EM 209
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Ori Ganor

Homework - Electrostatics

1 Water molecules

The dielectric constant of water is approximately 80. note that a single water molecule has an electric dipole moment whose magnitude is

$$|\vec{p}| = 6.2 \times 10^{-30} C \cdot m$$

Because of random orientations, the average vector dipole moment of each molecule is zero in a zero electric field. What is the ratio of the average vector dipole moment of each molecule to its magnitude $|\vec{p}|$ in an electric field of $\vec{E} = 1\text{V/m}$?

2 Quadrupole moment

In class we studied the contribution of a continuous density of pointlike dipoles $\vec{P}$ to the electric displacement field $\vec{D} = \vec{E} + 4\pi\vec{P}$. How much would a continuous density of pointlike quadrupoles contribute? (Recall that the quadrupole moment of a system is given by the tensor

$$Q_{ij} = \int \rho(\vec{y})(3y_i y_j - \delta_{ij}|\vec{y}|^2) d^3y, \qquad i, j = 1 \ldots 3.$$

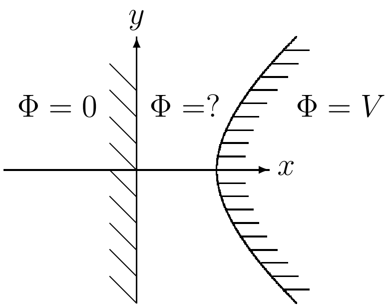
The integral is over the whole system and the system is assumed to be very small.)

3 A pointlike charge and a conducting ball

A pointlike charge Q is placed at a distance R from the center of a grounded conducting ball of radius a. (Assume that $a < R$.) Find the electrostatic potential everywhere in space.

4 Elliptic coordinates coordinates

Find the potential Φ in the space between the infinite conductors in the figure. The first conductor is at the potential $\Phi = 0$ and has the shape of an infinite plane, given analytically by $x = 0$. The second conductor is at $\Phi = V$ and has the shape of the branch of the hyperbola $x^2 - y^2 = a^2/2$ with $x > 0$. (Here a is a given constant.)



Hint: use elliptic coordinates, which are given by

$$x = a \cosh u \, \cos v, \quad y = a \sinh u \, \sin v, \quad z = z, \qquad 0 \le u < \infty, \quad -\pi \le v < \pi,$$

and a is a constant.

- Show that Laplace's equation in elliptical coordinates is

$$0 = \vec{\nabla}^2 \Phi = \frac{1}{a(\sinh^2 u + \sin^2 v)} \left(\frac{\partial^2 \Phi}{\partial u^2} + \frac{\partial^2 \Phi}{\partial v^2} \right) + \frac{\partial^2 \Phi}{\partial z^2}.$$

- Show that the location of conductors is given by $v = \pm\pi/2$ and $v = \pm\pi/4$, respectively.
- Look for a potential $\Phi = \Phi(v)$ that is independent of u and z.