Physics 250: Introduction to String Theory

Homework Assignment II

This assignment is worth 10 points, and is due in class on Thu, Feb 21.

1. (A slight extension of Problem 1.4 of [Polchinski, Vol. I]):
   (a): Show that the open string states at levels $m^2 = 1/\alpha'$ and $m^2 = 2/\alpha'$ form complete representations of $SO(D-1)$. You will need to work out the $SO(D-2)$ content of various symmetric and antisymmetric tensor representations of $SO(D-1)$.
   (b): Repeat (a) for the lowest massive level of the closed oriented bosonic string.
2. Consider closed oriented bosonic strings in flat, uncompactified spacetime $\mathbf{R}^{26}$. This theory has a $\mathbf{Z}_2$ symmetry Ω, induced on all world-sheet fields by the action of world-sheet parity

$$\Omega : (\tau, \sigma) \to (\tau, \ell - \sigma). \tag{1}$$

Consider the $\mathbf{Z}_2$ orientifold of this theory by Ω, and find the oscillator expansion of the twisted string sector on the orientifold, by solving the twisted periodicity conditions (indicating that the string in the twisted sector is "closed up to the action of Ω")

$$\begin{aligned} X^\mu(\tau, \sigma + \ell) &= \Omega[X^\mu(\tau, \sigma)], \\ X^\mu(\tau, \sigma - \ell) &= \Omega[X^\mu(\tau, \sigma)], \end{aligned} \tag{2}$$

with the action of Ω on $X^\mu(\tau, \sigma)$ defined by

$$\Omega[X^\mu(\tau, \sigma)] \equiv X^\mu(\Omega(\tau, \sigma)) \equiv X^\mu(\tau, \ell - \sigma). \tag{3}$$

What is the physical interpretation of the twisted sector? Given this physical interpretation, can you explain why both conditions in Eq. (2) (and not just, say, the first one) are necessary?

3. (Problem 1.9 of [Polchinski, Vol. I]):
   Consider the bosonic oriented closed string on $\mathbf{R}^{26}/\mathbf{Z}_2$, where the orbifold group $\mathbf{Z}_2$ acts as $X \to -X$ on the 26-th space-time dimension while leaving the other dimensions (and the world-sheet of the string) invariant. Find the mode expansion, the string mass spectrum, and the constraint following from σ-translation invariance in terms of the occupation numbers, in the twisted sector of this orbifold:

$$\begin{aligned} X(\tau, \sigma + \ell) &= -X(\tau, \sigma), \\ X^\mu(\tau, \sigma + \ell) &= X^\mu(\tau, \sigma) \quad \text{for } \mu = 0, \ldots 24. \end{aligned} \tag{4}$$