Physics 209
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Homework - Magnetic multipole expansion

1 Some vector spherical harmonics

We work in radial coordinates (r, θ, ϕ). Let $\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi$ be unit vectors in the radial, longitudinal and azymuthal (i.e. along a latitude line) directions respectively.

$$\hat{e}_r \equiv \sin\theta\cos\phi\hat{x}+\sin\theta\sin\phi\hat{y}+\cos\theta\hat{z}, \qquad \hat{e}_\theta \equiv \cos\theta\cos\phi\hat{x}+\cos\theta\sin\phi\hat{y}-\sin\theta\hat{z}, \qquad \hat{e}_\phi \equiv -\sin\phi\hat{x}+\cos\phi\hat{y}.$$

Expand the vector spherical harmonics $\vec{L}Y_{1,0}, \vec{L}Y_{1,1}, \vec{L}Y_{2,0}, \vec{L}Y_{2,1}, \vec{L}Y_{2,2}$ explicitly in $\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi$.

2 Symmetry

If the current configuration $\vec{J}(x, y, z)$ has some symmetry properties, then we can sometimes argue that certain magnetic multipole moments M_{lm} vanish. In each of the following four cases, find which M_{lm}'s vanish due to the symmetry.

1 Parity odd: $\vec{J}(x, y, z) = \vec{J}(-x, -y, -z)$;

2 Parity even: $\vec{J}(x, y, z) = -\vec{J}(-x, -y, -z)$;

3 Reflection: $J_x(x, y, z) = J_x(x, y, -z)$, $J_y(x, y, z) = J_y(x, y, -z)$, $J_z(x, y, z) = -J_z(x, y, -z)$;

4 90° rotation: $J_x(x, y, z) = J_y(-y, x, z)$, $J_y(x, y, z) = -J_x(-y, x, z)$, $J_z(x, y, z) = J_z(-y, x, z)$;

3 A magnetized cube

Find the magnetic mutipole moments with $l = 0, 1, 2, 3$ for a cube of dimensions $2a \times 2a \times 2a$ and constant magnetization $\vec{M} = A\hat{z}$. That is, assume that the region of space with $-a \leq x, y, z \leq a$ is filled with a uniform density of magnetic dipole moments pointing in the $\hat{z}$-direction, and calculate $M_{00}, M_{1,-1}, M_{10}, M_{11}, M_{2,-2}, M_{2,-1}, M_{20}, M_{21}, M_{22}, M_{3,-3}, M_{3,-2}, M_{3,-1}, M_{30}, M_{31}, M_{32}, M_{33}$.

Hint: Use the previous problems.

(If you didn't do the previous problems, do this exercise only for M_{l0}.).