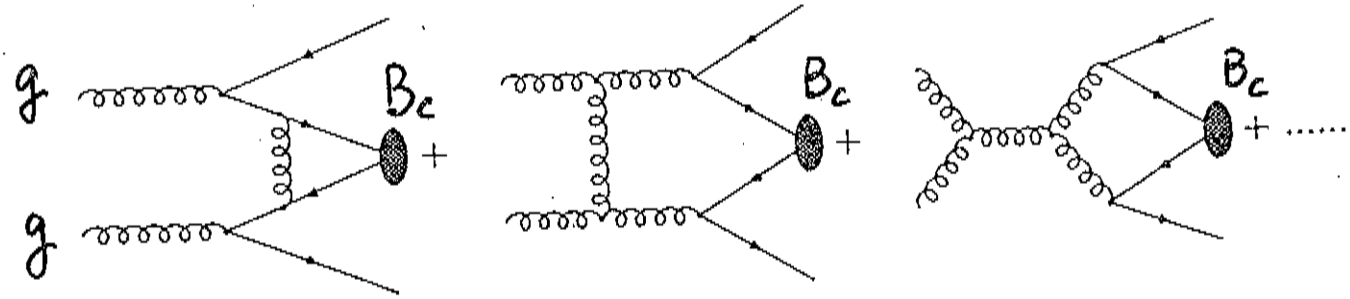
$$g + g \rightarrow B_c + \bar{c} + b$$

(a). "Fragmentation":



(a)

(b). "Combination"



(b)

6-1

Amplitude structure :

$$A(a,b;i,j) = \sum_{\alpha=1}^{6} C^{ab}_{\alpha;ij}\, M_\alpha(\varepsilon_1,\varepsilon_2,s_1,s_2)$$

$$C^{ab}_{1;ij} = (\lambda^c\cdot\lambda^c\cdot\lambda^a\cdot\lambda^b)_{;ij} = \frac{N^2-1}{N}(\lambda^a\cdot\lambda^b)_{;ij}$$

$$C^{ab}_{2;ij} = (\lambda^c\cdot\lambda^c\cdot\lambda^b\cdot\lambda^a)_{;ij} = \frac{N^2-1}{N}(\lambda^b\cdot\lambda^a)_{;ij}$$

$$C^{ab}_{3;ij} = (\lambda^c\cdot\lambda^a\cdot\lambda^c\cdot\lambda^b)_{;ij} = \frac{-1}{N}(\lambda^a\cdot\lambda^b)_{;ij}$$

$$C^{ab}_{4;ij} = (\lambda^c\cdot\lambda^b\cdot\lambda^c\cdot\lambda^a)_{;ij} = -\frac{1}{N}(\lambda^b\cdot\lambda^a)_{;ij}$$

$$C^{ab}_{5;ij} = (\lambda^c\lambda^a\lambda^b\lambda^c)_{;ij} = \delta_{ij}\,\mathrm{tr}(\lambda^a\lambda^b) - \frac{1}{N}(\lambda^a\lambda^b)_{;ij}$$

$$C^{ab}_{6;ij} = (\lambda^c\lambda^b\lambda^a\lambda^c)_{;ij} = \delta_{ij}\,\mathrm{tr}(\lambda^a\lambda^b) - \frac{1}{N}(\lambda^b\lambda^a)_{;ij}$$

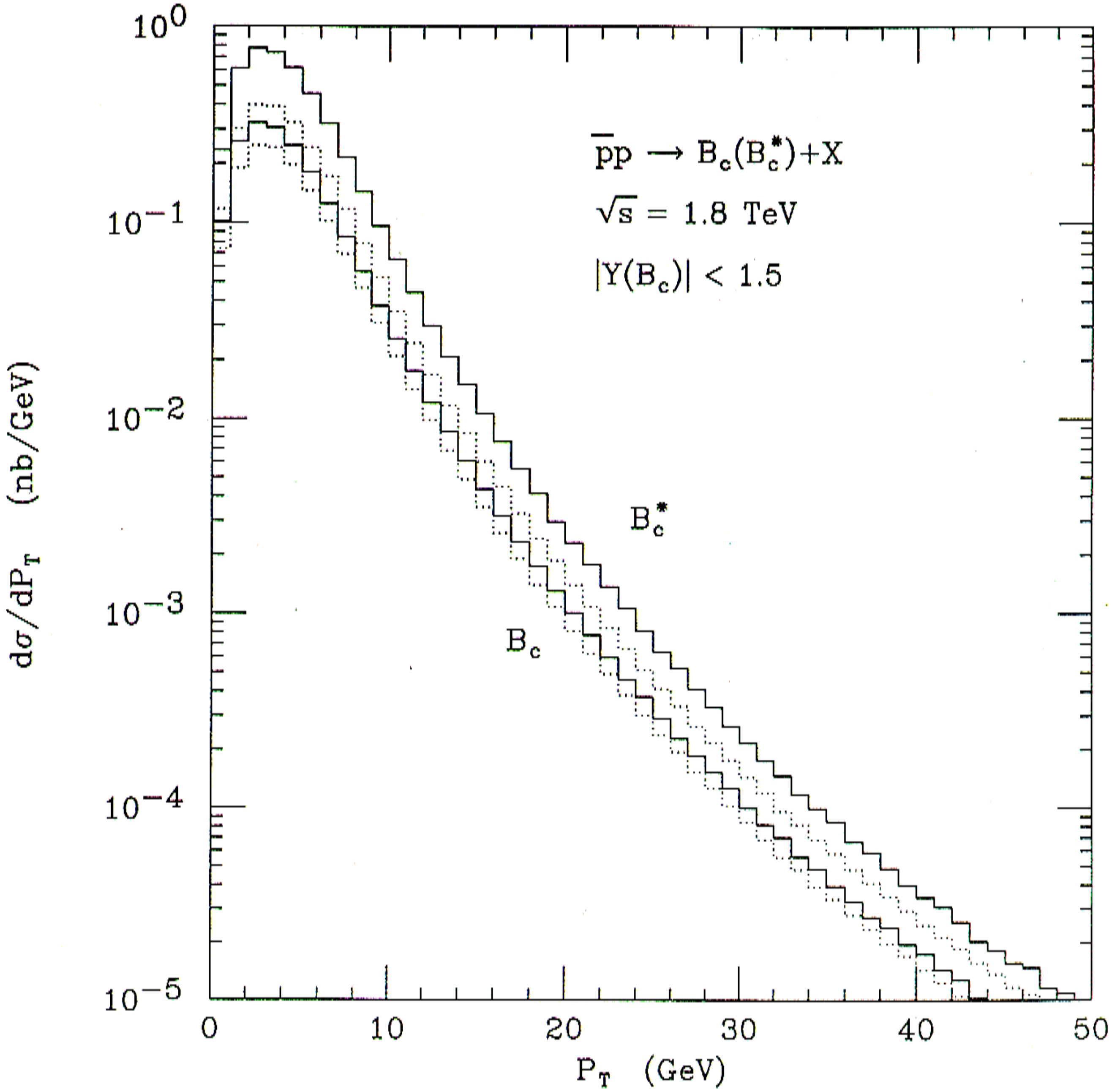
A relation :

$$C^{ab}_{3;ij} - C^{ab}_{5;ij} = C^{ab}_{4;ij} - C^{ab}_{6;ij}$$

Thus

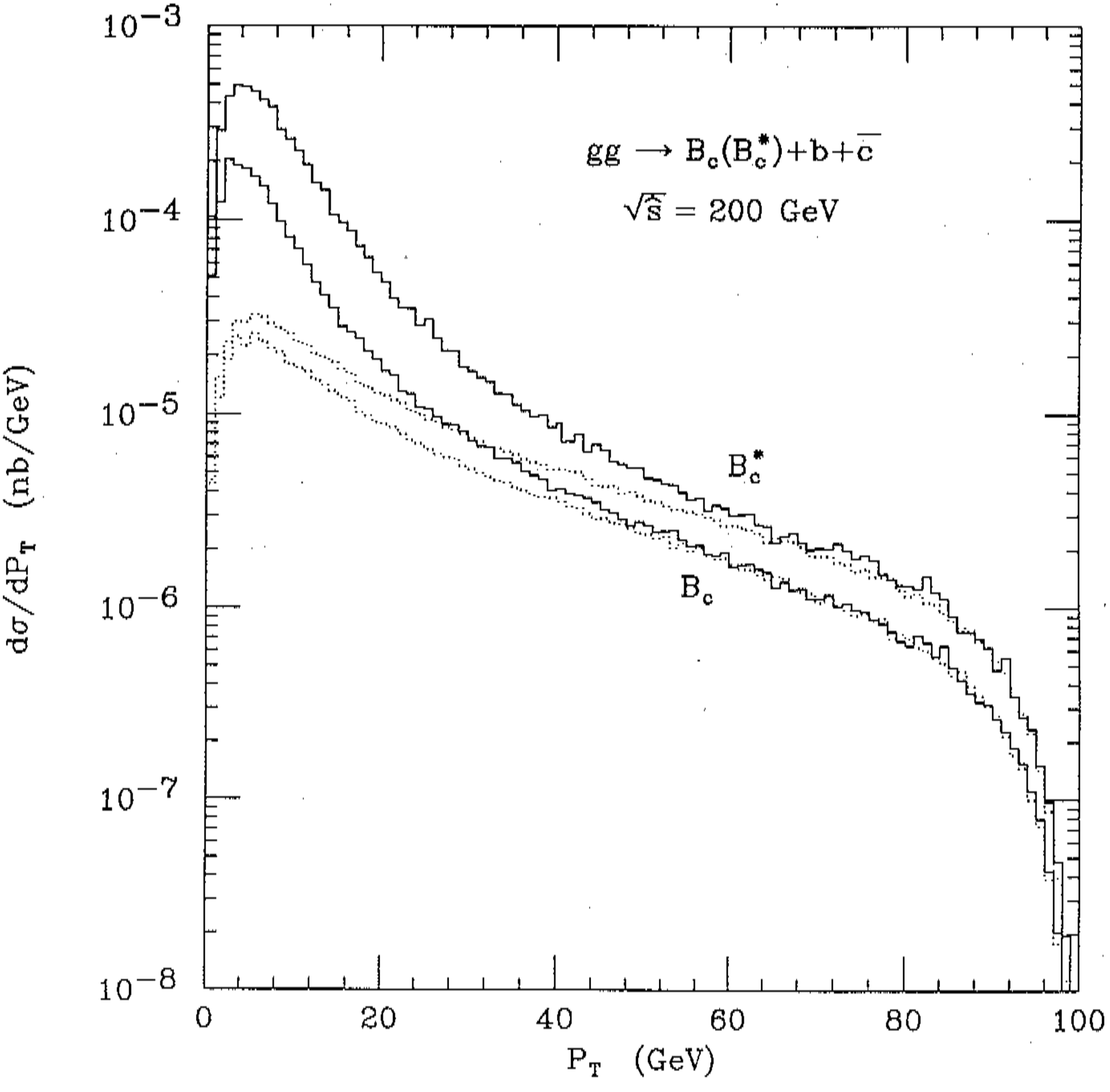
5 independent !

P.R. D54. 4344



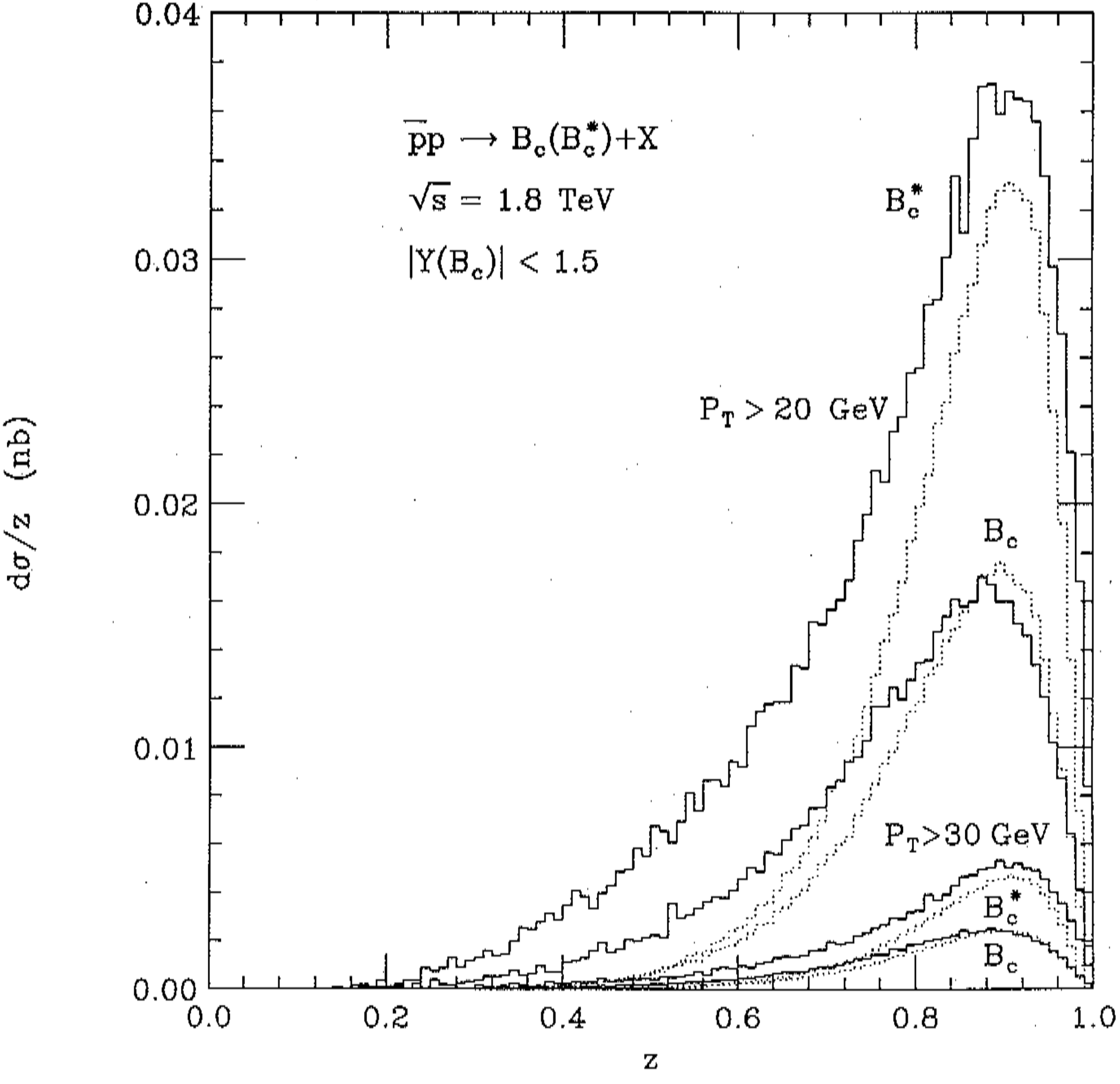
Fortuitous (accidental) consistancy

P.R. D54, 4344



$gg \longrightarrow B_c(B_c^*)+b+\bar{c}$
$\sqrt{s} = 200$ GeV
B_c^*
B_c
$d\sigma/dP_T$ (nb/GeV)
P_T (GeV)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
20
40
60
80
100

P.R. D54, 4344


$\bar{p}p \longrightarrow B_c(B_c^*)+X$
$\sqrt{s} = 1.8$ TeV
$|Y(B_c)| < 1.5$
B_c^*
$P_T > 20$ GeV
B_c
$P_T > 30$ GeV
B_c^*
B_c
$d\sigma/z$ (nb)
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0
z

P. L. B364, 78

$\alpha_s = 0.2$

1 Total cross sections (in unit nb)

otal cross sections for the productions of the B_c meson and its excited state B_c^*

Collision	Approach-I	
	$B_c\ (1^1S_0)$	$B_c^*\ (1^3S_1)$
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV})$	0.747(4)	1.23(1)
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV}, *)$	0.229(2)	0.389(3)
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV}, **)$	0.0331(9)	0.0570(6)
$pp(\sqrt{S} = 14\ \text{TeV})$	8.63(5)	14.0(1)
$pp(\sqrt{S} = 14\ \text{TeV}, *)$	3.07(3)	5.11(4)
$pp(\sqrt{S} = 14\ \text{TeV}, **)$	0.584(7)	0.986(10)
$gg(\sqrt{\hat{s}} = 20\ \text{GeV})$	$0.704(5) \cdot 10^{-2}$	$0.118(1) \cdot 10^{-1}$
$gg(\sqrt{\hat{s}} = 30\ \text{GeV})$	$0.678(8) \cdot 10^{-2}$	$0.103(1) \cdot 10^{-1}$
$gg(\sqrt{\hat{s}} = 60\ \text{GeV})$	$0.321(7) \cdot 10^{-2}$	$0.456(9) \cdot 10^{-2}$

Collision	Approach-II	
	$B_c\ (1^1S_0)$	$B_c^*\ (1^3S_1)$
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV})$	0.850(8)	2.07(2)
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV}, *)$	0.259(4)	0.646(6)
$p\bar{p}(\sqrt{S} = 1.8\ \text{TeV}, **)$	0.0373(1)	0.0894(3)
$pp(\sqrt{S} = 14\ \text{TeV})$	10.6(1)	26.4(3)
$pp(\sqrt{S} = 14\ \text{TeV}, *)$	3.71(6)	9.43(9)
$pp(\sqrt{S} = 14\ \text{TeV}, **)$	0.698(1)	1.69(4)
$gg(\sqrt{\hat{s}} = 20\ \text{GeV})$	$0.661(7) \cdot 10^{-2}$	$0.160(2) \cdot 10^{-1}$
$gg(\sqrt{\hat{s}} = 30\ \text{GeV})$	$0.949(8) \cdot 10^{-2}$	$0.244(3) \cdot 10^{-1}$
$gg(\sqrt{\hat{s}} = 60\ \text{GeV})$	$0.782(9) \cdot 10^{-2}$	$0.203(3) \cdot 10^{-1}$

ce at all in the above sensed $M_{B_c(B_c^*)} = 6.4$ GeV [2] are taken. Fu

we expect the results ac. the calculations the wave functions of

* : P_T cut = 5 GeV ** : P_T cut = 10 GeV

- Some desires (theoretical work)
  - more precise calculations than α_s^4 (resummation of L.L.)
  - sensitive observables to the mechanisms

    e.g. $C(z)\equiv\int dx_1 dx_2\, g(x_1) g(x_2) \frac{d\hat{\sigma}(\sqrt{\hat{s}})}{dz}$ & $z\equiv\frac{2(k_1+k_2)\cdot P}{\hat{s}}$

(Better understanding of B_c-production mechanisms will be helpful to understand: the mechanisms for J/ψ, ---- Υ, ---)

  - To cancel the uncertainties, the ratios to B meson but not B_c itself
  - ......

- **Decays:**

M. Lusignoli, M. Masetti; N. Isgur, D. Scora, B. Grinstein, M. Wise; D. Scora, N. Isgur; C.-h. Chang, Y.-Q. Chen; I. I. Bigi; M. Beneke, G. Buchalla; V. V. Kiselev, A. K. Likhoded, A. I. Onishchenko, ---

Focus to B_c ($\bar{B}_c$) the ground state decays only

—— weak decays

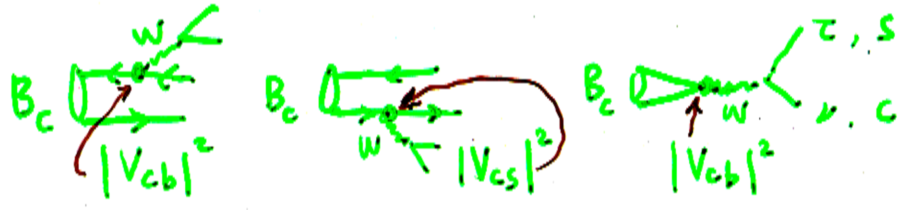
Three comparable "components":

decay probability

$$\frac{1}{\tau_{B_c}} = \bar{b}\text{-decay} + c\text{-decay} + c\bar{b}\text{-annihilation}$$

"spectator" consideration: naive considerations

$$\frac{1}{\tau_{B_c}} \simeq \frac{1}{\tau_B} + \frac{c}{\tau_D} + \Gamma_{ann}$$



⑧

$$\Gamma_{ann}^{f} \simeq \frac{G_F^2 |V_{cb}|^2}{8\pi} f_{B_c}^2 \, m_f^2 \, m_{B_c} \left(1 - \frac{m_f^2}{m_{B_c}}\right)\left(1 + O(\alpha)\right) \quad (f = \tau)$$

$$\Gamma_{ann} = \Gamma_{ann}^{cs} + \Gamma_{ann}^{\tau\nu} + \cdots$$

$$\tau_{D^0}^{exp} \simeq \tau_{D_s}^{exp} \simeq 0.45 \times 10^{-12} s$$

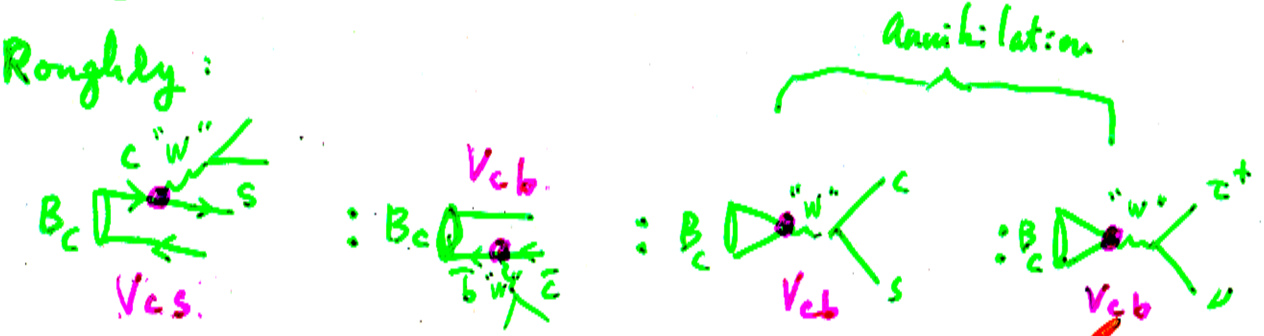
$$\tau_B^{exp} \simeq 1.6 \times 10^{-12} s$$

$$f_{B_c} \simeq 480 - 550 \text{ MeV}$$

if $c = 0.6$ for phase space and else ...

$$\tau_{B_c} \simeq 0.4 \times 10^{-12} s$$

Roughly :



$\simeq$ 54% : 26% : 15% : 5%

The three components are comparable !

We may study c-quark & b-quark decays with B_c decays

① The lifetime estimation P.R. D53, 4991

$\tau_{B_c} = (0.4 - 0.7) \times 10^{-12} s$ may be too conservative

$(1.4 \text{GeV} \leq m_c \leq 1.6 \text{ GeV})$ — should be fixed from the lifetime D^0 (D_s)

$\tau_{B_c} \simeq (0.35 \sim 0.55) \times 10^{-12} s$

② Semileptonic decays :

i) b-decay

Typical decay

$B_c \to J/\psi + \ell + \nu \qquad (\ell = e, \mu)$

$q^2_{max} \Rightarrow \frac{|\vec{v}_{J/\psi}|}{c} = 0 \qquad$ (J/ψ at rest in CMS of B_c)

$q^2_{min} \Rightarrow \frac{|\vec{v}_{J/\psi}|}{c} \simeq 0.63 \qquad$ (in CMS of B_c)

Thus

$J/\psi <$ relativistic moving / nonrelativistic bound state } recoil effects

Most of calculations:

done at q^2_{max} $(|\vec{v}|/c = 0)$

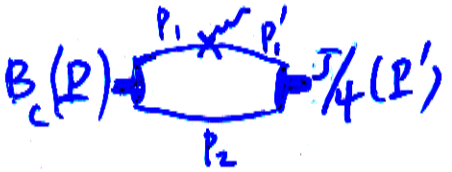
extrapolate to any q^2 with assumption(s).?

Our trick : P. R. D49, 3399

To start with B.S. equations for B_c and J/ψ. } Lorentz covariant
as well as Mandelstam formulation for transition matrix element

B.S. equations (wave function) ⟹ Schrödinger equation (wave fn)
(instantanious approximation)

The transition matrix element (under Mandelstam formulation)



$$\langle J/\psi | \Gamma^\mu | B_c \rangle = i \int \frac{d^4 q}{(2\pi)^4} \, tr[\bar{\chi}_{P'}(q') \Gamma^\mu \chi_P(q) (\not{p}_2 + m_2)]$$

$$P_1 = \frac{m_1}{m_1+m_2} P + q \qquad P_2 = \frac{m_2}{m_1+m_2} P - q$$

$$P_1' = \frac{m_1}{m_1+m_2} P' + q' \qquad P_2' = \frac{m_2}{m_1+m_2} P' - q' = P_2$$

$\Rightarrow$ $\oint \frac{dq_{\parallel}}{2\pi}$: $q_{\parallel} = \frac{P\cdot q}{M_{B_c}}$ — time like compoment

Generalized instantanious approximation $\nwarrow$ contour integration (break Lorentz Covariant)

The transition matrix element :

$$\langle J/\psi | \Gamma^{\mu} | B_c \rangle \simeq \int \frac{q_{\perp}^2\, dq_{\perp}\, ds}{(2\pi)^2}\, tr[\, \bar{\varphi}^{++}_{\underline{P}'}(q'_{p'_{\perp}})\, \Gamma_{\mu}\, \varphi^{++}_{\underline{P}}(q_{p_{\perp}}) \frac{\not{P}}{m_{B_c}}\,]\, \frac{\omega'_{2p}}{\omega_{2p}}$$

$\omega_{2p} = \sqrt{\vec{q}^{\,2} + m_2^2}$, $\omega_{2p'} = \frac{E'\omega_{2p} + (\vec{P} - \vec{P}')\cdot\vec{q}}{m_{J/\psi}}$, $q'_{p\perp} = \sqrt{\omega'^{2}_{2p} - m_2^2}$

$\bar{\varphi}^{++}_{\underline{P}}(q)$ (positive-positive) compoment of the B.S. wave function

$\Longrightarrow$ Schrödinger wave function
(straightforward)

Spin symmetry is obtained :

if $\langle \eta_c(P') | V_{\mu} | B_c(P) \rangle = \boxed{f_+}(P+P') + \boxed{f_-}(P-P')$

$\langle J/\psi(P',\varepsilon^*) | V_{\mu} | B_c(P) \rangle = i\,\boxed{g}\,\varepsilon_{\mu\nu\rho\sigma}\, \varepsilon^{*\nu}(P+P')^{\rho}(P-P')^{\sigma}$

$\langle J/\psi(P',\varepsilon^*) | A_{\mu} | B_c(P) \rangle = \boxed{f}\,\varepsilon^*_{\mu} + \boxed{a_+}(\varepsilon^* P)(P+P')_{\mu} + \boxed{a_-}(\varepsilon^* P)(P-P')_{\mu}$

the form factors : f_+, f_-, g, f, a_+, a_-

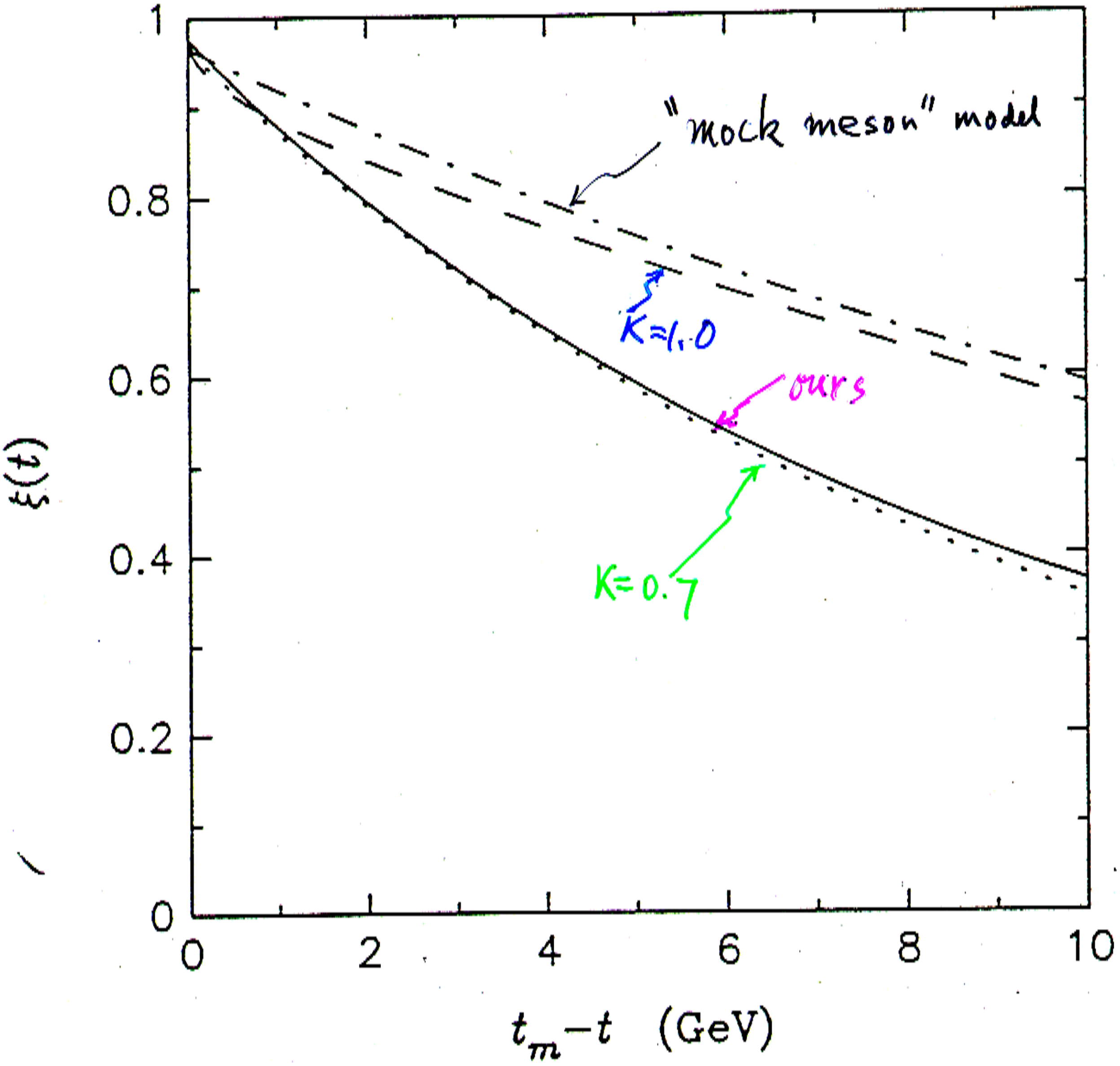
relate to only one form factor ξ with proper recoil factors

$$\xi = \left[\frac{2\omega'_2\, m_1^2\, m_2^2}{[(P'_1 \cdot P_1) + m_1 m'_1]\, \omega_1 \omega'_1 \omega_2} \right]^{1/2} \int \frac{d^3\vec{q}}{(2\pi)^3}\, \phi'_{\underline{P}'}(q'_{p'_{\perp}})\, \phi_{\underline{P}}(|q|)$$

ξ — behavior

form factors (f_+, f_-, g, f, a_+, a_-) are obtained

$\Rightarrow$ transition matrix element $\Rightarrow$ differential decay width $\Rightarrow$ ..



$\langle \eta_c(J/\psi) | J_\mu^W | B_c \rangle$

Comparison with ISGW model (N. Isgur, D. Scora, B. Grinstein and M.B. Wise, PRD <u>39</u> (89) 799).

$$\xi_{ISGW} \approx \left(\frac{2\beta\beta'}{\beta^2 + \beta'^2} \right)^{3/2} \exp\left\{ - \frac{m_2^2}{2MM'} \frac{t_m - t}{\kappa^2(\beta^2 + \beta'^2)} \right\}$$

(parameter κ is put into by hand!)

ii) c - decay

Typical decay

$B_c \to B_s + l + \nu$

$q^2_{max} \Rightarrow |\vec{v}_{B_s}|/c \simeq 0$

$q^2_{min} \Rightarrow |\vec{v}_{B_s}|/c \simeq 0.17$

but we may do the same.

$\Rightarrow$ form factors $\Rightarrow$ tran. matrix element $\Rightarrow d\Gamma(\cdots) \Rightarrow \Gamma$.

Numerical results

$B_r(B_c \to J/\psi + e + \nu) \sim 2\%$

$(\Gamma(B_c \to J/\psi + e + \nu) \simeq 34 \times 10^{-6}\,eV$ & $\tau_{B_c} \sim 0.4\,ps\,)$

③ Nonleptonic decays (two-body decays)

Typical decay

$B_c \to J/\psi + \pi^+$

$\quad\quad J/\psi + \rho^+$

$\quad\quad - - - -$

the recoil.

$B_c \to J/\psi + \pi^+$, $|\vec{v}_{J/\psi}|/c \simeq 0.62$

recoil !

, factorization $\Rightarrow$

with $q^2 = m_\pi^2$

The numerecal results

$B_r(B_c \to J/\psi\,\pi) \sim 0.17\%$

big: $B_r(B_c \to B_s + \pi)$, $B_r(B_c \to B_s + \rho)$, $B_r(B_c \to B_s^* + \pi)$, $B_r(B_c \to B_s^* + \rho)$

④ Pure leptonic decays and radiative leptonic decays
(decay constant f_{B_c} measurement)

$B_c \to \tau\nu$
$\mu\nu$
$e\nu$
} chiral suppresion $\Rightarrow$ $Br(B_c \to \tau\nu)$ sizable Ⓧ
$Br(B_c \to \mu\nu)$ small X
$Br(B_c \to e\nu)$ much small X

$B_c \to \mu\nu\gamma$
$e\nu\gamma$

Feynman diagrams & proportional to f_{B_c} (except L.D.)
can be used to measure f_{B_c}

$\Gamma(B_c \to \ell\nu) + \Gamma(B_c \to \gamma\ell\nu)$ radiative corrections

"short distance" interaction contribution : dominant

"long distance" " " : small

numerical result :
(see Tables)

$\frac{d\Gamma}{dE_\gamma}$ vs. E_γ

Accessible with challenges

⑤ Rare decays and $\not{CP}$.

1. Exclusive semileptonic decay width (in 10^{-6} eV) for various modes calculated

odel.

	Ours	ISGW	WSB	SR		
				I	II	III
$B_c \rightarrow \eta_c + e^+ \bar{\nu}_e$	14.2	10.6	16.5	20.4	15	11
$B_c \rightarrow J/\psi + e^+ \bar{\nu}_e$	34.4	38.5	21.8	37.3	44	33
$B_c \rightarrow D^0 + e^+ \bar{\nu}_e$	0.094					
$B_c \rightarrow D^{0*} + e^+ \bar{\nu}_e$	0.269					
$B_c \rightarrow \eta_c' + e^+ \bar{\nu}_e$	0.727					
$B_c \rightarrow \psi(2S) + e^+ \bar{\nu}_e$	1.45					
$B_c \rightarrow B_s + e^+ \bar{\nu}_e$	26.6					
$B_c \rightarrow B_s^* + e^+ \bar{\nu}_e$	44.0					
$B_c \rightarrow B^0 + e^+ \bar{\nu}_e$	2.30					
$B_c \rightarrow B^{0*} + e^+ \bar{\nu}_e$	3.32					

Some of typical decay channels of b-decay (non leptonic

Table 2. Exclusive two body nonleptonic decay rates (in 10^{-6} eV) with c spectator, For the modes including $c\bar{c}$ state, only η_c and J/ψ are contained.

		$a_1 = 1.26$ $a_2 = -0.51$
$B_c \to \eta_c + \pi^+$	$a_1^2 \cdot 2.07$	3.29
$B_c \to \eta_c + \rho$	$a_1^2 \cdot 5.48$	8.70
✓ $B_c \to J/\psi + \pi^+$	$a_1^2 \cdot 1.97$	3.14
$B_c \to J/\psi + \rho$	$a_1^2 \cdot 5.95$	9.45
$B_c \to \eta_c + K^+$	$a_1^2 \cdot 0.161$	0.256
$B_c \to \eta_c + K^{+*}$	$a_1^2 \cdot 0.286$	0.453
$B_c \to J/\psi + K^+$	$a_1^2 \cdot 0.152$	0.242
$B_c \to J/\psi + K^{+*}$	$a_1^2 \cdot 0.324$	0.514
$B_c \to D^+ + D^0$	$a_2^2 \cdot 0.664$	0.173
$B_c \to D^+ + D^{0*}$	$a_2^2 \cdot 0.695$	0.181
$B_c \to D^{+*} + D^0$	$a_2^2 \cdot 0.653$	0.170
$B_c \to D^{+*} + D^{0*}$	$a_2^2 \cdot 1.08$	0.281
$B_c \to D_s + D^0$	$a_2^2 \cdot 0.340 \times 10^{-1}$	8.85×10^{-3}
$B_c \to D_s + D^{0*}$	$a_2^2 \cdot 0.354 \times 10^{-1}$	9.20×10^{-3}
$B_c \to D_s^* + D^0$	$a_2^2 \cdot 0.334 \times 10^{-1}$	8.68×10^{-3}
$B_c \to D_s^* + D^{0*}$	$a_2^2 \cdot 0.564 \times 10^{-1}$	0.015
$B_c \to \eta_c + D^+$	$(a_1 \cdot 0.193 + a_2 \cdot 0.440)^2$	3.40×10^{-4}
$B_c \to \eta_c + D^{+*}$	$(a_1 \cdot 0.181 + a_2 \cdot 0.430)^2$	7.40×10^{-5}
$B_c \to J/\psi + D^+$	$(a_1 \cdot 0.177 + a_2 \cdot 0.442)^2$	0.382×10^{-6}
$B_c \to \eta_c + D_s$	$(a_1 \cdot 1.13 + a_2 \cdot 1.98)^2$	0.173
$B_c \to \eta_c + D_s^*$	$(a_1 \cdot 1.04 + a_2 \cdot 1.90)^2$	0.118
$B_c \to J/\psi + D_s$	$(a_1 \cdot 1.02 + a_2 \cdot 1.95)^2$	0.085
$B_c \to \eta_c' + \pi^+$	$a_1^2 \cdot 0.268$	0.426
$B_c \to \eta_c' + \rho$	$a_1^2 \cdot 0.622$	0.987
$B_c \to \psi(2S) + \pi^+$	$a_1^2 \cdot 0.251$	0.398
$B_c \to \psi(2S) + \rho$	$a_1^2 \cdot 0.710$	1.13
$B_c \to \eta_c' + K^+$	$a_1^2 \cdot 0.020$	0.032
$B_c \to \eta_c' + K^{+*}$	$a_1^2 \cdot 0.031$	0.049
$B_c \to \psi(2S) + K^+$	$a_1^2 \cdot 0.018$	0.029
$B_c \to \psi(2S) + K^{+*}$	$a_1^2 \cdot 0.038$	0.060
$B_c \to \eta_c' + D^+$	$(a_1 \cdot 0.220 + a_2 \cdot 0.403)^2$	5.06×10^{-3}
$B_c \to \eta_c' + D^{+*}$	$(a_1 \cdot 0.174 + a_2 \cdot 0.366)^2$	1.10×10^{-3}
$B_c \to \psi(2S) + D^+$	$(a_1 \cdot 0.174 + a_2 \cdot 0.373)^2$	8.56×10^{-4}
$B_c \to \eta_c' + D_s$	$(a_1 \cdot 1.31 + a_2 \cdot 1.84)^2$	0.502
$B_c \to \eta_c' + D_s^*$	$(a_1 \cdot 0.981 + a_2 \cdot 1.58)^2$	0.185
$B_c \to \psi(2S) + D_s$	$(a_1 \cdot 0.988 + a_2 \cdot 1.62)^2$	0.173

$B_r(B_c \to J/\psi\,\pi^+) \simeq 0.17\%$

Some of typical decay channels of c-decay (nonleptonic)

Table 3. Exclusive two body nonleptonic decay rates (in 10^{-6} eV) with b spectat

		$a_1 = 1.12$ $a_2 = -0.26$	
$B_c \to B_s + \pi^+$	$a_1^2 \cdot 58.4$	73.3	quite big
$B_c \to B_s + \rho$	$a_1^2 \cdot 44.8$	56.1	
$B_c \to B_s^* + \pi^+$	$a_1^2 \cdot 51.6$	64.7	
$B_c \to B_s^* + \rho$	$a_1^2 \cdot 150.$	188.	
$B_c \to B_s + K^+$	$a_1^2 \cdot 4.20$	5.27	
$B_c \to B_s^* + K^+$	$a_1^2 \cdot 2.96$	3.72	
$B_c \to B^+ + K^0$	$a_2^2 \cdot 96.5$	4.25	
$B_c \to B^+ + K^{0*}$	$a_2^2 \cdot 68.2$	3.01	
$B_c \to B^{+*} + K^0$	$a_2^2 \cdot 73.3$	3.23	
$B_c \to B^{+*} + K^{0*}$	$a_2^2 \cdot 141.$	6.23	
$B_c \to B^+ + \phi$	$a_2^2 \cdot 14.7$	0.650	
$B_c \to B^{+*} + \phi$	$a_2^2 \cdot 10.7$	0.471	
$B_c \to B^0 + \pi^+$	$a_1^2 \cdot 3.30$	4.14	
$B_c \to B^0 + \rho$	$a_1^2 \cdot 5.97$	7.48	
$B_c \to B^{0*} + \pi^+$	$a_1^2 \cdot 2.90$	3.64	
$B_c \to B^{0*} + \rho$	$a_1^2 \cdot 11.9$	15.0	
$B_c \to B^0 + K^+$	$a_1^2 \cdot 0.255$	0.320	
$B_c \to B^0 + K^{+*}$	$a_1^2 \cdot 0.180$	0.226	
$B_c \to B^{0*} + K^+$	$a_1^2 \cdot 0.195$	0.244	
$B_c \to B^{0*} + K^{+*}$	$a_1^2 \cdot 0.374$	0.469	
$B_c \to B^+ + \pi^0$	$a_2^2 \cdot 1.65$	0.0738	
$B_c \to B^+ + \rho$	$a_2^2 \cdot 2.98$	0.132	
$B_c \to B^{+*} + \pi^0$	$a_2^2 \cdot 1.45$	0.064	
$B_c \to B^{+*} + \rho$	$a_2^2 \cdot 5.96$	0.263	

FIGURES

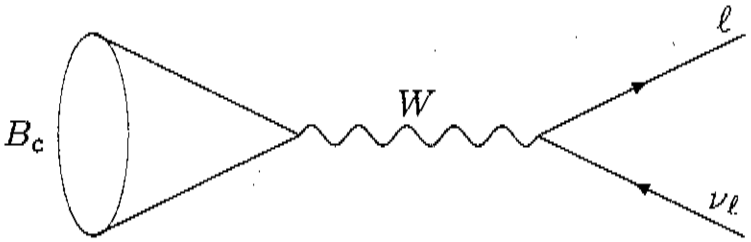


FIG. 1. **Tree diagram for** $B_c \longrightarrow \ell\nu_\ell$.

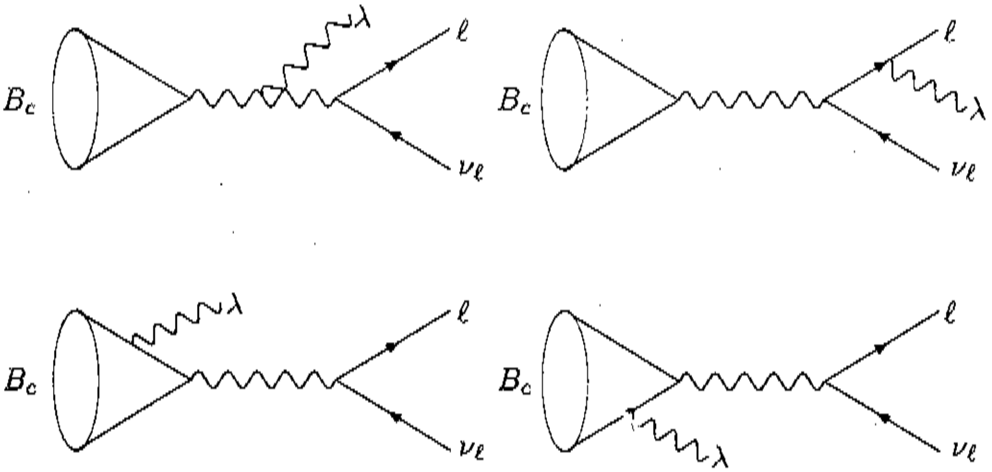
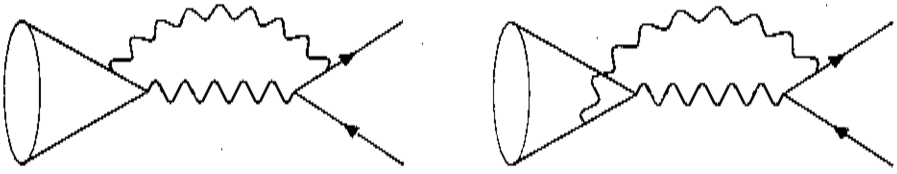


FIG. 2. **Diagrams for** $B_c \longrightarrow \ell\nu\gamma$.



(a) (b)

FIG. 3. 1. **Box-loop diagrams for** $B_c \longrightarrow \ell\nu$.

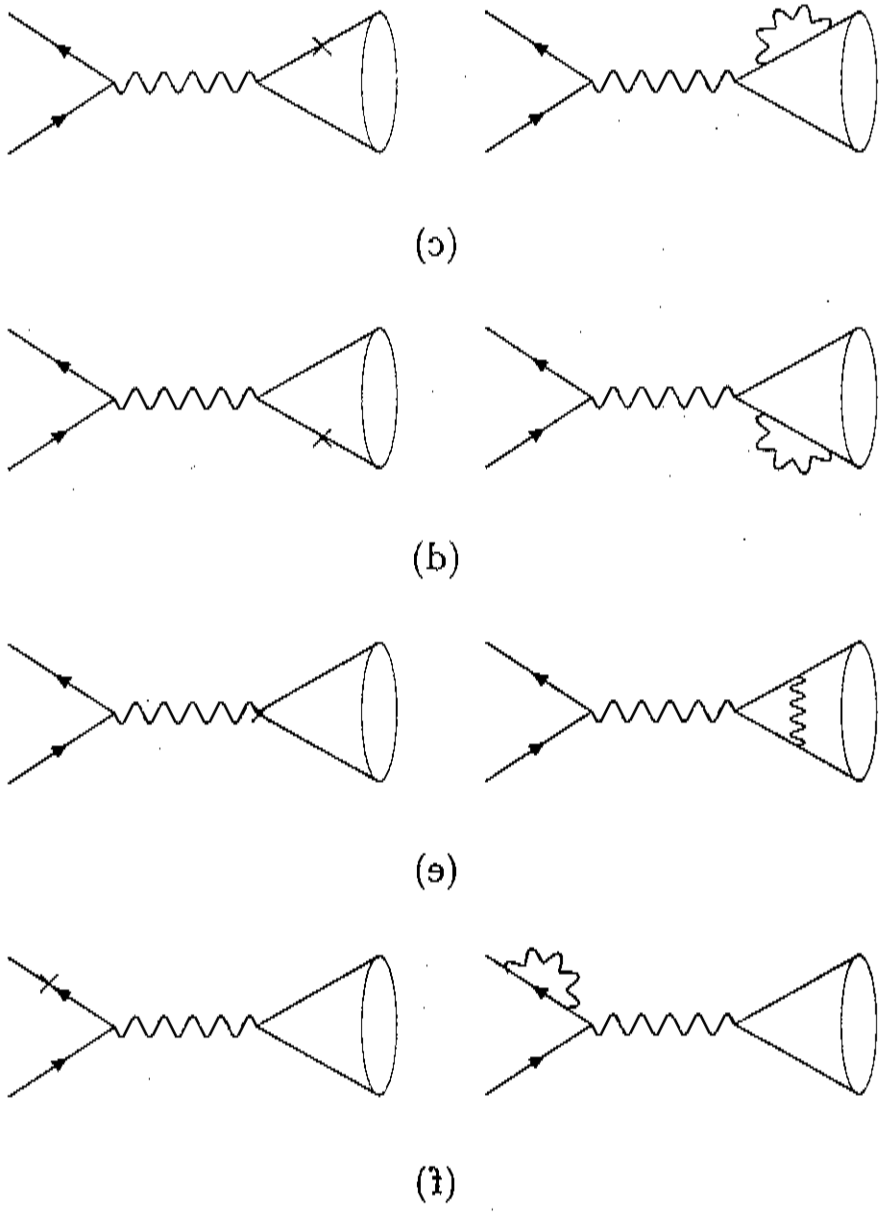


FIG. 3. 2. Self-energy and vertex diagrams for $B_c \longrightarrow \ell\nu$.

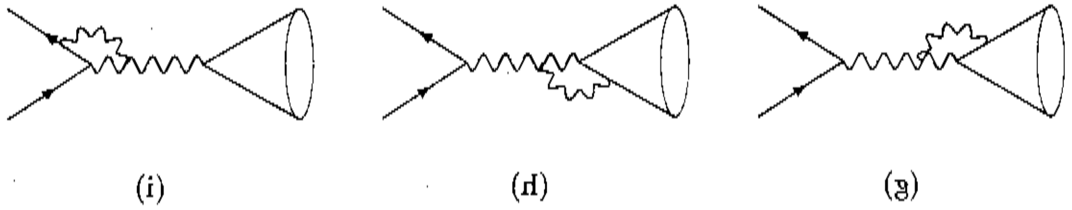
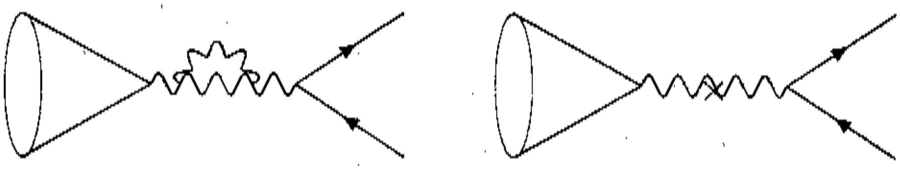
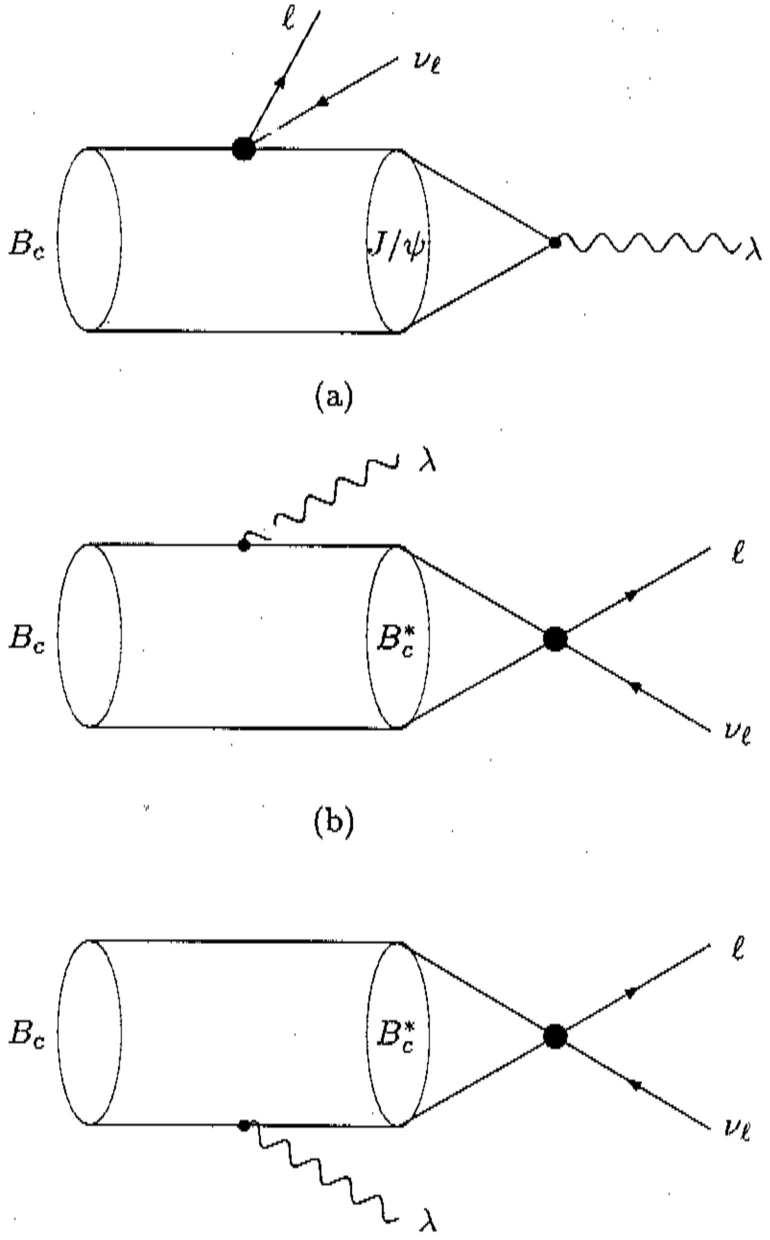


FIG. 3. 3. Vertex diagrams for $B_c \longrightarrow \ell\nu$.



(j)

FIG. 3. 4. Self-energy diagrams for $B_c \longrightarrow \ell\nu$.



(a)

(b)

(c)

Table (5): The Radiative Decay Widths (in unit 10^{-17}GeV)

ith cuts of the photon momentum and the angle between photon and lepton)

		(1)			(2)		
$k_{min(GeV)}$		5^0	15^0	30^0	5^0	15^0	30^0
0.1	Γ_e	6.384	6.370	6.297	6.832	6.819	6.752
0.2	Γ_e	6.317	6.303	6.242	6.762	6.750	6.693
0.5	Γ_e	5.931	5.918	5.883	6.351	6.340	6.307
1.0	Γ_e	4.807	4.800	4.790	5.151	5.143	5.136
0.1	Γ_μ	6.613	6.518	6.385	7.049	6.958	6.834
0.2	Γ_μ	6.484	6.412	6.306	6.920	6.850	6.753
0.5	Γ_μ	6.018	5.977	5.917	6.433	6.394	6.340
1.0	Γ_μ	4.843	4.824	4.802	5.184	5.165	5.146
0.1	Γ_τ	13.75	13.66	12.88	13.60	13.52	12.78
0.2	Γ_τ	10.87	10.82	10.34	10.86	10.81	10.36
0.5	Γ_τ	7.139	7.121	6.970	7.282	7.265	7.122
1.0	Γ_τ	4.169	4.165	4.146	4.340	4.335	4.318

Table (3) Branching Ratios of the 'Whole' Leptonic Decays

(short distance contributions)

	(1-a)	(2-a)	(1-b)	(2-b)
$B_e(10^{-5})$	5.09	5.45	4.5	4.82
$B_\mu(10^{-5})$	10.93	10.98	9.69	9.76
$B_\tau(10^{-2})$	1.477	1.407	1.306	1.246

Table (4) Tree Level Branching Ratios of The Pure Leptonic Decays

	(1-a)	(2-a)	(1-b)	(2-b)
$B_e(10^{-9})$	1.44	1.36	1.28	1.21
$B_\mu(10^{-4})$	0.62	0.586	0.55	0.52
$B_\tau(10^{-2})$	1.47	1.40	1.30	1.24

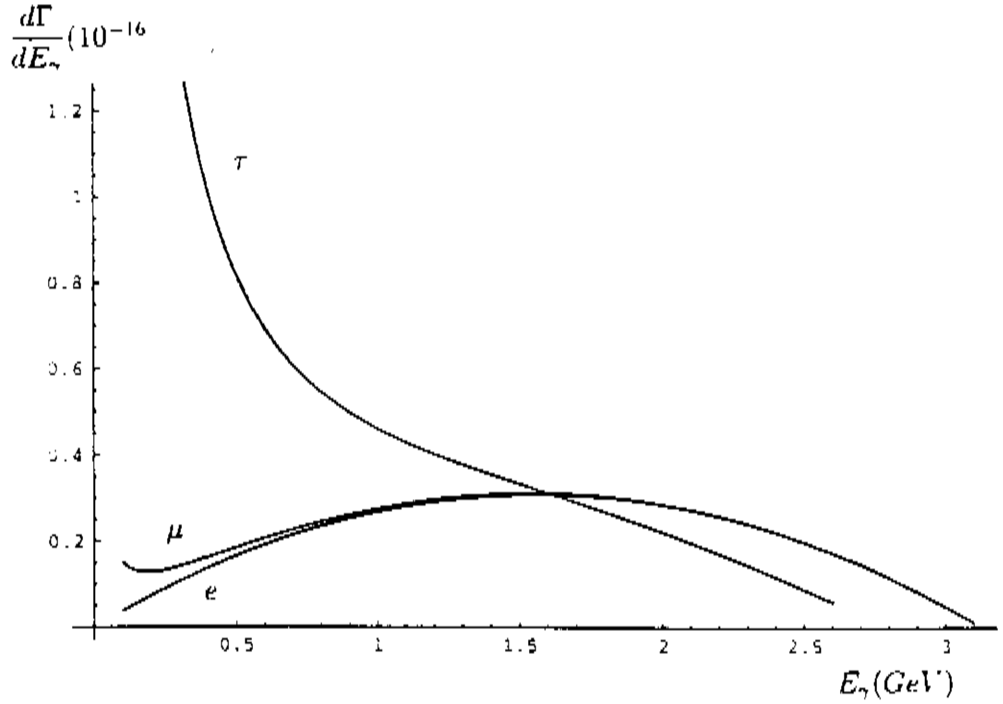


FIG. 6. **Photon energy spectra of radiative decays** $B_c \longrightarrow \ell\nu_\ell\gamma(\ell = e, \mu, \tau)$.

- **Outlooks**

Opportunities vs. Challenges for Run-II

A. Decays

① Mass (m_{B_c})

Measurements:

Simileptonic (ν-missing)

nonleptonic (exclusive, no missing)

$J/\psi\ \pi \Leftarrow$ } should be seen in Run-II
$J/\psi\ \rho \Leftarrow$

- - -

②. lifetime (τ_{B_c})

advatages: exclusive, no missing, - - -

through $J/\psi\ \pi$, $J/\psi\ \rho$, - -

③ More decay channels will be seen.

b-decay: $J/\psi\ \pi$, $J/\psi\ \rho$, - -

- - -

c-decay: $B_s \pi$, $B_s \rho$, $B_s^* \pi$, $B_s^* \rho$, ...

radiative leptonic decays

④ Form factors from semileptonic decays

ξ;

f_+, f_-, g, f, a_+, a_-

⑤ Decay constant f_{B_c} from radiative decay whereas, $B^{\pm} \to \gamma \ell \nu$ — background

$$\frac{N_{B_c}}{N_{B_u}} \simeq \frac{1}{4} \frac{f(b \to B_c)}{f(b - B_u)} \frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{f_{B_c}^2}{f_B^2} \left(\frac{m_{B_c}}{m_{B_u}}\right)^3 (X_b + X_c) \frac{m_u^2}{m_c^2}$$

$$X_b = \left(3 - \frac{m_{B_c}}{m_b}\right)^2, \quad X_c = \left(3 - 2\frac{m_{B_c}}{m_c}\right)^2$$

$$\simeq 0.8$$

$$\left\{ \begin{array}{ll} m_{B_c} \simeq 6.4\ \text{GeV} & \tau_{B_c} \simeq 0.4\ \text{ps} \\ m_B \simeq 5.278\ \text{GeV} & \tau_{B^+} \simeq 1.65\ \text{ps} \end{array} \right\}$$

⑥. strange

B. Production

High statistics
More decay channel $\Rightarrow$ Production Mechanism understanding

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Quite a lot of theories (models or approaches) will be tested and/or improved by Tevatron Run-II results.

Theories also need experimental data as input.