EM 209
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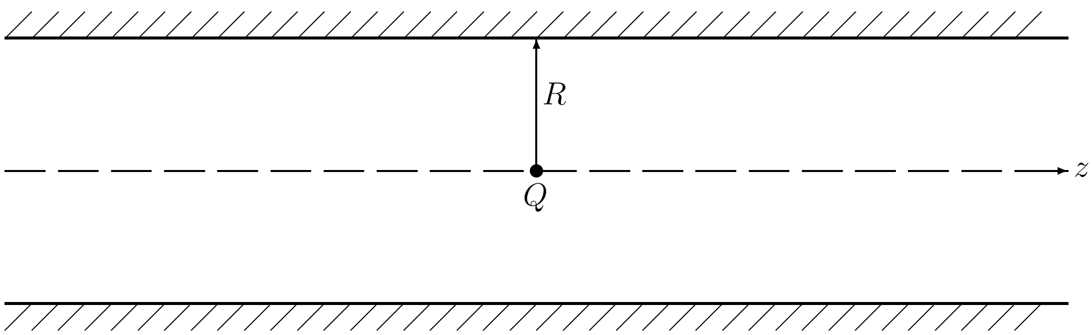
Homework - Bessel functions and spherical harmonics

1 Point particle in a conducting cylinder



A point particle of charge Q is placed on the axis of an infinite hollow grounded conducting cylinder of radius R.

- Find the electrostatic potential on the axis as an infinite sum involving Bessel functions.
- Show that for $z \gg R$ the potential falls off exponentially $\sim C\exp(-\alpha z)$ with the distance along the axis. Find the constant α.

You may need to use the following expansion of the δ-function in two dimensions in terms of Bessel functions:

$$\delta^{(2)}(\vec{r}) = \frac{1}{\pi R^2}\sum_{n=1}^{\infty}\frac{1}{J_1^2(x_n)}J_0\left(\frac{x_n}{R}\rho\right), \qquad \rho \le R,$$

where x_n is the n^{th} zero of the Bessel function $J_0(x_n)=0$, and we use polar coordinates

$$\vec{r} = (\rho\cos\theta, \rho\sin\theta).$$

[The formula for $\delta^{(2)}(\vec{r})$ follows from equations (3.96)-(3.97) of Jackson's textbook.]

2 Dielectric sphere in uniform electric field

An insulating dielectric sphere of radius R is placed in a uniform electric field so that far away from the sphere $\vec{E} = E\hat{z}$. The dielectric constant of the sphere is ϵ. Find the solution for the scalar potential Φ. (recall that it is defined so that $\vec{E} = -\vec{\nabla}\Phi$.) **Hint:** Use separate multipole expansions for $r < R$ and $r > R$.

3 Two halves

A hollow conducting sphere of radius R is cut into two equal hemispheres along the equatorial. The two halves are then glued together with a perfectly insulating glue and a battery is connected to the two halves that keeps the lower half at potential $\Phi = 0$ and the upper hemisphere at potential $\Phi = V$.

Find the potential inside the hollow sphere as an infinite multipole series. **Hint:** You can get the individual coefficients of the multipole expansion from the orthogonality formulas. You may have to use

$$\int_0^1 P_l(\lambda) d\lambda = \begin{cases} 0 & \text{for } l \text{ even} \\ (-1)^{\frac{l-1}{2}} \frac{(l-1)!}{2^l \left(\frac{l-1}{2}\right)! \left(\frac{l+1}{2}\right)!} & \text{for } l \text{ odd} \end{cases}$$

[See formula (3.23) in Jackson.]