Homework Assignment I

Physics 105, Fall 04, Instructor: Petr Hořava

This first assignment is due Friday, September 17. The purpose of the assignment is roughly two-fold: (1) to brush up on elementary operations with vectors and matrices, something that will be required throughout the semester as a part of the standard mathematical technology; and (2) to refresh your memory on how one solves elementary dynamical problems using Newtonian mechanics. Thus, in my view, this week's homework tests your undestanding of some of the pre-requisite material for this course, at the level of the Freshman course of Mechanics. However, if you do experience any serious difficulties solving this week's homework, you may want to quickly catch up on the basics of Newtonian mechanics by reading the first few chapters of Marion-Thornton, for example. (Also, if any of the terminology used in this week's problems looks unfamiliar to you, you are advised to consult that same source.)

So, here are the first five problems. Enjoy!

1. Consider three vectors, $\mathbf{v}$, $\mathbf{w}$ and bz in the three-dimensional vector space $\mathbf{R}^3$. Consider the *triple scalar product* $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z}$, where $\times$ is the cross- (i.e., "vector-") product and $\cdot$ denotes the inner- (=”dot”) product of vectors. Show that this triple scalar product can be written as

$$(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z} = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

where the straight vertical lines on the right-hand-side denote the determinant of the 3×3 matrix consisting of the components of the three vectors in an orthonormal basis.

2. Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \qquad \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}.$$

Find the following:

$$\text{(a) } |AB|, \quad \text{(b)} AC, \quad \text{(c)} ABC, \quad \text{(d)} AB - B^t A^t,$$

where A^t denotes the transposition of the matrix A, etc.

3. Find the position and velocity of a particle undergoing vertical motion in a medium having a retarding force proportional to the velocity. More precisely, consider a particle falling downward with an initial velocity v_0 from a height h in a constant gravitational field, and in the presence of a retarding force proportional to the particle's velocity. Write down the Newtonian equation of motion and solve it.

4. Study the motion of a projectile in two dimensions (i.e., vertical distance x and horizontal distance y) under the influence of constant gravitational force along y, without

considering air resistance. The projectile starts with an intitial velocity $\mathbf{v}$ at $y = 0$. Calculate the projectile's position, velocity and range (= the value of the x coordinate at the impact point where the projectile hits the ground at $y = 0$). As a voluntary bonus, you can add the influence of air resistance.

5. Consider an n-dimensional smooth manifold $\mathcal{M}$, and assume an arbitrary coordinate system x_i, $i = 1, \ldots n$ on it. What would be wrong with defining the distance on $\mathcal{M}$ by the following formula,

$$|\mathbf{x}|^2 = \sum_{i=1}^{n} (x_i)^2 \quad ?$$