Consequences of Fine-Tuning for Fifth-Force Searches

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Based on:
1807.11508

N Blinov, SE, A Hook

Berkeley, October 3, 2018

Outline

Introduction

Why light bosons?

— Light scalars:

- Extra dimensions/modifications of Gravity
- Broken scale invariance: Dilaton
- Ultralight bosonic Dark Matter
- Quintessence

— Light vectors:

- Ultralight dark photon Dark Matter (Not discussed further)

Light scalars and long-range forces

A light scalar with coupling to matter

$$\mathcal{L} \supset \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - y\phi\bar{\psi}\psi$$

Sources a potential of the form

$$V_\phi(r) = -\frac{y^2}{4\pi}\frac{e^{-mr}}{r}$$

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Recall gravitational potential:

$$V_G(r) = -\frac{G_N M_i M_j}{r}$$

Scalar leads to modifications of inverse-square law over distances

$$\lambda = 1/m$$

Light scalars and long-range forces

Modified potential is

$$V(r) = -\frac{G_N M_i M_j}{r}\boxed{\left(1 + \alpha\, e^{-mr}\right)}$$

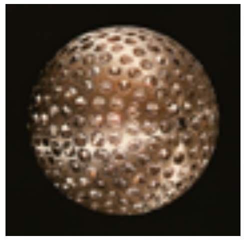
So that the modified force is

$$F(r) = \frac{G_N M_i M_j}{r^2}\left(1 + \alpha\,(1 + mr)\, e^{-mr}\right)$$

$$\alpha = \frac{y^2}{4\pi}\frac{M_{\rm pl}^2}{m_\psi^2}, \quad \beta = \frac{\sqrt{4\pi\alpha}}{M_{\rm pl}}$$

Many efforts since the 1970s to constrain $\{\alpha, m\}$

Light scalars and long-range forces



LAGEOS

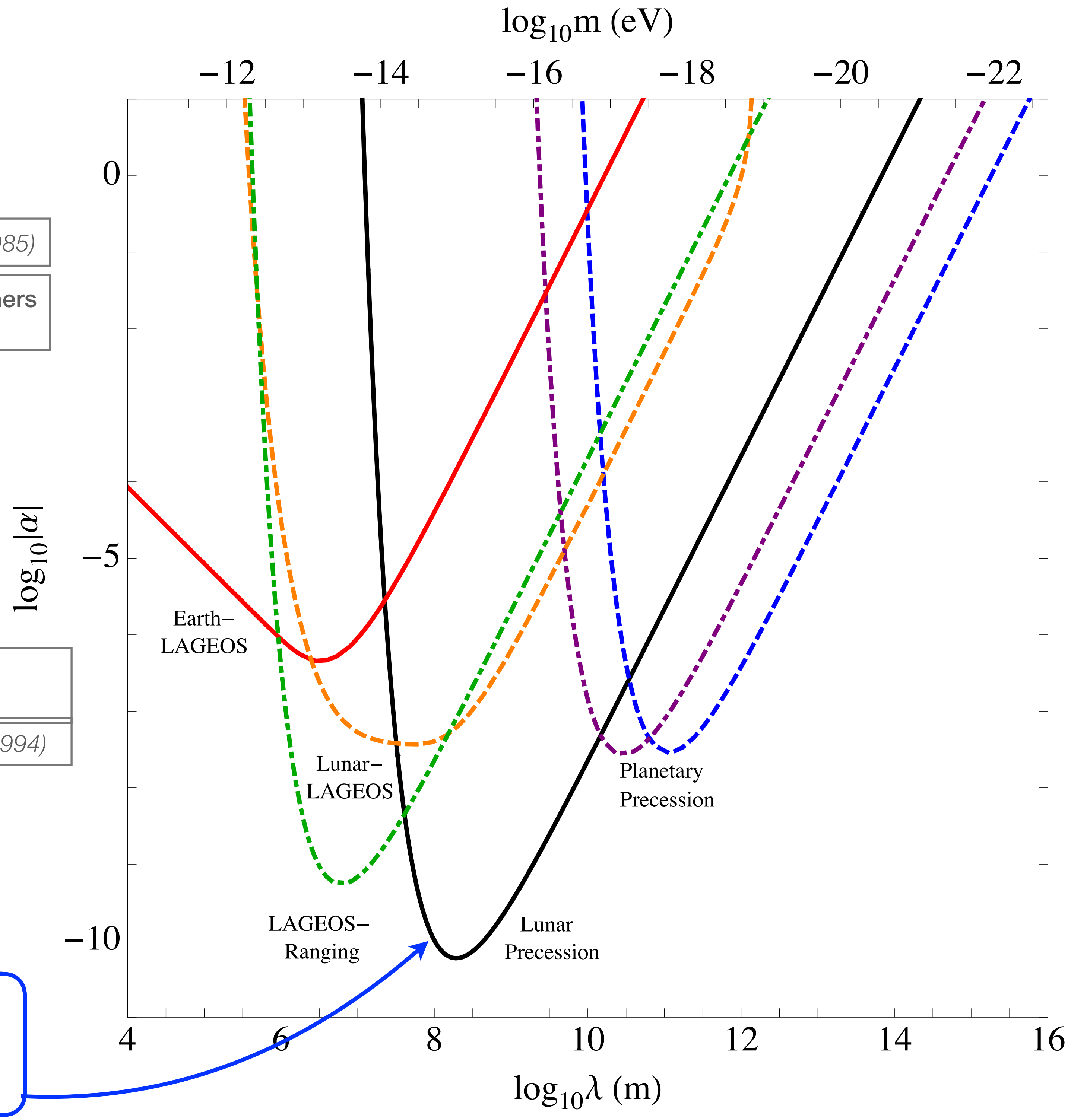
Smith et al *(1985)*

Lucchesi & others *(+ 2014)*

Lunar
Reflectors

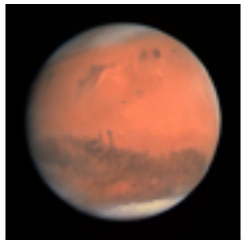
Talmadge et al *(1988)*

Dickey et al *(1994)*



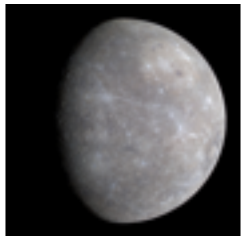
$\lesssim 10^{-10}$ strength of gravity!

Talmadge et al *(1988)*



Mars

Talmadge et al *(1988)*



Mercury

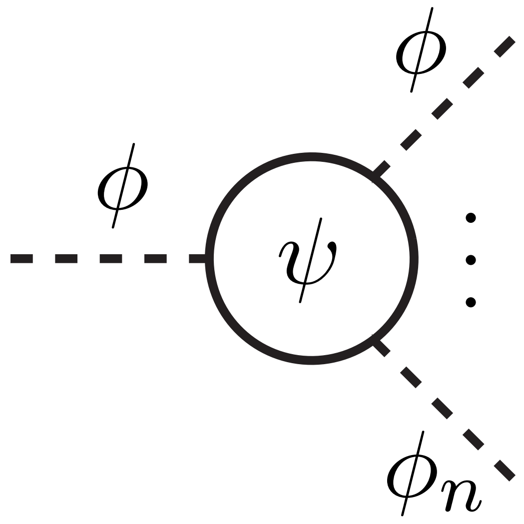
Fischbach & Talmadge *(1998)*
Adelberger, Heckel, Nelson *(2003)*

II. Fine-tuned, natural, or natural & fine-tuned?

Fine-tuned, natural, natural and fine-tuned?

Small mass $\Longrightarrow$ hierarchy problem $\Longrightarrow$ fine-tuning

Coupling to matter induces radiative corrections to scalar potential



$$\Rightarrow \quad V^{(1)} \simeq \frac{1}{16\pi^2}\left(\sum_n c_n \phi^n \Lambda^{4-n}\right) \; ,$$

Coleman & E. Weinberg *(1973)*

Correction to mass $\Longrightarrow$ **fine-tuned**

Fine-tuned, **natural,** natural and fine-tuned?

Dilaton: pNGB associated with broken scale invariance

Non-derivative self-interactions only generated proportional to explicit breaking parameter

$$V \simeq \frac{1}{2}m^2\varphi^2 + \frac{am^2}{f}\varphi^3 + \frac{bm^2}{f^2}\varphi^4 + \ldots$$

For breaking by $\mathcal{O} \sim \varphi^4(\varphi/f)^{\Delta-4}, \quad |\Delta - 4| \ll 1$

$$a = 5/6 \qquad b = 11/24$$

Therefore all parameters naturally small

See e.g.
Rattazzi & Zaffaroni *(2000)*
Goldberger, Grinstein, Skiba *(2007)*
Chacko & Mishra *(2012)*
Coradeschi et al *(2013)*

Fine-tuned, **natural,** natural and fine-tuned?

Z_N scalars: Z_N symmetry non-linearly realised on scalar as a shift symmetry and an exchange symmetry on N copies of particles.

Spurion ε breaks shift symmetry

$$\varphi \to \varphi + \theta \quad \underset{\varepsilon}{\Longrightarrow} \quad \varphi \to \varphi + 2\pi f$$

Scalar only appears as $\varepsilon \sin\left(\frac{\varphi}{f} + \vartheta\right)$

Hook (2018)

$$\mathcal{L} \sim \sum_k^N \varepsilon \sin\left(\frac{\varphi}{f} + \frac{2\pi k}{N}\right) \bar{\psi}_k \psi_k + \left(\frac{\varepsilon}{m_\psi}\right)^N m_\psi^4 \cos\frac{N\varphi}{f}$$

Self-interactions suppressed by ε^N and naturally small

Fine-tuned, natural, **natural and fine-tuned**?

Allow only fine-tuning of the mass, but natural potential otherwise

$$V(\varphi) = \boxed{\frac{1}{2}m^2\varphi^2} + \boxed{\frac{1}{3}\kappa\varphi^3 + \frac{1}{4}\epsilon\varphi^4} + y\varphi\bar{\psi}\psi$$

Tuned **Natural**

Couplings at 1-loop:

$$\kappa(\mu) \simeq \kappa(\mu_0) + \frac{3y^3}{2\pi^2}m_\psi \ln\frac{\mu}{\mu_0} \quad \Longrightarrow \quad \boxed{|\kappa| \gtrsim \frac{3y^3}{2\pi^2}m_\psi}$$

$$\epsilon(\mu) \simeq \epsilon(\mu_0) - \frac{y^4}{2\pi^2}\ln\frac{\mu}{\mu_0} \quad \Longrightarrow \quad \boxed{|\epsilon| \gtrsim \frac{y^4}{2\pi^2}}$$

Fine-tuned, natural, **natural and fine-tuned**?

Alternatively consider Coleman-Weinberg potential

$$V_{CW} = -\frac{1}{16\pi^2} m_\psi(\varphi)^4 \left(\ln \frac{m_\psi(\varphi)^2}{\mu^2} - \frac{3}{2} \right)$$

With $m_\psi(\varphi) = m_\psi - y\varphi$

Then characteristic self-interaction of $\mathcal{O}(\varphi^n)$

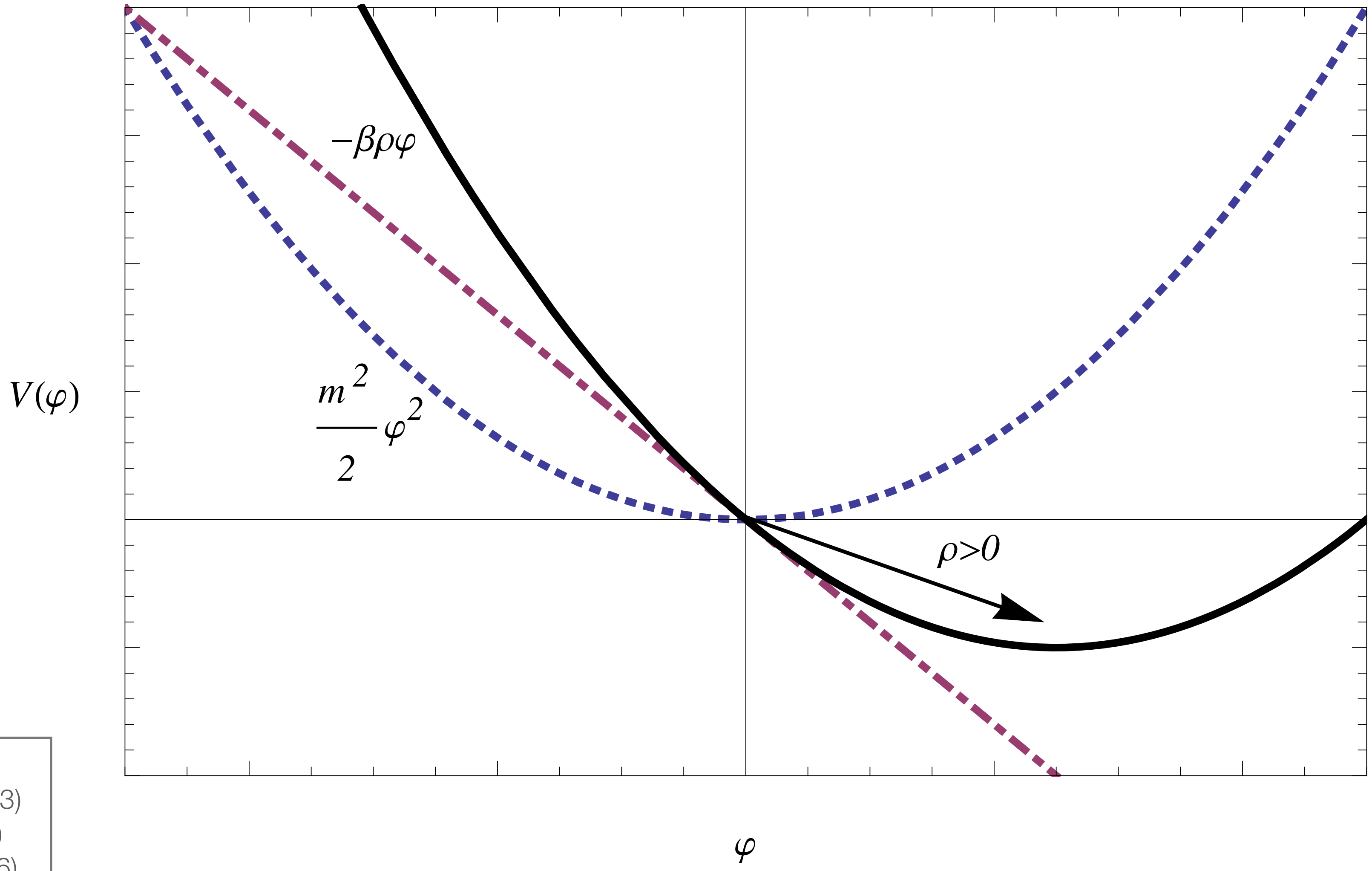
$$\boxed{g_{(n)} \sim \frac{y^n m_\psi^{4-n}}{16\pi^2}}$$

Non-renormalisable operators generated as well

III. Screening

Essence of Screening

Linear coupling to matter shifts vacuum inside dense object

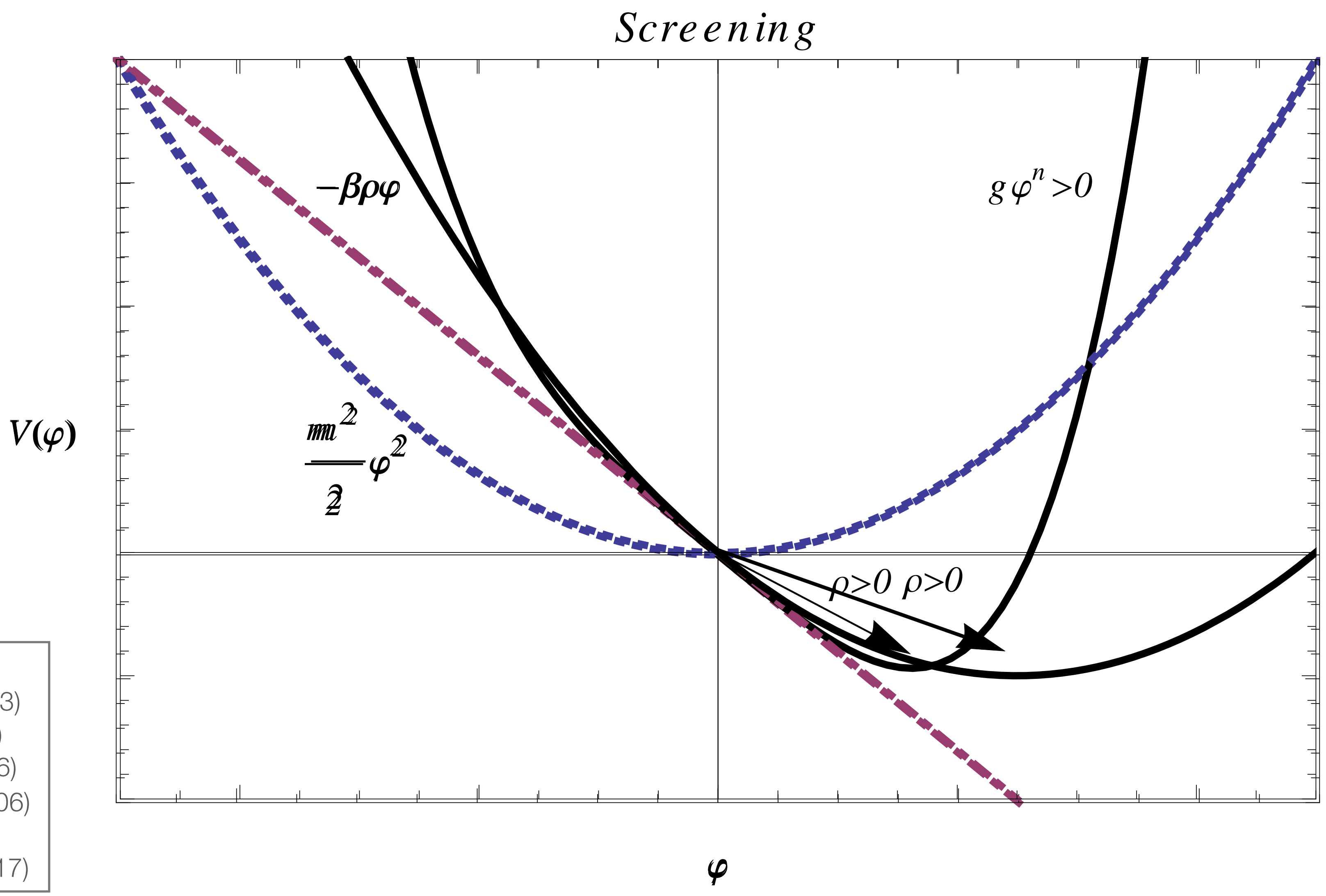


See e.g.
Khoury & Weltman (2003)
Gubser & Khoury (2004)
Feldman & Nelson (2006)
Mota & Shaw (2006, 2006)
See also
Burrage & Sakstein (2017)

Essence of Screening

Include natural self-interactions:

$$m_{\text{eff}}^2 = V''(\varphi) \simeq m^2 + \sum_n n(n-1) g_{(n)} \varphi_{\text{min}}^{n-2}$$



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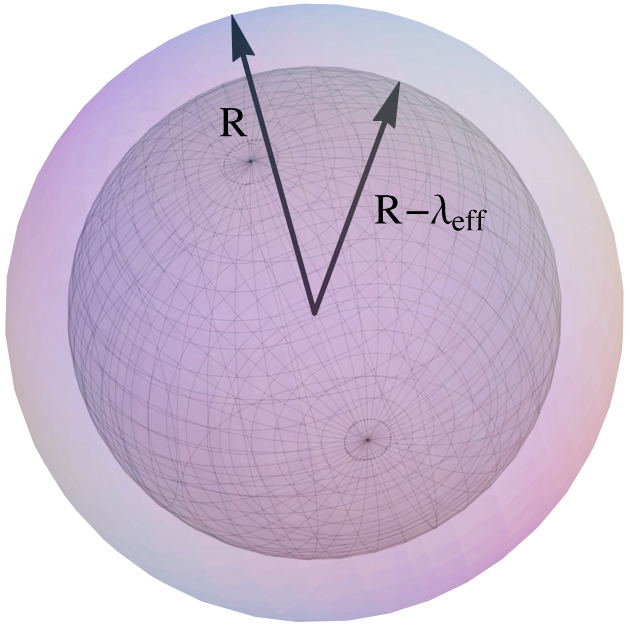
See also
Burrage & Sakstein (2017)

Essence of Screening

Effective mass larger than bare mass

$$m_{\text{eff}} > m \quad \Leftrightarrow \quad \lambda_{\text{eff}} < \lambda$$



Screening condition: $m_{\text{eff}} R > 1$

Two ways of thinking about screening:

Effective range smaller than radius, so only a shell sources the field

Field has reached its in-medium minimum, and so ceases to change

Essence of Screening

EoM for scalar determines field profile, and therefore strength of force:

$$\varphi'' + \frac{2}{r}\varphi' = V'(\varphi) - \beta\rho\,\theta(r - R),$$

In the screened regime, highly non-linear

Field profile approximately

$$\varphi \sim \frac{Q}{r}, \quad Q = \beta M \boxed{\gamma}$$

Screening Parameter

Essence of Screening

Estimate size of screening parameter at r ~ R:

$$\varphi'' + \frac{2}{r}\varphi' \sim \frac{2Q}{r^3} \approx g\varphi^{n-1} \sim g\frac{Q^{n-1}}{r^{n-1}}$$

$$\Longrightarrow \quad \boxed{\gamma \sim \left(\frac{g_c}{g}\right)^{1/(n-2)}}\,, \qquad g_c = \frac{2R^{n-4}}{(\beta M)^{n-2}}$$

Notice screening parameter dependent on:

Strength of coupling to matter: $\beta = \frac{y}{m_\psi} = \frac{\sqrt{4\pi\alpha}}{M_{\rm pl}}$

Geometry of object: $M, \ R$

Screening from natural potential

When self-interactions dominate:

$$m_{\text{eff}}^2 \sim \beta^{2n-2} m_\psi^4 \left(\rho_i R_i^2\right)^{n-2}$$

$$\beta = \frac{y}{m_\psi} = \frac{\sqrt{4\pi\alpha}}{M_{\text{pl}}}$$

Recall α is strength relative to gravity

Screening condition translates into a critical α:

$$\alpha_c^{(n)} \sim \frac{M_{\text{pl}}^2}{R^2 \left(m_\psi^4 \rho^{n-2}\right)^{1/(n-1)}}$$

Screening from natural potential

Object	$\alpha_c^{(3)}$	$\alpha_c^{(4)}$	$\alpha_c^{(5)}$
Earth (⊕)	10^{2}	$10^{4.1}$	10^{5}
Moon (☾)	$10^{3.2}$	$10^{5.4}$	$10^{6.3}$
Mercury (☿)	$10^{2.8}$	10^{5}	$10^{5.9}$
Mars (♂)	$10^{2.6}$	$10^{4.8}$	$10^{5.7}$
LAGEOS (L)	10^{17}	10^{19}	10^{20}
Sun (⊙)	$10^{-1.8}$	$10^{0.44}$	$10^{1.4}$
Pulsar (P)	$10^{0.48-0.55}$	$10^{0.34-0.35}$	$10^{0.09-0.13}$
Inner Dwarf (D_i)	$10^{-0.53}$	$10^{0.92}$	$10^{1.5}$
Outer Dwarf (D_o)	$10^{-0.64}$	$10^{0.70}$	$10^{1.2}$

Cubic **Quartic** **Quintic**

Consequences

Consider screening condition for cubic self-interaction, for Earth, Moon and LAGEOS

Object	$\alpha_c^{(3)}$
Earth ($\oplus$)	10^2
Moon (☾)	$10^{3.2}$
LAGEOS (L)	10^{17}

All different values

Effective violation of the Equivalence Principle

Consequences

Consider screening condition for cubic self-interaction, for Earth, Moon and LAGEOS

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LAGEOS (L)	10^{17}

All different values

Effective violation of the Equivalence Principle

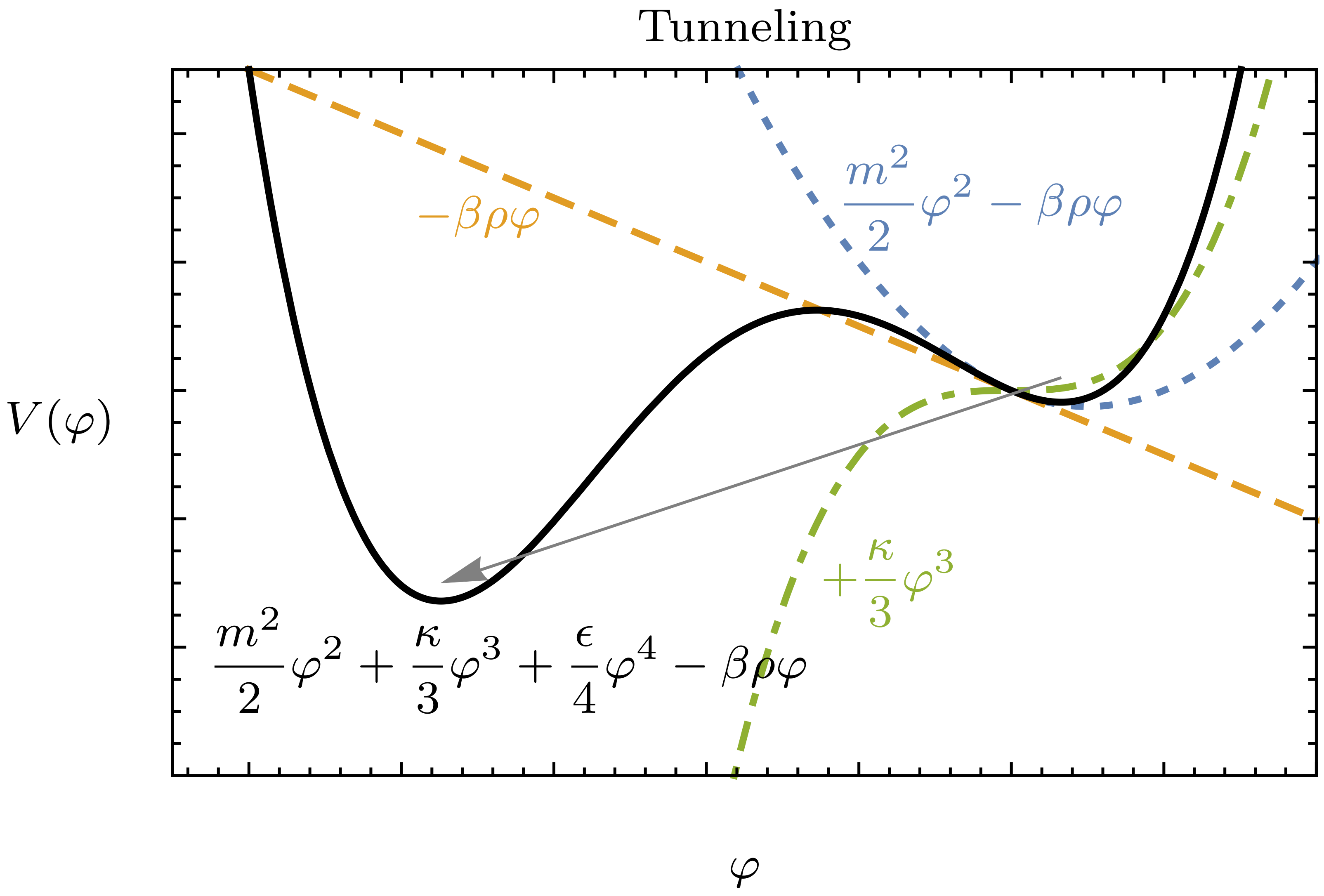
EP-violation searches can apply

III b. Beyond Screening

Tunneling

Minimum near $\varphi = 0$ can be metastable

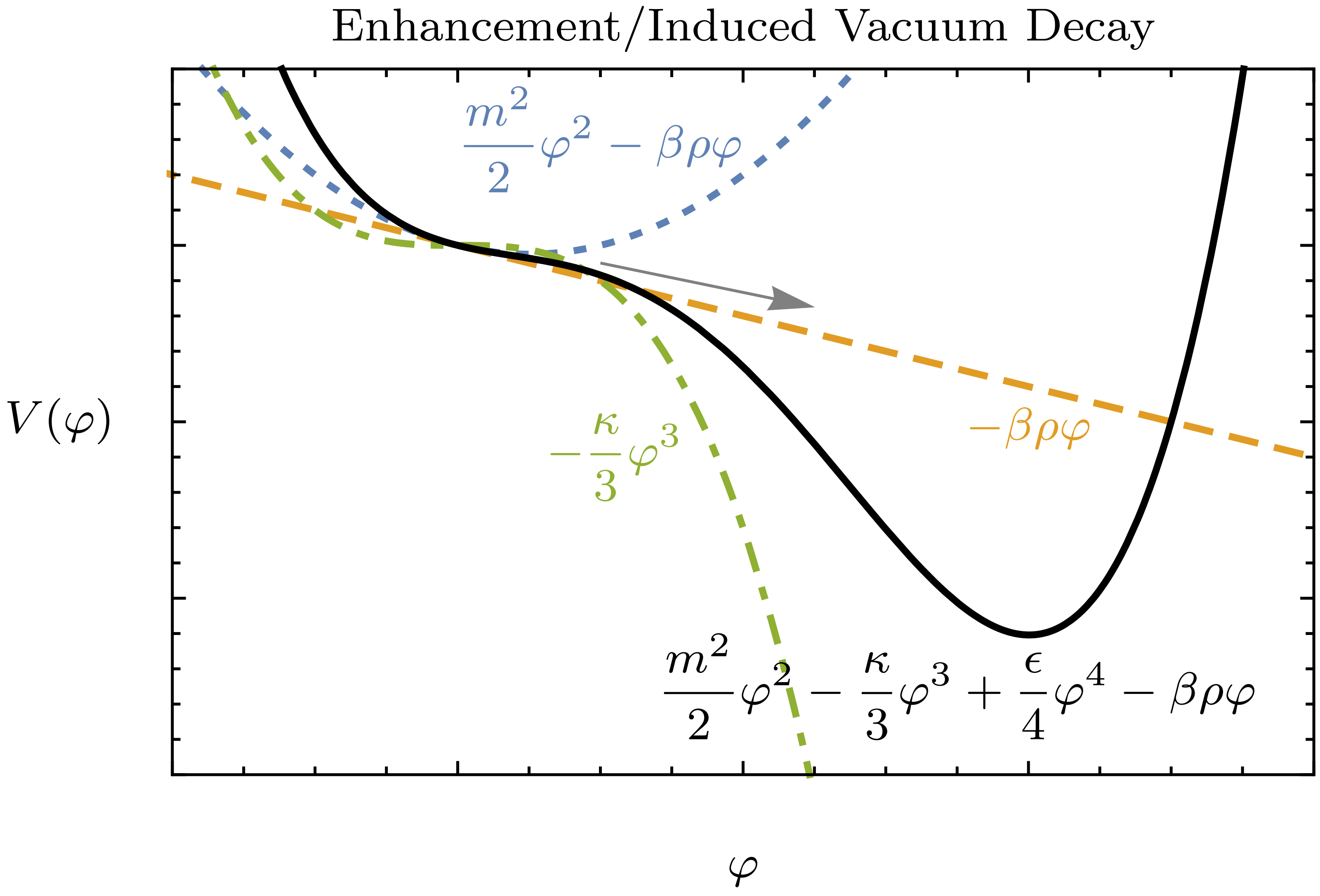
Potential with cubic and small stabilising quartic:



$\boxed{\beta\kappa > 0}$

Enhancements

Minimum near $\varphi = 0$ can be unstable



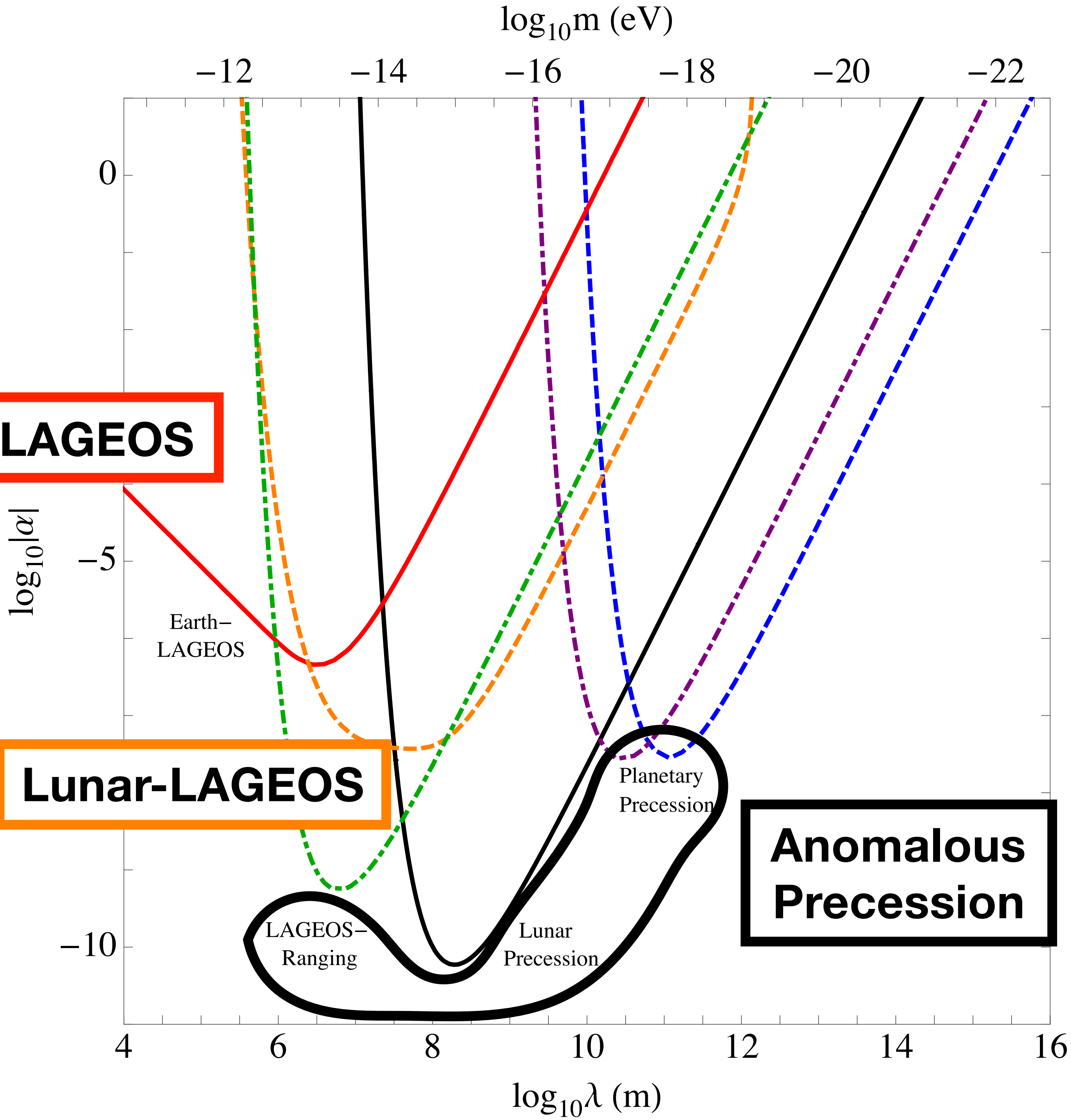
$$\beta\kappa < 0$$

φ will rapidly evolve classically towards global minimum

C.f. "Spontaneous scalarisation"
Damour & Esposito-Farese (1993)

IV. Constraints

Constraints on EP-preserving forces



Earth-LAGEOS

Lunar-LAGEOS

Anomalous Precession

Talmadge et al *(1988)*
Dickey et al *(1994)*
Fischbach & Talmadge *(1998)*
Adelberger, Heckel, Nelson *(2003)*
Lucchesi & others *(+ 2014)*

Anomalous precession

Motion under influence of a central force:

$$u = 1/r$$

$$u(\theta) = u_p + u_e \cos\omega(\theta - \theta_0)$$

Semi-major axis a_p

$$1/u_p = a_p(1-\epsilon^2)$$

No 5th force

Precession rate: $\omega = 1$

Pericenter: $\theta - \theta_0 = 2\pi n, \quad n \in \mathbb{Z}$

Anomalous precession

Motion under influence of a central force:

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Semi-major axis a_p

$$1/u_p = a_p(1 - \epsilon^2)$$

No 5th force

Precession rate: $\omega = 1$

Pericenter: $\theta - \theta_0 = 2\pi n, \quad n \in \mathbb{Z}$

5th force present

Precession rate shift:

$$\boxed{\frac{\delta\omega}{\omega} \simeq \frac{\alpha}{2}(m a_p)^2 e^{-m a_p}}$$

Anomalous precession

Constraints:

$$\frac{\delta\omega}{\omega} \simeq \frac{\alpha}{2}(m a_p)^2 e^{-m a_p}$$

$$\left.\frac{\delta\omega}{\omega}\right|_L = (1.4 \pm 22 \pm 270) \times 10^{-13}, \quad \left.\frac{\delta\omega}{\omega}\right|_{☾} = (-3.0 \pm 8.0) \times 10^{-12},$$

LAGEOS satellite Lucchesi & others *(+ 2014)*

Moon Talmadge et al *(1988)*; Dickey et al *(1994)*

$$\left.\frac{\delta\omega}{\omega}\right|_{☿} = (-13 \pm 33) \times 10^{-9}, \quad \left.\frac{\delta\omega}{\omega}\right|_{♂} = (-21 \pm 29) \times 10^{-9}$$

Mercury Talmadge et al *(1988)*

Mars Talmadge et al *(1988)*

Lunar-LAGEOS

Measurements of $\mu_\oplus(r) = G_N(r)M_\oplus$ at LAGEOS and on lunar surface

$$\eta_{LL} = \frac{\mu_\oplus(r_L) - \mu_\oplus(r_{\oplus-☾})}{\left(\mu_\oplus(r_L) + \mu_\oplus(r_{\oplus-☾})\right)/2}$$

Constraint:

$$\eta_{LL,\text{ meas.}} = (-1.8 \pm 1.6) \times 10^{-8}$$

Fischbach & Talmadge *(1998)*

Without a fifth force:

$$\eta_{LL} = 0$$

Lunar-LAGEOS

When a fifth force is present:

$$\eta_{5,LL} = 2\alpha \boxed{\gamma_\oplus} \left\{ \frac{\boxed{\gamma_L}\,\mathbb{G}_\oplus(R_L,m) - \boxed{\gamma_{\leftmoon}}\,\mathbb{G}_\oplus(R_{\oplus-\leftmoon},m)(R_{\oplus-\leftmoon}/R_L)^2}{2\mu_\oplus/R_L^2 + \alpha\boxed{\gamma_\oplus}\left(\boxed{\gamma_L}\,\mathbb{G}_\oplus(R_L,m) + \boxed{\gamma_{\leftmoon}}\,\mathbb{G}_\oplus(R_{\oplus-\leftmoon},m)(R_{\oplus-\leftmoon}/R_L)^2\right)} \right\}$$

Earth **LAGEOS** **Moon**

Modified acceleration:

$$\mathbb{G}_i(r,m) = G_N M_i\,(1+mr)\left(\frac{e^{-mr}}{r^2}\right)\underline{F_i\,(mR_i)}$$

Form factor: $F_i(x) = \frac{3}{x^3}\left(x\cosh x - \sinh x\right)$

Lunar-LAGEOS

When a fifth force is present:

$$\eta_{5,LL} = 2\alpha \boxed{\gamma_\oplus} \left\{ \frac{\boxed{\gamma_L}\, \mathbb{G}_\oplus(R_L, m) - \boxed{\gamma_\leftmoon}\, \mathbb{G}_\oplus(R_{\oplus-\leftmoon}, m)(R_{\oplus-\leftmoon}/R_L)^2}{2\mu_\oplus/R_L^2 + \alpha \boxed{\gamma_\oplus} (\boxed{\gamma_L}\, \mathbb{G}_\oplus(R_L, m) + \boxed{\gamma_\leftmoon}\, \mathbb{G}_\oplus(R_{\oplus-\leftmoon}, m)(R_{\oplus-\leftmoon}/R_L)^2)} \right\}$$

Earth **LAGEOS** **Moon**

In $m \to 0$ limit:

$$\eta_{5,LL} \sim \alpha\gamma_\oplus(\boxed{\gamma_\leftmoon} - \boxed{\gamma_L}) + \mathcal{O}(r_i^2 m^2)$$

If $\gamma_\leftmoon - \gamma_L \neq 0$, effective EP violation

Earth-LAGEOS

Measurements of $\mu_\oplus(r) = G_N(r) M_\oplus$ at LAGEOS and on Earth's surface

$$\eta = \frac{g_\oplus(R_\oplus) - g_L(R_\oplus)}{g_L(R_\oplus)}$$

Constraint: $$\eta = (-2 \pm 5) \times 10^{-7}$$

Fifth force:

$$\eta_5 = \left(\frac{\alpha \bar{\gamma}_\oplus (R_\oplus) R_\oplus^2 \, \mathbb{G}_\oplus(R_\oplus, m) - \alpha \bar{\gamma}_\oplus (R_L) \gamma_L R_L^2 \, \mathbb{G}_\oplus(R_L, m)}{\mu_\oplus(R_L) + \alpha \bar{\gamma}_\oplus (R_L) \gamma_L R_L^2 \, \mathbb{G}_\oplus(R_L, m)} \right)$$

Earth-LAGEOS: a closer look

Fifth force:

$$\eta_5 = \left(\frac{\alpha \boxed{\bar{\gamma}_\oplus}(R_\oplus) R_\oplus^2 \, \mathbb{G}_\oplus(R_\oplus, m) - \alpha \boxed{\bar{\gamma}_\oplus}(R_L) \gamma_L R_L^2 \, \mathbb{G}_\oplus(R_L, m)}{\mu_\oplus(R_L) + \alpha \boxed{\bar{\gamma}_\oplus}(R_L) \gamma_L R_L^2 \, \mathbb{G}_\oplus(R_L, m)} \right)$$

Why $\bar{\gamma}_\oplus(R_\oplus)$? Measurement done on Earth's surface

$$\alpha \bar{\gamma}_\oplus(r) \mathbb{G}_\oplus(r, m) = -\beta \varphi'(r)$$

$$\varphi'' \sim \epsilon \varphi^3 - \beta \rho \theta(r - R_i)$$

$$\varphi'(R_\oplus) \sim -2^{1/6} \pi^{1/3} \left(\frac{\rho_\oplus}{m_n} \right)^{2/3}$$

$$\boxed{\bar{\gamma}_\oplus(R_\oplus) \propto \alpha^{-1/2}}$$

Constraint grows with α

Other constraints

EP tests:

- Stellar triple system *PSR J0337+1715*:

$$\eta = \frac{a_{\rm NS} - a_{{\rm WD},I}}{(a_{\rm NS} + a_{{\rm WD},I})/2} < 2.6 \times 10^{-6}$$

Archibald et al *(2018)*

- Earth-Moon-Sun system:

$$\eta = \frac{|a_\oplus - a_{☾}|}{(a_\oplus + a_{☾})/2} < 1.8 \times 10^{-13}$$

Williams et al *(2004)*

General constraints:

- Light deflection (Cassini)

$$\gamma_{\rm PPN} - 1 \approx -\frac{2\beta\varphi(b_{\rm min})}{\Phi_N(b_{\rm min})} = (2.1 \pm 2.5) \times 10^{-5}$$

Bertotti et al *(2003)*

- Cooling of SN1987A:

$$\alpha \lesssim 10^{17}$$

Cooling of HB and RG stars:

$$\alpha \lesssim 2 \times 10^{13}$$

Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

V. Quartic self-interactions

Quartic self-interaction

Far away from source, force is Yukawa-like

$$F_{ij} = \frac{GM_iM_j}{r^2}\left[1 + \alpha_{\rm eff}\left(1 + mr\right)e^{-mr}\right]$$

Where $$\alpha_{\rm eff} = \alpha\gamma_i(g)\gamma_j(g)$$

Natural size of coupling

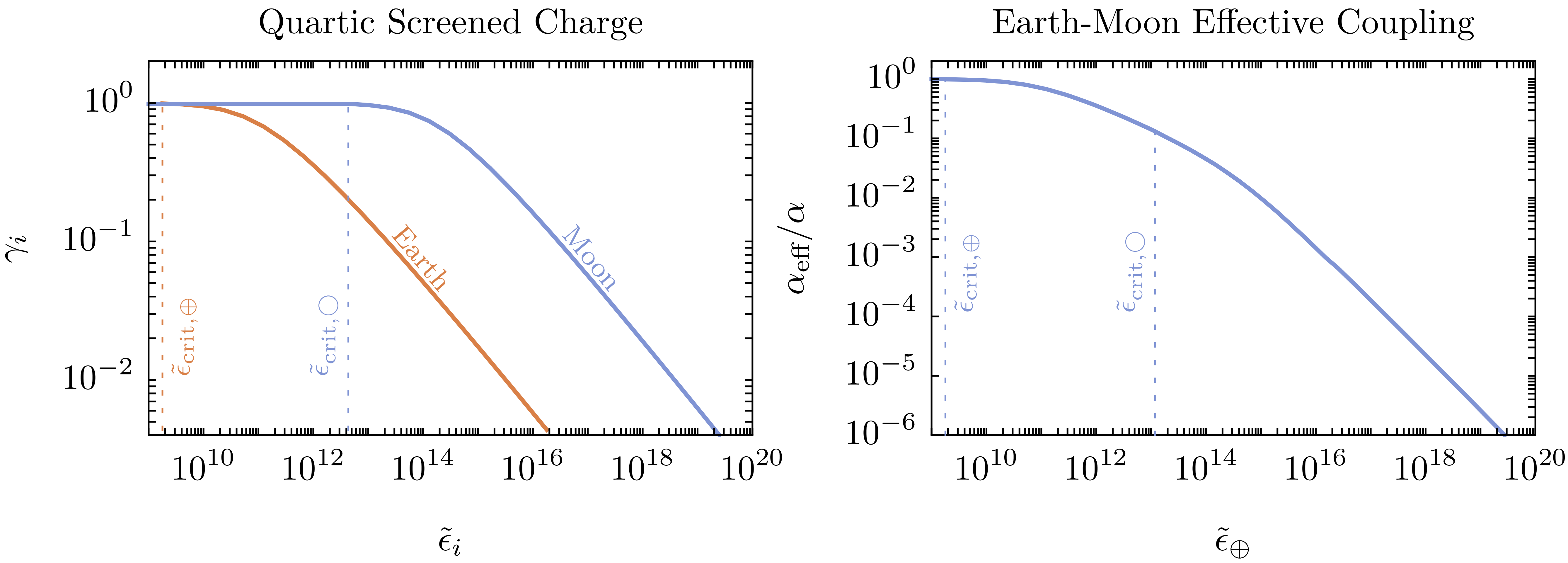
$$g = \epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$

Screening parameter $\gamma \propto \alpha^{-3/2}$

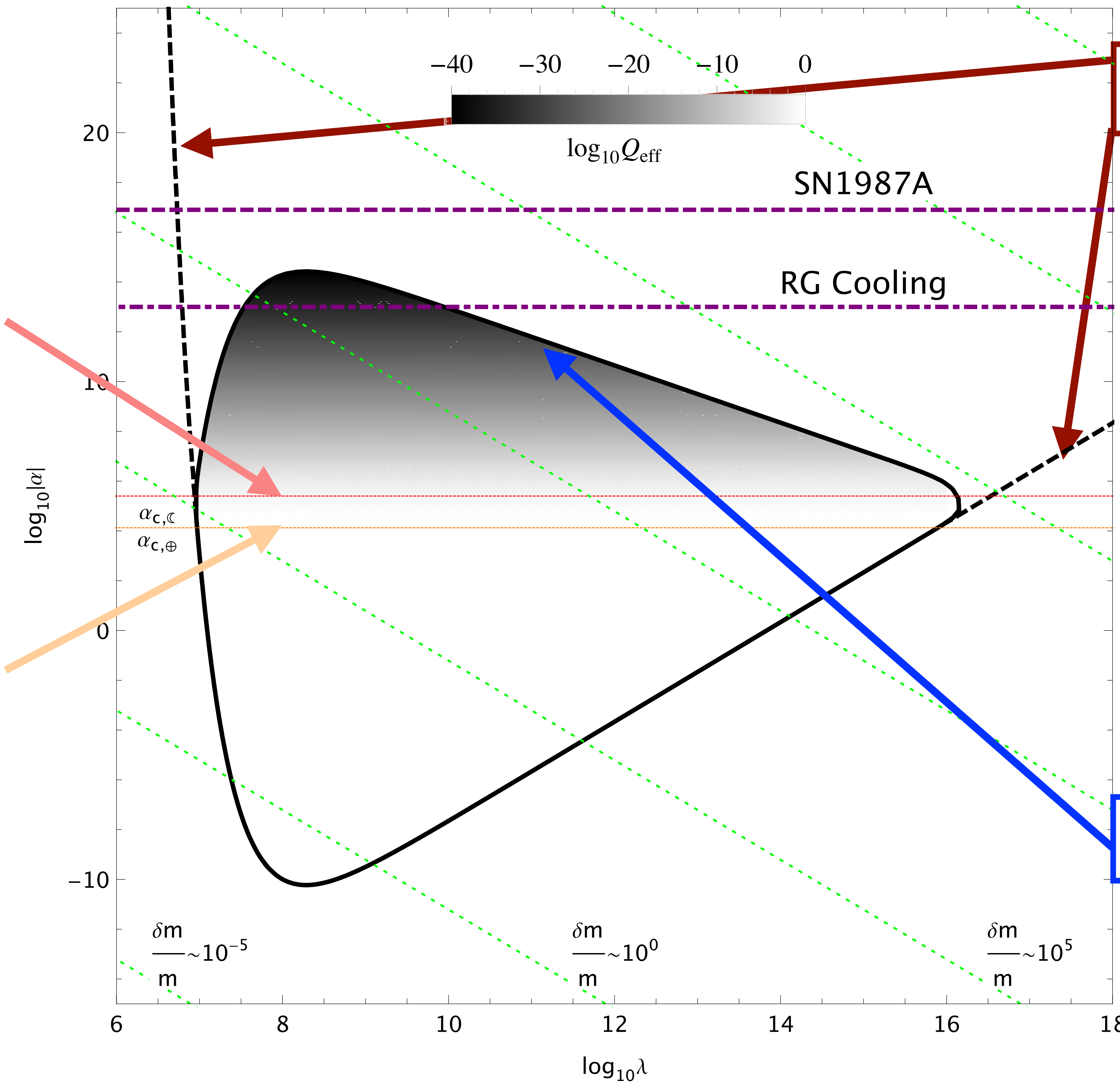
Effective charge

Above $\alpha_{c,i}^{(n)}$, charge is screened.

Consider Earth-Moon system:



Anomalous precession of Moon



$\alpha_c^{(4)}{}_{☾} \sim 10^5$

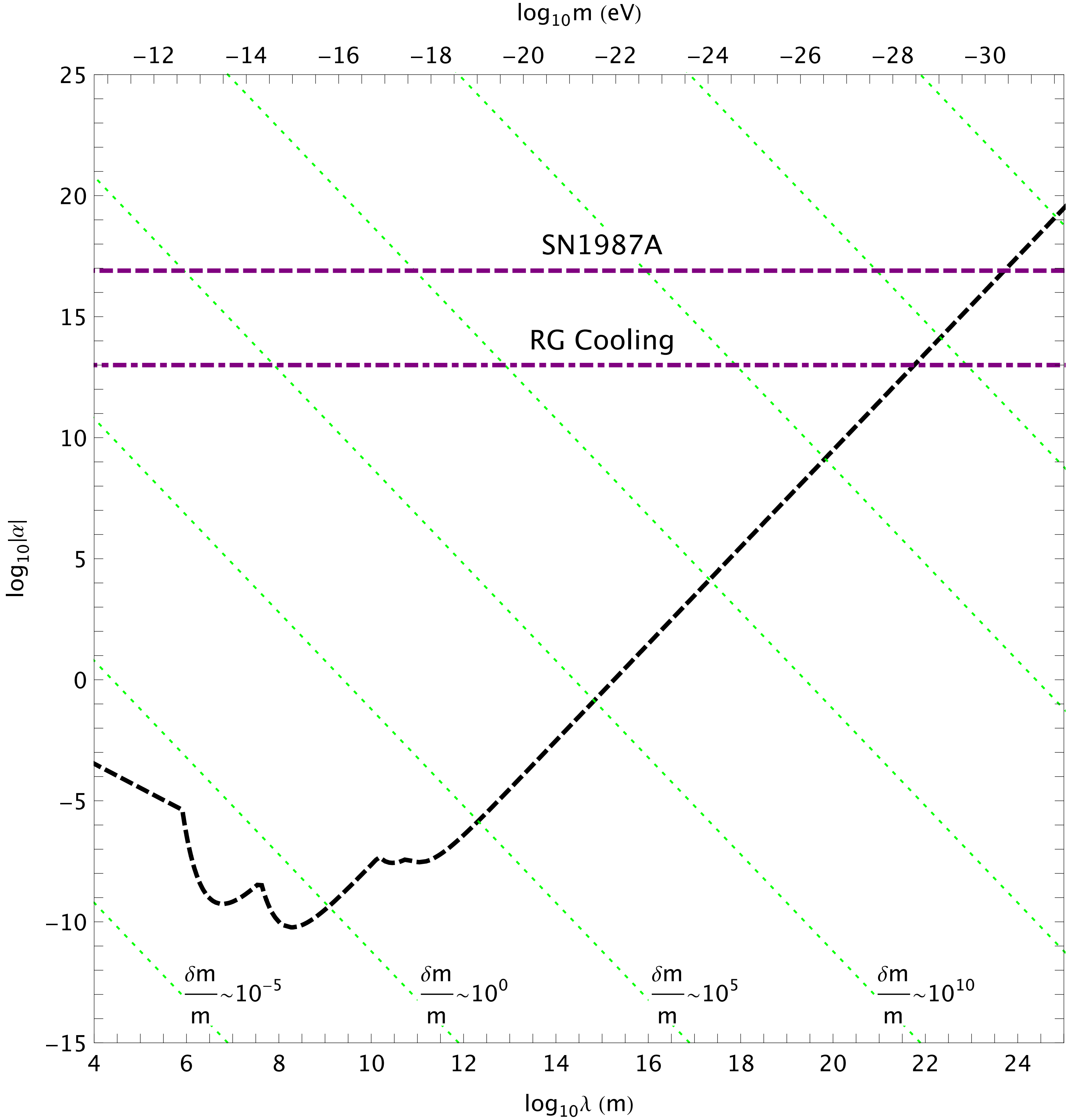
$\alpha_c^{(4)}{}_{\oplus} \sim 10^4$

Old constraint

New constraint

Quartic — Old constraints

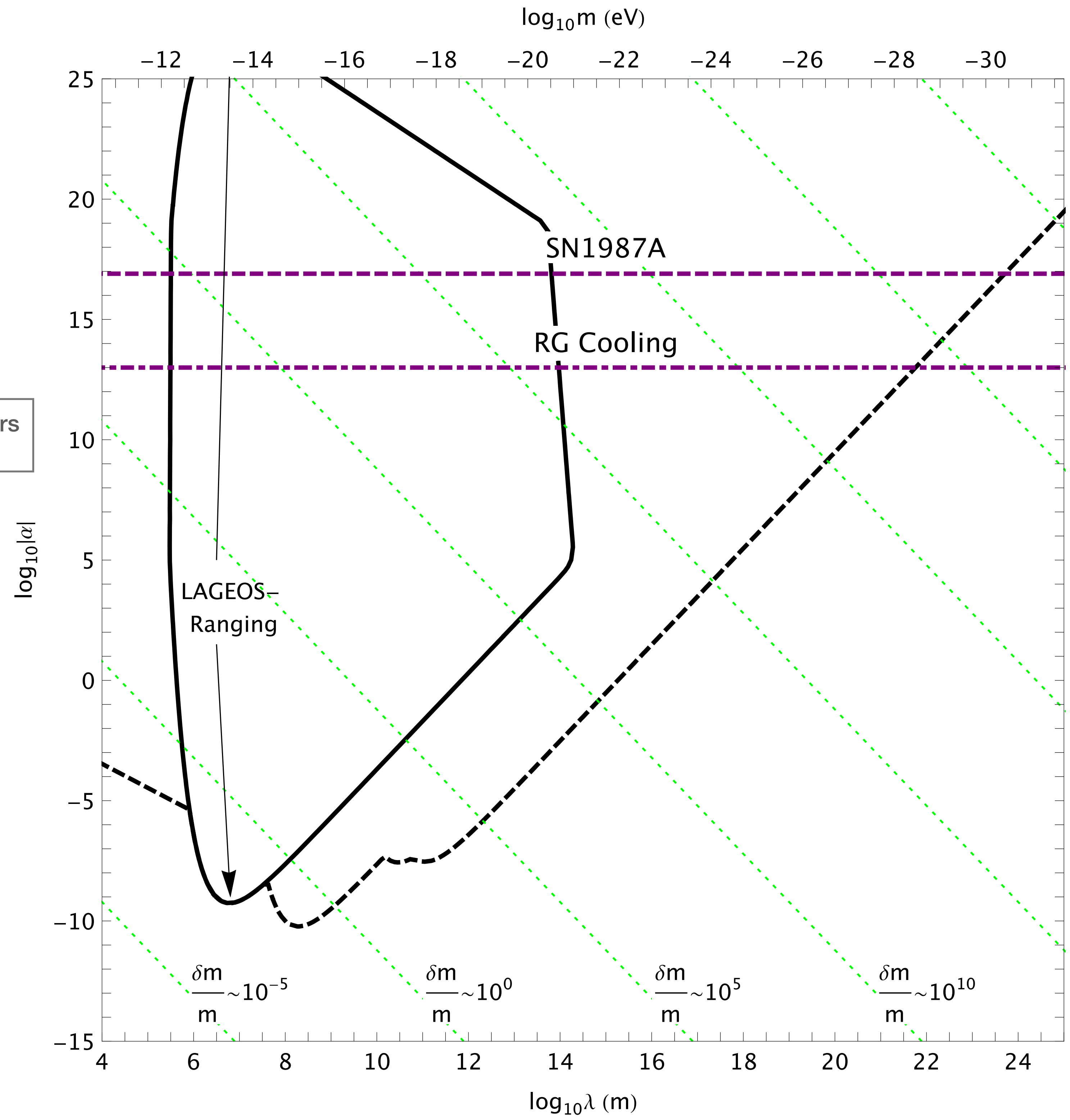
$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$



Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Quartic — LAGEOS anomalous precession $\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$

Lucchesi & others *(+ 2014)*

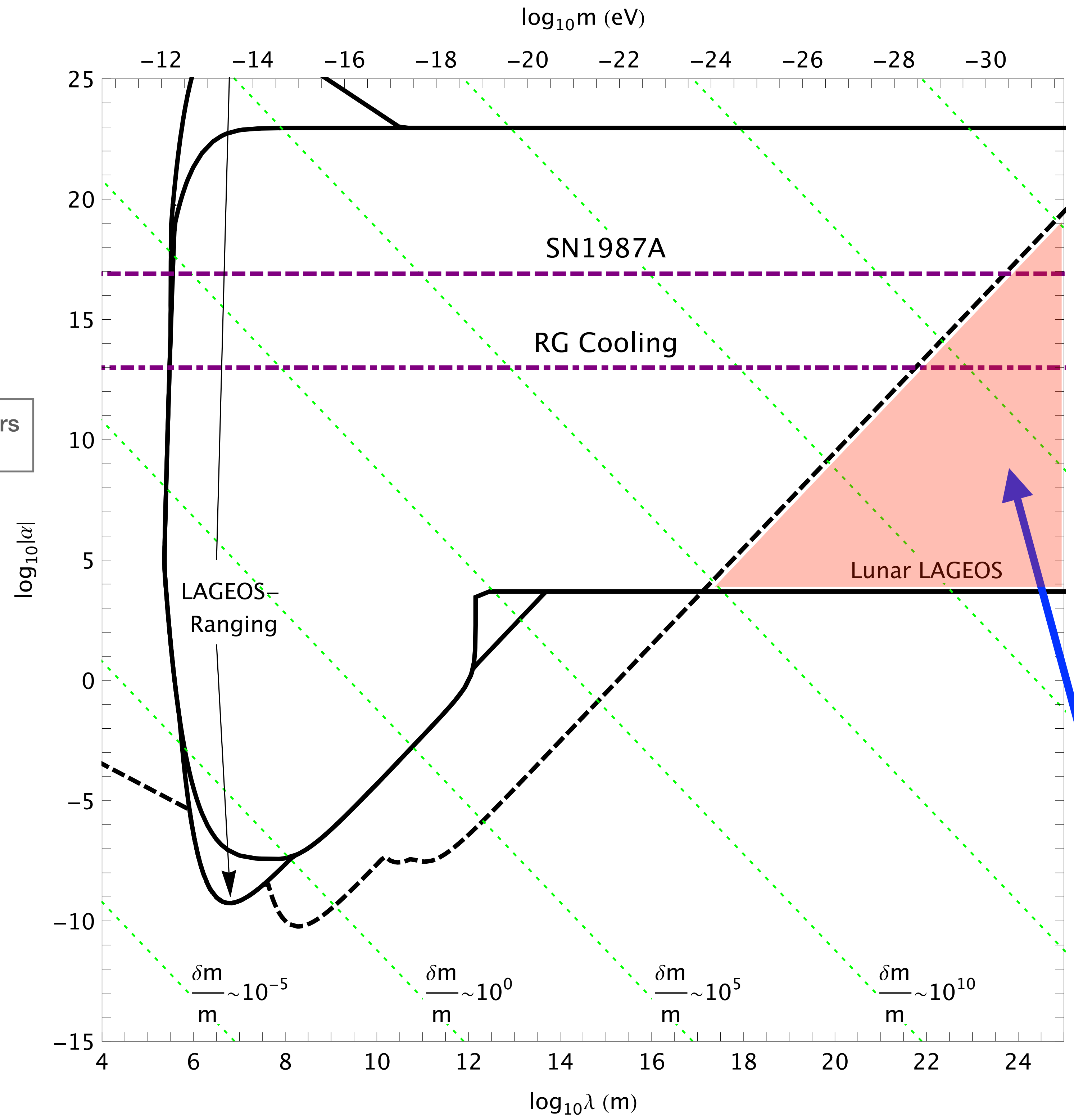


Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Quartic — Lunar-LAGEOS

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$

Lucchesi & others
(+ 2014)



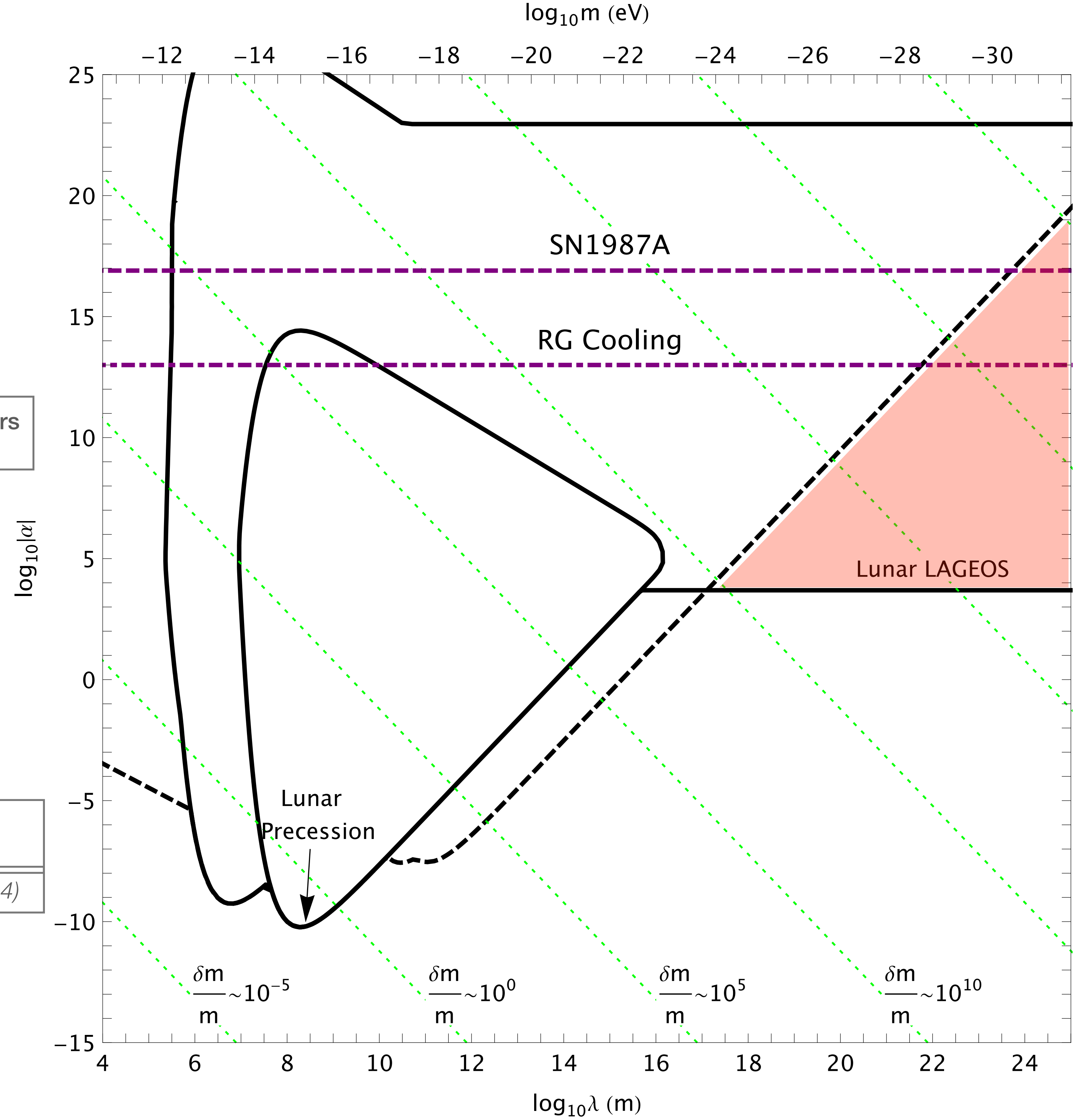
Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Fischbach & Talmadge *(1998)*

New constraint due to effective EP violation

Quartic — Lunar precession

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$



Hardy & Lasenby *(2016)*
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Lucchesi & others
(+ 2014)

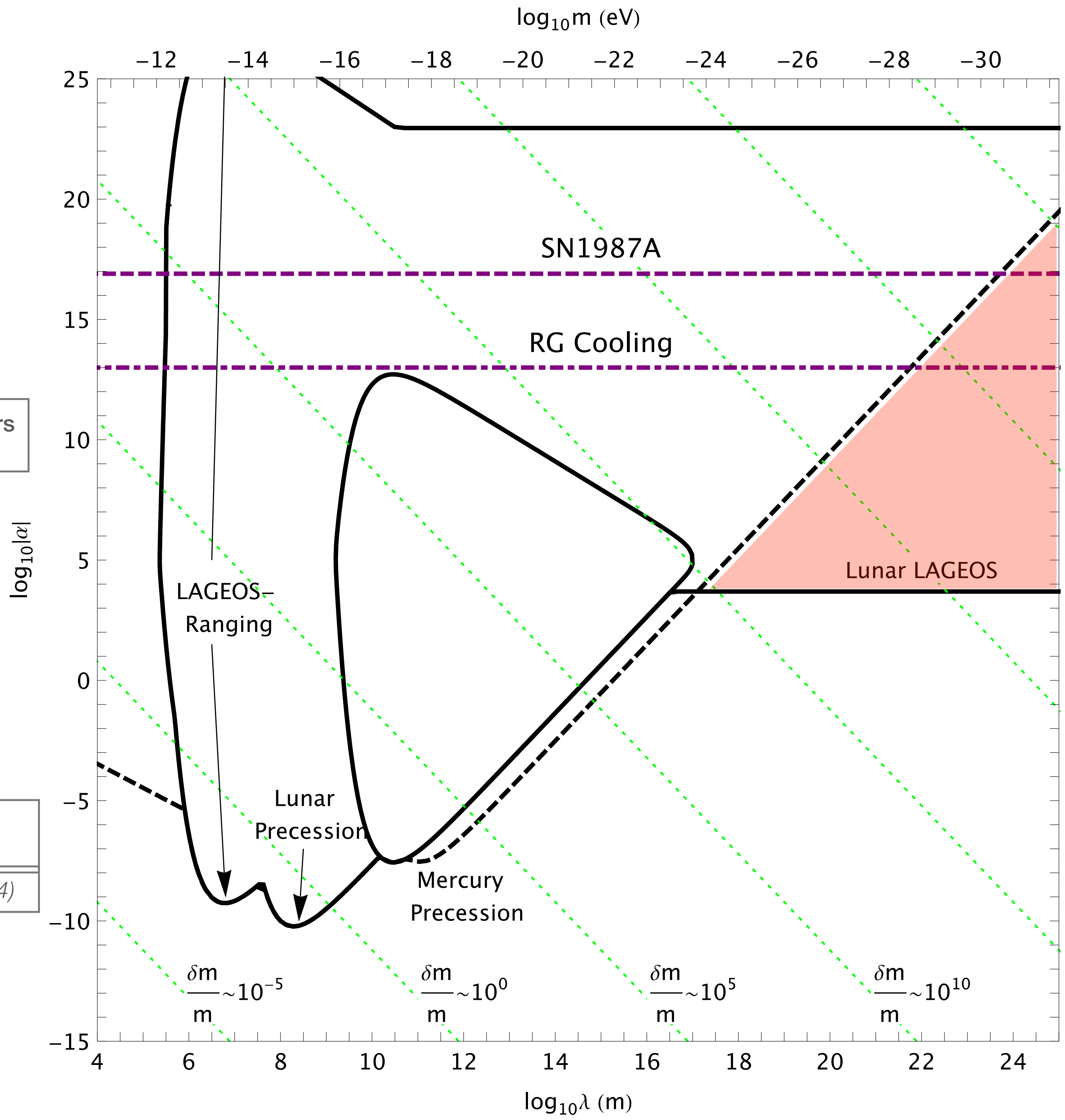
Fischbach & Talmadge *(1998)*

Talmadge et al
(1988)

Dickey et al *(1994)*

Quartic — Mercury precession

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$



Lucchesi & others *(+ 2014)*

Talmadge et al *(1988)*

Dickey et al *(1994)*

Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Fischbach & Talmadge *(1998)*

Talmadge et al *(1988)*

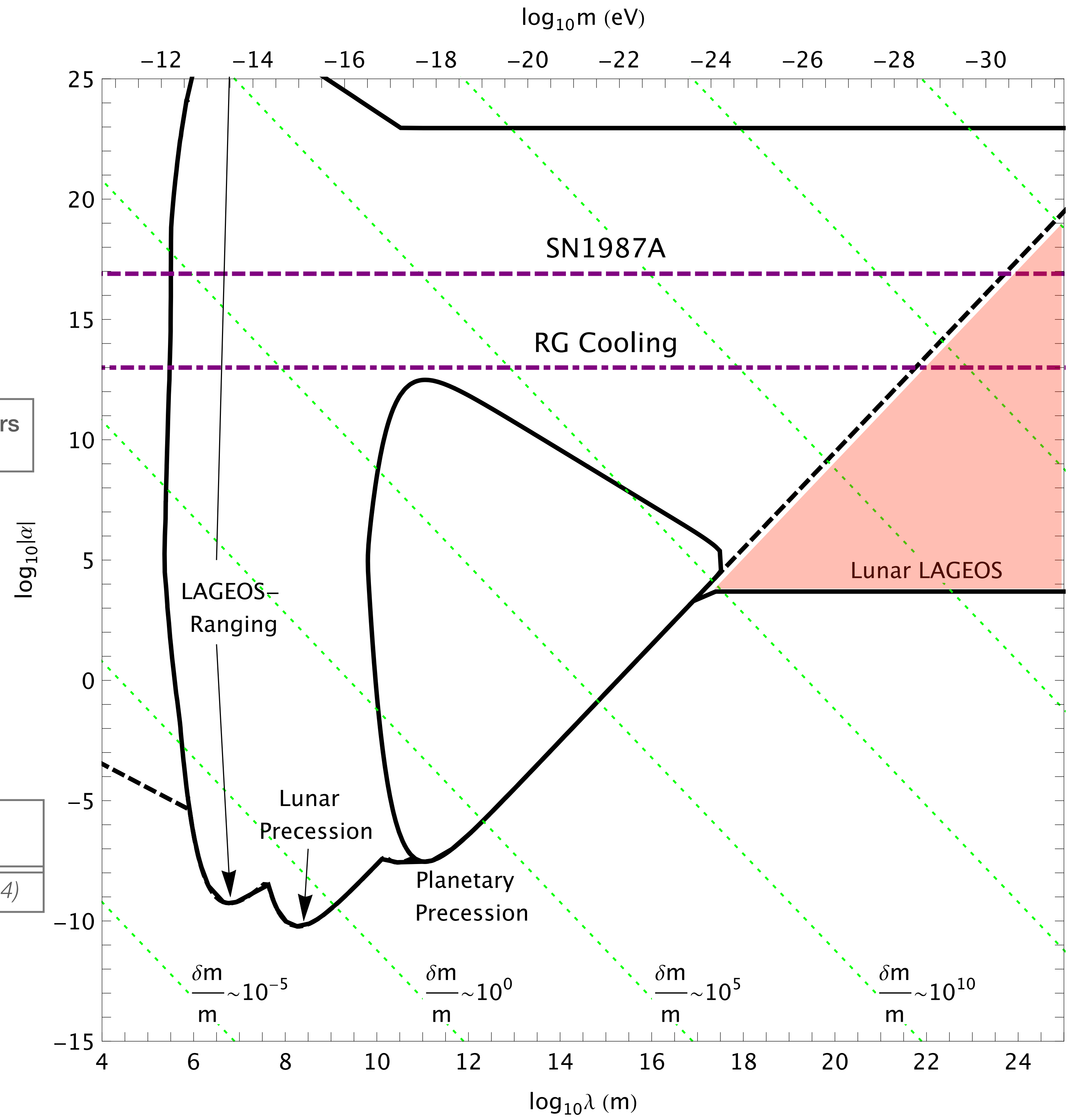
Quartic — Mars precession

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$

Lucchesi & others *(+ 2014)*

Talmadge et al *(1988)*

Dickey et al *(1994)*



Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Fischbach & Talmadge *(1998)*

Talmadge et al *(1988)*

Quartic — Earth-LAGEOS

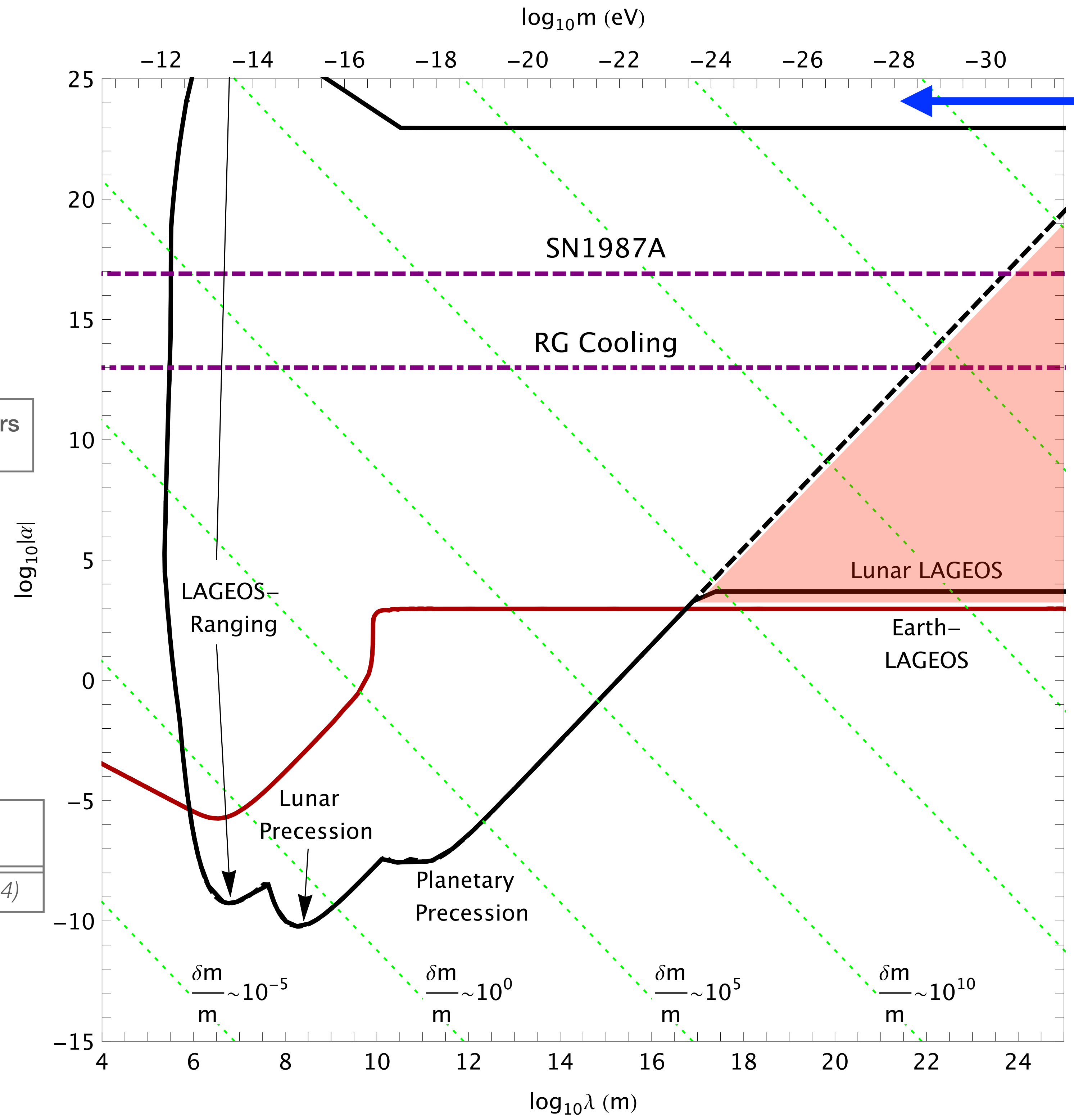
$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$

Closed off

Lucchesi & others (+ 2014)

Talmadge et al (1988)

Dickey et al (1994)



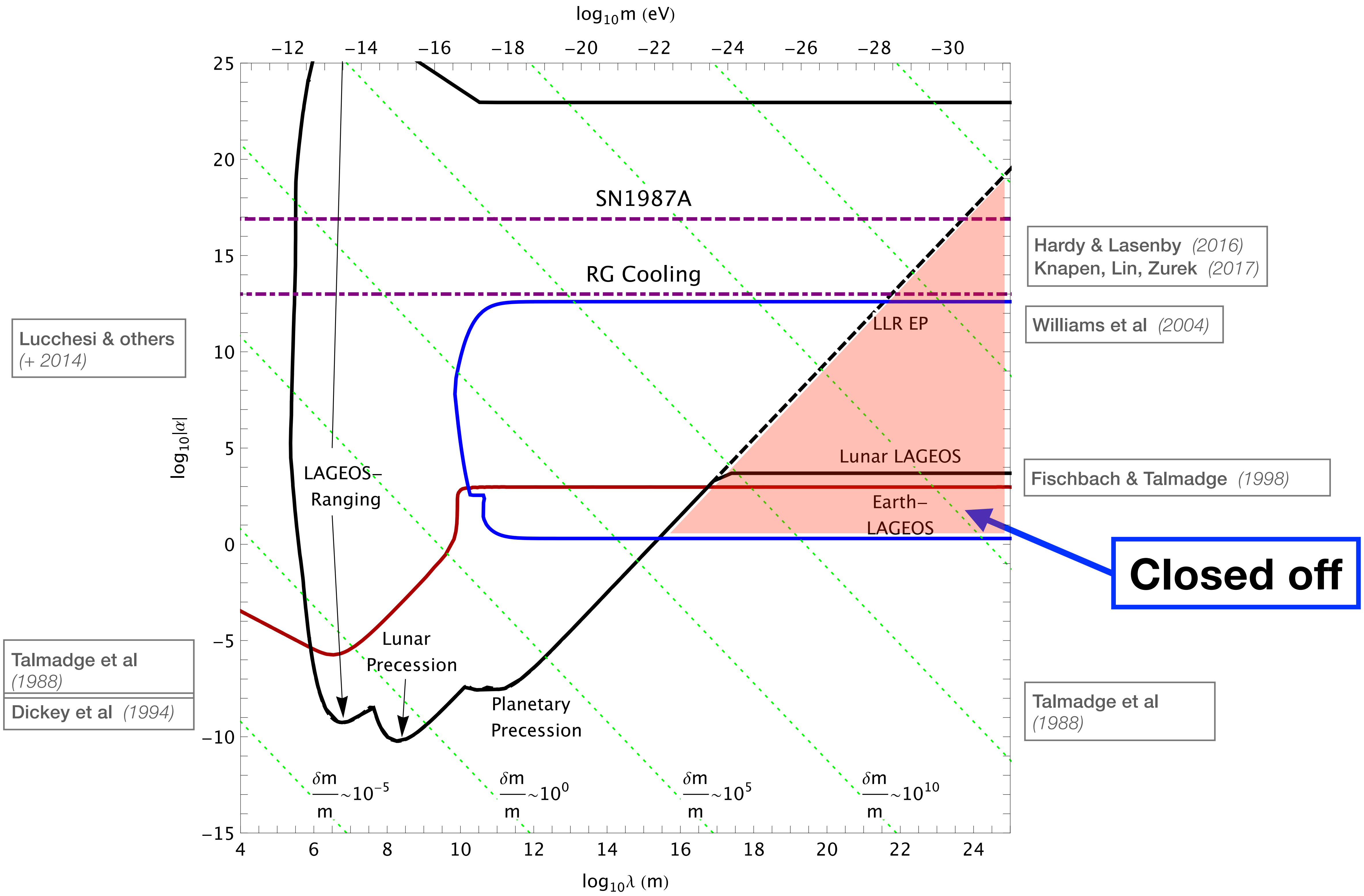
Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Fischbach & Talmadge *(1998)*

Talmadge et al *(1988)*

Quartic — Earth-Moon-Sun (LLR-EP)

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$



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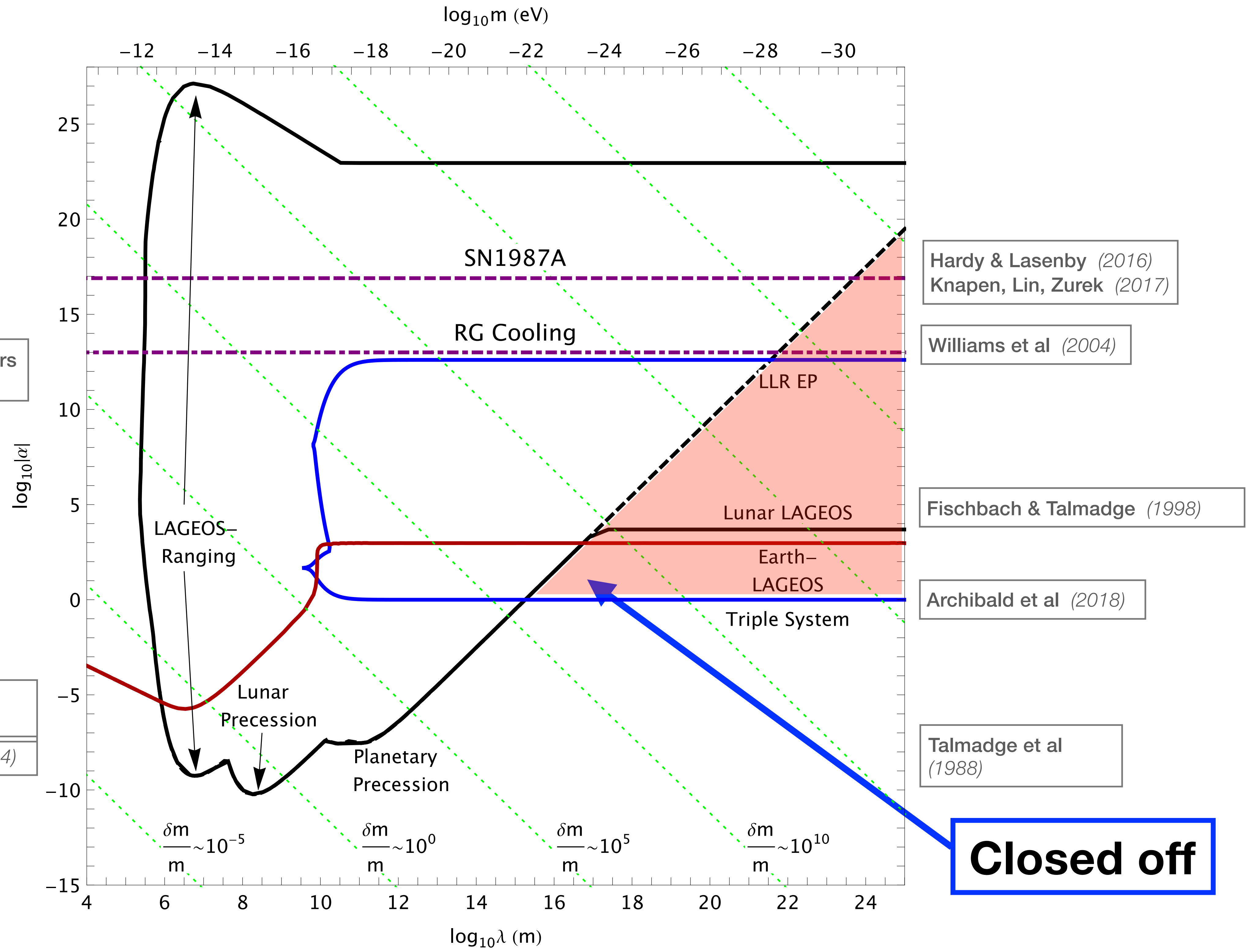
Williams et al *(2004)*

Fischbach & Talmadge *(1998)*

Talmadge et al *(1988)*

Closed off

Quartic — Triple System *PSR J0337+1715* $\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$



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Fischbach & Talmadge *(1998)*

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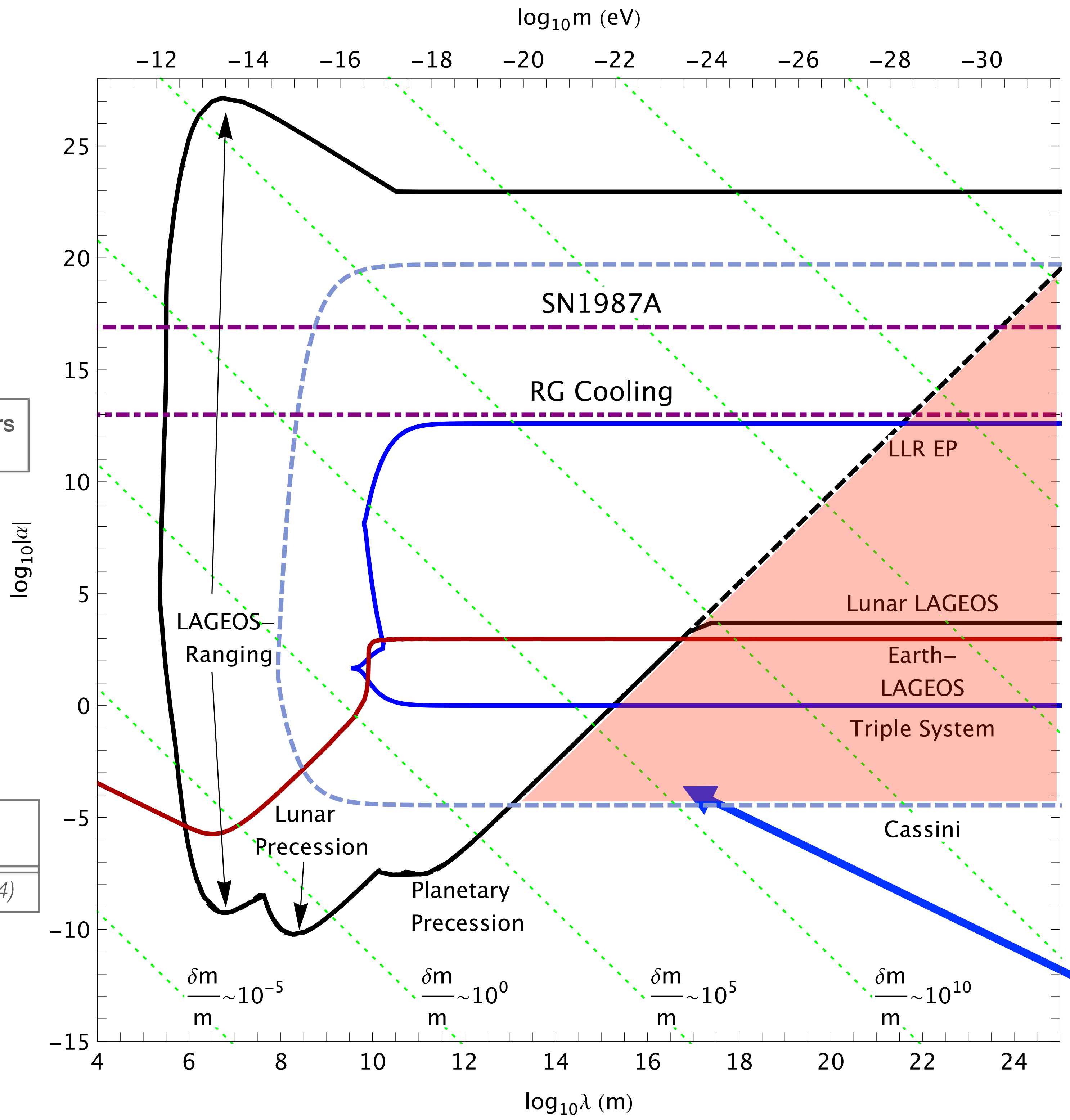
Talmadge et al *(1988)*

Closed off

Quartic — Cassini

$$\epsilon \sim \frac{\alpha^2 m_n^4}{M_{\rm pl}^4}$$



Hardy & Lasenby *(2016)*
Knapen, Lin, Zurek *(2017)*

Williams et al *(2004)*

Lucchesi & others
(+ 2014)

Fischbach & Talmadge *(1998)*

Archibald et al *(2018)*

Talmadge et al
(1988)

Dickey et al *(1994)*

Bertotti et al *(2003)*

Talmadge et al
(1988)

Closed off

VI. Higher-dimensional self-interactions

O(n>4) self-interaction

Higher-dim. interactions: insight into quantum gravity

Consider d=5 self-interaction, EoM:

$$\nabla^2\varphi = \frac{\varphi^4}{\Lambda}$$

Imagine force discovered with 1/r behaviour.

Op. of d=5 will not cause deviations as long as

$$\frac{1}{r^2}\frac{Q}{r} \gtrsim \frac{1}{\Lambda}\frac{Q^4}{r^4}$$

e.g. Sun: $Q_\odot \lesssim 10^{35}, \; r \sim 10^{11} \text{ m} \implies \boxed{\Lambda \gtrsim 10^{60} M_{\rm pl}}$

O(n>4) self-interaction

More generally, O(n>4) self-interaction:

$$\nabla^2\varphi = \frac{\varphi^{n-1}}{\Lambda^{n-4}}$$

Measurement of 1/r force implies:

$$\Lambda > \frac{Q^{\frac{n-2}{n-4}}}{r}$$

For n=5,6 constraint is super-Planckian

n≥7 sub-Planckian

O(n=5) self-interaction in detail

Alternative approach: consider impact of tree-level self-interaction for n=5 self-interaction

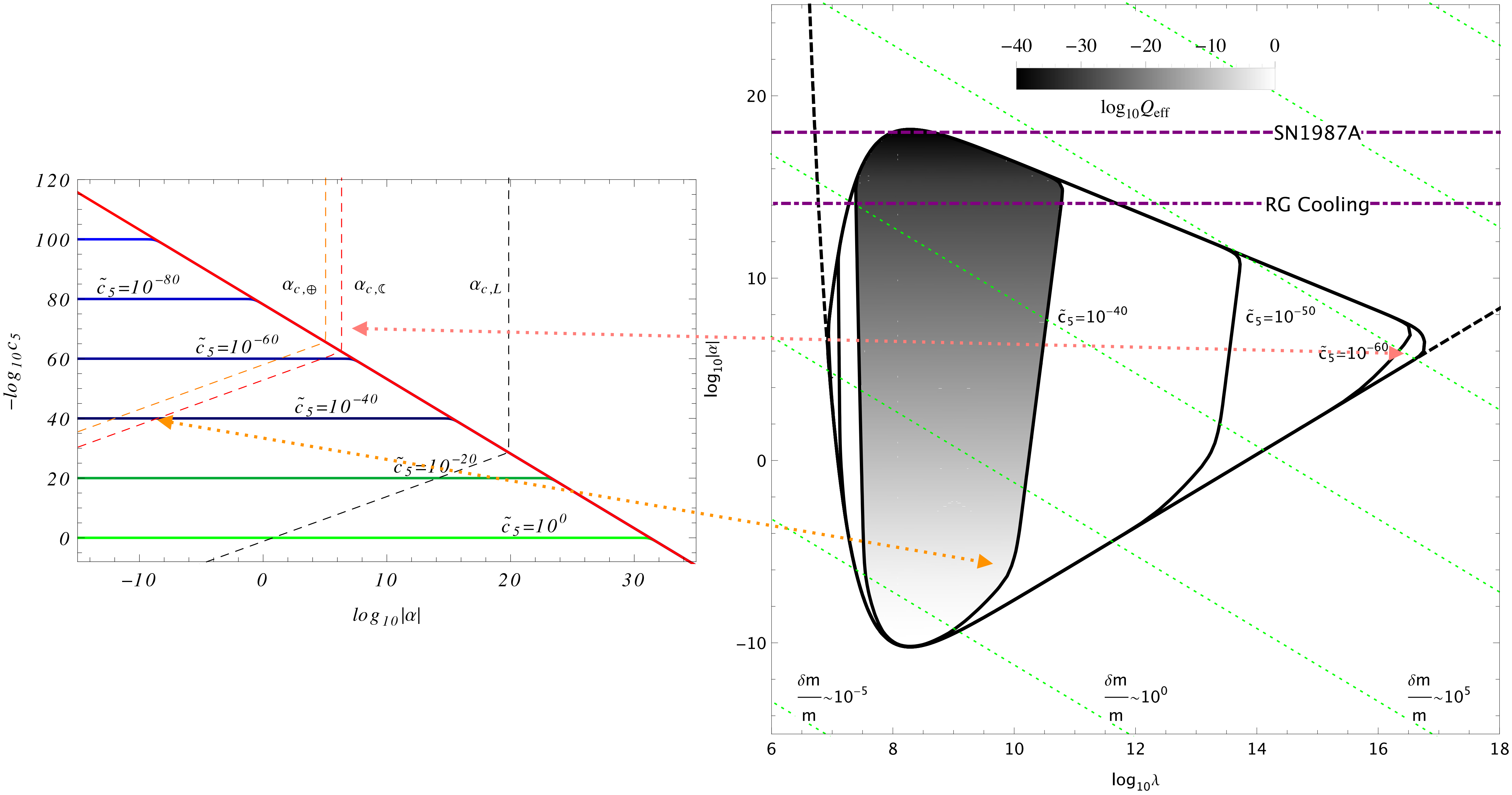
$$\frac{c_5}{\Lambda} \sim \frac{(4\pi\alpha)^{5/2}}{16\pi^2}\left(\frac{m_n}{M_{\rm pl}}\right)^4 \frac{1}{M_{\rm pl}} \sim 10^{-76}\frac{\alpha^{5/2}}{M_{\rm pl}}$$

Add tree-level contribution

$$\frac{c_5}{\Lambda} \sim 10^{-76}\frac{\alpha^{5/2}}{M_{\rm pl}} + \frac{\tilde{c}_5}{M_{\rm pl}}$$

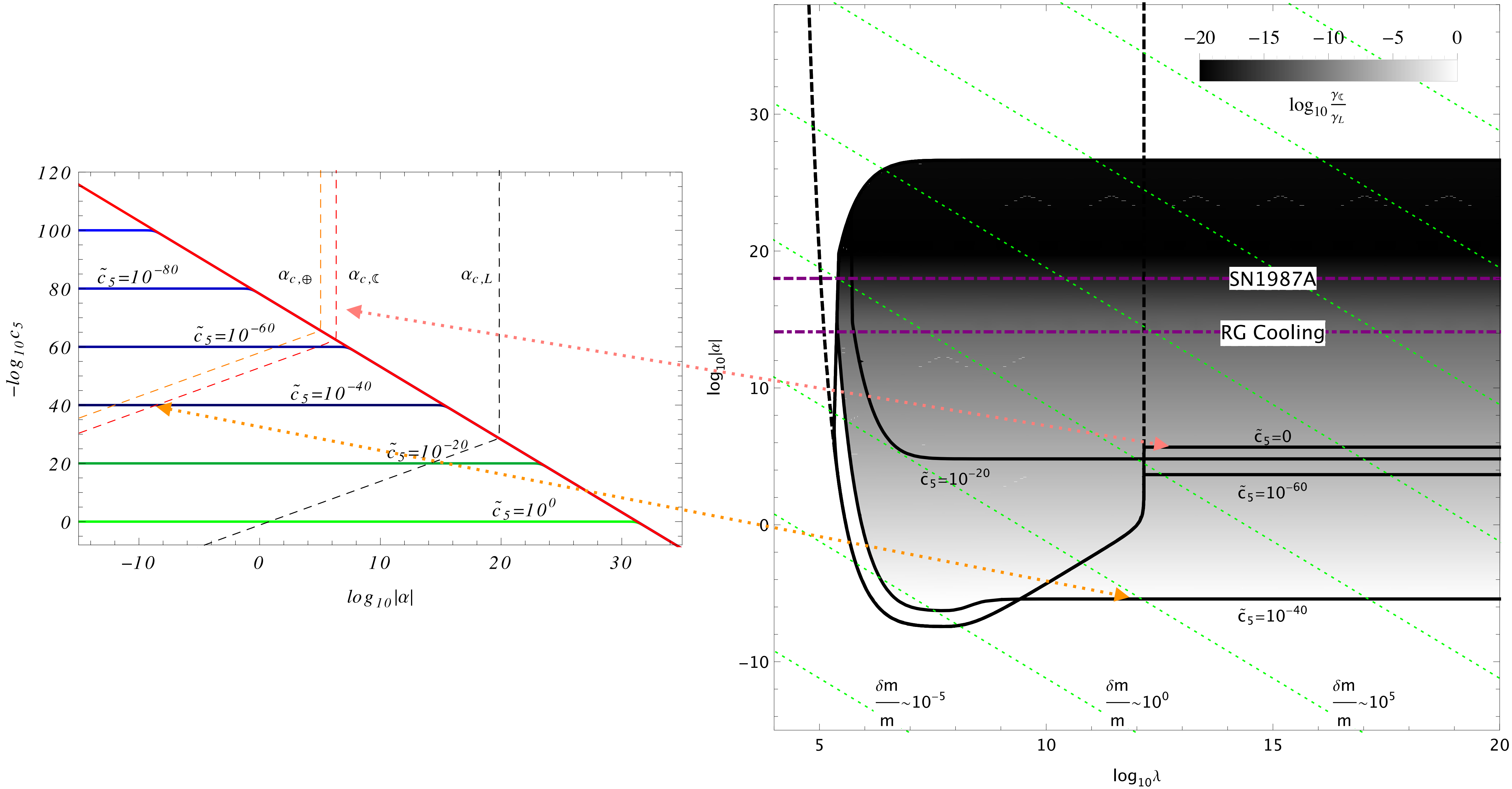
Impact of tree-level d=5 self-interaction

Example of lunar precession bound:



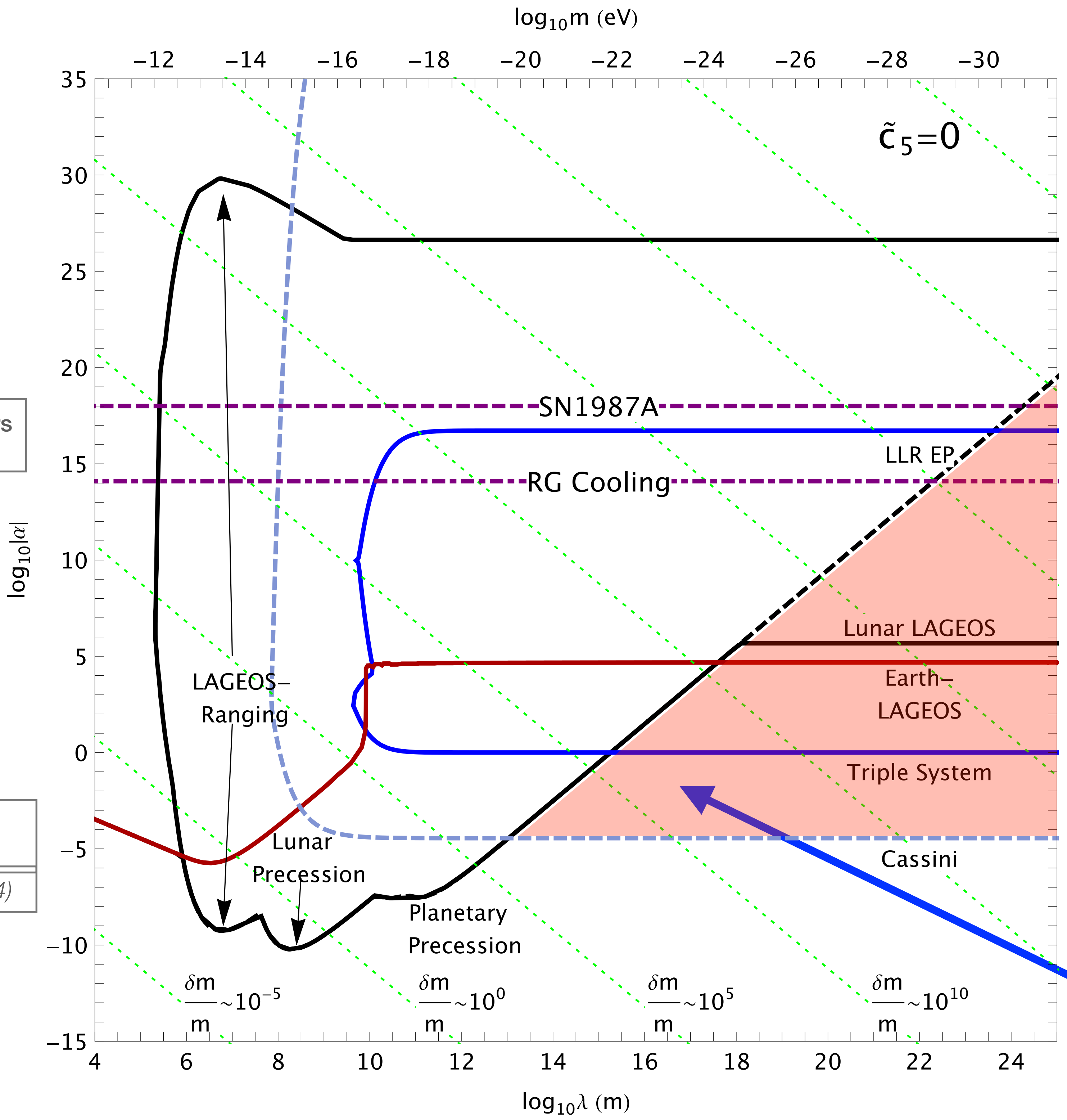
Impact of tree-level d=5 self-interaction

Example of Lunar-LAGEOS bound:



Quintic — natural-only

$$\frac{c_5}{\Lambda} \sim \frac{(4\pi\alpha)^{5/2}}{16\pi^2}\left(\frac{m_n}{M_{\rm pl}}\right)^4 \frac{1}{M_{\rm pl}}$$



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Closed off

VII. Cubic self-interactions

Cubic self-interaction

Qualitatively different — relevant operator w/ characteristic distance

$$\frac{1}{r_c^2}\frac{Q}{r_c} \sim \kappa\frac{Q^2}{r_c^2} \Rightarrow r_c \sim \frac{1}{\kappa Q}$$

Possible regimes:

$$\begin{aligned} r < r_c : &\quad \varphi \sim \frac{Q}{r} \\ 1/m > r > r_c : &\quad \varphi \sim \frac{1}{\kappa r^2} \\ r > 1/m : &\quad \varphi \sim \frac{e^{-mr}}{r} \end{aligned}$$

Cubic self-interaction

Account for this by modifying potential

$$V_{5,ij}(r,m) = \frac{G_N M_i M_j}{r}\left(1 + \alpha\,\gamma_i\,\gamma_j\,e^{-mr}\left(1 + \boxed{\frac{f(\kappa)}{r}}\right)\right)$$

Function $f(\kappa)$ encodes different regimes

$$f(\kappa) \xrightarrow{\kappa\to 0} 0$$

$$f(\kappa) \xrightarrow{\text{large } \kappa} \kappa^{-1}$$

Caveat: potential above is not solution of EoM, but gives good fit to numerical solutions

Cubic self-interaction: Vacuum decay

Existence of large cubic means vacuum is metastable

Tunneling constraint:

$$S_E \approx \frac{205 m^2}{\kappa^2}$$

$$\Rightarrow \qquad \kappa < \mathcal{O}(1)\, m$$

$$\Rightarrow \qquad \boxed{\alpha < 10^{25}\left(\frac{10^6\,\mathrm{m}}{\lambda}\right)^{2/3}}$$

Spontaneous

Cubic self-interaction: Vacuum decay

If $\beta\kappa < 0$, bubble of true vacuum can nucleate

$$|\kappa|\varphi(R_c)^3 \sim \frac{\varphi(R_c)^2}{R_c^2} \Rightarrow R_c \sim \frac{1}{\varphi_0 R_0 |\kappa|}$$

$$\varphi(R) \sim \frac{\varphi_0 R_0}{R}$$

Requiring we be starting near the origin:

$$\varphi(R_c) \lesssim m^2/\kappa$$

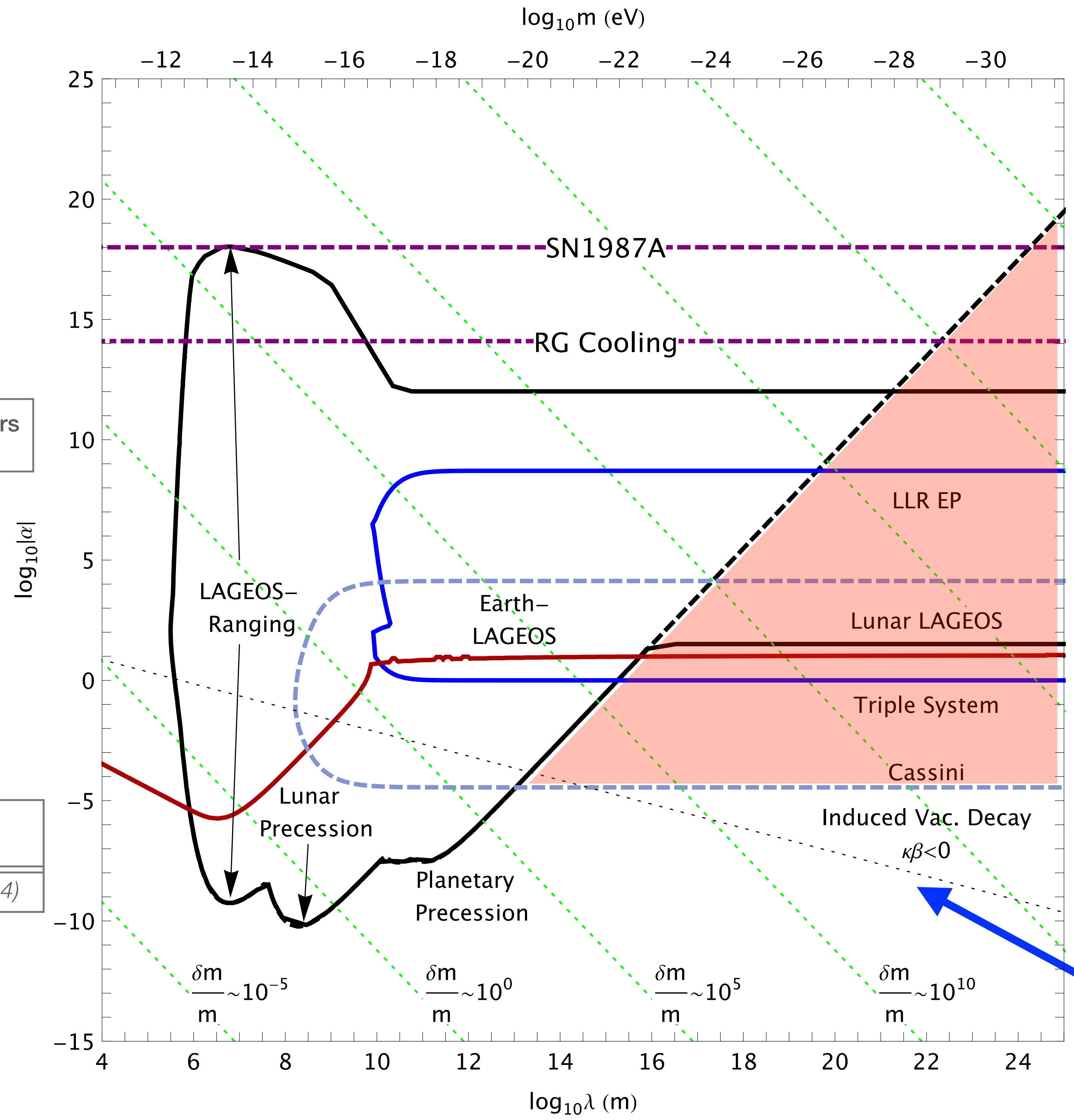
$$\Rightarrow \qquad |\kappa| < \frac{m}{\varphi_0 R_0}$$

Induced

Consider Neutron Star: $|\kappa| < \frac{m}{\beta M_{\text{NS}}} \Rightarrow \boxed{\alpha < 0.7 \left(\frac{10^6\ \text{m}}{\lambda}\right)^{1/2}}$

Cubic

$$\kappa \sim \frac{(4\pi\alpha)^{3/2} m_n}{16\pi^2}\left(\frac{m_n}{M_{\rm pl}}\right)^3$$



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Induced Decay

Summary

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Screening occurs in finite density media

Screening expected to weaken bounds

Natural self-interactions result in **stronger** bounds:

- Earth-LAGEOS closes large α parameter space

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- Earth-LAGEOS closes large α parameter space
- Effective EP violation constrains $m \to 0$ limit
- Strong bound on natural scalars from Cassini
- Higher-dim. ops: discovery of 5th force would place super-Planckian constraint
- Cubic operator: new constraints from vacuum decay