Semileptonic Decays in Run II

??

Inclusives	Exclusives

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??

Inclusives

Set aside question of whether accesible here. Learn anything? (beyond B-factories).

$\Lambda_b \to X e\nu$ (moments, spectrum)

Moreover, theory discussed in

- BaBar book
- Wise's talk (?)

Exclusives

3 bodies*, full list

$B \to D^{(*)} \ell\nu$

$B \to \pi \ell\nu$

* $B \to \rho \ell\nu$

at B-fac's

$B_s \to D_s^{(*)} \ell\nu$

$B_s \to K^{(*)} \ell\nu$

* $B_c \to B_{s,d}^{(*)} \ell\nu$

$B_c \to D^{0(*)} \ell\nu$

$B_c \to (\eta_c, \psi)^{(*)} \ell\nu$

* $\Lambda_b \to \Lambda_c \ell\nu$

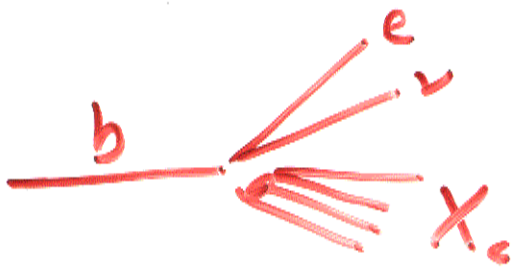
* hadron is (almost) stable

Moments: determine $\bar{\Lambda}$ & λ_1.

Kinematics for $B \to X_c \ell \nu$

$$s = (p_B - p_e - p_\nu)^2 = \text{invariant hadronic mass}$$

$$E = \frac{1}{m_B} p_B \cdot (p_B - p_e - p_\nu) = \text{hadronic energy in the } B\text{-frame}$$



$p_{X_c} = p_B - p_e - p_\nu$

Define moments using

$$\langle \cdot \rangle = \frac{1}{\Gamma} \int \frac{d^2\Gamma}{ds\,dE} \cdot$$

Falk et al, PRD 53, 2491 & 6316

Gremm & Kapustin PRD 55, 6924

with $\bar{m}_D = \frac{m_D + 3m_{D^*}}{4}$

$$\langle (s - \bar{m}_D^2) \rangle = m_B^2 \left[0.051 \frac{\alpha_s}{\pi} + 0.23 \frac{\bar{\Lambda}}{m_B} \left(1 + 0.43 \frac{\alpha_s}{\pi}\right) + 0.26 \frac{\bar{\Lambda}^2}{m_B^2} + 1.01 \frac{\lambda_1}{m_B^2} - 0.31 \frac{\lambda_2}{m_B^2} \right]$$

$$\langle (s - \bar{m}_D^2)^2 \rangle = m_B^4 \left[0.0053 \frac{\alpha_s}{\pi} + 0.067 \frac{\alpha_s}{\pi} \frac{\bar{\Lambda}}{m_B} + 0.065 \frac{\bar{\Lambda}^2}{m_B^2} - 0.14 \frac{\lambda_1}{m_B^2} \right]$$

- Extract $\bar{\Lambda}$ & λ_1 from these (in principle)
  → use in Γ → $|V_{cb}|$.
- Practice: modify to allow only $E_e > 1.5$ GeV
  Do also E moments as check.

☆☆ → Use also for $b \to u l \nu$ (in $\frac{d\Gamma}{ds}$)

- More checks from inclusive D decays.

Moreover, this is relevant to $B \to D^* l \nu$:
δ_{1/m^2} is constrained by sum-rules:

$$|\mathcal{F}_{D^*}(1)|^2 \le \eta_A - \frac{\lambda_2}{m_c^2} + \left(\frac{\lambda_1 + 3\lambda_2}{4}\right)\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3 m_c m_b}\right)$$

$$+ \frac{\alpha_s}{\pi} X_A + \frac{\alpha_s^2}{\pi^2} Y_A + \cdots$$

$$0 \le \frac{\lambda_2}{m_c^2} - \left(\frac{\lambda_1 + 3\lambda_2}{4}\right)\left[\frac{1}{m_c^2} + \frac{1}{m_b^2} - \frac{2}{3 m_c m_b}\right] + \frac{\alpha}{\pi} X_V + \frac{\alpha^2}{\pi^2} Y_V + \cdots$$

The 2nd sum rule can be used to put bounds on λ_1 ($\lambda_1 \lesssim 0.03\,\mathrm{GeV}^2$). The first gives

$$|\mathcal{F}_{D^*}(1)|^2 \le 1 - 0.064 + \frac{\lambda_1}{3 m_c^2}$$

where I've used $m_b = 3 m_c$ for simplicity.

For $\lambda_1 \simeq -0.8\,\mathrm{GeV}^2$, $|\mathcal{F}_{D^*}(1)|^2 \le 0.82$!

$\lambda_1 \simeq -0.2\,\mathrm{GeV}^2$ $|\mathcal{F}_{D^*}(1)|^2 \le 0.91$.

refs:

Moments in Inclusive $b \to c\ell\nu$ Decays

- Moments of experimental observables can aid in the extraction of $|V_{cb}|$ by constraining non-perturbative HQET parameters in the OPE:

$$\Gamma_{b\to c\ell\nu} = \frac{G_F^2 |V_{cb}|^2 M_B^5}{192\pi^3} 0.369 \left[1 - 1.54\frac{\alpha_s}{\pi} - 1.65\frac{\overline{\Lambda}}{M_B}\left(1 - 0.87\frac{\alpha_s}{\pi}\right) - 0.95\frac{\overline{\Lambda}^2}{M_B^2} - 3.18\frac{\lambda_1}{M_B^2} + 0.02\frac{\lambda_2}{M_B^2} \right]$$

- λ_1: squared average momentum of b quark inside meson (K.E.)
- λ_2: energy of hyperfine interaction of b quark with light d.o.f.
  - $\lambda_2 \cong 0.12$ (GeV/c^2)^2, from $B^* - B$ mass difference

$\overline{\Lambda}$: relates b quark and meson masses: $\overline{\Lambda} = M_B - M_b + \frac{\lambda_1 + 3\lambda_2}{2M_b} + \cdots$

Double expansions in $1/M_B$ and $\alpha_s(M_b)$ relate λ_1 and $\overline{\Lambda}$ to the first and second moments in hadronic-recoil mass and lepton energy

[Falk, Luke, and Savage, Phys. Rev. D **53**, 2491 (1996); *ibid.* D **53**, 6316 (1996); Voloshin, Phys. Rev. D **51**, 4934 (1995); Gremm *et al.*, hep-ph/9603448]

Hadronic Moments Measurement

- *e.g.*, second hadronic mass-squared moment expansion for $E_\ell > 1.5$ GeV:

$$\left\langle \left(M_{X_c}^2 - \overline{M}_D^2\right)^2 \right\rangle = M_B^4 \left[0.00148 \frac{\alpha_s}{\pi} + 0.038 \frac{\overline{\Lambda}}{M_B} \frac{\alpha_s}{\pi} + 0.0535 \frac{\overline{\Lambda}^2}{M_B^2} - 0.12 \frac{\lambda_1}{M_B^2} \right]$$

- Reconstruct mass of the X_c recoil system in $B \to X_c \ell \nu$ decay:

$$M_{X_c}^2 = (E_B - E_\ell - E_\nu)^2 - (\vec{P}_B - \vec{P}_\ell - \vec{P}_\nu)^2$$

$$= M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu} + 2|\vec{P}_B||\vec{P}_{\ell\nu}|\cos\theta_{\ell\nu,B}$$

- Candidate neutrino kinematics are “reconstructed” by exploiting:
  - the CLEO detector’s hermeticity for tracks and showers
  - the well-known beam energy
- In practice, since $\vec{P}_B$ is small in magnitude (~ 300 MeV/c) and its direction is unknown, we define:

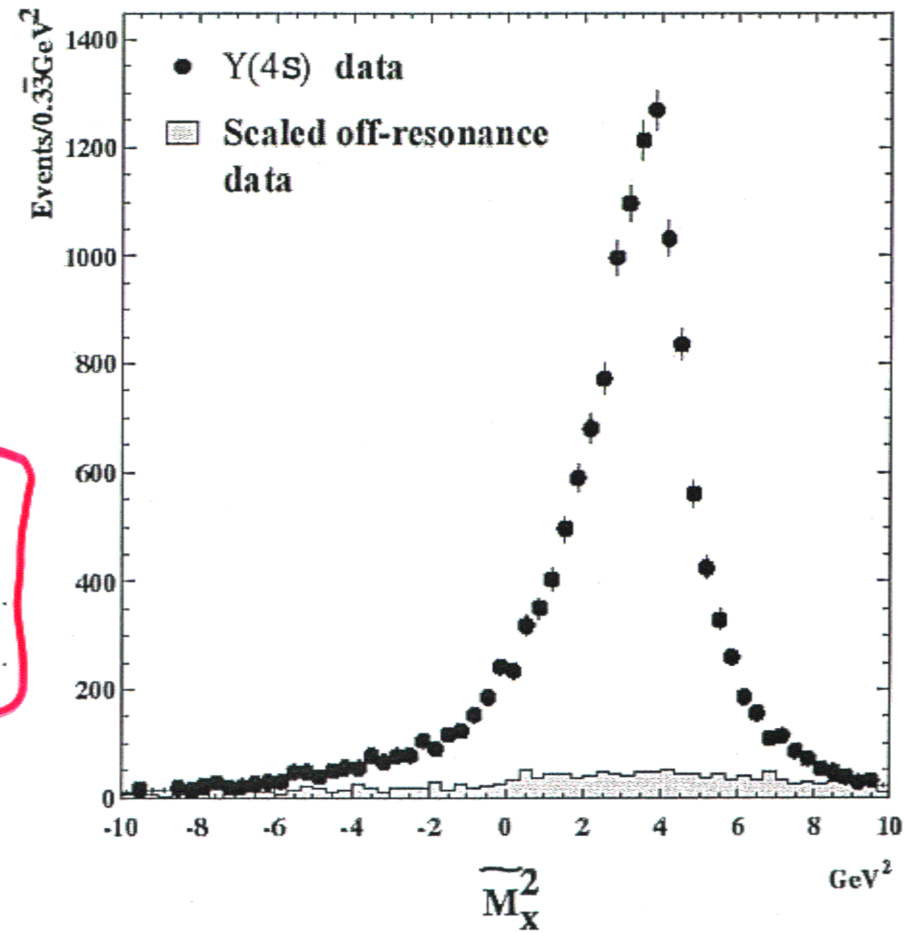
$$\tilde{M}_{X_c}^2 \equiv M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu}$$

Hadronic-Mass Moments Results

- Monte Carlo calculations estimate:
  - $b \to u\ell\nu$ contributions (~1%)
  - $b \to c \to s\ell\nu$ and J/ψ {and ψ(2S)} lepton contributions (~3%)
  - $\tilde{M}^2_{X_c} - M^2_{X_c}$ difference
  - asymmetric neutrino-momentum resolution
- Measure moments relative to
  $$\overline{M}^2_D = (1.975\,\mathrm{GeV}/c^2)^2$$
- First and second mass moments:

$$\langle M^2_{X_c} - \overline{M}^2_D \rangle = 0.286 \pm 0.023 \pm 0.080\,(\mathrm{GeV}/c^2$$
$$\langle (M^2_{X_c} - \overline{M}^2_D)^2 \rangle = 0.911 \pm 0.066 \pm 0.309\,(\mathrm{GeV}/c^2$$

~ 3.3×10^6 $B\overline{B}$ candidates



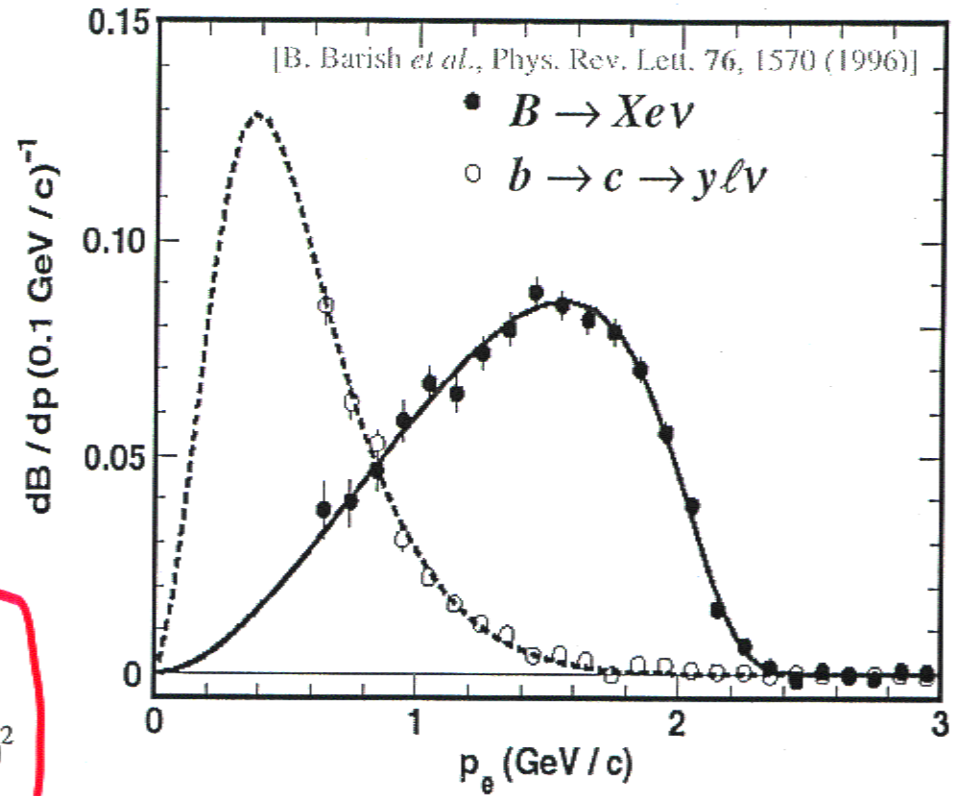
Lepton-Energy Moments Results

- B-meson tag using >1.4 GeV/c leptons
- Primary and secondary lepton distributions are identified using angular and charge correlations with tag lepton
- Correct lepton E_ℓ distribution for:
  - E. M. radiative corrections
  - experimental resolution (including *Bremsstrahlung*)
  - boost to B-meson rest frame
- Lepton energy moments:

$$\langle E_\ell \rangle = 1.36 \pm 0.01 \pm 0.02\ \mathrm{GeV}$$

$$\left\langle \left(E_\ell - \langle E_\ell \rangle\right)^2 \right\rangle = 0.190 \pm 0.004 \pm 0.005\ (\mathrm{GeV})^2$$

$\sim 2\times10^6\ B\overline{B}$ candidates



$b \to c \ell \nu$ HQET Parameter Results

ons:

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ent?

blem in the theory?

ass moments:

0.08 GeV/c^2

$\pm 0.06 \left(\text{GeV}/c^2\right)^2$

ding required

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and

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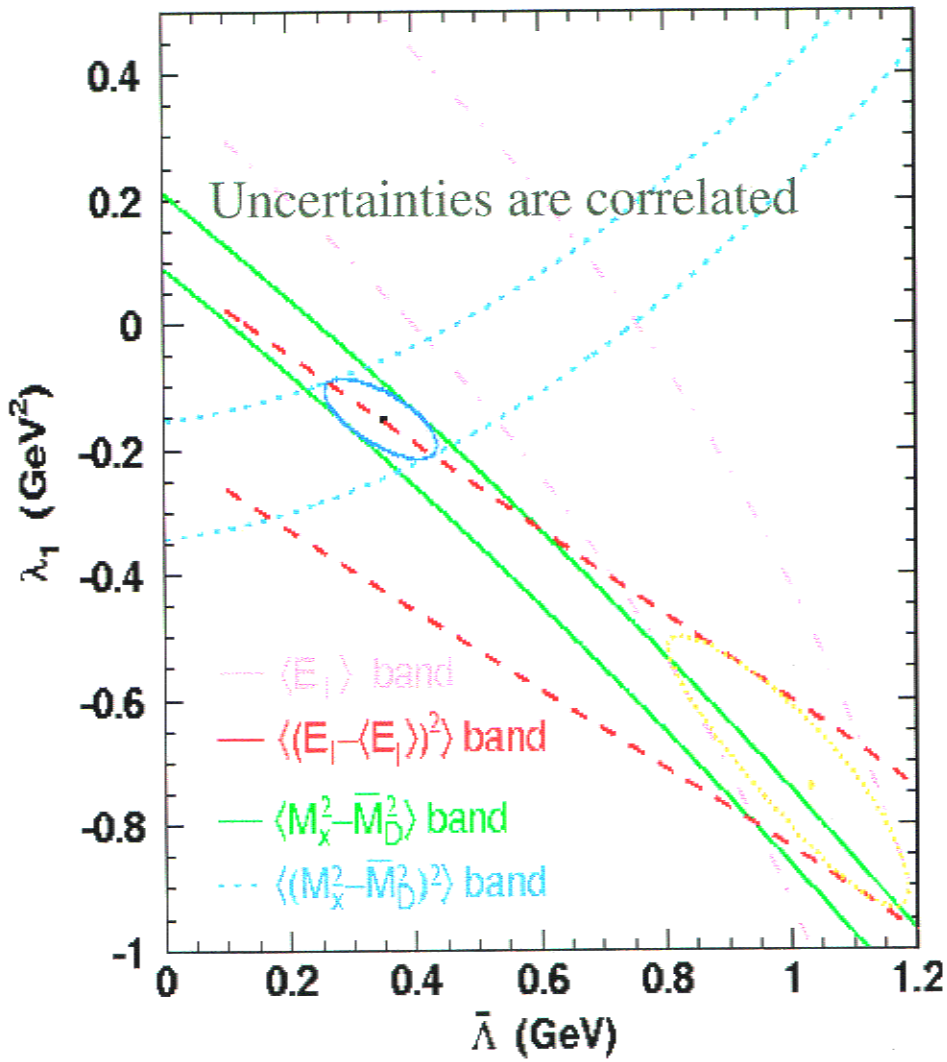

urement

$E_\ell > 1.5$ GeV:

$0535 \frac{\bar{\Lambda}^2}{M_B^2} - 0.12 \frac{\lambda_1}{M_B^2} \Big]$

y:

oiting:

and its direction is unknown,

$\Lambda_b \to \Lambda_c \ell \nu$

Form factors

$$\langle \Lambda_c(v', s') | V^\mu | \Lambda_b(v, s) \rangle =$$

$$\bar{u}^{(s')}(v') \left[F_1(w) \gamma^\mu + F_2(w) v^\mu + F_3(w) v'^\mu \right] u^{(s)}(v)$$

$$\langle \Lambda_c(v', s') | A^\mu | \Lambda_b(v, s) \rangle =$$

$$\bar{u}^{(s')}(v') \left[G_1(w) \gamma^\mu \gamma_5 + G_2(w) v^\mu \gamma_5 + G_3(w) v'^\mu \gamma_5 \right] u^{(s)}(v)$$

At $v' = v$ (use $\slashed{v} u = u$)

$$\langle \Lambda_c(v, s') | V^\mu | \Lambda_b(v, s) \rangle =$$

$$\left[F_1(1) + F_2(1) + F_3(1) \right] \bar{u}^{s'}(v) u^s(v) \, v^\mu$$

$$\langle \Lambda_c(v, s') | A^\mu | \Lambda_b(v, s) \rangle = G_1(1) \, \bar{u}^{s'}(v) \gamma^\mu \gamma_5 u^s(v)$$

At $M \to \infty$ all ff's in terms of one IW-function.

To order $(1+\frac{\bar\Lambda}{m})(1+\alpha_s(m))$ still only one independent form factor.

cho & BG PLB285 ('92) 153

$$\frac{F_1}{G_1} = 1 + \left(\frac{\bar\Lambda}{2m_c} + \frac{\bar\Lambda}{2m_b}\right)\frac{2}{w+1} + \frac{4}{3}\frac{\alpha_s}{\pi} r + \frac{4\alpha_s}{3\pi}\frac{\bar\Lambda}{2m_c}\frac{2(1+r-wr)}{w+1}$$

$$\frac{F_2}{G_1} = \frac{G_2}{G_1} = -\frac{\bar\Lambda}{2m_c}\frac{2}{w+1} - \frac{4\alpha_s}{3\pi} - \frac{4\alpha_s}{3\pi}\frac{\bar\Lambda}{2m_c}\frac{2(1+r-wr)}{w+1}$$

$$\frac{F_3}{G_1} = -\frac{G_3}{G_1} = -\frac{\bar\Lambda}{2m_b}\frac{2}{w+1}$$

For all $w \rightarrow$ $F_1 + F_2 + F_3 = G_1$

At $w=1$ $$G_1(1) = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25}(1 + \underbrace{\ldots}_{O(\alpha_s)})$$

Here

$\alpha_s = \alpha_s(m_c)$ $\qquad r = r(w) = \frac{1}{\sqrt{w^2-1}}\ln(w+\sqrt{w^2-1})$

$\bar\Lambda = m_{\Lambda_Q} - m_Q$ (should call it something else?!)

Shape of ff's? constrained by QCD
+ unitarity/analiticity/crossing. (Lebed & Boyd ph/9512363)
(They did not combine results of constraints
with HQET rel's). Else →
Models, eg Shih/Lee/Li (ph 9908346)
PQCD model predict asymmetry (due to
polarization) ....

$|V_{ub}|$

Ligeti, (Stewart), Wise ph/9512225
ph/9711 248

Idea: HQ + LF symmetries $\to$

$$\frac{f^{B\to\rho}}{f^{B\to K^*}} = \frac{f^{D\to\rho}}{f^{D\to K^*}}\left(1 + O\left(\frac{m_s}{m_c} - \frac{m_s}{m_b}\right)\right)$$

$\sim 10\%$

Use $f^{B\to\rho}$ to extract $|V_{ub}|$. Need to measure

$f^{D\to K^*}$ $\to$ from $D\to K^* \ell\nu$

$f^{D\to\rho}$ $\to$ from $D\to\rho\,\ell\nu$

$f^{B\to K^*}$ $\to$ from $B\to K^* e^+e^-$ $\leftarrow$ not at B-factories (rate)
or $B\to K^* \nu\bar{\nu}$

Def's:

$$\langle \rho(p',\epsilon)|\bar{q}\gamma_\mu Q|B(p)\rangle = 2ig\,\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p'^{\lambda}p^{\sigma}$$

$$\langle \rho(p',\epsilon)|\bar{q}\gamma_\mu\gamma_5 Q|B(p)\rangle = f\epsilon^*_\mu + a_+\epsilon^*\cdot p\,(p+p')_\mu + a_-\epsilon^*\cdot p\,(p-p')_\mu$$

For any of these

$$F^{(B\to V)}(y) = \left(\frac{m_B}{m_D}\right)^{1/2}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25} F^{(D\to V)}(y)$$

where $y \equiv \frac{-q^2+m_B^2+m_V^2}{2m_Bm_V}$ (like $v\cdot v'$)

and $V = \rho$ or K^*.

Limited kinematics in relation.

$$\frac{d\Gamma}{dy}(B\to\rho\ell\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{48\pi^3} m_B^3 r^2 S(y)$$

$$r = m_\rho / m_B$$

$$S(y) = \sqrt{y^2-1}\,(2+y^2-6yr+3r^2)\cdot$$
$$|f^{B\to\rho}(y)|^2 \left[1+\delta^{B\to\rho}(y)\right]$$

$\delta^{B\to\rho}$ depends on a_+/f and g/f

$|\delta^{B\to\rho}| < 5\%$ in $1<y<1.5$

$|\delta^{B\to K^*}| < 8\%$ in $1<y<1.5$

Aim

$$|f^{B\to\rho}|^2(1+\delta^{B\to\rho}) = |f^{B\to K^*}|^2(1+\delta^{B\to K^*})\left|\frac{f^{D\to\rho}}{f^{D\to K^*}}\right|$$

Now $B\to K^* e^+ e^-$ treated fully in rare decays section. Won't repeat here.

Write

$$\frac{d\Gamma}{dy}(B\to K^* \ell \bar{\ell}) = G_F^2 |V_{ts} V_{tb}|^2 \left(\frac{\alpha}{4\pi}\right)^2 m_B^3 r^2 \left[|\tilde{C}_9|^2 + |C_{10}|^2\right]$$

$$\times |f^{B\to K^*}(y)|^2 (1+\delta^{B\to K^*})(1+\Delta(y))\sqrt{y^2-1}\,(2+y^2-6yr+3r^2)$$

where $\Delta(y)$ contains dependence on C_7

(recall $O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} b_R$),

and $\tilde{C}_9 = C_9$ + contrib from 'light' (u, c) quarks in loop

---

$\Delta(y)$ is independent of ff's at $y=1$, and

$\Delta(1) \simeq -0.14$, $\Delta(1.5) \simeq -0.18$ almost linearly

(obtained from $\frac{g^{B\to K^*}}{f^{B\to K^*}} = \frac{g^{D\to K^*}}{f^{D\to K^*}} \cdots$).

$|V_{ub}|$ from $B_c \to D^{0(*)} \ell \nu$

Also $B_c \to B_{d,s} \ell \nu$ & $B_c \to (\eta_c, J/\psi) \ell \nu$.

Jenkins et al ph/9204238

$$\langle B_a^{(*)}(v,q) | \bar{q}_a \Gamma c | B_c, v \rangle = -\sqrt{m_{B_c} m_{B_a}} \cdot$$

Hilbert space of v

momentum there

$$\mathrm{Tr}\left(\bar{H}_a^{\bar{b}} \, \Omega(v, a_0 q) \, \Gamma \, H^{(c\bar{b})} \right)$$

Bohr radius

$$\Omega(v, a_0 q) = \Omega_1(a_0 q) + a_0 \Omega_2(a_0 q) \not{q}$$

$$H^{(c\bar{b})} = \left(\frac{1+\not{v}}{2}\right)\left[B_c^{*\mu} \gamma_\mu - B_c \gamma_5\right]\left(\frac{1-\not{v}}{2}\right)$$

$$\Rightarrow \langle B_a | V_\mu | B_c \rangle = \sqrt{2 m_{B_c} 2 m_{B_a}} \left[\Omega_1 v_\mu + a_0 \Omega_2 q_\mu \right]$$

$$\langle B_a^* | V_\mu | B_c \rangle = 2(-i) \sqrt{m_{B_c} m_{B_a}} \, a_0 \Omega_2 \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha v^\beta$$

$$\langle B_a^* | A_\mu | B_c \rangle = 2\sqrt{m_{B_c} m_{B_a}} \left[\Omega_1 \epsilon^*_\mu + a_0 \Omega_2 \epsilon^* \cdot q \, v_\mu \right]$$

Near zero recoil, and $V_\mu = \bar{b}\gamma_\mu u$ ...

$$\langle D^0 | V_\mu | B_c \rangle = 2\sqrt{m_{B_c} m_D}\,\left[\Sigma_1 v_\mu + a_0 \Sigma_2 q_\mu \right]$$

$$\langle D^{0*} | V_\mu | B_c \rangle = -i\,2\sqrt{m_{B_c} m_D}\; a_0 \Sigma_2\, \epsilon_{\mu\nu\alpha\beta}\, \epsilon^{*\nu} q^\alpha v^\beta$$

$$\langle D^{0*} | A_\mu | B_c \rangle = 2\sqrt{m_{B_c} m_D}\,\left[\Sigma_1 \epsilon^*_\mu + a_0 \Sigma_2\, \epsilon^* \cdot q\, v_\mu \right]$$

and for $V_\mu = \bar{b}\gamma_\mu c$

$$\langle \eta_c | V_\mu | B_c \rangle = 2\sqrt{m_{B_c} m_{\eta_c}}\;\Delta\; v_\mu$$

$$\langle \psi | A_\mu | B_c \rangle = 2\sqrt{m_{B_c} m_\psi}\;\Delta\; \epsilon^*_\mu$$

So at zero recoil expect

$$\sim f_{M_Q} \cdot f_{B_c}$$

with scale of variations a_0^{-1}

$$\Omega_1 \simeq \frac{1}{\sqrt{2}} f_{B_c} \sqrt{m_{B_c}} \int d^3x \, e^{i\vec{q}\cdot\vec{x}} \, \Psi_{B_c}(x)$$

for coulomb w.f.

$$= \frac{1}{\sqrt{2}} f_{B_c} \sqrt{m_{B_c}} \; \frac{8\sqrt{\pi}\, a_0^{3/2}}{(1 + a_0^2 \vec{q}^{\,2})^2}$$

Similarly

$$\Sigma_1 = \frac{1}{\sqrt{2}} f_D \sqrt{m_D} \; \frac{8\sqrt{\pi}\, a_0^{3/2}}{(1 + a_0^2 \vec{q}^{\,2})^2}$$

$$\Delta = 16 \, \frac{a_0^{3/2} a_\eta^{3/2}}{(a_0 + a_\eta)^3} \left[1 + \frac{\vec{q}^{\,2} a_0^2 a_\eta^2}{4(a_0 + a_\eta)^2} \right]^{-2}$$

For V_{ub} use ratios as $\dfrac{B_c \to D l \nu}{B_c \to B_s l \nu}$

Colangelo & DeFazio ph/9909423

TABLE II. Semileptonic B_c^+ decay widths and branching fractions.

Channel	$\Gamma(10^{-15}$ GeV)	$\Gamma_L(10^{-15}$ GeV)	$\Gamma_T(10^{-15}$ GeV)	BR
$B_c^+ \to B_s e^+ \nu$	11.1(12.9)	-	-	$0.8(0.9) \times 10^{-2}$
$B_c^+ \to B_s^* e^+ \nu$	33.5(37.0)	19.1(21.4)	7.2(7.8)	$2.3(2.5) \times 10^{-2}$
$B_c^+ \to B_d e^+ \nu$	0.9(1.0)	-	-	$0.06(0.07) \times 10^{-2}$
$B_c^+ \to B_d^* e^+ \nu$	2.8(3.2)	1.6(1.8)	0.6(0.8)	$0.19(0.22) \times 10^{-2}$
$B_c^+ \to \eta_c e^+ \nu$	2.1(6.9)	-	-	$0.15(0.5) \times 10^{-2}$
$B_c^+ \to J/\psi e^+ \nu$	21.6(48.3)	13.2(33.2)	4.2(7.6)	$1.5(3.3) \times 10^{-2}$
$B_c^+ \to \eta_c' e^+ \nu$	0.3(0.3)	-	-	$0.02(0.02) \times 10^{-2}$
$B_c^+ \to \psi' e^+ \nu$	1.7(1.7)	1.1(1.1)	0.3(0.3)	$0.12(0.12) \times 10^{-2}$
$B_c^+ \to D^0 e^+ \nu$	0.005(0.03)	-	-	$0.0003(0.002) \times 10^{-2}$
$B_c^+ \to D^{*0} e^+ \nu$	0.12(0.5)	0.08(0.35)	0.02(0.05)	$0.008(0.03) \times 10^{-2}$

relativistic
quark model

- Can these be observed?
- Are there model independent predictions for the ratio(s) $\frac{B_c \to D}{B_c \to B}$? (in some parametric sense).
- If so, what are the corrections?