Ulrich Nierste (Fermilab)

B_d and B_s mixing

A theory overview

workshop on B physics
at the Tevatron
Run-II and beyond

Outline

1 General formalism

2 Mass differences

Δm_{B_d}, Δm_{B_s}

3 Width differences

$\Delta \Gamma_{B_s}$, $\Delta \Gamma_{B_d}$

4 Preliminary Conclusions

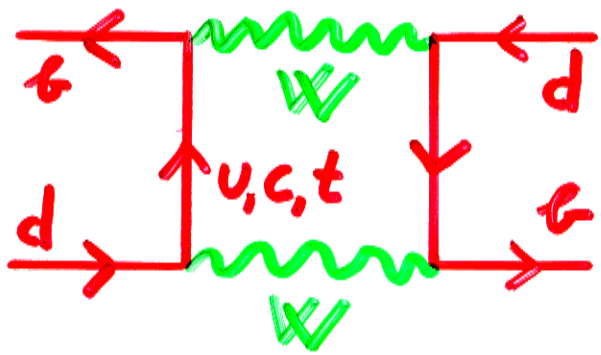
General formalism

flavor eigenstates

$|B_d^0\rangle \sim \bar{b}d, \; |\overline{B_d^0}\rangle \sim b\bar{d}$

$\neq$ mass eigenstates

due to $B^0 - \overline{B^0}$-mixing



$$i\frac{d}{dt}\begin{pmatrix}|B^0\rangle \\ |\overline{B^0}\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|B^0\rangle \\ |\overline{B^0}\rangle\end{pmatrix}$$

mass matrix

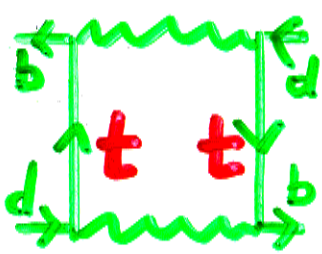
decay matrix

$M^\dagger M = \Gamma^\dagger \Gamma = 1$

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{12}^* & m_{22} \end{pmatrix}$$ mass matrix

$$m_{11} = m_{22} = 5.3 \text{ GeV}$$

CPT symmetry conserved
(until Saturday...)



$$|m_{12}| = 150\ \mu\text{eV} \text{ for } B_d$$

$(\approx \Delta m/2)$ $(|m_{12}| \approx 5 \text{ meV f. } B_s)$

13 orders of magnitude!

$$\Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$ decay matrix

$$\Gamma_{11} \overset{CPT}{=} \Gamma_{22} \approx 440\ \mu\text{eV}$$

$$|\Gamma_{12}| \approx \begin{cases} 2\ \mu\text{eV} & \text{for } B_d \\ 50\ \mu\text{eV} & \text{for } B_s \end{cases}$$

$(\approx \Delta\Gamma/2)$

Mass eigenstates:

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$$|p|^2 + |q|^2 = 1$$

like K_L and K_S in Kaon physics

= eigenvectors of $M - i\Gamma/2$

Eigenvalues: $\boxed{M_H - i\frac{\Gamma_H}{2}}\ \boxed{M_L - i\frac{\Gamma_L}{2}}$

$$\Delta m = M_H - M_L$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H$$

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4\left(|m_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2\right)$$

$$\Delta m \cdot \Delta\Gamma = -4\,\mathrm{Re}\left[m_{12}\Gamma_{12}^*\right]$$

Expand in $\Delta\Gamma/\Delta m$, Γ_{12}/m_{12} :

$$\Delta m = 2\,|m_{12}|$$

$$\Delta\Gamma = -2\cdot\frac{\mathrm{Re}\left[m_{12}\Gamma_{12}^*\right]}{\Delta m}$$

$$= -2\cdot|\Gamma_{12}|\cdot\cos\phi$$

where $m_{12}\Gamma_{12}^* = |m_{12}\Gamma_{12}|e^{i\phi}$

$$\frac{q}{p} = -\frac{m_{12}^*}{|m_{12}|}\left[1-\frac{1}{2}\,\mathrm{Im}\,\frac{\Gamma_{12}}{m_{12}}\right]$$

$$= -\frac{m_{12}^*}{|m_{12}|}\left[1-\frac{1}{2}\left|\frac{\Gamma_{12}}{m_{12}}\right|\sin\phi\right]$$

with corrections of order $\left|\frac{\Gamma_{12}}{m_{12}}\right|^2 \lesssim 10^{-4}$

Time evolution:

$$|B_H(t)\rangle = e^{-iM_H t} \cdot e^{-\Gamma_H t/2} |B_H(0)\rangle$$

$$|B_L(t)\rangle = e^{-iM_L t} \cdot e^{-\Gamma_L t/2} |B_L(0)\rangle$$

A B-meson tagged as B^0 at time $t=0$ evolves as

$$\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_H) = \Gamma_{11} = \Gamma_{22}$$
$$M = \frac{1}{2}(M_L + M_H) = M_{11} = M_{22}$$

$$|B^0(t)\rangle = g_+(t)\,|B^0\rangle - \frac{m_{12}^*}{|m_{12}|}\left[1 - \frac{1}{2}\,\mathrm{Im}\,\frac{\Gamma_{12}}{m_{12}}\right] g_-(t)\,|\overline{B^0}\rangle$$

$$|\overline{B^0}(t)\rangle = g_+(t)\,|\overline{B^0}\rangle - \frac{m_{12}}{|m_{12}|}\left[1 + \frac{1}{2}\,\mathrm{Im}\,\frac{\Gamma_{12}}{m_{12}}\right] g_-(t)\,|B^0\rangle$$

$$g_\pm(t) = \frac{1}{2} e^{-iMt - \frac{\Gamma}{2}t} \cdot \left[e^{\frac{t}{2}(i\Delta m - \Delta\Gamma)} \pm e^{-\frac{t}{2}(i\Delta m - \Delta\Gamma)} \right]$$

In studies of tagged B's one can safely neglect $\Delta\Gamma$ in $g_\pm(t)$:

$$g_\pm(t) = e^{-iMt-\frac{\Gamma}{2}t} \cdot \begin{cases} \cos\left(\frac{\Delta m}{2}t\right) \\ i\sin\left(\frac{\Delta m}{2}t\right) \end{cases}$$

[Exception: Precision measurement of Δm]

The time evolution of **untagged** samples is governed by $\Delta\Gamma$:

Example: Flavor-specific mode:

$$B^0 \to f, \quad \overline{B^0} \not\to f$$

$$\Gamma_{untagged}(t) = \Gamma(B^0(t)\to f) + \Gamma(\overline{B^0}(t)\to f)$$

$$\propto |\langle f|B^0(t)\rangle|^2 + |\langle f|\overline{B^0}(t)\rangle|^2$$

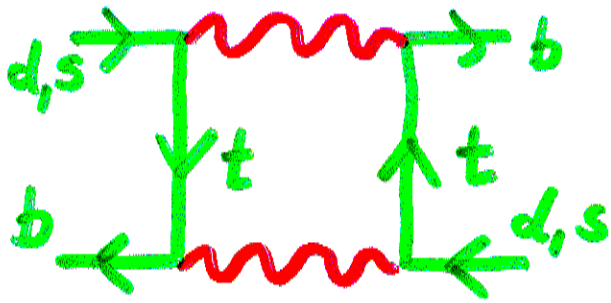
$$= |\langle f|B^0\rangle|^2 \cdot [|g_+|^2 + |g_-|^2] + O\left(\frac{\Delta\Gamma}{\Delta m}\right)$$

$$= \frac{1}{2}|\langle f|B^0\rangle|^2 \cdot e^{-\Gamma t} \cdot \left[e^{-\frac{\Delta\Gamma}{2}t} + e^{\frac{\Delta\Gamma}{2}t}\right]$$

need 2-exponential fit

Δm for B_d and B_s

Standard Model:



short distance dominated

$\Rightarrow$ sensitive to new physics

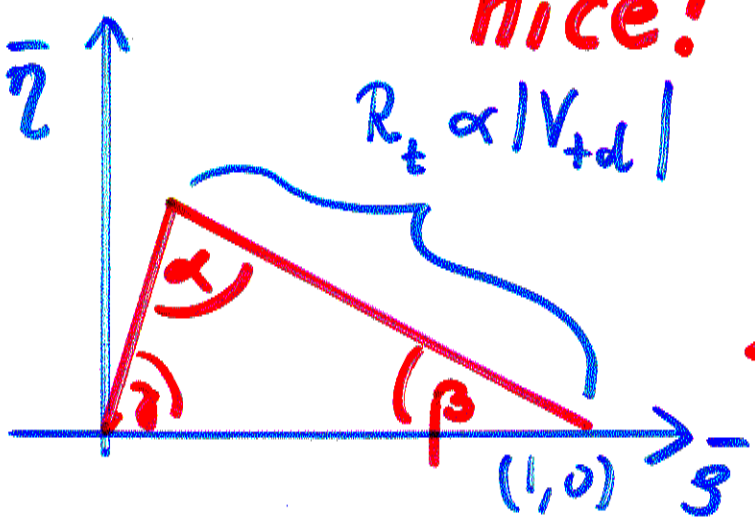
$\rightarrow$ JoAnne Hewett's talk

$$\underbrace{\left|\langle \bar{B}^0_d | \; t \; t \; | B^0_d \rangle\right|}_{|m_{12}| = \frac{\Delta m}{2}} = |V_{td}|^2 \cdot C \cdot \langle \bar{B}^0_d | \bar{d}\, b\, \bar{b}\, d \, | B^0_d \rangle$$

calculable Wilson coefficient (Buras, Jamin, Weisz)

nice!

hadronic physics (difficult to calculate)



$\leftarrow$ unitarity triangle

$$Q = \bar{b} d_{V-A} \, \bar{b} d_{V-A}$$

$\Delta B = 2$ operator

$$\langle \overline{B_d^0} | Q | B_d^0 \rangle = \frac{8}{3} M_{B_d}^2 \cdot f_{B_d}^2 \cdot B_d$$

$$\left.\begin{aligned} f_{B_d} &= 210 \pm 30 \text{ MeV} \\ B_d &= 0.80 \pm 0.15 \end{aligned}\right\} \text{LaHice '99 (Hashimoto)}$$

$= B_d^{\overline{MS}}(m_b)$

$\Delta m_{B_d} = 0.481 \pm 0.017 \ ps^{-1}$ LEP

$\Rightarrow$ 20% theoretical uncertainty in $|V_{td}|$.

Δm_{B_s} : Same formula except

$$|V_{td}| \longrightarrow |V_{ts}| \simeq |V_{cb}|$$

$$f_{B_d}^2 B_{B_d} \longrightarrow f_{B_s}^2 B_{B_s}$$

$$\frac{\Delta m_{B_d}}{\Delta m_{B_s}} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \cdot \underbrace{\frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 B_{B_s}}}_{(1.16 \pm 0.04)^2}$$

lattice'99

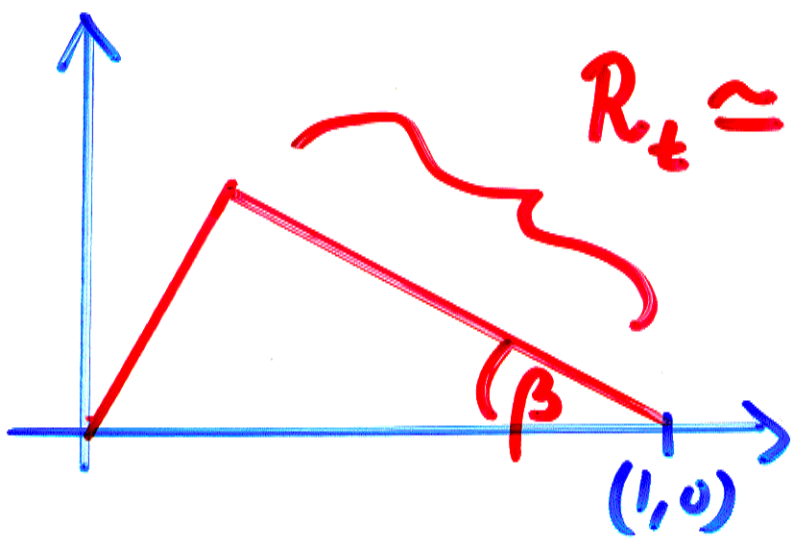
$\Delta m_{B_s} > 14.3\, ps^{-1}$ LEP

4% th. uncertainty in $\left|\frac{V_{td}}{V_{ts}}\right| \simeq \frac{R_t}{\lambda}$!

Two Run-II measurements:

$\sin 2\beta$ (expect 0.71 ± 0.13)

Δm_{B_s}



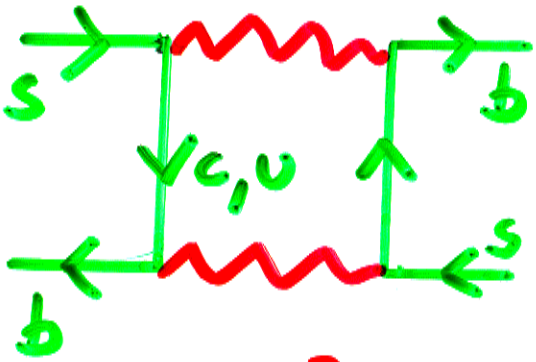
$$R_t \simeq \left|\frac{V_{td}}{V_{ts}}\right| \frac{1}{\lambda}$$

→ unitarity triangle

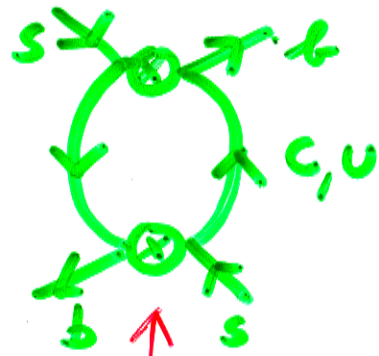
more important for unitarity triangle

Most new physics cancels in $\frac{\Delta m_{B_d}}{\Delta m_{B_s}}$, but not all.....

$\Delta\Gamma$ for B_d and B_s



$\rightarrow \Gamma_{12}$



$\Delta B = 1$ operators

$\Delta\Gamma$ comes from final states common to B and $\bar{B}$:

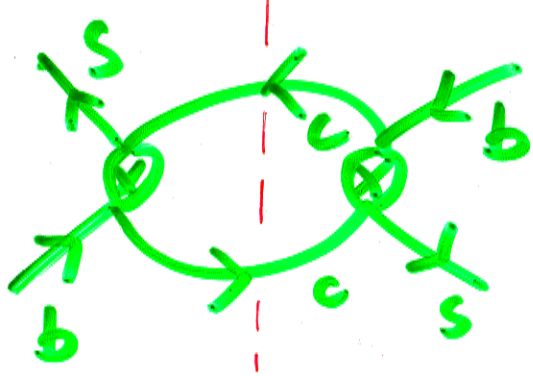
Standard model:

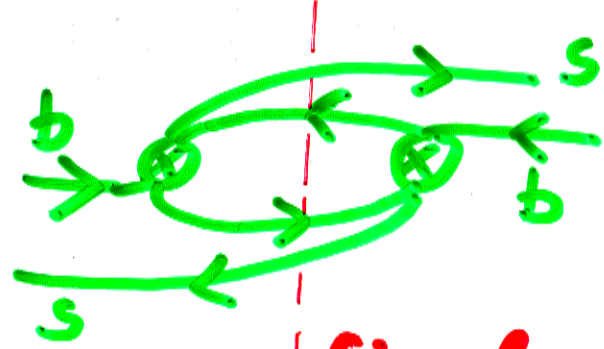
CP in $B_s - \bar{B}_s$ mixing negligible

$$|B_{H,L}\rangle = \frac{1}{\sqrt{2}}\left[|B_s^0\rangle \pm |\bar{B}_s^0\rangle\right]$$

$$\Delta\Gamma \propto \sum_f \left\{ |\langle f|B_H\rangle|^2 - |\langle f|B_L\rangle|^2 \right\}$$

$$= 2\sum_f \langle \bar{B}_s^0|f\rangle\langle f|B_s^0\rangle$$





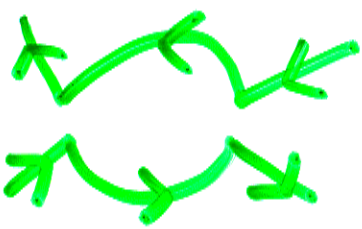
$$\sum_f \langle f|\overline{B_s^0}\rangle \langle B_s^0|f\rangle$$

final state $c\bar{c}s\bar{s}$

phase space integration over $(c,\bar{c})$ final state.

$$\frac{\Delta\Gamma}{\Delta m} = O\left(\frac{m_b^2}{m_t^2}\right) \ll 1$$

Use Heavy Quark Expansion to calculate $\Delta\Gamma$



$= C_1 \cdot$

$+ C_2$

Q_s

short distance physics

$\sim O(m_b)$

new:

$Q_s = \bar{s}\, b_{s-p}\, \bar{s}\, b_{s-p}$

$\Delta\Gamma_{B_s}$ is known to next-to-leading order in QCD

Beneke, Buchalla, Greub, Lenz, U.N.

$$\frac{\Delta\Gamma_{B_s}}{\Gamma} = \left(\frac{f_{B_s}}{210\,\text{MeV}}\right)^2 \cdot \left[0.15\, B_s - 0.063\right]$$

Beneke, Buchalla, Dunietz

$$\langle B_s^0 | Q_s | \bar{B}_s^0 \rangle \simeq f_{B_s}^2 \cdot M_{B_s}^2 \left(-\frac{5}{3}\right) \cdot B_s$$

$$B_s \approx 1.19 \pm 0.20$$

$$f_{B_s} \approx 245 \pm 30\,\text{MeV}$$

Lattice '99

$$\frac{\Delta\Gamma_{B_s}}{\Gamma} = 0.16 \pm \underset{f_B}{0.03} \pm \underset{B_s}{0.04}$$

$\Rightarrow$ large!

Two ways to measure $\Delta\Gamma_{B_s}$:

(Bigi)

1] $B^0, \bar{B}^0 \longrightarrow$ flavor-specific f

2-exponential fit

$$\Gamma(B \to f, t) \propto e^{-\Gamma_L t} + e^{-\Gamma_H t}$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H$$

untagged sample

or similarly with $B \to a\ell\ell$

2] $B^0, \bar{B}^0 \longrightarrow$ CP eigenstate f_{CP}

$f_{CP} = J/\psi\, \phi$ (mostly CP even)

In SM B_L is CP-even and $B_H \not\to J/\psi\, \phi$.

Hence $\Gamma(B \to f_{CP=+1})$ measures Γ_L! Compare with average B_s lifetime to find $\Delta\Gamma/2$.

How about new physics?

Grossman

$$\Delta\Gamma = -2\,|\Gamma_{12}|\cos\phi$$

$$M_{12}\Gamma_{12}^{*} = |M_{12}|\,|\Gamma_{12}|\cdot e^{i\phi}$$

$\phi \approx 0$ in SM

almost insensitive to new physics

$$\Rightarrow \Delta\Gamma \leq \Delta\Gamma_{SM}$$

$\Rightarrow$ affects method 2:

B_L, B_H no more CP-eigenstates (due to $\phi \neq 0$)

$\rightarrow f_{CP=1}$ allowed

$$\frac{\Delta\Gamma_{CP\text{-method}}}{\Delta\Gamma_{2\text{-expon.}}} = \cos\phi$$

measuring $\Delta\Gamma$ allows to predict ΔM better and vice versa:

In $\frac{\Delta\Gamma_{B_s}}{\Delta M_{B_s}}$ the factor $f_{B_s}^2$ cancels. CKM-factors known.

$\Rightarrow$ Could see new physics in ΔM_{B_s}.

Nice: Can we measure

$$\Delta\Gamma_{B_d} = O(1\%)\cdot\Gamma_{B_d} \; ?$$

$\Rightarrow$ $\frac{\Delta\Gamma_{B_d}}{\Delta\Gamma_{B_s}}$ is theoretically quite clean.

$\hookrightarrow$ information on V_{ub} in $\Delta\Gamma_{B_d}$.

- CP-method should work!

Untagged $B \to J/\psi K_S$ samples, compare with $\bar{\tau}(B_d) \Rightarrow \Delta\Gamma_{B_d} \cdot \cos[2(\beta+\delta)]$

$$\Gamma_{12} = |\Gamma_{12}| e^{\pm i\delta}$$

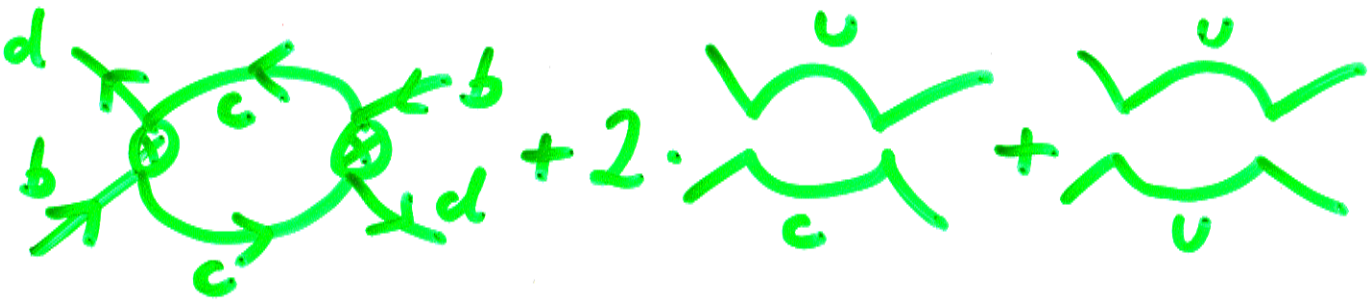
calculable (in terms of γ)

"Free lunch" from $\sin 2\beta$ measurement?

- Related: Semilept. CP-asymm.

$$a_{CP} \propto \mathrm{Im}\, \frac{\Gamma_{12}}{M_{12}} .$$

- Other ways to measure Γ_{12} in B_d system?



$\Gamma^{cc} \propto (V_{cb} V_{cd}^*)^2$ $\quad \Gamma^{uc} \propto V_{cb} V_{cd}^* V_{ub} V_{ud}^*$ $\quad \Gamma^{uu} \propto (V_{ub} V_{ud}^*)^2$

From $Br[B \to \text{no charm}, t]$ one could infer Γ^{uu}.

Side result from

Beneke, Buchalla, Dunietz

$A_{CP}^{mix,incl.}(B \to \text{no charm}, t)$?

(untagged sample can be used for Γ^{uu}.)

$\sin 2\alpha$ measured from $A_{CP}^{mix,incl}$.

Preliminary Conclusions

B_s- and B_d-mixing is an exciting field with many **hidden treasures**:

→ UT construction: • $\frac{\Delta m_{B_d}}{\Delta m_{B_s}}$

→ new physics: • $\frac{\Delta m_{B_s}}{\Delta \Gamma_{B_s}}$

• $\Delta \Gamma_{B_s}$ (new CP')

→ test of HQE: $\Delta \Gamma_{B_s}$, $\Delta \Gamma_{B_d}$

→ $|V_{ub}|$, γ: • $\Delta \Gamma_{B_d}$, Γ_{12}^{uu}, $\frac{\Delta \Gamma_{B_d}}{\Delta \Gamma_{B_s}}$

Final conclusions:
After the 2nd workshop!