Physics 250: Introduction to String Theory

Homework Assignment VI

This assignment consists of two parts, the first is worth 10 points, while the other is purely optional. This assignment is due in class on Thu, May 9. The main theme are tree-level amplitudes for string scattering.

1. (Exercise 6.8 of [Polchinski I]):

   Derive the tree-level amplitude for three massless vector bosons with momenta k_i (with $i = 1, \ldots 3$), polarizations e_i and Chan-Paton factors λ^{a_i} (in the adjoint representation of $SU(N)$) in the open bosonic string theory, as given by eqn. (6.5.15) of [Polchinski I],

$$\begin{aligned} &S_{D_2}(k_1, a_1, e_1; k_2, a_2, e_2; k_3, a_3, e_3) \\ &= i g_o'(2\pi)^{26}\delta^{26}(\sum_i k_i)\left(e_1 \cdot k_{23}\, e_2 \cdot e_3 + e_2 \cdot k_{31}\, e_3 \cdot e_1 + e_3 \cdot k_{12}\, e_1 \cdot e_2 \right. \\ &\qquad \left. + \frac{\alpha'}{2} e_1 \cdot k_{23}\, e_2 \cdot k_{31}\, e_3 \cdot k_{12}\right) \mathrm{Tr}\left(\lambda^{a_1}[\lambda^{a_2}, \lambda^{a_3}]\right), \end{aligned}$$

   where $k_{ij} = k_i - k_j$. How does it compare to the amplitude in Yang-Mills gauge theory described by the spacetime action

$$S = -\frac{1}{4 g_o'^2}\int d^{26}x\, \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}),$$

   where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, and $A_\mu = A^a_\mu \lambda^a$?

2. **(OPTIONAL!)** If you find yourself bored by the simplicity of the previous problem, and/or are interested in seeing more of the Yang-Mills structure appearing from string theory, try exercise 6.9 of [Polchinski I], in which the generalization of the previous problem to the four-point function of massless vectors at tree level is considered.