EM 209
Dec 4, 2005
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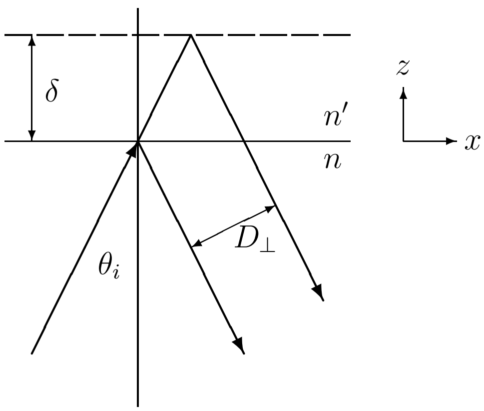
Homework – Goos Hänchen effect [**]



The Goos-Hänchen effect has to do with internal reflection at the interface between a medium with index of refraction n and a medium with index of refraction $n' < n$. Assume that the magnetic permeabilities are the same for both mediums, $\mu = \mu'$. A wavepacket appears to be reflected not from the surface but from an imaginary surface a distance δ into the forbidden region. Let D be the transverse distance between the reflected ray and the ray predicted by geometrical optics. The first order expressions for D for the two states of linear polarizations are

$$D_\perp = \frac{\lambda}{\pi}\frac{\sin\theta_i}{\sqrt{\sin^2\theta_i - \sin^2\theta_{i_0}}} \qquad D_\parallel = D_\perp \cdot \frac{\sin^2\theta_{i_0}}{\sin^2\theta_i - \cos^2\theta_i \cdot \sin^2\theta_{i_0}}$$

Derive the first expression by considering an incident wave-packet

$$\vec{E}_\perp(\vec{r},t) = \hat{y}\int A(\kappa_x,\kappa_z)e^{i\kappa_x x + i\kappa_z z - i\sqrt{\kappa_x^2+\kappa_z^2}ct}d\kappa_x d\kappa_y,$$

where $A(\kappa_x,\kappa_z)$ is sharply peaked around $(\kappa_x = k\sin\theta_i, \kappa_z = k\cos\theta_i)$. Use Fresnel's equations to determine the phase change of the reflected amplitude, and then use the principle of stationary phase to find $D_\perp$. You may assume that $A(\kappa_x,\kappa_z)$ is real and positive so that the incident wave packet passes through the origin $(x,z) = (0,0)$ at time $t = 0$.

Recall: According to the principle of stationary phase, the absolute value of an integral of the form

$$F(x,z) \equiv \int f(\kappa_x,\kappa_z;x,z)e^{i\varphi(\kappa_x,\kappa_z;x,z)}d\kappa_x d\kappa_z,$$

(where f is real and slowly-varying as a function of κ_x,κ_z and peaked near $\vec{\kappa} = \vec{k}$) is maximal when

$$0 = \left.\frac{\partial\varphi}{\partial\kappa_x}\right|_{\vec{\kappa}=\vec{k}} = \left.\frac{\partial\varphi}{\partial\kappa_x}\right|_{\vec{\kappa}=\vec{k}}$$

(See *Jackson*'s problem 7.7 for more clues.)