Lattice Results (?)

for

Bag Parameters

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Matrix Elements

$\langle \bar{B}_q | \bar{b}\gamma^\mu(1-\gamma_5)q\,\bar{b}\gamma_\mu(1-\gamma_5)q | B_q \rangle$ $\quad B_B$, B_{B_s}

$\langle \bar{B}_s | \bar{b}(1-\gamma_5)s\,\bar{b}(1-\gamma_5)s | B_s \rangle$ $\quad B_S$

$\langle B | \bar{b}\gamma_\mu(1-\gamma_5)q\,\bar{q}\gamma_\mu(1-\gamma_5)b | B \rangle$ $\quad B_1$

$\langle B | \bar{b}(1-\gamma_5)q\,\bar{q}(1+\gamma_5)b | B \rangle$ $\quad B_2$

$\langle B | \bar{b}\gamma^\mu(1-\gamma_5)T^a q\,\bar{q}\gamma_\mu(1-\gamma_5)T^a b | B \rangle$ $\quad \varepsilon_1$

$\langle B | \bar{b}(1-\gamma_5)T^a q\,\bar{q}(1+\gamma_5)T^a b | B \rangle$ $\quad \varepsilon_2$

$\langle \Lambda | \bar{b}\gamma^\mu(1-\gamma_5)q\,\bar{q}\gamma_\mu(1-\gamma_5)b | \Lambda \rangle$ $\quad L_1$

$\langle \Lambda | \bar{b}\gamma^\mu(1-\gamma_5)T^a q\,\bar{q}\gamma_\mu(1-\gamma_5)T^a b | \Lambda \rangle$ $\quad L_2$

$\Delta m_{B-\bar{B}}$ $\qquad \dfrac{\Delta\Gamma}{\Gamma}$ $\qquad \dfrac{\tau(B^-)}{\tau(B^0)}$ $\qquad \dfrac{\tau(\Lambda_b)}{\tau(B^0)}$

Lattice QCD

Brute force, "first principles" calculation of these matrix elements.

Need careful analysis of systematic errors

Errors

[1] Discretization Errors

- finite lattice spacing a $(1 < a^{-1} < 3\ \mathrm{GeV})$
- simplest action $O(a)$

[2] Matching, Perturbative Errors

- matching coefficients (1-loop)
- expansion parameter

[3] Light Quarks

- cannot directly simulate u, d
  (chiral extrapolation)

[4] Heavy Quarks

- cannot simulate b with the simplest action

[5] Excited State Contamination

- wavefunctions are not known exactly

[6] Finite Volume Errors

- problem for light particles

[7] Quenching

- no sea quarks

[8] Statistical

- Monte Carlo estimate of exact functional integral

Discretization and Heavy Quarks

Glue: Wilson Action $\mathcal{O}(a^2)$ errors

Light Quarks: Wilson Action $\mathcal{O}(a)$ errors

SW Action $\mathcal{O}(\alpha a)$, $\mathcal{O}(a^2)$ errors

Heavy Quarks: Relativistic Actions

- nonrelativistic normalization (KLM)
- kinetic mass $\leftrightarrow$ pole mass

NRQCD

- $\mathcal{O}(\frac{1}{m})$, $\mathcal{O}(\frac{1}{m^2})$, ...

Static

- simple

* discretization errors reduced

* you can simulate the b quark directly.

Light Quarks

Chiral Extrapolation

- extrapolate to point where $m_\pi = 0$ (or physical K mass)

$a^{-1} \sim 2\,\text{GeV}$

$\kappa_c = 0.15711(7)$

Matching and Lattice Perturbation Theory

Choice of α_s^{latt} (tadpole improved)

$$\alpha^{latt}(a^{-1}) = \frac{6}{4\pi\beta u_0^4}$$

$\alpha_V^{latt}(q^*)$ from plaquette $\qquad \frac{1}{a} \leq q^* \leq \frac{\pi}{a}$

Matching Coefficients

- 1-loop perturbation theory $\quad \mathcal{O}(\alpha^2)$
- calculated in the $\left\{ \begin{array}{l} \text{static} \\ \text{massless} \end{array} \right\}$ limits
- work in progress for NRQCD and massive quarks
- new nonperturbative calculations (APE)

Excited State Contamination

Smooth-out Operators

- smearing
- fuzzing
- Coulomb gauge wavefunctions

Fitting

- multi-state fits

Quenching

$f_B \qquad 170\,(20)$ MeV $\longrightarrow$ $210\,(30)$ MeV

* becoming important (and possible)

B_B

Wilson
SW – Static

1) Kentucky ('98)

- fitting range 3%
- α^{latt} 1%
- matching 20%
- statistical 4%

2) UKQCD ('98)

- statistical 5%
- α^{latt} 4%
- matching 3%

3) Giménez, Martinelli ('98)

- statistical 6%
- α^{latt} 4%
- matching 3%

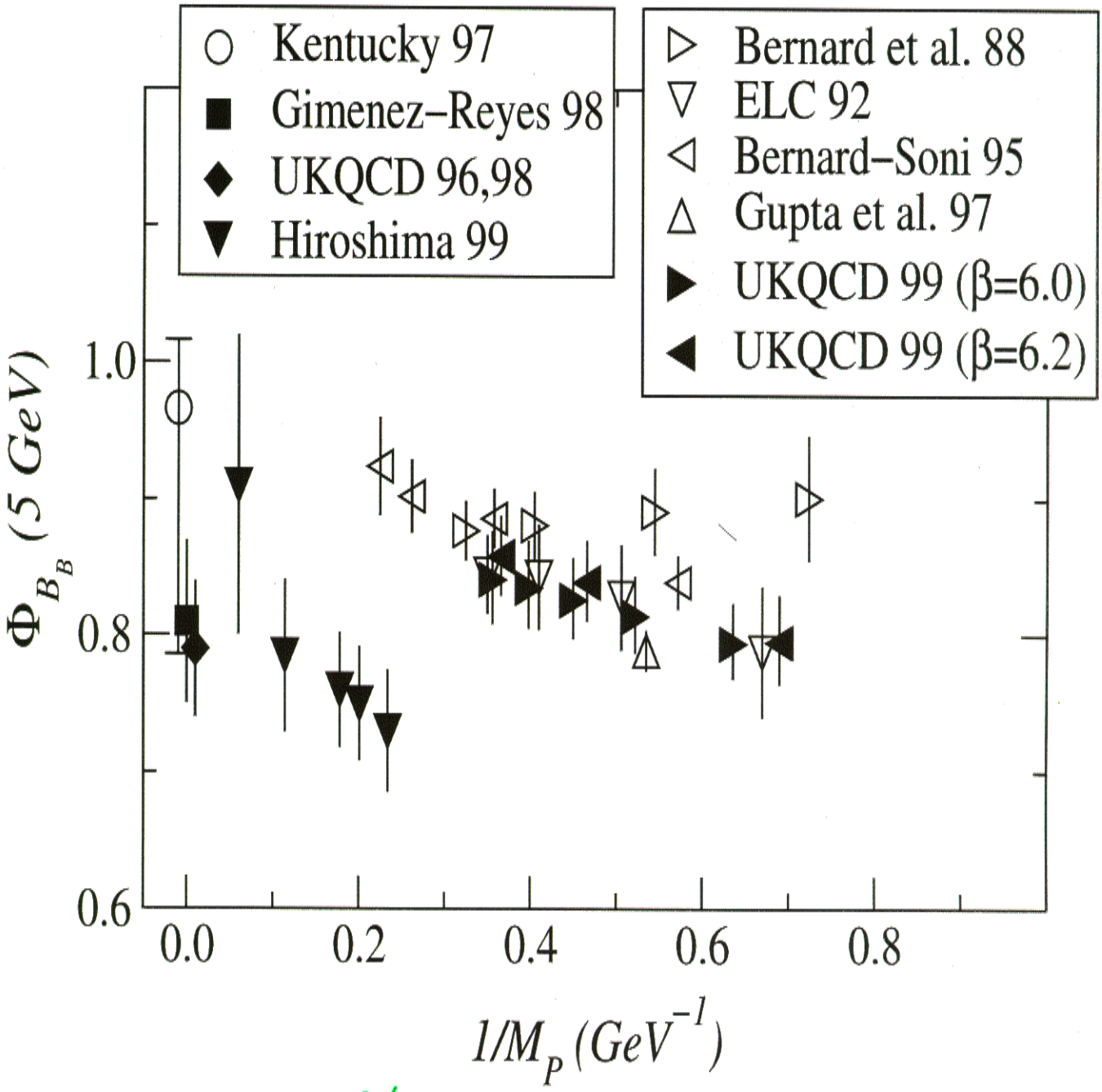
(2) and 3) reanalyzed by Giménez, Reyes)

$$B_B(4.33) = 0.98(4)^{+3}_{-18}$$

$$B_B(5) = 0.81(5)^{+4}_{-4}$$

$$B_B(5) = 0.79(4)^{+4}_{-4}$$

Hashimoto (Lattice '99)



$$\Phi_{B_B}(\mu) \equiv \left[\frac{\alpha_s(M_P)}{\alpha_s(M_B)}\right]^{2/\beta_0} B_B(\mu)$$

Wilson - Wilson

Bernard, Blum, Soni ('98)

- continuum limit extrapolation
- statistical 13 %
- massless matching coefficients

SW - SW

UKQCD (unpublished)

- statistical 4 %
- α^{latt} 3 %
- massless matching coefficients
- linear chiral, heavy quark extrapolation

SW - NRQCD

Hashimoto et.al. ('99)

* $\frac{1}{m}$, $\frac{1}{m^2}$ actions

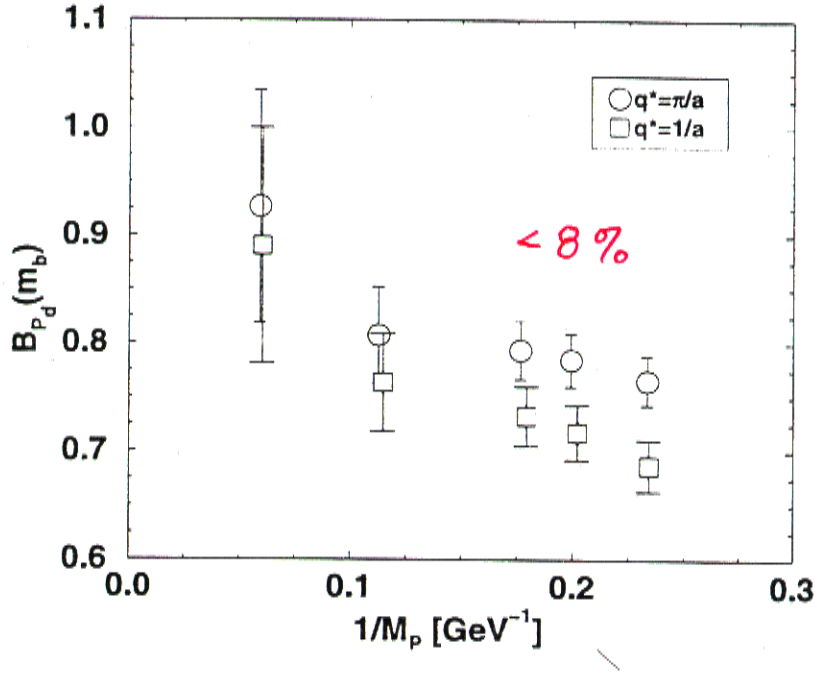
- static matching coefficients 10 %
- chiral extrapolation 3 % ($2\kappa_\ell$)
- α^{latt} 7 %
- statistical 4 %
- discretization 5 %

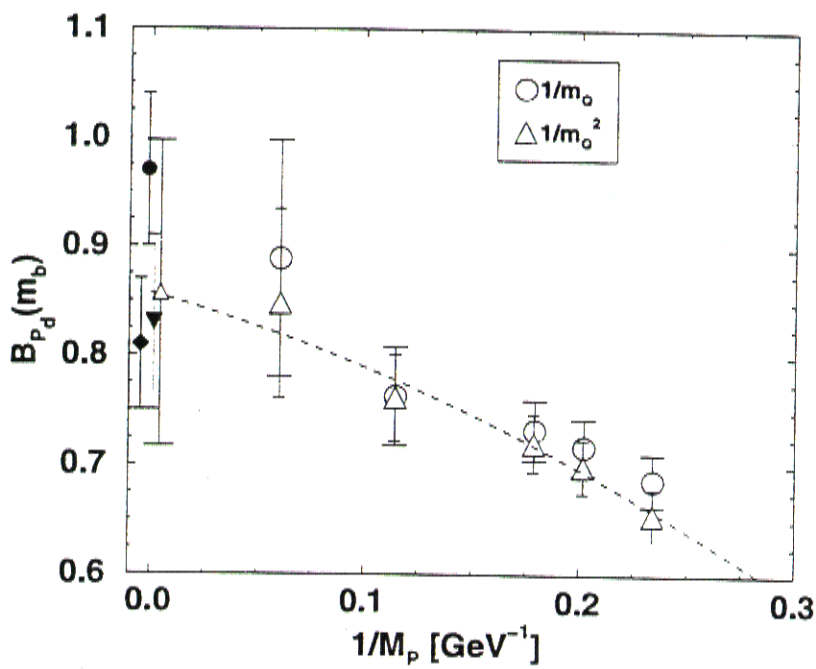
* $\frac{1}{m} \sim 10$ to $15\,\%$

$\frac{1}{m^2} \sim 2\,\%$

$B_B(m_b) = 0.75(3)(12)$

Hiroshima ('99)





$\frac{1}{m_Q}$ ~15 %

$\frac{1}{m_Q^2}$ ≲ 2 %

$$\boxed{B_{B_s} \qquad B_{B_s}/B_B}$$

Wilson-Wilson

BBS (98)

$$r_{sd} = \frac{\langle \bar{B}_s | \bar{b}\gamma^\mu(1-\gamma_5)s\,\bar{b}\gamma_\mu(1-\gamma_5)s | B_s \rangle}{\langle \bar{B}_d | \bar{b}\gamma^\mu(1-\gamma_5)d\,\bar{b}\gamma_\mu(1-\gamma_5)s | B_d \rangle}$$

- continuum limit extrapolation 10 %
  - constant 1.54 (13)
  - linear 1.72 (67)
- heavy quark extrapolation 10 %
- α^{latt} 20 %
- chiral extrapolation 4 %
- statistical 6 %

$$r_{sd} = 1.76\,(10)^{+57}_{-42}$$

SW-NRQCD

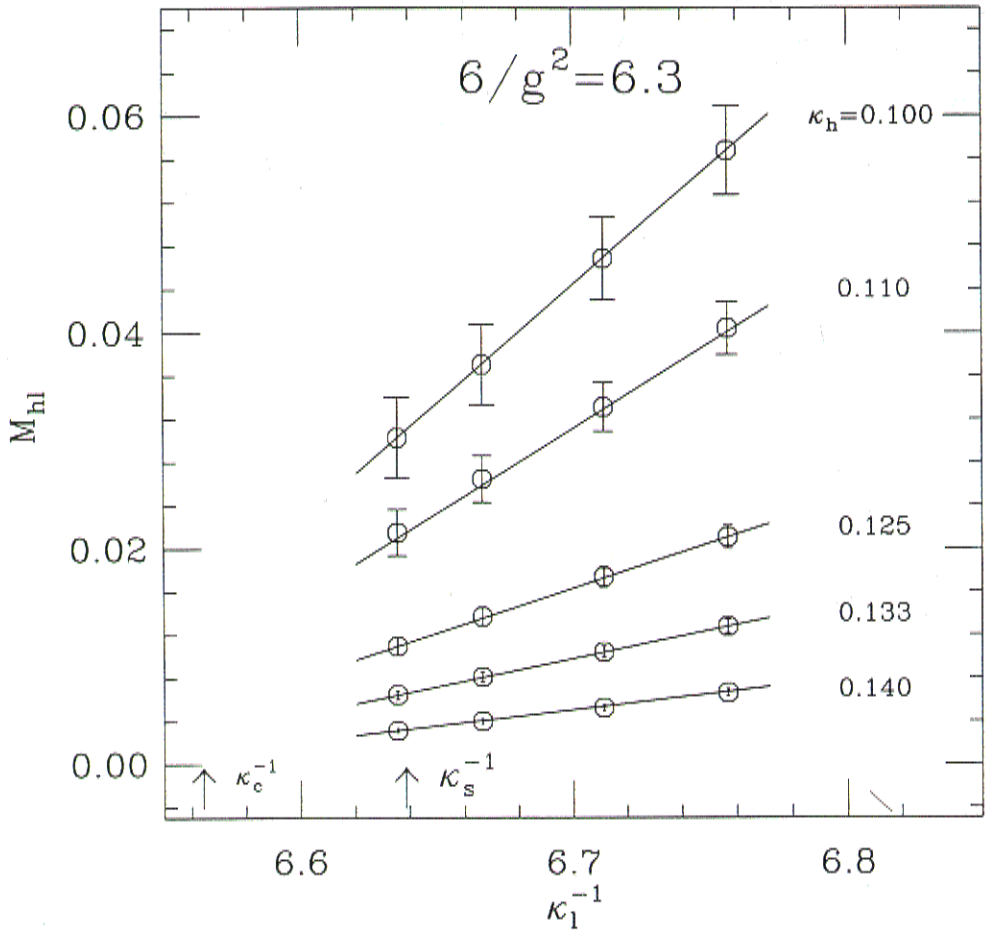
Hashimoto et.al. ('99)

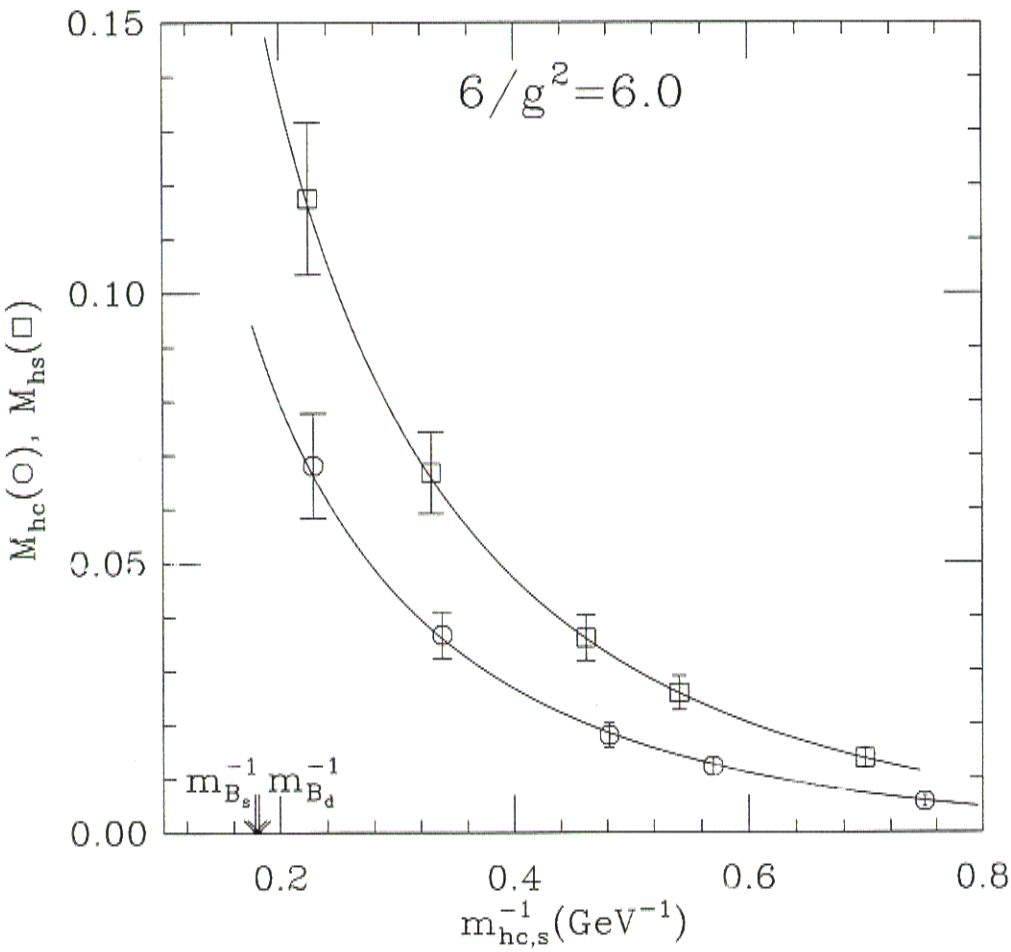
$$\frac{B_{B_s}}{B_B}$$

- systematical errors cancel
- statistical 1 %
- chiral extrapolation 3 %

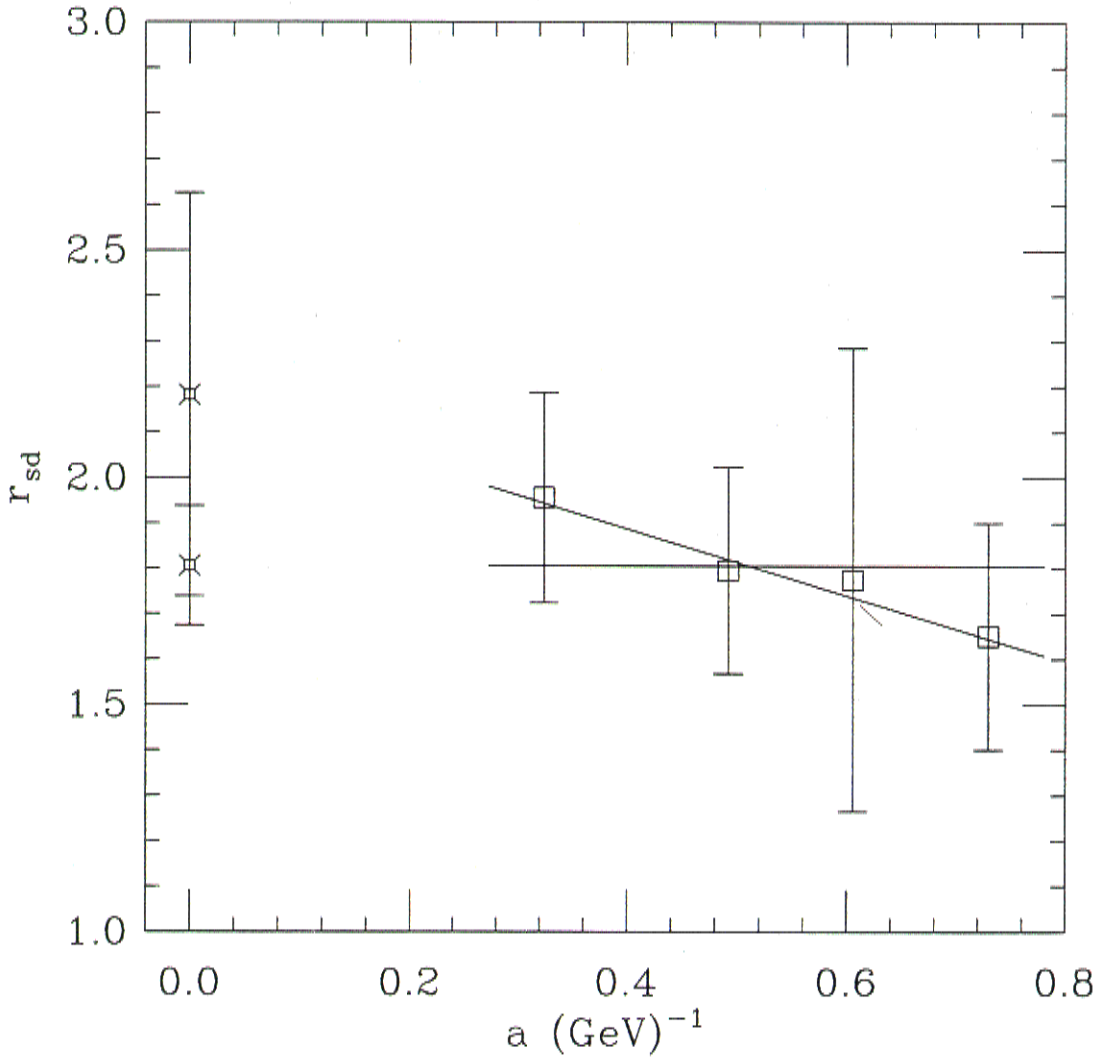
$$\frac{B_{B_s}}{B_B} = 1.01\,(1)(3)$$

Bernard
Blum
Soni ('98)





Bernard, Blum, Soni ('98)



$$r_{sd} = \frac{\langle \bar{B}_s | \bar{b}\gamma_\mu(1-\gamma_5)s\,\bar{b}\gamma_\mu(1-\gamma_5)s | B_s \rangle}{\langle \bar{B}_d | \bar{b}\gamma_\mu(1-\gamma_5)d\,\bar{b}\gamma_\mu(1-\gamma_5)d | B_d \rangle}$$

B_S

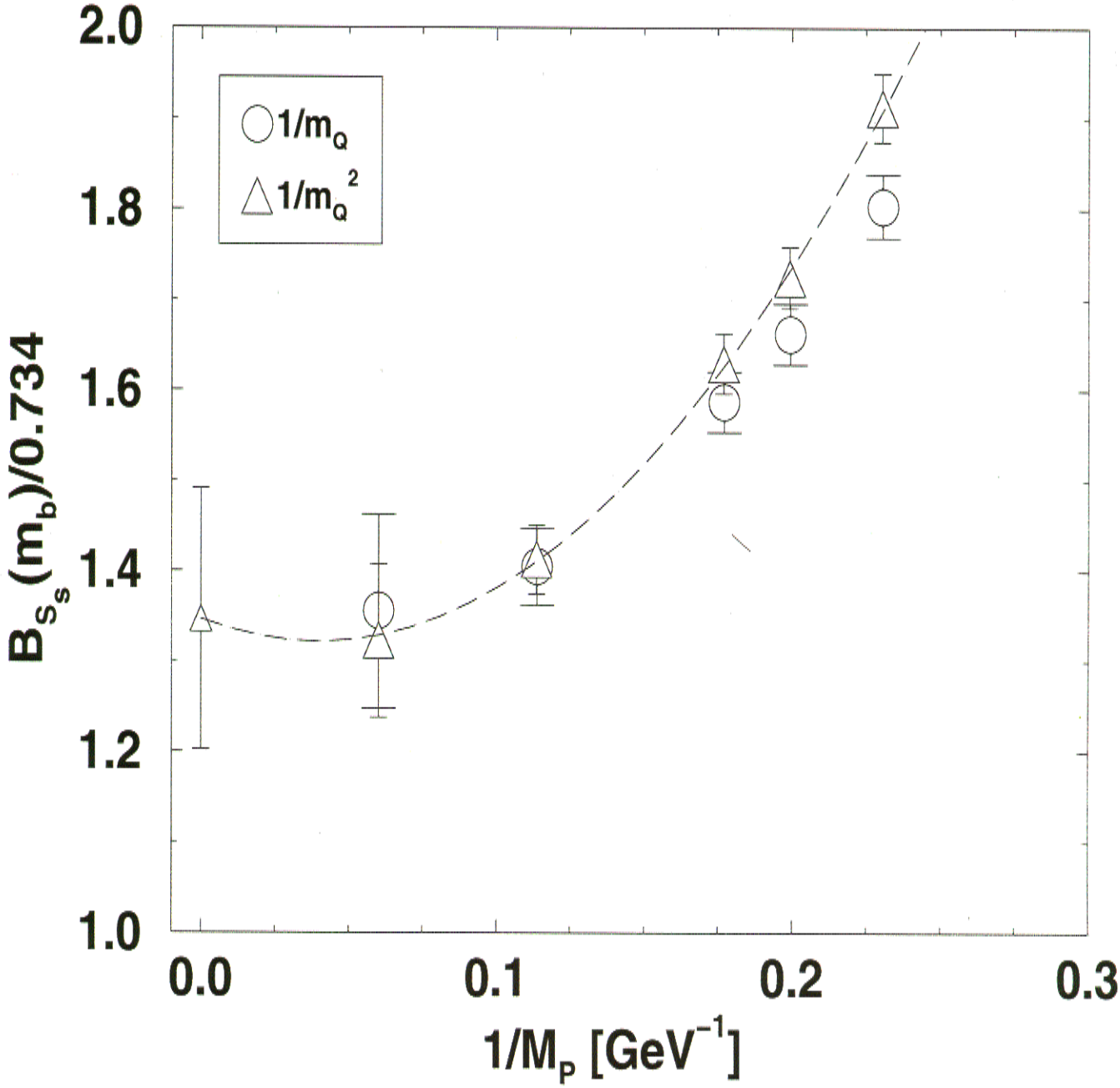
SW-NRQCD

Yamada et.al. (Lattice '99)

- similar analysis (B_B)
- $\frac{1}{m} \sim 20\%$
- $\frac{1}{m^2} \sim 3\%$
- statistical 2%

$B_S^{(m_b)} = 1.19(2)(20)$

Hiroshima (Lattice '99)



$$\boxed{\frac{\tau(B^-)}{\tau(B_d^0)}}$$

SW-Static

Di Pierro, Sachrajda ('98)

- matching coefficients 1-loop
- Wilson coefficients tree-level

} 3 %

- chiral extrapolation with 3 m_q
- statistical 2 %
- $\mathcal{O}(\alpha a)$

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.03(2)(3)$$

$$\boxed{\frac{\tau(\Lambda_b)}{\tau(B_d)}}$$

SW-Static

Di Pierro, Sachrajda, Michael ('99)

- Exploratory study
- similar analysis
- 2 "light" quarks
- $a^{-1} \sim 1$ GeV
- stochastic methods
- ground state extraction

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 - 0.02 - 0.05(1)$$

$m_u = 75$ MeV

Improvements

UKQCD

- final SW-SW results

Hiroshima, JLQCD

- 1-loop matching coefficients in NRQCD
- smearing
- increased statistics
- unquenching ?

FNAL

- 1-loop matching coefficients for massive quarks
- simulation at the b mass
- a dependence (continuum limit)

APE

- non perturbative matching

spectator effects

- 1 loop Wilson coefficients
- chiral extrapolations
- a dependence