String Theory 250
Feb 15, 2005
Ori Ganor

Problem set # 2

1 The spectrum at levels $2, 3$

Assemble the spectrum of the open string at leven $N = 2$ into $SO(D-1)$ irreducible representations. Repeat for level 3. **Hint:** Representations of $SO(n)$ can be described by tensors $T_{i_1 \ldots i_k}$ that are: (i) traceless ($\delta^{i_r i_s} T_{i_1 \ldots i_k} = 0$ for $1 \le r < s \le k$) and (ii) correspond to an irreducible representation of the permutation group S_k. For levels $2, 3$ the last condition can be taken to mean that the tensor is either completely symmetric in the indices $i_1, \ldots, i_k$ or completely antisymmetric. (For level ≥ 4 we need to consider higher dimensional representations of S_k that are labeled by Yang-Diagrams.)

2 Deriving $D = 26$ and $A = -1$ [**]

(See [2].) Derive the commutation relation for the Lorentz anomaly of the open string in lightcone gauge $\langle 0|\alpha_m^k [J^{i-}, J^{j-}]\alpha_{-m}^l|0\rangle$ and show that it is zero only if $D = 26$ and $A = -1$. You will need to use the following identities

$$
\begin{aligned}
\langle 0|\alpha_m^- \alpha_{-m}^-|0\rangle &= \frac{D-2}{12}(m^3 - m) - 2mA, \\
\langle 0|\alpha_m^- \sum_{n=1}^{m} \frac{1}{n}\alpha_{-n}^j \alpha_{n-m}^l|0\rangle &= 2\alpha' p^j p^l + \frac{1}{2}\delta^{jl} m(m-1), \\
\langle 0|\sum_{n=1}^{m} \frac{1}{n}\alpha_{m-n}^k \alpha_n^i \sum_{r=1}^{m} \frac{1}{r}\alpha_{-r}^j \alpha_{r-m}^l|0\rangle - (i \leftrightarrow j) &= (m-1)(\delta^{il}\delta^{jk} - \delta^{jl}\delta^{ik}),
\end{aligned}
$$

3 Related to Problem (1.5) of [1]

Modify the worldsheet theory of a closed string as follows. Denote $Z(\sigma,\tau) \equiv X^1(\sigma,\tau) + iX^2(\sigma,\tau)$ and take the boundary conditions to be $Z(\sigma + l, \tau) = e^{2\pi i\theta} Z(\sigma,\tau)$ (for some constant θ). Find the analog of the mode expansion [eqn (1.3.22)] and the analog of the constant A [equation (1.3.35)]. Formally you need to calculate $\sum_1^\infty (n - \theta)$ to find A. Do this using the *string-bits* regularization that we studied in class.

References

[1] J. Polchinski, “String Theory,” Cambridge University Press.

[2] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory,” Cambridge University Press.