EM 209
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Homework - Magnetism

1 Vector potential

Find the vector potential $\vec{A}$ for the magnetic field produced by a straight and infinite wire at the origin of the $\hat{y}-\hat{z}$ plane, carrying current I in the $\hat{x}$ direction. Convert it to the Coulomb gauge. (**Note:** don't forget δ-functions when you check for Coulomb gauge.)

2 Gauge transformation

Given the vector potential from class,

$$\vec{A} = B_3 x_1 \hat{x}_2 + (B_1 x_2 - B_2 x_1)\hat{x}_3,$$

find a gauge transformation Λ such that

$$\vec{A} + \vec{\nabla}\Lambda = \frac{1}{2}\vec{B}\times\vec{r},$$

where

$$\vec{B} = B_1\hat{x}_1 + B_2\hat{x}_2 + B_3\hat{x}_3$$

is a constant magnetic field.

3 Identities

Let $\vec{C}$ be a constant vector. Use the charge conservation equation $\vec{\nabla}\cdot\vec{J} = 0$ to show that

$$\int \vec{J}(\vec{r})d^3\vec{r} = 0$$

and

$$\int (\vec{C}\cdot\vec{r})\vec{J}d^3\vec{r} = -\frac{1}{2}\vec{C}\times\int(\vec{r}\times\vec{J})d^3\vec{r},$$

where the integrals are over all space. Finally, show that

$$\int(\vec{r}\cdot\vec{J})d^3\vec{r} = 0.$$

Hint: It is convenient to use component notation and the Einstein summation convention. Integrate by parts the following integrals

$$0 = \int x_i \partial_k J_k d^3x, \qquad 0 = \int x_i x_j \partial_k J_k d^3x.$$

4 Wire near a paramagnet or diamagnet

In this problem we work in Cartesian x, y, z coordinates. Let the upper half space $z > 0$ be filled with a material with magnetic permeability μ and the lower half space $z < 0$ be filled with vacuum. The interface layer is the $z = 0$ plane. Place a straight thin wire carrying current I parallel to the $z = 0$ plane, at position $z = -a, y = 0$. The current is flowing in the positive $\hat{x}$ direction (out of the page, in the picture below).

- Write down the boundary conditions on the magnetic field $\vec{H}$ at $z = 0$.
- Solve for $\vec{H}$. **Hint:** use the method of images. Place an image wire with current I' somewhere in the upper half plane $z > 0$.
- Find the magnetic force per unit length on the wire.