EM 209
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Ori Ganor

Homework – Radiation

1 Elliptical polarization

A plane electromagnetic wave is given by

$$\vec{E} = \Re\{\vec{E}_0 e^{ikz-i\omega t}\}, \qquad k = \frac{\omega}{c}.$$

Calculate $\vec{E}$ explicitly for $\vec{E}_0 = E_0(\cos\chi\,\hat{x} + i\sin\chi\,\hat{y})$. Imagine plotting the vectors $\vec{E}(\vec{r},t)$ for fixed $\vec{r}$ and varying t. What curve do the endpoints of $\vec{E}(\vec{r},t)$ trace? Calculate the Poynting vector $\vec{S}$. What are the minimum and maxmimum values of its $\hat{z}$-component S_z over one cycle? What is the average?

2 An infinitesimal ring [*]

A very small ring of radius a carrying a DC current I is centered at the origin. The current is running clockwise. The ring is rotating with angular velocity ω around an axis that passes through the origin and is in the plane of the ring. Calculate the average radiated power profile in the radiation zone.

[Please turn the page.]

3 Conducting ball in a plane wave [**]

Complete the exercise from the lecture notes. A conducting ball of radius a is in the path of a plane wave with

$$\vec{E} = E_0 \hat{x} e^{ikz}.$$

Find the spherical wave decomposition of the reflected wave. Go through the following steps:

- First we need to expand e^{ikz} in spherical harmonics. You can do this by taking the limit of

$$\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} = 4\pi ik \sum_{l=0}^{\infty} j_l(kr_<) h_l^{(1)}(kr_>) \sum_{m=-l}^{l} Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi).$$

  with $\vec{r}' = R\hat{z}$ and $R \to -\infty$. You will need to use

$$Y_{lm}(\pi, 0) = (-1)^l \delta_{m0} \sqrt{\frac{2l+1}{4\pi}}.$$

- What we actually need is the expansion of $\vec{E} \cdot \vec{r}$ and $\vec{B} \cdot \vec{r}$ in spherical harmonics. We can do this by noting that xe^{ikz} and ye^{ikz} are components of $\vec{L}e^{ikz}$. Complete the expansion in the form

$$E_0 e^{ikz} \hat{x} = \sum_{l,m} \left[\frac{i}{k} b_E(l,m) \vec{\nabla} \times j_l(kr) \vec{X}_{lm} + b_M(l,m) j_l(kr) \vec{X}_{lm} \right], \qquad \vec{X}_{lm} \equiv \vec{L} Y_{lm}.$$

- From the expansion in the previous step, find the TM and TE modes $b_E(l,m)$ and $b_M(l,m)$ of the plane wave.
- Now add an outgoing wave, and solve the boundary conditions at $r = a$ to get the components of the outgoing wave (as was done in class).
- What do you get in the limiting case $ka \ll 1$?
- How much power will be reflected off the ball in this limiting case? Compare this to the amount of power that the plane wave carries through a cross section of πa^2.