Inclusive $|V_{cb}|$: what's the limit?

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- Introduction
- The OPE and HQET parameters
- The determination of $|V_{cb}|$
  ... Theoretical and experimental status
  ... Present results and possible future limitations
- Conclusions

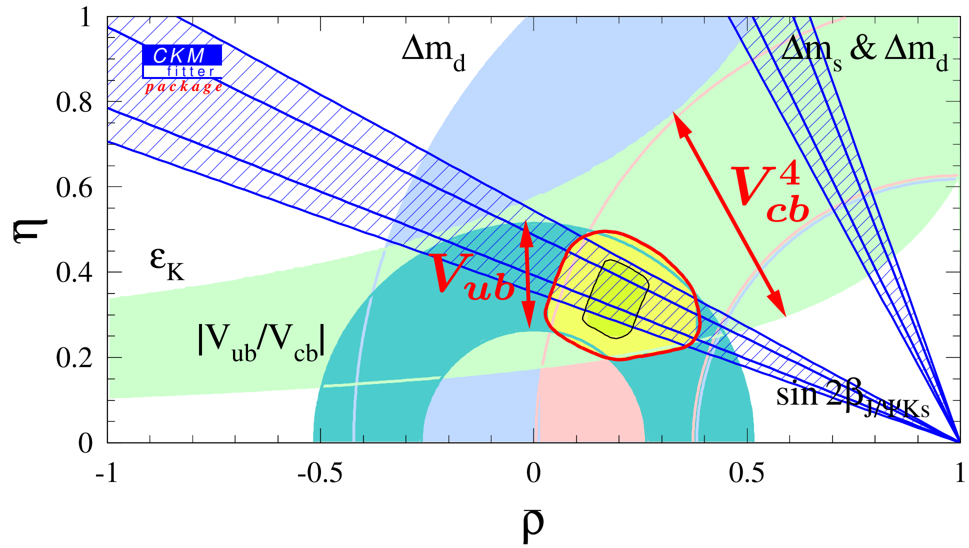
see: C. Bauer, Z.L., M. Luke, A. Manohar, PRD **67** 054012 (2003) [hep-ph/0210027]

related work: M. Battaglia *et al.*, PLB **556** 41 (2003) [hep-ph/0210319]

Why care about $|V_{cb}|$?

Error of $|V_{cb}|$ is a large part of the uncertainty in the ϵ_K constraint, and in $K \to \pi\nu\bar{\nu}$ when it is measured

How well OPE works for $b \to c$ spectra may affect what we believe about accuracy of $|V_{ub}|$ using phase space cuts



Inclusive decays mediated by $b \to s\gamma$, $b \to s\,\ell^+\ell^-$, and $b \to s\,\nu\bar{\nu}$ transitions are sensitive probes of the SM; theoretical tools for semileptonic and rare decays are similar — understanding accuracy of theory affects sensitivity to new physics

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The players...

1.) Inclusive semileptonic $B \to X_c \ell \bar{\nu}$ width sensitive to $|V_{cb}|$

2.) Shape variables (largely) independent of CKM elements:

– Photon energy moments in $B \to X_s \gamma$

$$T_1(E_0) = \langle E_\gamma \rangle \Big|_{E_\gamma > E_0} \qquad T_2(E_0) = \Big\langle (E_\gamma - \langle E_\gamma \rangle)^2 \Big\rangle \Big|_{E_\gamma > E_0}$$

– Hadronic invariant mass moments in $B \to X_c \ell \bar{\nu}$

$$S_1(E_0) = \langle m_X^2 - \overline{m}_D^2 \rangle \Big|_{E_\ell > E_0} \qquad S_2(E_0) = \Big\langle (m_X^2 - \langle m_X^2 \rangle)^2 \Big\rangle \Big|_{E_\ell > E_0}$$

– Lepton energy moments in $B \to X_c \ell \bar{\nu}$

$$R_0(E_0, E_1) = \frac{\displaystyle\int_{E_1} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\,\mathrm{d}E_\ell}{\displaystyle\int_{E_0} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\,\mathrm{d}E_\ell} \qquad R_n(E_0) = \frac{\displaystyle\int_{E_0} E_\ell^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\,\mathrm{d}E_\ell}{\displaystyle\int_{E_0} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\,\mathrm{d}E_\ell}$$

Goals of a global fit

- It is often stated that semileptonic B width gives a precise determination of $|V_{cb}|$

  The devil is hidden (as always) in the details:

  – What are the values of m_b and λ_1? Determine them in same analysis as $|V_{cb}|$

  – Theoretical correlations between different observables $\Rightarrow$ Include them

  – Size of theoretical uncertainties? Investigate them (incl. duality) experimentally

  – All observables fit using a consistent scheme $\Rightarrow$ study sheme dependence

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  – Are there tensions between measurements? If yes, which one(s)?

  – Optimal use of data $\Rightarrow$ reduce uncertainties

The OPE

Operator product expansion

- Consider semileptonic $b \to c$ decay: $O_{bc} = -\frac{4G_F}{\sqrt{2}} V_{cb} \underbrace{(\bar{c}\,\gamma^\mu P_L\, b)}_{J^\mu_{bc}} \underbrace{(\bar{\ell}\,\gamma_\mu P_L\, \nu)}_{J_{\ell\mu}}$

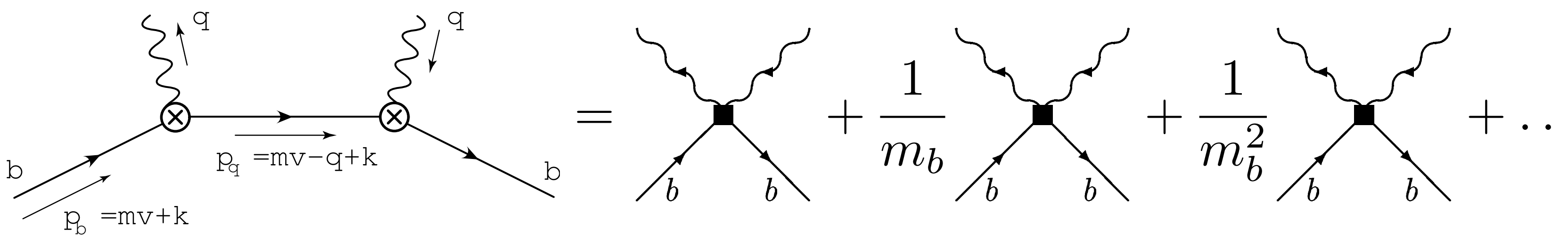
Decay rate: $\Gamma(B \to X_c \ell \bar{\nu}) \sim \sum_{X_c} \int \mathrm{d}[\mathrm{PS}] \left| \langle X_c \ell \bar{\nu} | O_{bc} | B \rangle \right|^2$

Factor to: $B \to X_c W^*$ and $W^* \to \ell \bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X) \left| \langle B | J^{\mu\dagger}_{bc} | X_c \rangle \langle X_c | J^\nu_{bc} | B \rangle \right|^2$$

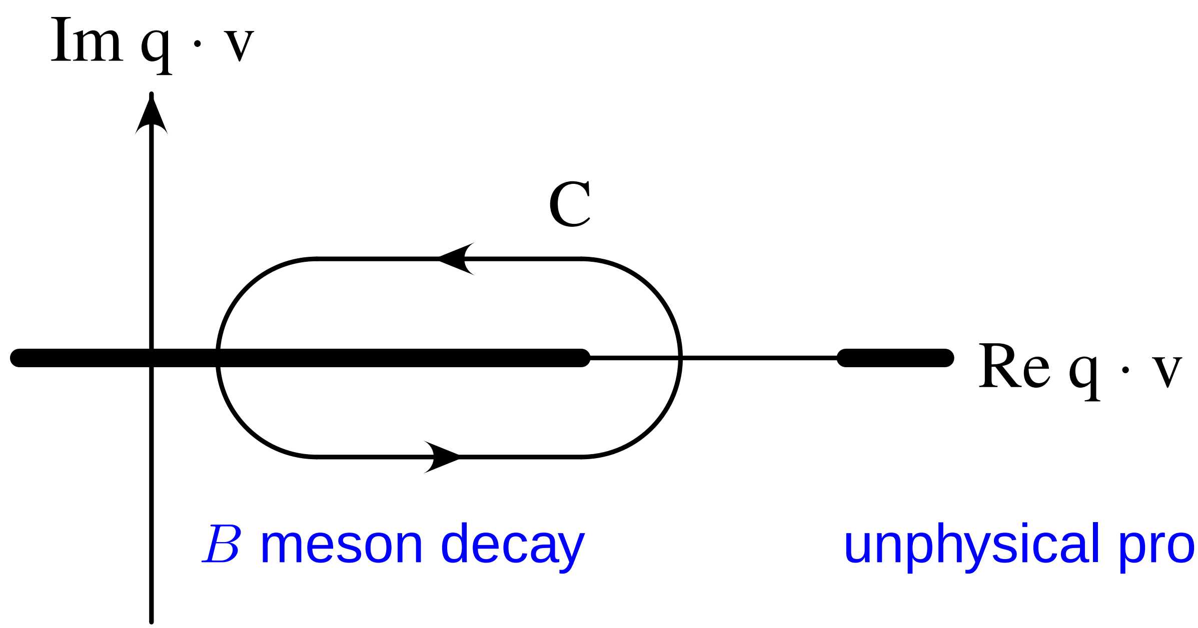
(optical theorem) $\sim \mathrm{Im} \int \mathrm{d}x\, e^{-iq\cdot x} \langle B | T\{ J^{\mu\dagger}_{bc}(x)\, J^\nu_{bc}(0) \} | B \rangle$

In $m_b \gg \Lambda_{\mathrm{QCD}}$ limit, time ordered product dominated by $x \ll \Lambda^{-1}_{\mathrm{QCD}}$



Cuts — semileptonic decays

Analytic structure of $T^{\mu\nu}$ in the complex $q \cdot v = q^0$ plane, with q^2 fixed:



$$q^0 < (m_B^2 + q^2 - m^2_{X_q^{\min}})/2m_B \qquad q^0 > (m^2_{X_{\bar{q}bb}^{\min}} - m_B^2 - q^2)/2m_B$$

To compute any observable, integration contour must cross the cut — do not know even formally the uncertainty induced, and its dependence on phase space cuts

Result of OPE

- The $m_b \to \infty$ limit is given by free quark decay, $\langle B|\, \bar{b}\, \gamma^\mu b\, |B\rangle = 2p_B^\mu = 2m_B\, v^\mu$

  No $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections

  Order $\Lambda_{\rm QCD}^2/m_b^2$ corrections depend on two hadronic matrix elements

$$\lambda_1 = \frac{1}{2m_B}\,\langle B|\,\bar{b}\,(iD)^2\,b\,|B\rangle \qquad \lambda_2 = \frac{1}{6m_B}\,\langle B|\,\bar{b}\,\frac{g}{2}\,\sigma_{\mu\nu}\,G^{\mu\nu}\,b\,|B\rangle$$

not well-known $\qquad \lambda_2 = (m_{B^*}^2 - m_B^2)/4$

- OPE predicts decay rates in an expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$

$$\mathrm{d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

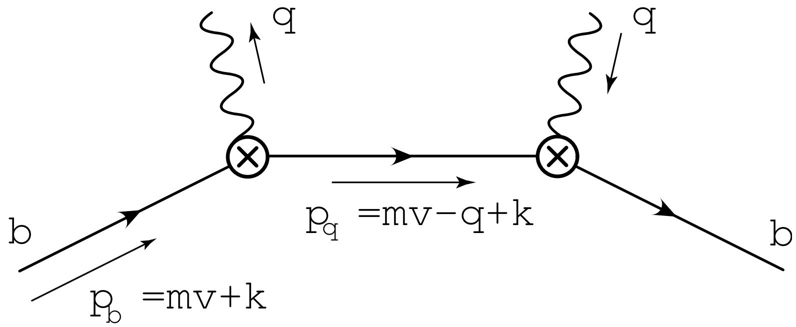
  Interesting quantities computed to order α_s, $\alpha_s^2\beta_0$, and $1/m^3$

  When can the results be trusted?

Inclusive decay rates

- In which regions of phase space can we expect the OPE to converge?

Can think of the OPE as an expansion in $k \sim \Lambda_{\rm QCD}$



$$\frac{1}{(m_b v + k - q)^2 - m_q^2} = \frac{1}{(m_b v - q)^2 - m_q^2 + 2k \cdot (m_b v - q) + k^2}$$

Need to allow: $m_X^2 - m_q^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$

Implicit assumption: quark-hadron duality valid once $m_X \gg m_q$ allowed

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The analysis

Theoretical calculations

- Typical OPE result for shape variables:

$$\langle X\rangle = \langle X\rangle_{\rm parton} + \frac{0}{m_b} F_\Lambda + \frac{\lambda_i}{m_b^2} F_{\lambda_i} + \frac{\rho_i}{m_b^3} F_{\rho_i} + \ldots$$

$\langle X\rangle_{\rm parton}$ and each F_i has an expansion in α_s and depends on m_c/m_b

- Compute to order: $1,\quad \Lambda_{\rm QCD}^2/m_b^2,\quad \Lambda_{\rm QCD}^3/m_b^3,\quad \alpha_s,\quad \alpha_s^2\beta_0$

(For hadronic moments, $\alpha_s\Lambda_{\rm QCD}/m_b$ terms only known without lepton energy cut)

- Parameters: $|V_{cb}|,\quad m_b,\quad m_c,\quad \lambda_{1-2},\quad \rho_{1-2},\quad \mathcal{T}_{1-4}$ (11)

Use $\overline{m}_B - \overline{m}_D$ to eliminate m_c; $m_{B^*} - m_B$ and $m_{D^*} - m_D$ to fix λ_2 and $\rho_2 - \mathcal{T}_2 - \mathcal{T}_4$

Rates depend on $\mathcal{T}_1 + 3\mathcal{T}_2$ and $\mathcal{T}_2 + \mathcal{T}_4$; masses depend on $\mathcal{T}_1 + \mathcal{T}_3$ and $\mathcal{T}_2 + \mathcal{T}_4$

$\Rightarrow$ Fit for: $|V_{cb}|,\quad m_b,\quad \lambda_1,\quad \rho_1,\quad \mathcal{T}_1 - 3\mathcal{T}_4,\quad \mathcal{T}_2 + \mathcal{T}_4,\quad \mathcal{T}_3 + 3\mathcal{T}_4$ (7)

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Mass schemes

Use 4 mass schemes for comparison — do all fits completely in each

Pole mass

- renormalon ambiguity of order Λ_{QCD}
- perturbation series poorly behaved
- these problems may be related — asymptotic nature of perturbation series related to nonperturbative corrections

$\overline{\mathrm{MS}}$ mass

$1S$ mass using the upsilon expansion

PS mass (and some other schemes): require introducing a factorization scale μ_f that enters linearly, $m_{\mathrm{pole}} = m_{\mathrm{PS}} + \ldots + \mu_f$

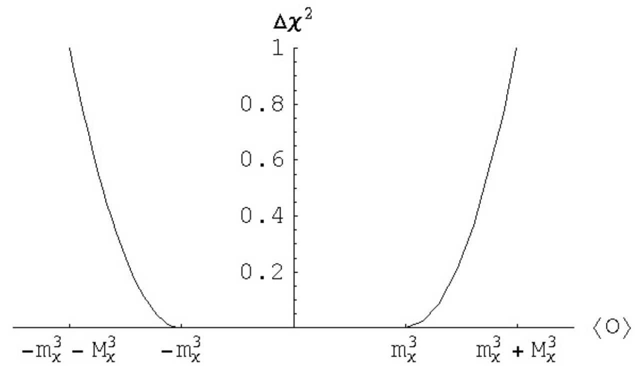
Theoretical uncertainties

Define theoretical uncertainties, so it is not judged case-by-case and a posteriori

Avoid large weight to an accurate measurement that cannot be computed reliably

- Unknown $1/m_b^3$ matrix elements — $\mathcal{O}(\Lambda_{\rm QCD}^3)$ but no preferred value $\Rightarrow$ add in fit:

$$\Delta\chi^2(m_\chi, M_\chi) = \begin{cases} 0\,, & |\langle\mathcal{O}\rangle| \le m_\chi^3 \\ \left[|\langle\mathcal{O}\rangle| - m_\chi^3\right]^2 / M_\chi^6\,, & |\langle\mathcal{O}\rangle| > m_\chi^3 \end{cases}$$



Take $M_\chi = 0.5$ GeV, and vary 0.5 GeV $< m_\chi < 1$ GeV

- Uncomputed higher order terms — estimate using naive dimensional analysis:
  - $(\alpha_s/4\pi)^2 \sim 0.0003$
  - $(\alpha_s/4\pi)(\Lambda_{\rm QCD}^2/m_b^2) \sim 0.0002$
  - $\Lambda_{\rm QCD}^4/(m_b^2 m_c^2) \sim 0.001$

Use relative error: $\sqrt{(0.001)^2 + (\text{last-computed}\,/\,2)^2}$

Observables — recall definitions

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– Photon energy moments in $B \to X_s \gamma$

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$$R_0(E_0, E_1) = \frac{\displaystyle\int_{E_1} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\, \mathrm{d}E_\ell}{\displaystyle\int_{E_0} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\, \mathrm{d}E_\ell} \qquad R_n(E_0) = \frac{\displaystyle\int_{E_0} E_\ell^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\, \mathrm{d}E_\ell}{\displaystyle\int_{E_0} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\, \mathrm{d}E_\ell}$$

Experimental data

- Photon energy moments in $B \to X_s \gamma$

CLEO '01:

$$T_1(2\,\text{GeV}) = (2.346 \pm 0.034)\,\text{GeV}$$
$$T_2(2\,\text{GeV}) = (0.0226 \pm 0.0069)\,\text{GeV}^2$$

- Hadronic invariant mass moments in $B \to X_c \ell \bar{\nu}$

CLEO '01:

$$S_1(1.5\,\text{GeV}) = (0.251 \pm 0.066)\,\text{GeV}^2$$
$$S_2(1.5\,\text{GeV}) = (0.576 \pm 0.170)\,\text{GeV}^4$$

BABAR '02:

$$S_1(1.5\,\text{GeV}) = (0.354 \pm 0.080)\,\text{GeV}^2$$
$$S_1(0.9\,\text{GeV}) = (0.694 \pm 0.114)\,\text{GeV}^2$$

DELPHI '02:

$$S_1(0) = (0.553 \pm 0.088)\,\text{GeV}^2$$
$$S_2(0) = (1.26 \pm 0.23)\,\text{GeV}^4$$

Experimental data (cont.)

- Lepton energy moments in $B \to X_c \ell \bar{\nu}$

CLEO '02:

$$R_0(1.5\,\text{GeV}, 1.7\,\text{GeV}) = 0.6187 \pm 0.0021$$
$$R_1(1.5\,\text{GeV}) = (1.7810 \pm 0.0011)\,\text{GeV}$$
$$R_2(1.5\,\text{GeV}) = (3.1968 \pm 0.0026)\,\text{GeV}^2$$

DELPHI 02:

$$R_1(0) = (1.383 \pm 0.015)\,\text{GeV},$$
$$R_2(0) - R_1(0)^2 = (0.192 \pm 0.009)\,\text{GeV}^2$$

- Average semileptonic decay width of $B^{\pm}$ and B^0

PDG '02:

$$\Gamma(B \to X \ell \bar{\nu}) = (42.7 \pm 1.4) \times 10^{-12}\,\text{MeV}$$

Cannot use average including B_s and Λ_b

Error analysis

- Included:
  - Conservative estimate of $1/m^3$ uncertainties
  - Best estimate of perturbative uncertainties
  - Best estimate of uncomputed $1/m^4$ and α_s/m^2 terms
  - All publicly available experimental uncertainties and correlations
- Not included:
  - Unknown experimental correlations
  - Uncertainties from “duality violation”

Results in $1S$ scheme

- Do fits both excluding (top) and including (bottom) BABAR data

m_χ [GeV]	χ^2	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	5.0	40.8 ± 0.9	4.74 ± 0.10
1.0	3.5	41.1 ± 0.9	4.74 ± 0.11
0.5	12.9	40.8 ± 0.7	4.74 ± 0.10
1.0	8.5	40.9 ± 0.8	4.76 ± 0.11

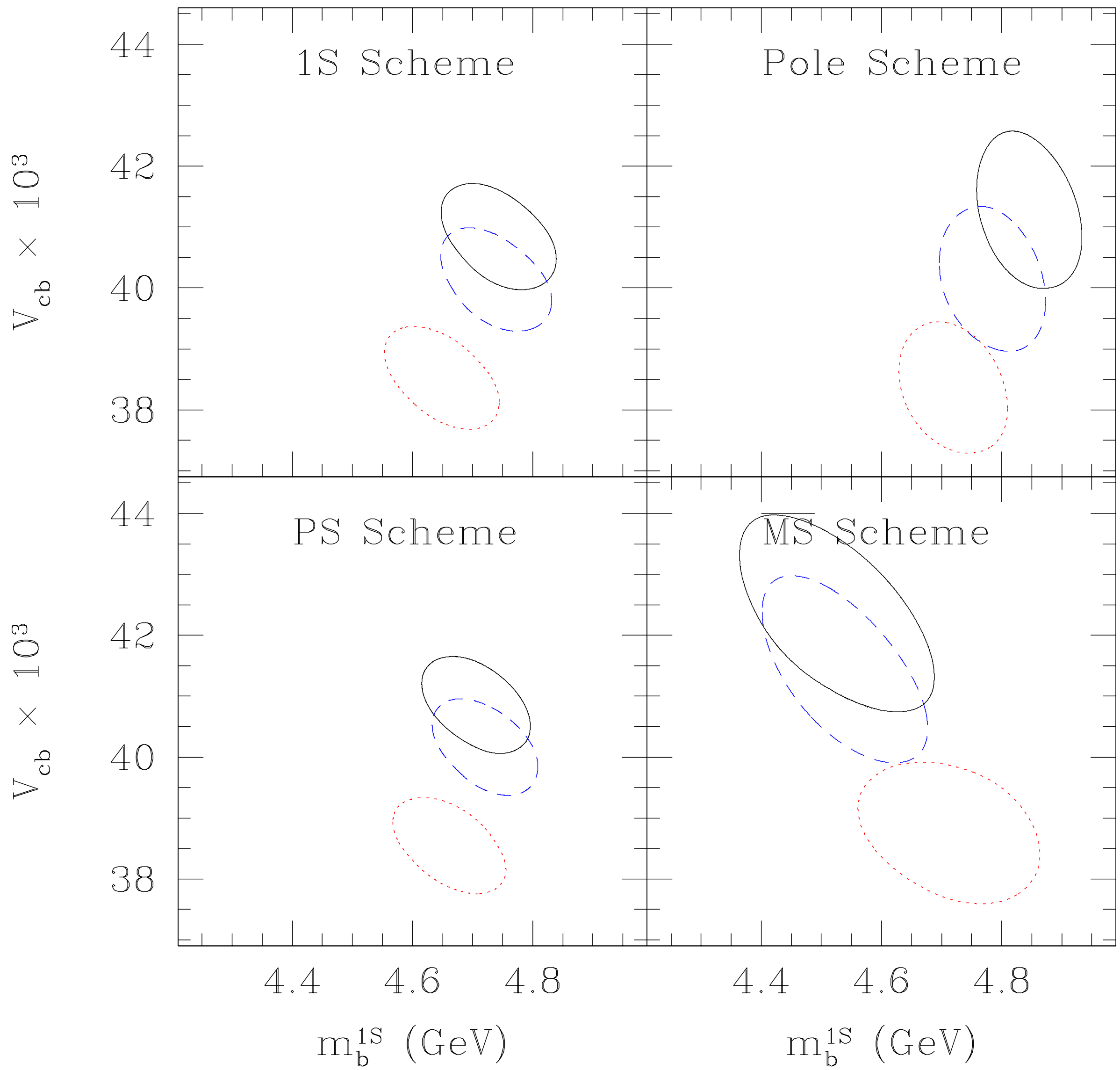
Sensitivity to m_χ is small ($1/m^3$ errors significant, but so are their correlations)

BABAR data increases χ^2/d.o.f. significantly — more later

Theoretical uncertainties important — neglecting them gives $\chi^2 = 81$ for 9 d.o.f.
Including only $1/m^3$ terms gives $\chi^2 = 21$ for 5 d.o.f.; much better (but still bad) fit

Results in different mass schemes

tree level, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2\beta_0)$



better convergence in 1S and PS schemes than in pole or $\overline{\text{MS}}$

More on fit results

- Theoretical correlations:

m_χ [GeV]	χ^2	λ_1 [GeV^2]	$\lambda_1 + \frac{\mathcal{T}_1+3\mathcal{T}_2}{m_b}$ [GeV^2]
0.5	5.0	-0.22 ± 0.38	-0.31 ± 0.17
1.0	3.5	-0.40 ± 0.26	-0.31 ± 0.22
0.5	12.9	-0.14 ± 0.13	-0.29 ± 0.10
1.0	8.5	-0.22 ± 0.25	-0.17 ± 0.21

- Can fit $1/m^3$ matrix elements consistently, but they are not well-determined:

m_χ [GeV]	ρ_1 [GeV^3]	ρ_2 [GeV^3]	$\mathcal{T}_1 + \mathcal{T}_3$ [GeV^3]	$\mathcal{T}_1 + 3\mathcal{T}_2$ [GeV^3]
0.5	0.15 ± 0.12	-0.01 ± 0.11	-0.15 ± 0.84	-0.45 ± 1.11
1.0	0.16 ± 0.18	-0.05 ± 0.16	0.41 ± 0.40	0.45 ± 0.49
0.5	0.17 ± 0.09	-0.04 ± 0.09	-0.34 ± 0.16	-0.66 ± 0.32
1.0	0.08 ± 0.18	-0.12 ± 0.15	0.11 ± 0.33	0.23 ± 0.47

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Experimental uncertainties

- Importance of correlations: increase all errors (except $\Gamma_{\rm sl}$) by a factor of 2

	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
Original fit	40.8 ± 0.9	4.74 ± 0.10
$2 \times$ errors	40.8 ± 1.2	4.74 ± 0.24

second fit has $\chi^2/\mathrm{d.o.f.} < 1$ and error of $\lvert V_{cb}\rvert$ does not increase dramatically

- Theoretical limitations: setting all experimental errors to zero, we would obtain

$\sigma(\lvert V_{cb}\rvert) \times 10^3$	$\sigma(m_b^{1S})$
± 0.35	± 35 MeV

Bauer-Trott moments

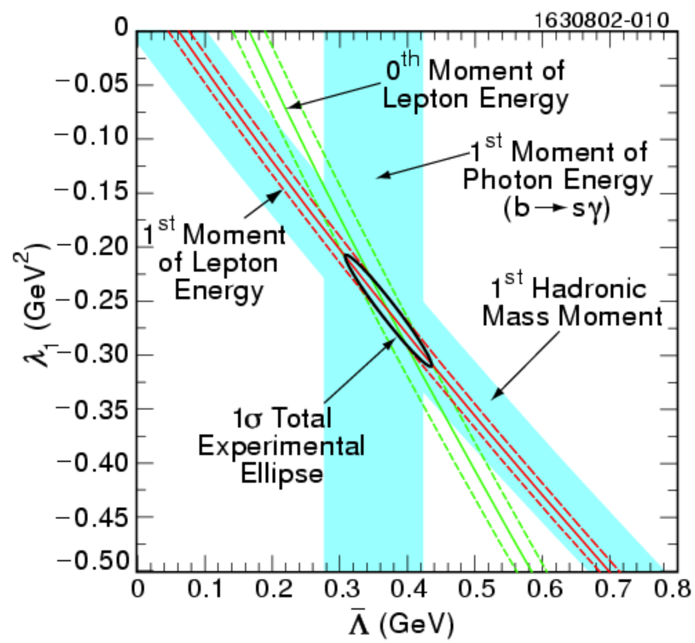
- Constructed to suppress (enhance) sensitivity to certain matrix elements

R_{3a}	R_{3b}	R_{4a}	R_{4b}	D_3	D_4
0.302 ± 0.003	2.261 ± 0.013	2.127 ± 0.013	0.684 ± 0.002	0.520 ± 0.002	0.604 ± 0.002
above is our prediction, below is CLEO measurement (hep-ex/0212051)					
0.3016 ± 0.0007	2.2621 ± 0.0031	2.1285 ± 0.0030	0.6833 ± 0.0008	0.5193 ± 0.0008	0.6036 ± 0.0006

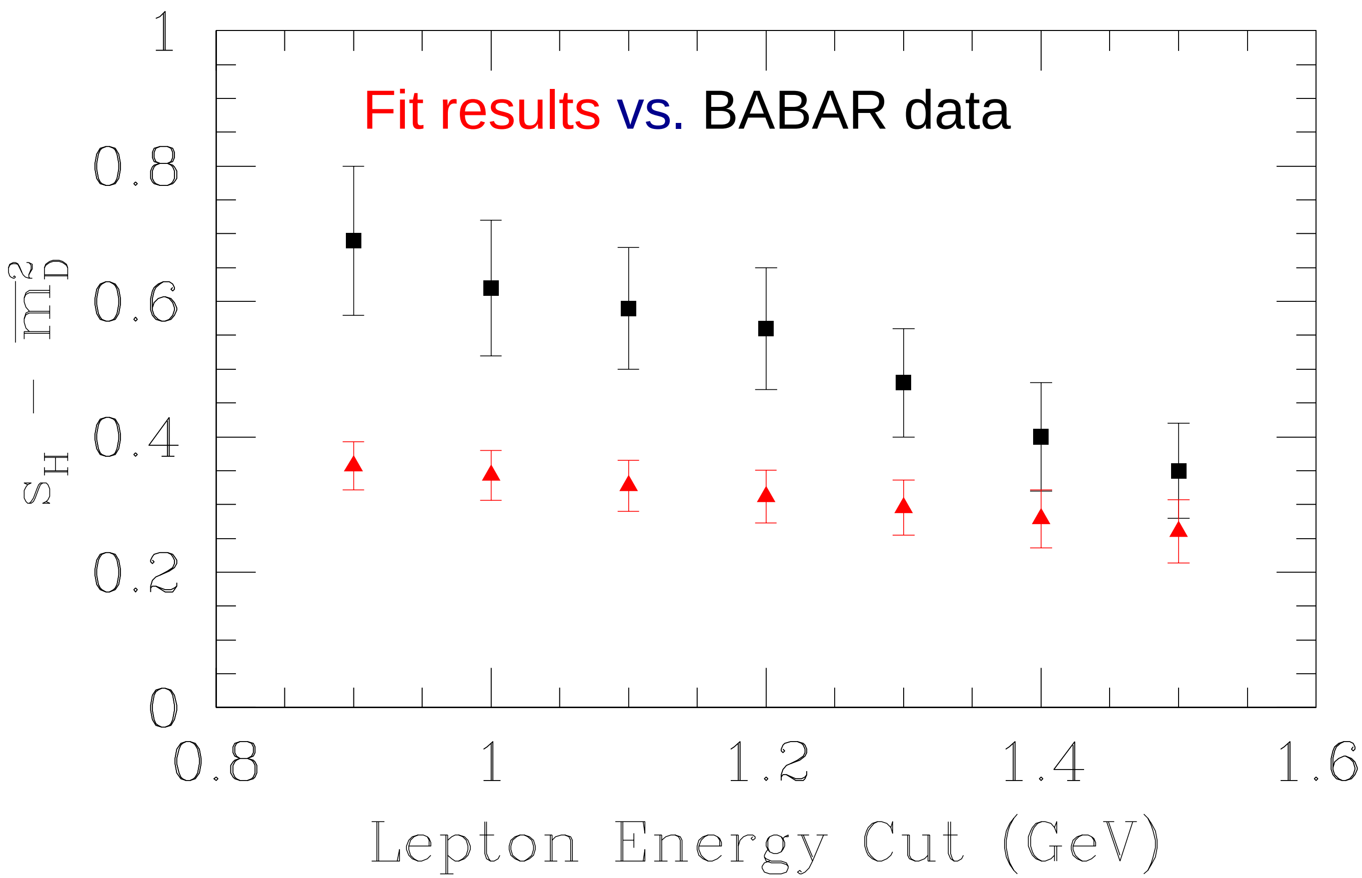
Predictions insensitive to m_χ and whether BABAR data is included in the fit

CLEO results are beautifully consistent within themselves

Note: excited D states make small contribution in regions studied by CLEO



Caveat 1: Hadronic moments for $E_\ell < 1.5$ GeV?



Difference appears to be significant

Measurement has implicit model dependence that can be eliminated

Caveat 2: “Gremm-Kapustin puzzle”?

Assuming negligible non-resonant contribution between D^* and D^{**}:

Prediction (fit result) for $S_1(0)$ implies that excited charm states contribute less than 25% to $B \to X_c \ell \bar{\nu}$ decay

In conflict with $\mathcal{B}(B \to X_c \ell \bar{\nu}) - \mathcal{B}(B \to D^{(*)} \ell \bar{\nu})$, which indicates that $\sim 35\%$ of semileptonic rate goes to excited states

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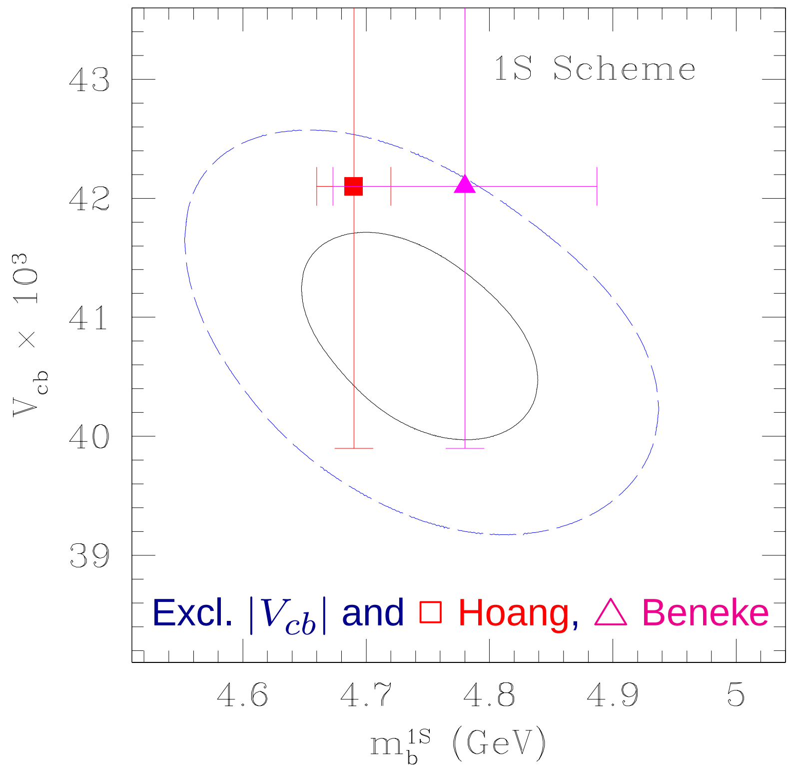
Either the assumption that low-mass nonresonant channels are negligible could be wrong, or some of the measurements or the theory have to be several σ off

Problem may disappear? Precise experimental $D_{u,d,s}$ spectroscopy is essential!
BELLE observed 0^+ D state at 2290 MeV, significantly below most predictions
BABAR’s D_s state at 2320 (most probably the 0^+) is also lighter than expected

$\Rightarrow$ Crucial to precisely and model independently measure the m_{X_c} distribution

Conclusions

Conclusions



We obtained:

$$|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$$
$$m_b^{1S} = (4.74 \pm 0.10)\,\text{GeV}$$
$$\overline{m}_b(\overline{m}_b) = (4.22 \pm 0.09)\,\text{GeV}$$

Battaglia *et al.*:

$$|V_{cb}| = (41.9 \pm 1.1) \times 10^{-3}$$
$$m_b(1\,\text{GeV}) = (4.59 \pm 0.08)\,\text{GeV}$$
$$\Rightarrow\ m_b^{1S} \simeq 4.69\,\text{GeV}$$

- Since theoretical uncertainties dominate, their correlations are essential when fitting many observables to determine hadronic parameters and $|V_{cb}|$
- Error of $|V_{cb}|$ may be reduced to $\sim 2\%$ level if all caveats resolved
- Nevertheless, important to pursue both inclusive and exclusive

Extra slides

Andre asked to generate discussion...

Uraltsev @ Durham workshop:

Bottle neck: 'Hardness' too low with the cut on E_ℓ

CLEO's extraordinary accuracy cannot be even nearly used

For total width $\mathcal{Q} \simeq m_b - m_c$ with the cut ?

+ backup from HQ symmetry

Generally $\mathcal{Q} \lesssim \omega_{\rm max}$ $\omega_{\rm max}$ is the threshold energy at which the process disappears if $m_b \to m_b - \omega$

In semileptonic decays

$$\mathcal{Q} \simeq m_b - E_{\rm min} - \sqrt{{E_{\rm min}}^2 + m_c^2}$$

This is only about $1.25\,{\rm GeV}$ for cut at $E_\ell = 1.5\,{\rm GeV}$

and below $1\,{\rm GeV}$ for $E_\ell > 1.7\,{\rm GeV}$

marginal $\mathcal{Q} \simeq 2\,{\rm GeV}$ for $E_\ell > 1\,{\rm GeV}$

A complementary consideration suggests the expansion for M_X^2 loses sense for $E_{\rm cut} \gtrsim 1.7\,{\rm GeV}$

In $b \to s + \gamma$ $\mathcal{Q} \simeq M_B - 2E_{\rm min} \simeq 1.2\,{\rm GeV}$

if the cut is at $E_\gamma = 2\,{\rm GeV}$

Reliability of theory is questionable ...

"Hardness" is not a physical parameter, that describes the accessible final states

OPE: The relevant question is the range of hadronic final states summed over and how they are weighted; e.g.:

$1.5\,$GeV $< E_\ell$ cut allows $m_{X_c} \le 3.47\,$GeV