Annihilation in $B \to M_1 M_2$

Zoltan Ligeti

Capri, May 30, 2006

- Introduction
- SCET and the “zero-bin”

  A new approach to factorization

  [Manohar & Stewart hep-ph/0605001]
- Factorization for weak annihilation

  [Arnesen, ZL, Rothstein, Stewart, hep-ph/0606nnn]

Motivation

- Many observables sensitive to NP — can we disentangle from hadronic physics?
  - $B \to \pi\pi, K\pi$ branching ratios and CP asymmetries (related to α, γ in SM)
  - Transverse polarization in charmless $B \to VV$ decays
  - α from $B \to \pi\pi$ using SCET vs. α from CKM fit

  Dozens, if not hundreds of papers... (2σ-type effects at present)

- Various power suppressed contributions to amplitudes have been argued to be large, and often described by complex parameters

  E.g., "annihilation" and "chirally enhanced" terms

- First derive correct expansion in $m_b \gg \Lambda_{\rm QCD}$ limit, then worry about predictions
  - Need to test accuracy of expansion (even in $B \to \pi\pi$, $|\vec{p}_q| \sim 1\,\mathrm{GeV}$)
  - Sometimes model dependent additional inputs needed

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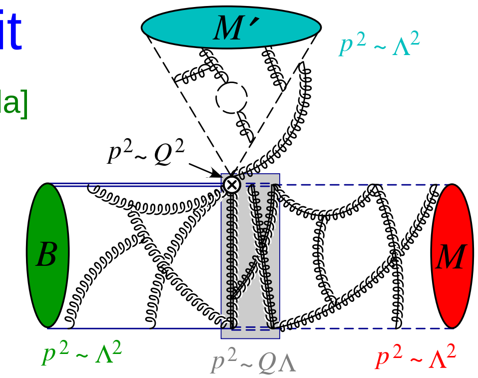
Charmless $B \to M_1 M_2$ (a month ago)

- Some (dis)agreements about implications of heavy quark limit

[Bauer, Pirjol, Rothstein, Stewart; Chay, Kim; Beneke, Buchalla, Neubert, Sachrajda]



$$A = A_{c\bar{c}} + N \bigg[f_{M_2}\, \zeta^{BM_1} \int \mathrm{d}u\, T_{2\zeta}(u)\, \phi_{M_2}(u) + f_{M_2} \int \mathrm{d}z\mathrm{d}u\, T_{2J}(u,z)\, \zeta_J^{BM_1}(z)\, \phi_{M_2}(u) + (1 \leftrightarrow 2) \bigg]$$

- $\zeta_J^{BM_1} = \int \mathrm{d}x\mathrm{d}k_+\, J(z,x,k_+)\, \phi_{M_1}(x)\, \phi_B(k_+)$ also appears in $B \to M_1$ form factors

  $\Rightarrow$ Relations to semileptonic decays do not require expansion in $\alpha_s(\sqrt{\Lambda Q})$

- Charm penguins: suppression of long distance part argued, not proven

  Lore: “charming penguins”, “long distance charm loops”, “$D\overline{D}$ rescattering” all related (unknown) physics, could yield strong phases, etc. [Ciuchini et al.; ...]

- SCET: fit both ζ’s and ζ_J’s, calculate T’s; QCDF: fit ζ’s, use factorization to calculate $\zeta_J \sim \phi_B \phi_M$; PQCD: $k_\perp$ dependent soft form factor is suppressed

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SCET & zero-bin

HQET vs. SCET

- HQET: nonperturbative interactions do not change four-velocity of heavy quark

$p_b^\mu = m_b v^\mu + k^\mu$ — once we fix v, superselection rule; v label, k residual momenta

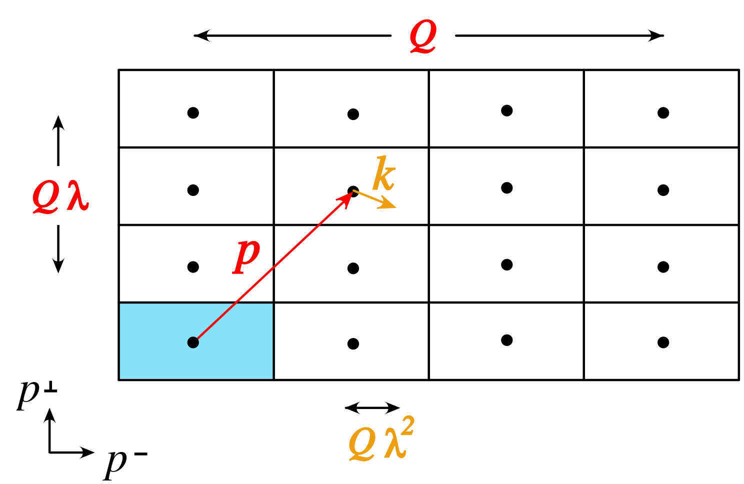
$$b(x) = \sum_v e^{-i m_b v\cdot x}\left[\tfrac{1}{2}(1+\not{v})\underbrace{h_v^{(b)}(x)}_{\text{large}} + \tfrac{1}{2}(1-\not{v})\underbrace{\widetilde{h}_v^{(b)}(x)}_{\text{small}}\right]$$

- SCET: light-cone momentum of collinear partons change via $\mathcal{O}(1)$ interactions

Collinear quark in n direction: $p^- = \bar{n}\cdot p$ and $p_\perp$ are labels, but no superselection rule (label conserv.)

$$p^\mu = \tfrac{1}{2}(\bar{n}\cdot p)n^\mu + p_\perp^\mu + \tfrac{1}{2}(n\cdot p)\bar{n}^\mu\,,\quad n^2 = \bar{n}^2 = 0\,,\ n\cdot\bar{n} = 2$$

$$\psi(x) = \sum_p e^{-ip\cdot x}\left[\tfrac{1}{4}\not{n}\not{\bar{n}}\underbrace{\xi_{n,p}(x)}_{\text{large}} + \tfrac{1}{4}\not{\bar{n}}\not{n}\underbrace{\widetilde{\xi}_{n,p}(x)}_{\text{small}}\right]$$



- Need multiple fields to describe same particle

Minimal SCET details

- Effective theory for processes involving energetic hadrons, $E \gg \Lambda$

  [Bauer, Fleming, Luke, Pirjol, Stewart, + . . .]

  Introduce distinct fields for relevant degrees of freedom, power counting in λ

modes	fields	$p = (+, -, \perp)$	p^2
collinear	$\xi_{n,p},\, A^\mu_{n,q}$	$E(\lambda^2, 1, \lambda)$	$E^2\lambda^2$
soft	$q_q,\, A^\mu_s$	$E(\lambda, \lambda, \lambda)$	$E^2\lambda^2$
usoft	$q_{us},\, A^\mu_{us}$	$E(\lambda^2, \lambda^2, \lambda^2)$	$E^2\lambda^4$

  SCET$_{\rm I}$: $\lambda = \sqrt{\Lambda/E}$ — jets $(m{\sim}\Lambda E)$

  SCET$_{\rm II}$: $\lambda = \Lambda/E$ — hadrons $(m{\sim}\Lambda)$

  Match QCD $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$

- Can decouple ultrasoft gluons from collinear Lagrangian at leading order in λ

$$\xi_{n,p} = Y_n\, \xi^{(0)}_{n,p} \qquad A_{n,q} = Y_n\, A^{(0)}_{n,q}\, Y_n^\dagger \qquad Y_n = \mathrm{P}\exp\Big[ig \int_{-\infty}^x \mathrm{d}s\, n\cdot A_{us}(ns)\Big]$$

  Nonperturbative usoft effects made explicit through factors of Y_n in operators

  New symmetries: collinear / soft gauge invariance

- Simplified / new $(B \to D\pi, \pi\ell\bar\nu)$ proofs of factorization theorems [Bauer, Pirjol, Stewart]

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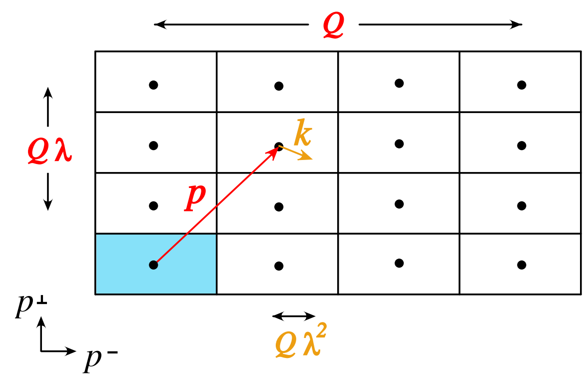
Modes, overcounting, zero-bin

- In a Wilsonian OPE, in the sense of scale separation, everything has to factorize

  May be difficult to see with continuum methods, factored objects can be complicated; e.g., strong phase in factorization for $B \to D^{(*)0} M_2^0$ decay [Mantry, Pirjol, Stewart]

- Often have to sum over collinear fields' labels and integrate over residual momenta

  $\sum_p \int \mathrm{d}k \to \int \mathrm{d}p$ sum excludes zero-bin ($p = 0$), where physics is described by soft mode

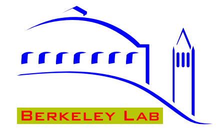


  Straightforward with hard cutoff, but less so in continuum (want to use dim. reg., etc.)

- MS explained how to add and subtract zero-bin to make computations convenient and avoid double counting:

$$\sum_{\text{labels}\neq 0} \int \mathrm{d}k \to \int \mathrm{d}p - \left(\int \mathrm{d}k \right)_{p=0}$$

[Manohar & Stewart, hep-ph/0605001]

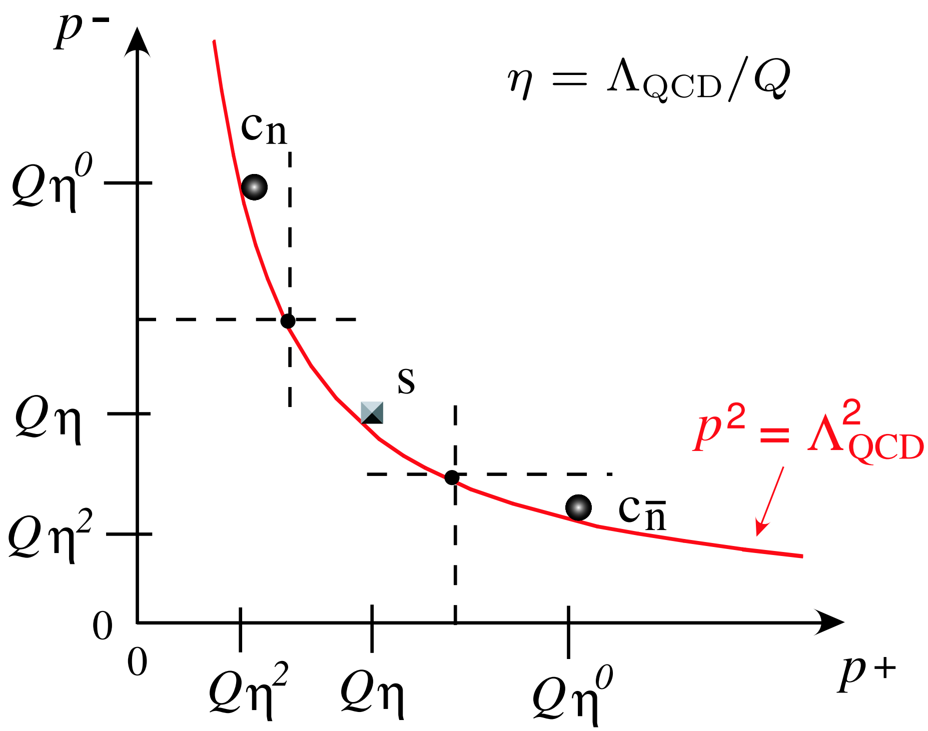


MS (zero-bin) factorization in $\mathrm{SCET}_{\mathrm{II}}$

- IR divergencies in perturbation theory cancel or cut off by nonperturbative physics

  When several fields describe same particle, special care to avoid double counting

- Modes and momentum regions in $\mathrm{SCET}_{\mathrm{II}}$



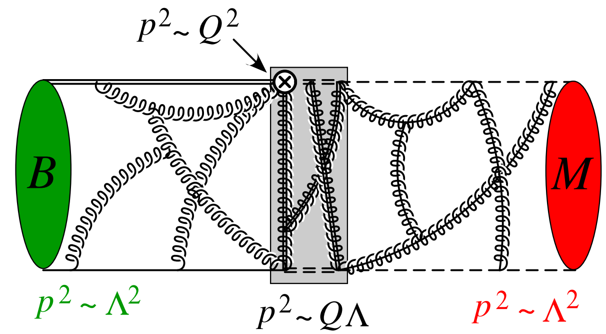
Possible double counting: zero-bin of collinear modes described by soft fields

The c_n, s, $c_{\bar{n}}$ modes have comparable invariant masses, but only interact via larger p^2 modes ("rapidity gaps")

Distinguish between modes by p^-/p^+ and keep boost inversion symmetry

Semileptonic $B \to \pi\ (\rho)$ form factors

- Old issues: endpoint singularities, Sudakov effects, etc.



At leading order in Λ/Q, to all orders in α_s, form factors for $q^2 \ll m_B^2$ written as ($Q = E, m_b$; omit μ-dep's)

[Beneke & Feldmann; Bauer, Pirjol, Stewart; Becher, Hill, Lange, Neubert]

$$F(Q) = C_k(Q)\,\zeta_k(Q) + \frac{m_B f_B f_M}{4E^2} \int \mathrm{d}z\mathrm{d}x\mathrm{d}r_+\, T(z,Q)\, J(z,x,r_+,Q)\, \phi_M(x)\phi_B(r_+)$$

Matrix elements of distinct $\int \mathrm{d}^4x\, T\big[J^{(n)}(0)\, \mathcal{L}_{\xi q}^{(m)}(x)\big]$ terms (turn spectator $q_{us} \to \xi$)

- Symmetry: first term obeys form factor relations ($10 \Rightarrow 3$ universal fn's) [Charles *et al.*]

Relative size? SCET: 1st $\sim$ 2nd QCDF: 2nd $\sim \alpha_s \times$(1st) PQCD: 1st ~ 0

- Zero-bin factorization: $\zeta_k(Q) \sim J_{ij}(x, z_k, k_\ell^+) \otimes \phi_\pi^i(z_k)\, \phi_B^j(k_\ell^+)$

$\Rightarrow$ ζ_k formally contains $\alpha_s(\mu_i)$, just like second term [Manohar & Stewart, hep-ph/0605001]

Charmless B decays

Charmless $B \to M_1 M_2$ decays

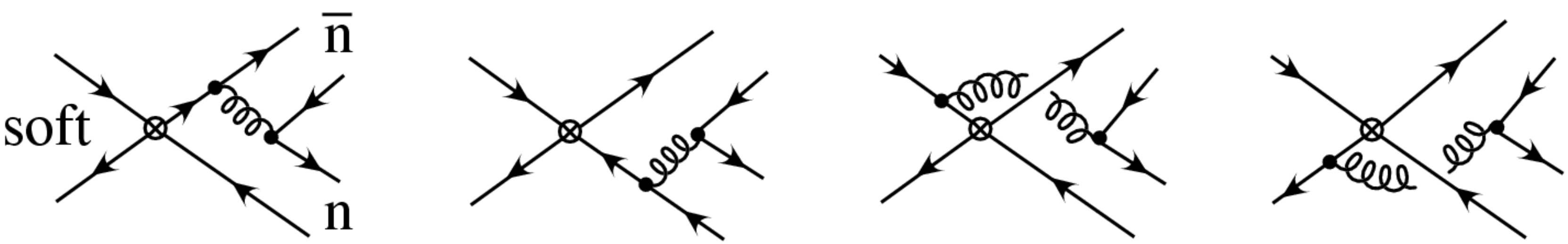
- BBNS (QCDF) factorization proposal:

$$\langle \pi\pi | O_i | B \rangle \sim \; F_{B\to\pi}\, T(x) \otimes \phi_\pi(x) \; + \; T(\xi, x, y) \otimes \phi_B(\xi) \otimes \phi_\pi(x) \otimes \phi_\pi(y)$$

The KLS (pQCD) formulae involve only ϕ_B, ϕ_{M_1}, ϕ_{M_2}, with $k_\perp$ dependence

- SCET: $\langle \pi\pi | O_i | B \rangle \sim \sum_{ij} T(x, y) \otimes \left[J_{ij}(x, z_k, k_\ell^+) \otimes \phi_\pi^i(z_k)\, \phi_B^j(k_\ell^+) \right] \otimes \phi_\pi(y)$

- Weak annihilation (WA) gives power suppressed (Λ/E) corrections



Yields convolution integrals of the form: $\displaystyle\int_0^1 \frac{\mathrm{d}x}{x^2}\, \phi_\pi(x)\,, \qquad \phi_\pi(x) \sim 6x(1-x)$

- BBNS: interpret as IR sensitivity $\Rightarrow$ modelled by complex parameters

KLS: rendered finite by $k_\perp$, but sizable and complex contributions

Subtractions for divergent convolutions

- Choose interpolating field for pion to be made of collinear quarks $(p_i^- \neq 0)$

$$\langle \pi_n^+(p_\pi)|\bar{u}_{n,p_1^-}\,\bar{n}\!\!\!/\gamma_5\,d_{n,-p_2^-}|0\rangle = -if_\pi\,\delta(\bar{n}\cdot p_\pi - p_1^- - p_2^-)\,\phi_\pi(x_1,x_2,\mu)$$

Zero-bin: $p_i^- \neq 0$ (collinear quark with $p_i^- = 0$ is not a collinear quark)

Divergence in $\int_0^1 \phi_\pi(x)/x^2$ related to one of the quarks becoming soft near $x = 0$

- Zero-bin ensures there is no contribution from $x_i = p_i^-/(\bar{n}\cdot p_\pi) \sim 0$

Subtractions implied by zero-bin depend on the singularity of integrals, e.g.:

$$\int_0^1 \mathrm{d}x\,\frac{1}{x^2}\,\phi_\pi(x,\mu) \quad \Rightarrow \quad \int_0^1 \mathrm{d}x\,\frac{\phi_\pi(x,\mu) - x\,\phi_\pi'(0,\mu)}{x^2} + \phi_\pi'(0,\mu)\ln\left(\frac{\bar{n}\cdot p_\pi}{\mu_-}\right)$$

$= \text{finite}$

Weak annihilation

- Match onto six-quark operators of the form (only hard contributions, no jet scale):

$$O_{1d}^{(ann)} = \sum_q \underbrace{\left[\bar{d}_s \Gamma_s\, b_v\right]}_{\text{gives } f_B} \underbrace{\left[\bar{u}_{\bar{n},\omega_2} \Gamma_{\bar{n}}\, q_{\bar{n},\omega_3}\right]}_{\pi \text{ in } \bar{n} \text{ direction}} \underbrace{\left[\bar{q}_{n,\omega_1} \Gamma_n\, u_{n,\omega_4}\right]}_{\pi \text{ in } n \text{ direction}}$$

[Arnesen, ZL, Rothstein, Stewart]

  Similar to leading order contributions to the amplitude

- At leading nonvanishing order in Λ/m_b and α_s:
  - Real, because there is no way for these matrix elements to be complex
  - Calculable, and do not introduce nonperturbative inputs beyond those that occur in leading order factorization formula

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- Constrain parameters in QCDF and pQCD to be real, which have been taken to be complex $\Rightarrow$ fewer unknowns

- Can try to disentangle charm penguin amplitudes from weak annihilation, etc.

“Chirally enhanced” terms

- Terms proportional to $m_\pi^2/(m_u+m_d)$ or $m_K^2/(m_u+m_s)$ (from using the Dirac eq.)

  Isolating these terms relies on assumptions about three-body wave functions

- Can be understood in SCET$_{\rm II}$ as operators with a $\not{\mathcal{P}}_\perp$ between collinear quarks $\Gamma_s \otimes \Gamma_{\bar n} \otimes \Gamma_n \mathcal{P}_\perp^\beta$

- Chirally enhanced WA power suppressed (compared to leading WA) and depends on the intermediate jet scale

  Can indeed cause some transverse polarization in $B \to VV$

  Real at leading order (same holds for chirally enhanced hard scattering)

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Conclusions

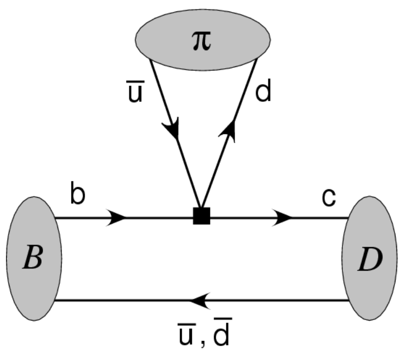
- Theory of charmless two-body decays continues to develop rapidly
- Zero-bin factorization $\Rightarrow$ no divergent convolutions
  Annihilation and “chirally enhanced” terms are calculable and real
- More work & experience with data needed to understand behavior of expansions
  Why some predictions work at $\lesssim 10\%$ level, while others receive $\gtrsim 30\%$ corrections
- We have the tools to try to address the questions

Backup slides

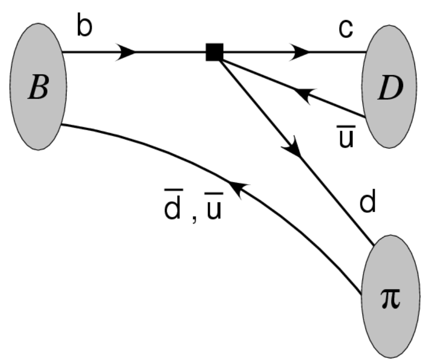
$B \to D^{(*)}\pi$ decays in SCET

- Decays to $\pi^\pm$: proven that $A \propto \mathcal{F}^{B\to D}\, f_\pi$ is the leading order prediction
  Also holds in large N_c, works at 5–10% level, need precise data to test mechanism

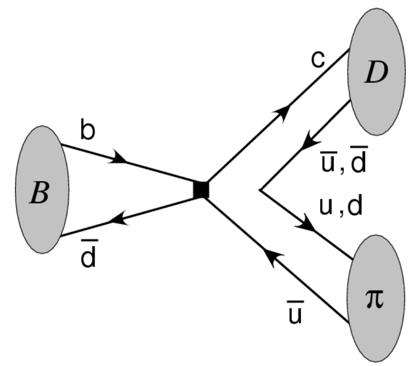
$\overline{B}^0 \to D^+\pi^-$
$B^- \to D^0\pi^-$



$B^- \to D^0\pi^-$
$\overline{B}^0 \to D^0\pi^0$



$\overline{B}^0 \to D^+\pi^-$
$\overline{B}^0 \to D^0\pi^0$



SCET: $\mathcal{O}(1)$ $\quad$ $\mathcal{O}(\Lambda_{\rm QCD}/Q)$ $\quad$ $\mathcal{O}(\Lambda_{\rm QCD}/Q)$ $\quad$ $Q = \{E_\pi, m_{b,c}\}$

- Predictions:

$$\frac{\mathcal{B}(B^- \to D^{(*)0}\pi^-)}{\mathcal{B}(\overline{B}^0 \to D^{(*)+}\pi^-)} = 1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\,,$$

data: $\sim 1.8 \pm 0.2$ (also for ρ)
$\Rightarrow\ \mathcal{O}(30\%)$ power corrections

[Beneke, Buchalla, Neubert, Sachrajda; Bauer, Pirjol, Stewart]

$$\frac{\mathcal{B}(\overline{B}^0 \to D^0\pi^0)}{\mathcal{B}(\overline{B}^0 \to D^{*0}\pi^0)} = 1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\,,$$

data: $\sim 1.1 \pm 0.25$

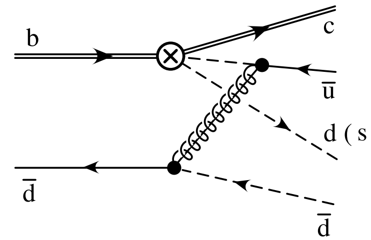
Unforeseen before SCET

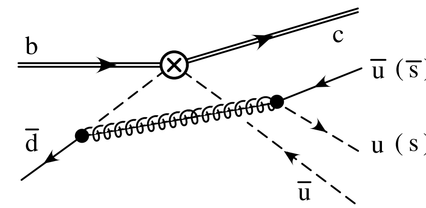
[Mantry, Pirjol, Stewart]

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed SCET$_\mathrm{I}$ operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}_{\xi q}^{(1)}, \mathcal{L}_{\xi q}^{(1)}\}$

[Mantry, Pirjol, Stewart]



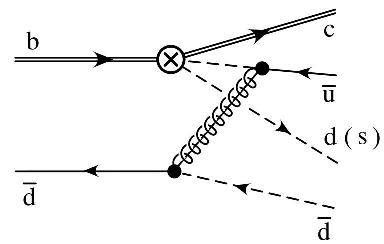


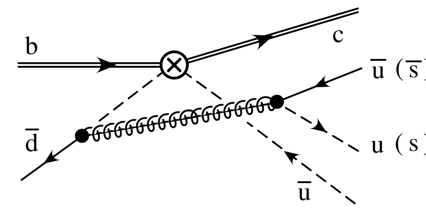
$$A(D^{(*)0}M^0) = N_0^M \int \mathrm{d}z\, \mathrm{d}x\, \mathrm{d}k_1^+ \mathrm{d}k_2^+\, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, \underbrace{S^{(i)}(k_1^+, k_2^+)}_{\text{complex} - \text{nonpert. strong phase}}\, \phi_M(x) + \ldots$$

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed SCET$_\mathrm{I}$ operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$

  [Mantry, Pirjol, Stewart]





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- Not your garden variety factorization formula... $S^{(i)}(k_1^+, k_2^+)$ know about n

$$S^{(0)}(k_1^+, k_2^+) = \frac{\langle D^0(v')|(\bar{h}_{v'}^{(c)} S)\, \not{n} P_L (S^\dagger h_v^{(b)})(\bar{d}S)_{k_1^+}\, \not{n} P_L (S^\dagger u)_{k_2^+} |\bar{B}^0(v)\rangle}{\sqrt{m_B m_D}}$$

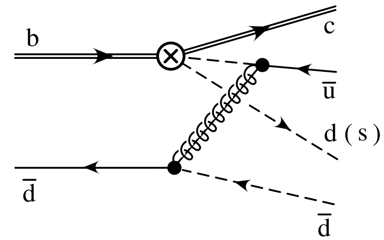
  Separates scales, allows to use HQS without $E_\pi/m_c = \mathcal{O}(1)$ corrections

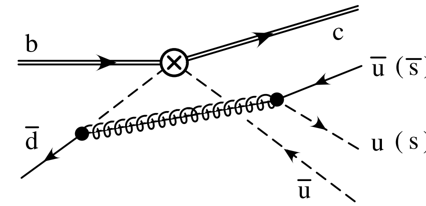
  ($i = 0, 8$ above)

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed $\mathrm{SCET_I}$ operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$

[Mantry, Pirjol, Stewart]





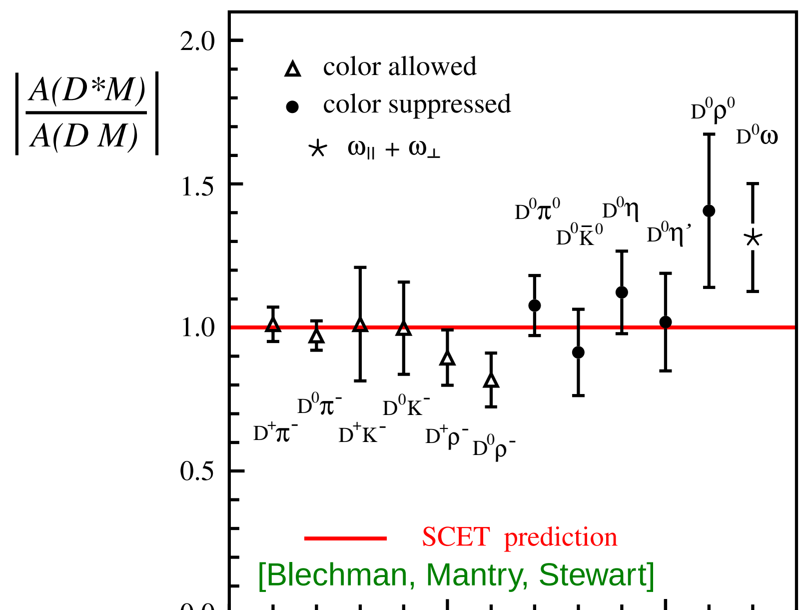
$$A(D^{(*)0}M^0) = N_0^M \int \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}k_1^+\mathrm{d}k_2^+\, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, \underbrace{S^{(i)}(k_1^+, k_2^+)}_{\text{complex} - \text{nonpert. strong phase}}\, \phi_M(x) + \ldots$$

- Ratios: the $\triangle = 1$ relations follow from naive factorization and heavy quark symmetry

The $\bullet = 1$ relations do not — a prediction of SCET not foreseen by model calculations

Also predict equal strong phases between amplitudes to $D^{(*)}\pi$ in $I = 1/2$ and $3/2$

Data: $\delta(D\pi) = (30 \pm 5)^\circ$, $\delta(D^*\pi) = (31 \pm 5)^\circ$



Baryons: $\Lambda_b \to \Lambda_c \pi$ and $\Sigma_c \pi$

- Factorization: holds for $m_Q \gg \Lambda_{\rm QCD}$ (not in large N_c)

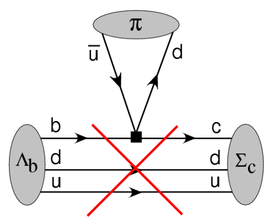
$$\frac{\Gamma(\Lambda_b \to \Lambda_c \pi^-)}{\Gamma(\overline{B}^0 \to D^{(*)+}\pi^-)} \simeq 1.8 \left(\frac{\zeta(w^{\Lambda}_{\rm max})}{\xi(w^{D^{(*)}}_{\rm max})}\right)^2$$

CDF: $\Gamma(\Lambda_b \to \Lambda_c^+ \pi^-)/\Gamma(\overline{B}^0 \to D^+\pi^-) \approx 2$

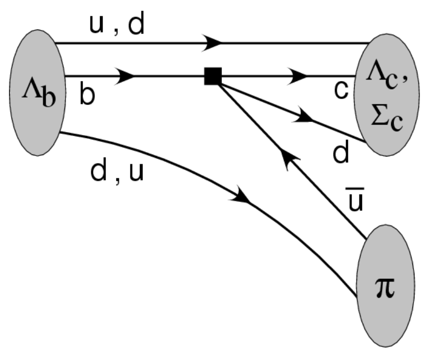
multiplets	s_l	$I(J^P)$
Λ_c	0	$0(\frac12^+)$
$\Sigma_c,\, \Sigma_c^*$	1	$1(\frac12^+),\, 1(\frac32^+)$

$\Sigma_c(2455),\, \Sigma_c^*(2520)$

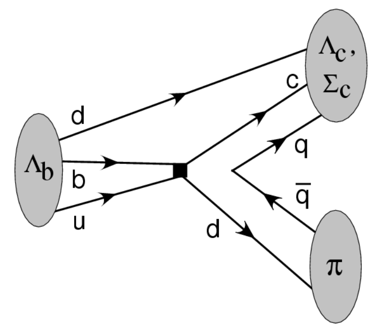
- Can't address in naive factorization, since $\Lambda_b \to \Sigma_c$ form factor vanishes by isospin



[Leibovich, ZL, Stewart, Wise]



$\mathcal{O}(\Lambda_{\rm QCD}/Q)$



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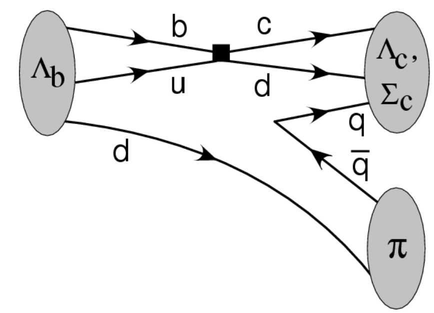

$\mathcal{O}(\Lambda^2_{\rm QCD}/Q^2)$

- Prediction: $\displaystyle \frac{\Gamma(\Lambda_b \to \Sigma_c^* \pi)}{\Gamma(\Lambda_b \to \Sigma_c \pi)} = 2 + \mathcal{O}\big[\Lambda_{\rm QCD}/Q\,,\ \alpha_s(Q)\big] = \frac{\Gamma(\Lambda_b \to \Sigma_c^{*0}\rho^0)}{\Gamma(\Lambda_b \to \Sigma_c^{0}\rho^0)}$

Can avoid π^0's from $\Lambda_b \to \Sigma_c^{(*)0}\pi^0 \to \Lambda_c \pi^-\pi^0$ or $\Lambda_b \to \Sigma_c^{(*)+}\pi^- \to \Lambda_c \pi^0\pi^-$

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