$D^0 - \bar{D}^0$ **Mixing**

Zoltan Ligeti, Lawrence Berkeley Lab

Rencontres de Moriond, Les Arcs, 3/10/02

- Introduction

  ... Why do we care about D mixing?

  ... Different measurements; present data

- Theoretical status

  ... $SU(3)$ analysis & OPE predictions for Δm and $\Delta\Gamma$

  ... $\Delta\Gamma$ from $SU(3)$ breaking in phase space

- Conclusions

see: A. Falk, Y. Grossman, Z.L., A. Petrov, PRD **65** (2002) 054034 [hep-ph/0110317]

S. Bergmann, Y. Grossman, Z.L., Y. Nir, A. Petrov, PLB **486** (2000) 418 [hep-ph/0005181]

Introduction

Dictionary: CPV $= CP$ violation

SM $=$ standard model

NP $=$ new physics

CA $=$ Cabibbo allowed

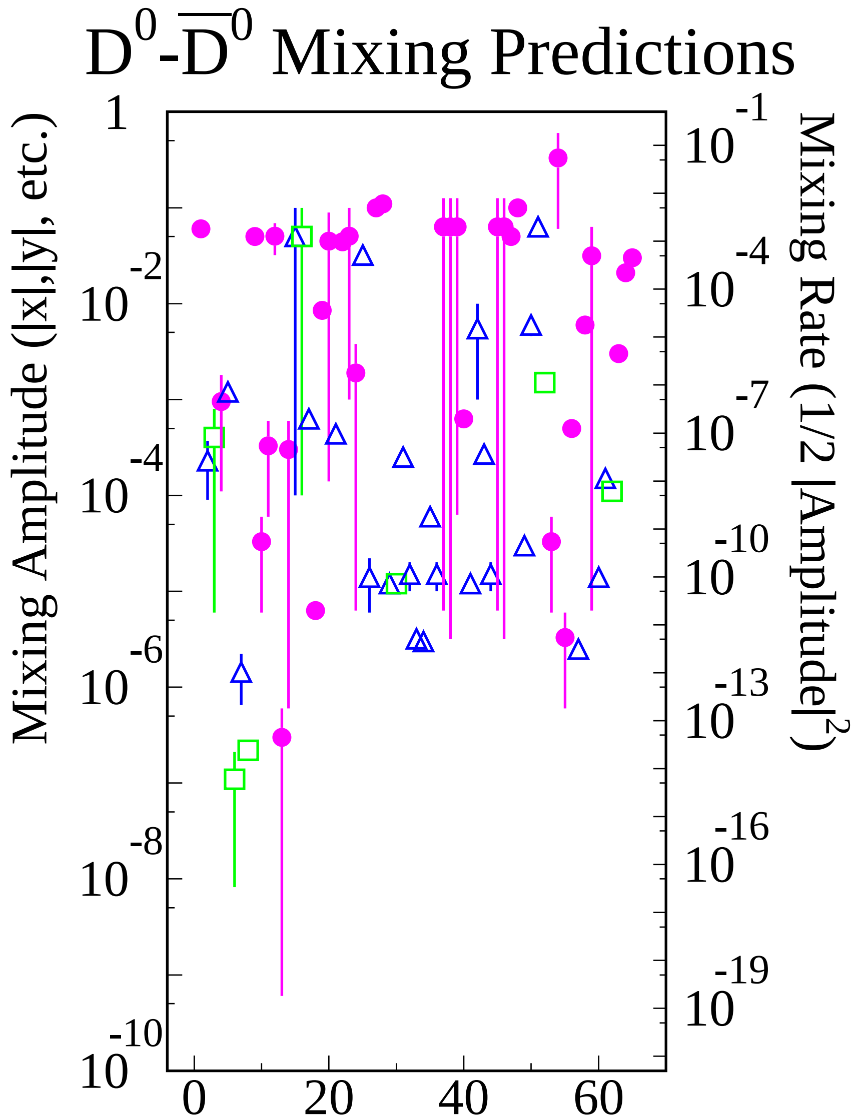
DCS $=$ doubly Cabibbo suppressed

Why is $D-\bar{D}$ mixing important?

Of the neutral meson systems $D-\bar{D}$ mixing is unique in that:

- The only meson system where mixing has not been observed
- The only meson where the mixing is generated by the down type quarks
- Expected to be small in the SM: $\Delta M, \Delta\Gamma \lesssim$ few $\times\, 10^{-3}\,\Gamma$, since it is DCS and vanishes in the flavor $SU(3)$ symmetry limit
- It involves only the first two generations: If CPV $\gg 10^{-3}$ is observed — unambiguously new physics
- Sensitive to new physics: NP can easily enhance ΔM but would not affect $\Delta\Gamma$
  If D mixing is large: $\Delta\Gamma \gtrsim \Delta M$ — probably large flavor $SU(3)$ breaking
  $\Delta M \gg \Delta\Gamma$ — probably new physics
- There were two recent measurements with signals at the $\sim 2\sigma$ level

D mixing predictions within and beyond the SM



(H. Nelson, hep-ex/9908021)

● : NP predictions for x

△ : SM predictions for x

□ : SM predictions for y

Definitions: $x \equiv \dfrac{\Delta M}{\Gamma}$

$$y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

$$|D_{L,S}\rangle = p\,|D^0\rangle \pm q\,|\bar{D}^0\rangle$$

Recent measurements of $D^0 - \bar{D}^0$ mixing (1)

- Measure D lifetime in decays to a CP eigenstate, e.g., K^+K^-, and a flavor eigenstate, e.g., π^+K^-, fitting exponential time-dependences:

$$y_{CP} = \frac{\Gamma(CP \text{ even}) - \Gamma(CP \text{ odd})}{\Gamma(CP \text{ even}) + \Gamma(CP \text{ odd})} = \frac{\hat\tau(D \to \pi^+K^-)}{\hat\tau(D \to K^+K^-)} - 1 = y\cos\phi - x\sin\phi\,\frac{A_m}{2}$$

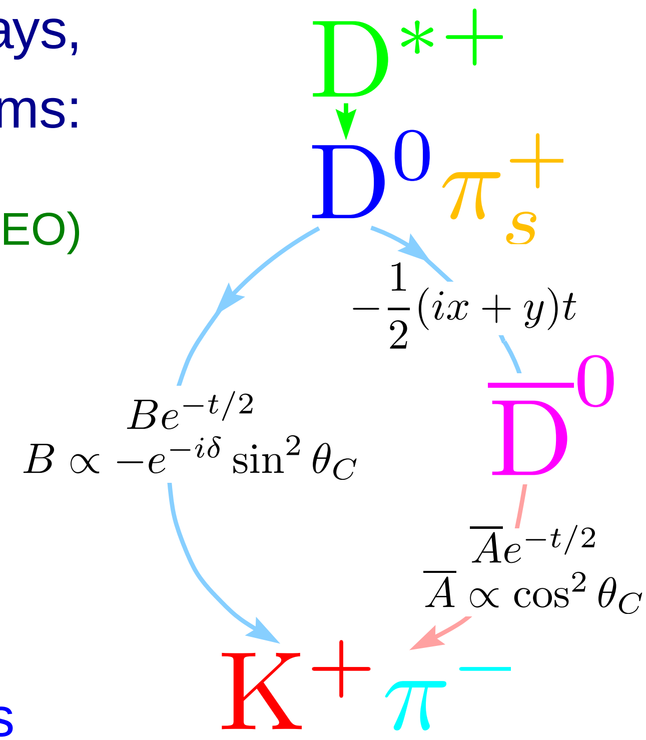
$A_m = |q/p|^2 - 1$ and $\phi = \arg(q/p)$ parameterize CPV in mixing – very small in SM

Data: $$y_{CP} = \begin{cases} 0.8 \pm 3.1\% & \text{(E791)} \\ 3.4 \pm 1.6\% & \text{(FOCUS)} \\ -1.1 \pm 2.9\% & \text{(CLEO)} \\ -0.5 \pm 1.3\% & \text{(BELLE)} \\ -1.0 \pm 2.8\% & \text{(BABAR)} \end{cases}$$

World average:
$y_{CP} = 0.65 \pm 0.85\%$

Large y_{CP} could be explained by large y or large x, A_m, and ϕ

Recent measurements of $D^0 - \bar{D}^0$ mixing (2)



Measure time dependence of "wrong sign" DCS decays, e.g.: $D^0(t) \to K^+\pi^-$ and $\bar{D}^0(t) \to K^-\pi^+$. Fit three terms:

$$e^{-\Gamma t}\{\text{(dir-DCS)} + (\Gamma t)\text{(int-CS)} + (\Gamma t)^2\text{(mix-CA)}\} \quad \text{(CLEO)}$$

$$x' = (0.0 \pm 1.5) \times 10^{-2} \qquad y' \cos\phi = (-2.5^{+1.4}_{-1.6}) \times 10^{-2}$$

$$R = (0.48 \pm 0.13) \times 10^{-2} \qquad A_m = 0.23^{+0.63}_{-0.80}$$

$$R \equiv \mathcal{B}(D^0 \to K^+\pi^-)/\mathcal{B}(D^0 \to K^-\pi^+) \ll 1$$

$$x' = x\cos\delta + y\sin\delta \qquad y' = y\cos\delta - x\sin\delta$$

δ is the strong phase between CA and DCS amplitudes

Large $-y'$ could be explained by $y \sim -10^{-2}$ and $\delta \ll 1$ or $x \sim 10^{-2}$ and $\delta \sim 1$

The central values of the FOCUS and CLEO data together implied last year that both y and δ would have to be large independent of x [in $SU(3)$ limit $\delta = 0$]

Theoretical status

$SU(3)$ analysis & OPE predictions for D mixing

Dependence of sensitivity to NP on y and x

Allowed range of y (and x) in the SM

$SU(3)$ analysis of D mixing

- Want to study: $\langle \bar{D}^0 | T\{H_w, H_w\} | D^0 \rangle = \langle 0 | D\, T\{H_w, H_w\} D | 0 \rangle$

  the field operator $D \in 3$ creates a D^0 or annihilates a $\bar{D}^0$

  $H(\Delta C = -1) = (\bar{q}_i\, c)(\bar{q}_j\, q_k) \in 3 \times \overline{3} \times \overline{3} = \overline{15} + 6 + \overline{3} + \overline{3}$

If 3rd generation is neglected, only $\overline{15}$ and 6 appear in H

$SU(3)$ breaking is introduced by $\mathcal{M}^i_j = \text{diag}(m_u, m_d, m_s) \sim \text{diag}(0, 0, m_s)$

A pair of D operators is symmetric, so $D_i D_j \in 6$

A pair of H's is symmetric, so $H^{ij}_k H^{lm}_n \in \big[(\overline{15} + 6) \times (\overline{15} + 6)\big]_S \to \overline{60} + 42 + 15'$

0. Since there is no $\overline{6}$ in $H_w H_w \;\Rightarrow\; D^0$ mixing vanishes in $SU(3)$ limit

1. $DD\mathcal{M} \in 6 \times 8 = 24 + \overline{15} + 6 + \overline{3} \;\Rightarrow\;$ no invariants with $H_w H_w$ at order m_s

2. $DD\mathcal{M}\mathcal{M} \in 6 \times (8 \times 8)_S = 6 \times (27 + 8 + 1) = 60 + \overline{24} + \overline{15'} + \ldots$

$\Rightarrow D^0 - \bar{D}^0$ mixing only arises at order m_s^2

Operator product expansion for D mixing

- It is very hard to estimate x and y in the SM — they vanish in the $SU(3)$ limit and are doubly Cabibbo suppressed: $x, y \sim \sin^2\theta_C\, \epsilon^2_{SU(3)}$

  Short distance box diagram: $x \propto \dfrac{m_s^2}{m_W^2} \times \dfrac{m_s^2}{m_c^2} \to 10^{-5}$

  y has additional m_s^2/m_c^2 helicity suppression

- Higher order terms in the OPE are suppressed by fewer powers of m_s: (Georgi '92)

	4-quark	6-quark	8-quark
$\dfrac{\Delta M}{\Delta M_{\rm box}}$	1	$\dfrac{\Lambda^2}{m_s m_c}$	$\dfrac{\Lambda^4}{m_s^2 m_c^2}\, \dfrac{\alpha_s}{4\pi}$
$\dfrac{\Delta\Gamma}{\Delta M}$	$\dfrac{m_s^2}{m_c^2}$	$\dfrac{\alpha_s}{4\pi}$	$\dfrac{\alpha_s}{4\pi}\,\beta_0$

(Bigi & Uraltsev '00 — argue that $x, y \propto m_s$ is possible; this is in conflict with our general proof from group theory)

With some assumptions about the matrix elements ($\Lambda \sim 4\pi f_\pi$): $\boldsymbol{x, y \lesssim 10^{-3}}$

Long distance contributions to D mixing

- May be large, but extremely hard to estimate — $SU(3)$ breaking has been argued to be $\mathcal{O}(1)$, based on $\mathcal{B}(D^0 \to K^+K^-)/\mathcal{B}(D^0 \to \pi^+\pi^-) \simeq 2.8$

  Cancellations sensitively depend on poorly known strong phases and DCS rates:

  $$y \approx \mathcal{B}(D \to \pi^+\pi^-) + \mathcal{B}(D \to K^+K^-) - 2\cos\delta\,\sqrt{\mathcal{B}(D \to K^-\pi^+)\,\mathcal{B}(D \to K^+\pi^-)}$$

  Experimental central values yield: $y \approx (5.76 - 5.29\cos\delta) \times 10^{-3}$

  Assuming $\cos\delta \sim 1$ [the $SU(3)$ limit] and that these intermediate states are representative, it is often stated that $x \lesssim y <$ few $\times 10^{-3}$

- The most important long distance effect may be due to phase space:
  - Contrary to $SU(3)$ breaking in matrix elements, this source of $SU(3)$ violation is calculable model independently with only mild assumptions
  - Negligible for lightest PP final states; important for states with mass near m_D

What if $y \gg x$? The case of $|M_{12}/\Gamma_{12}| \ll 1$

- It is possible that the long distance contributions and $SU(3)$ breaking are larger for y than for x, and could significantly enhance the OPE estimate of y

  This may be unique, since in the K system $M_{12} \sim \Gamma_{12}$, in the B system $\Gamma_{12} \ll M_{12}$

  New physics can significantly modify x, in particular, with new CPV phases

  - It is very unlikely for NP to significantly modify y
  - Observing $\phi \neq 0$ may be the best hope to find NP

- Solving the eigenvalue equation:

  - If $x \gg y$, the CPV phase can be **LARGE**: $\phi = \arg(M_{12}) + \mathcal{O}(\Gamma_{12}^2/M_{12}^2)$
  - If $y \gg x$, the CPV phase is **SMALL**: $\phi = \mathcal{O}(M_{12}^2/\Gamma_{12}^2) \times \sin[2\arg(M_{12}/\Gamma_{12})]$!

- If $y \gg x$ then even if new physics dominates M_{12}, the sensitivity of any physical observable to it is suppressed by x/y

$\Delta\Gamma$ from $SU(3)$ breaking in phase space

- Phase space difference between final states containing fewer or more strange quarks is a calculable source of $SU(3)$ breaking — these are "threshold effects"

- Let F_R denote final state F in representation R (e.g., PP can be in 8 or 27)

Define:
$$y_{F,R} = \frac{\sum_{n\in F_R} \langle \bar{D}^0 | H_w | n \rangle \rho_n \langle n | H_w | D^0 \rangle}{\sum_{n\in F_R} \Gamma(D^0 \to n)}$$

the "would-be" value of y, if D only decayed to the states F in representation R

- If the decay rates to all representations are known, we can reconstruct the value of y from $y_{F,R}$:

$$y = \frac{1}{\Gamma} \sum_{F,R} y_{F,R} \left[\sum_{n\in F_R} \Gamma(D^0 \to n) \right]$$

Simplest example: $D^0 \to PP$

- PP must be in $(8\times 8)_S = 27+8+1$ — possible amplitudes:
  - PP in 27 and $\mathcal{H}_w$ in $\overline{15}$: $A_{27}\,(PP_{27})^{km}_{ij}\,H^{ij}_k\,D_m$
  - PP in 8 and $\mathcal{H}_w$ in $\overline{15}$: $A_8^{\overline{15}}\,(PP_8)^k_i\,H^{ij}_k\,D_j$
  - PP in 8 and $\mathcal{H}_w$ in 6: $A_8^6\,(PP_8)^k_i\,H^{ij}_k\,D_j$

  (the last two:) proportional to each other

  So effectively there are only two amplitudes — for example, we obtain for $y_{PP,8}$:

$$s_1^2\left[\frac{1}{2}\Phi(\eta,\eta)+\frac{1}{2}\Phi(\pi^0,\pi^0)+\frac{1}{3}\Phi(\eta,\pi^0)+\Phi(\pi^+,\pi^-)+\Phi(K^+,K^-)-\frac{1}{6}\Phi(\eta,K^0)\right.$$
$$\left.-\frac{1}{6}\Phi(\eta,\bar K^0)-\Phi(K^+,\pi^-)-\Phi(K^-,\pi^+)-\frac{1}{2}\Phi(K^0,\pi^0)-\frac{1}{2}\Phi(\bar K^0,\pi^0)\right]$$
$$\times\left[\frac{1}{6}\Phi(\eta,\bar K^0)+\Phi(K^-,\pi^+)+\frac{1}{2}\Phi(\bar K^0,\pi^0)+\mathcal{O}(s_1^2)\right]^{-1}$$

- Result is explicitly proportional to $s_1^2 \equiv \sin^2\theta_C$ and vanishes in $SU(3)$ limit (as m_s^2)

Two–body final states

Final state representation		$y_{F,R}$ (%)
(PP)s-wave	$\mathbf{8}$	-0.018
	$\mathbf{27}$	-0.0034
(PV)p-wave	$\mathbf{8}_S$	0.15
	$\mathbf{8}_A$	0.15
	$\mathbf{10}$	0.10
	$\overline{\mathbf{10}}$	0.08
	$\mathbf{27}$	0.19
(VV)s-wave	$\mathbf{8}$	-0.39
	$\mathbf{27}$	-0.30
(VV)p-wave	$\mathbf{8}$	-0.48
	$\mathbf{27}$	-0.70
(VV)d-wave	$\mathbf{8}$	2.5
	$\mathbf{27}$	2.8

Results for lightest multiplets, assuming no $SU(3)$ breaking in matrix elements

Contribution of PP final states is "anomalously" small

Widths of vector mesons are important and taken into account (straightforward)

PV and VV channels effectively include resonant part of 3– and 4–body final states

Larger $SU(3)$ breaking expected in heavier multiplets when some final states are not allowed at all

Multi–body final states

Final state representation		$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	-2.3
	27	-0.54
$(3P)_{p\text{-wave}}$	8	-5.5
	27	-0.36
$(3P)_{\text{form-factor}}$	8	-2.1
	27	-0.64
$4P$	8	16
	27	9.2
	$27'$	11

Consider simplest representations only

Smaller representations tend to give larger effects

Assuming a "form factor suppression" in the matrix element, $\Pi_{i\neq j}(1-m_{ij}^2/Q^2)^{-1}$, where $m_{ij}^2 = (p_i + p_j)^2$ and $Q = 2\,\text{GeV}$, changes the results only moderately

For $4P$, only consider fully symmetric final state

- For many final state representations, especially those close to threshold, "large" effects are possible, i.e., $y_{F,R}$ at the percent level is not unusual

Conclusions from our analysis

- The 2–, 3–, and 4–body final states account for a large fraction of the D width
  Rounded to nearest 5%:

Final state	fraction
PP	5%
PV	10%
$(VV)_{s\text{-wave}}$	5%
$(VV)_{d\text{-wave}}$	5%
$3P$	5%
$4P$	10%

  There are other large rates near threshold, e.g.: $\mathcal{B}(D^0 \to K^- a_1^+) = (7.3 \pm 1.1)\%$

- Morals: There are final states that can contribute to y at the 1% level
  $\Rightarrow$ It would require cancellations to suppress y_{CP} much below $\sim 1\%$

Conclusions

- For the first time there is a concrete calculation without ad hoc assumptions showing that $\Delta\Gamma/\Gamma$ can naturally be at the 1% level

  Next: calculate ΔM from this source of $SU(3)$ breaking using dispersion relation

- Standard Model predictions of Δm and $\Delta\Gamma$ remain uncertain $\Rightarrow$ measurements of nonzero Δm or $\Delta\Gamma$ alone cannot be interpreted unambiguously as new phsyics

---

- To disentangle New Physics from Standard Model contribution, it's crucial to try to

  ... Improve measurements of both Δm and $\Delta\Gamma$

  ... Extract strong phase simultaneously in $K^-\pi^+\pi^0$ Dalitz plot analysis

  ... Look for CP violation, which remains a potentially robust signal of new physics

BaBar and Belle should be able to do this!