Physics 209
Oct 30, 2005
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Homework - Special Relativity and Electromagnetism II

1 Conservation of canonical momentum

A relativistic particle is moving in a constant magnetic field $\vec{B} = B\hat{z}$. Using the Hamiltonian formalism developed in class, write down the conservation of energy and canonical momentum and use that to solve for the trajectory. Ignore synchrotron radiation (which we haven't learnt yet!) **Hint:** Choose an appropriate gauge for momentum in the $\hat{x}$ direction and then switch to another gauge for momentum in the $\hat{y}$ direction.

2 Relativistic Ohm's law

Find the relativistic form of Ohm's law for a conductor with (rest-frame) conductivity σ and velocity 4-vector U^α. [This is problem 11.16 on p572 of *Jackson.*]

Follow these steps:

- In the rest frame of the conductor we have $\vec{J} = \sigma\vec{E}$.
- We are looking for a linear relation between the current 4-vector J^α and the field-strength tensor $F^{\alpha\beta}$ that reduces to $\vec{J} = \sigma\vec{E}$ in the rest frame.
- Write down the general 4-vector expression that can be constructed linearly from $F^{\alpha\beta}$ and J^α and can involve U^α (not necessarily linearly).
- Use the antisymmetry of $F^{\alpha\beta}$ and the form of the expression in the rest frame to find the coefficients.

After you find the covariant expression, take the velocity of the conductor to be $v = \beta c$ in the $\hat{x}$ direction. Rewrite the expression that you got in terms of 3-vectors $\vec{J}$, $\vec{E}$, $\vec{B}$ and $\vec{\beta}$, and expand up to first order in v. What is the physical meaning of the expression that you found?

3 Field gradients [*]

Assume that the electric dipole moment (EDM) of a neutron is exactly zero. (Although theory predicts a tiny EDM, it is beyond our current detection capabilities. The current limit is less than $10^{-25}e \times cm$.) The magnetic dipole moment of the neutron is

$$\vec{\mu} = \frac{g_n|e|}{2m_n c}\vec{s},$$

where $\vec{s}$ is the spin of the neutron, e is the charge of the electron, m_n is the mass of the neutron and $g_n = -3.826\ldots$

The acceleration of a neutron at rest in a background electromagnetic field is given by Newton's law

$$m_n \vec{a} = \vec{\nabla}(\vec{\mu} \cdot \vec{B}) - \frac{1}{c}\frac{d}{dt}(\vec{E} \times \vec{\mu}).$$

The first term is the magnetostatic force that we studied in class. The second term is the rate of change of the momentum stored in the electromagnetic field. [See the discussion after equation (5.69) on p189 of *Jackson.*]

Assuming for simplicity that $d\vec{s}/dt$ can be neglected, generalize this equation to a covariant expression for $dU^\alpha/d\tau$ that is linear in the spin 4-vector S^α and the first derivatives $\partial_\gamma F^{\alpha\beta}$ of the field-strength tensor. [Don't forget to include the dual field-strength tensor $\widetilde{\mathcal{F}}^{\alpha\beta}$!]