Physics 209
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Homework - Special Relativity

1 Acceleration [*]

Find the worldline of a particle that experiences constant acceleration a in its MCRF.

2 Spaceship [**]

Assume that a futuristic spaceship is powered by *anti-matter*. Let's calculate how fast it can go. Assume that its total (rest) mass is m and that a fraction f of it is fuel, in the form of equal amounts of matter and anti-matter. Let's say that the engines fuse matter with anti-matter (with equal proportion) and convert it to photons, which are then expelled backwards against the direction of motion, thus producing the thrust. Suppose the ship starts with velocity $v = 0$ in some inertial reference frame $\mathcal{K}$. How fast will it be going when all the fuel is consumed?

3 Transformation of a tensors

The components of an antisymmetric $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ tensor F can be parameterized as:

$$\begin{pmatrix} 0 & F_{01} & F_{02} & F_{03} \\ -F_{01} & 0 & F_{12} & F_{13} \\ -F_{02} & -F_{12} & 0 & F_{23} \\ -F_{03} & -F_{13} & -F_{23} & 0 \end{pmatrix}$$

Find the components in a reference frame that moves with velocity v in the $\hat{x}$ direction. Do the same for a symmetric tensor that can be parameterized as

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{01} & T_{11} & T_{12} & T_{13} \\ T_{02} & T_{12} & T_{22} & T_{23} \\ T_{03} & T_{13} & T_{23} & T_{33} \end{pmatrix}.$$

4 Tensor contraction

For the tensors $F_{\alpha\beta}$ and $T_{\alpha\beta}$ above, calculate $F_{\alpha\beta}F^{\alpha\beta}$ and $T_{\alpha\beta}T^{\alpha\beta}$.