

# New Lorentz Violating Nonlocal QFTs

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UC Berkeley and LBNL

November 11, 2006

M-theory in the City

Talk at Queen Mary College

## Based on

- OG, “A New Lorentz Violating Nonlocal Field Theory From String-Theory,” [[arXiv:hep-th/0609107](#)]
- Aki Hashimoto, Sharon Jue, Bom Soo Kim, Anthony Ndirango and OG, to appear.

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**DISCLAIMER:** This talk represents my own perspective on the project, and the more suspiciously looking parts of this talk do not necessarily represent the views of my collaborators.

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- String theory construction?
- $c > 1$ ?



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[Amelino-Camelia, Ellis, Mavromatos, Nanopoulos and Sarkar, 1998]

$$\frac{\Delta c}{c} < \left| \frac{E}{10^{17} \text{GeV}} \right|, \quad 1 \text{MeV} < E < 17 \text{MeV}.$$

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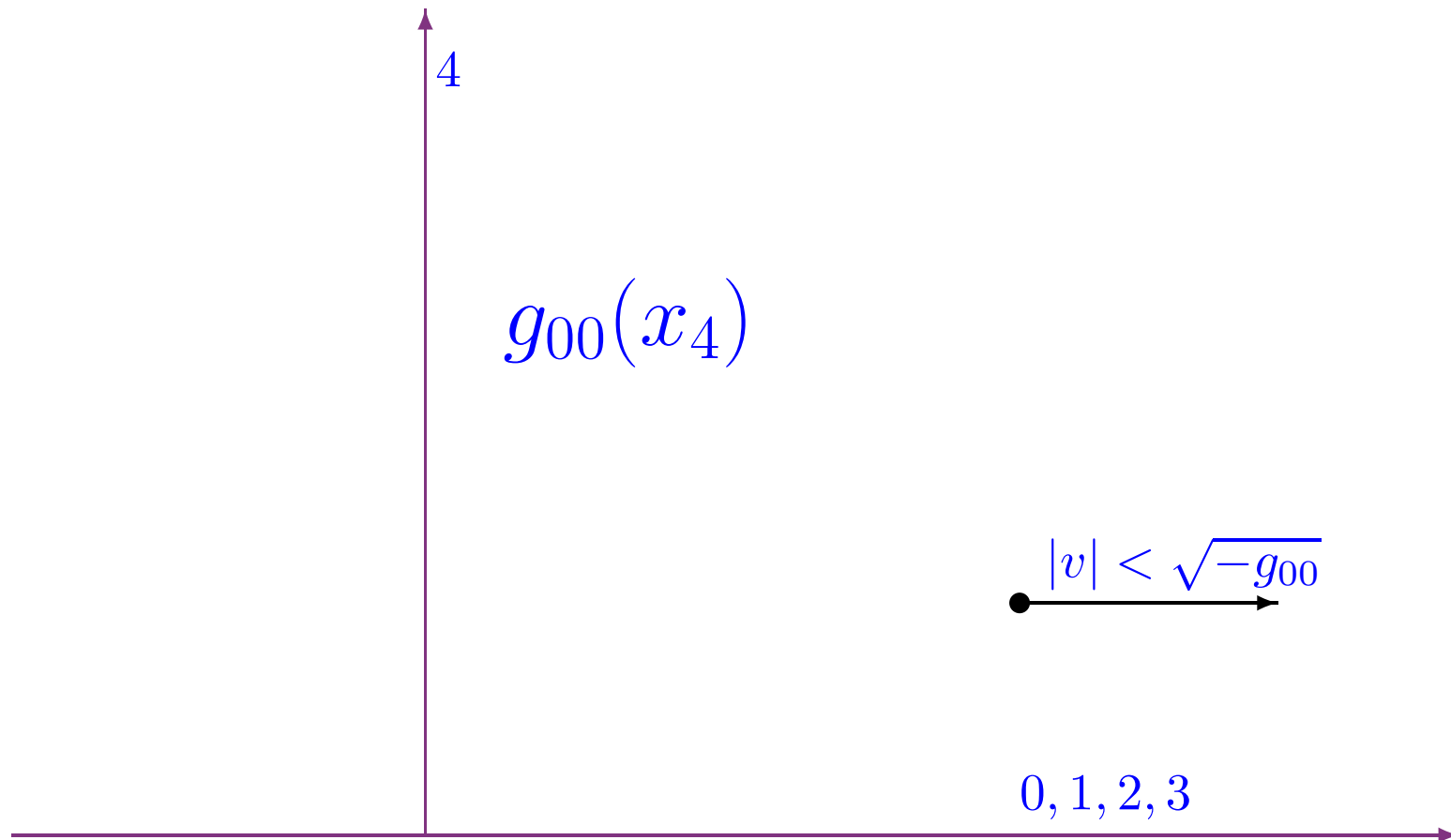
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Preserve Lorentz invariant photon dispersion relation.

# Lorentz Violation & Extra Dimensions



[Csaki, Erlich and Grojean, 2000]

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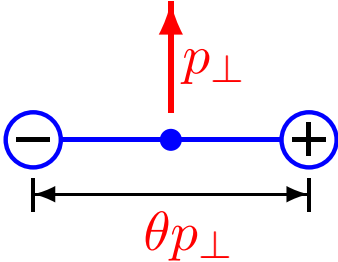
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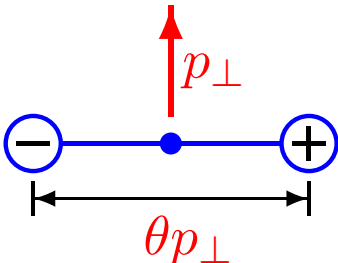
The diagram shows a horizontal blue line segment with a central dot. At the left end of the segment is a circle containing a minus sign, and at the right end is a circle containing a plus sign. A red arrow labeled  $p_\perp$  points vertically upwards from the central dot. Below the horizontal segment, a double-headed arrow indicates its length, labeled  $\theta p_\perp$ .

$$\Phi_1 \star \Phi_2 = \left( \exp \frac{i}{2} \theta^{ij} \partial_i^{(1)} \partial_j^{(2)} \right) [\Phi_1 \Phi_2]$$



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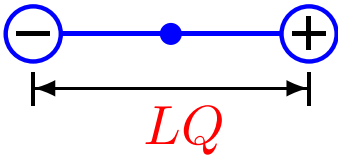


A diagram showing a horizontal line segment with a blue circle containing a minus sign at the left end and a blue circle containing a plus sign at the right end. A blue dot is located at the midpoint of the segment. A red arrow labeled  $p_\perp$  points vertically upwards from the blue dot. A horizontal double-headed arrow below the segment is labeled  $\theta p_\perp$ .

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- Dipole theories.

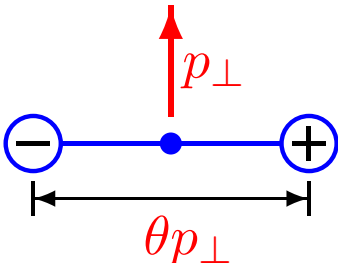
$Q = \text{R-charge}$



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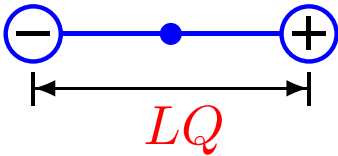


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$$D_\mu \Phi(x) = \partial_\mu \Phi(x) - i A_\mu(x - \frac{1}{2}L) \Phi(x) + i \Phi(x) A_\mu(x + \frac{1}{2}L)$$

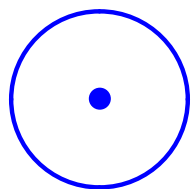
# Nonlocality - Volume

- In this talk:



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$$\text{Volume} \propto Q$$

# Deformations of $N = 4$ SYM

$$L = L_{N=4} + \zeta \mathcal{O}^{(\Delta+4)} + \dots$$

$$[\zeta] = \text{Length}^\Delta$$

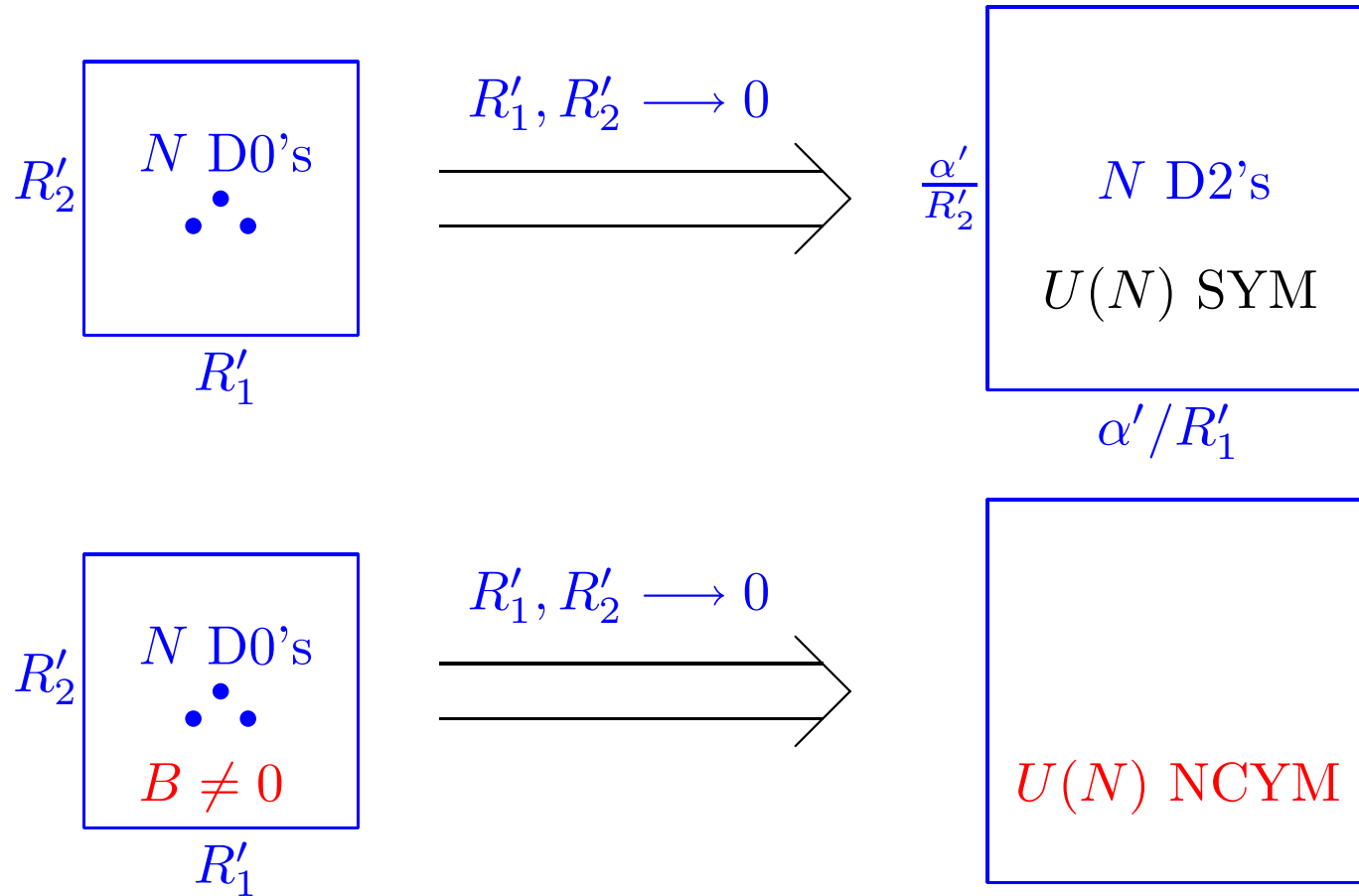
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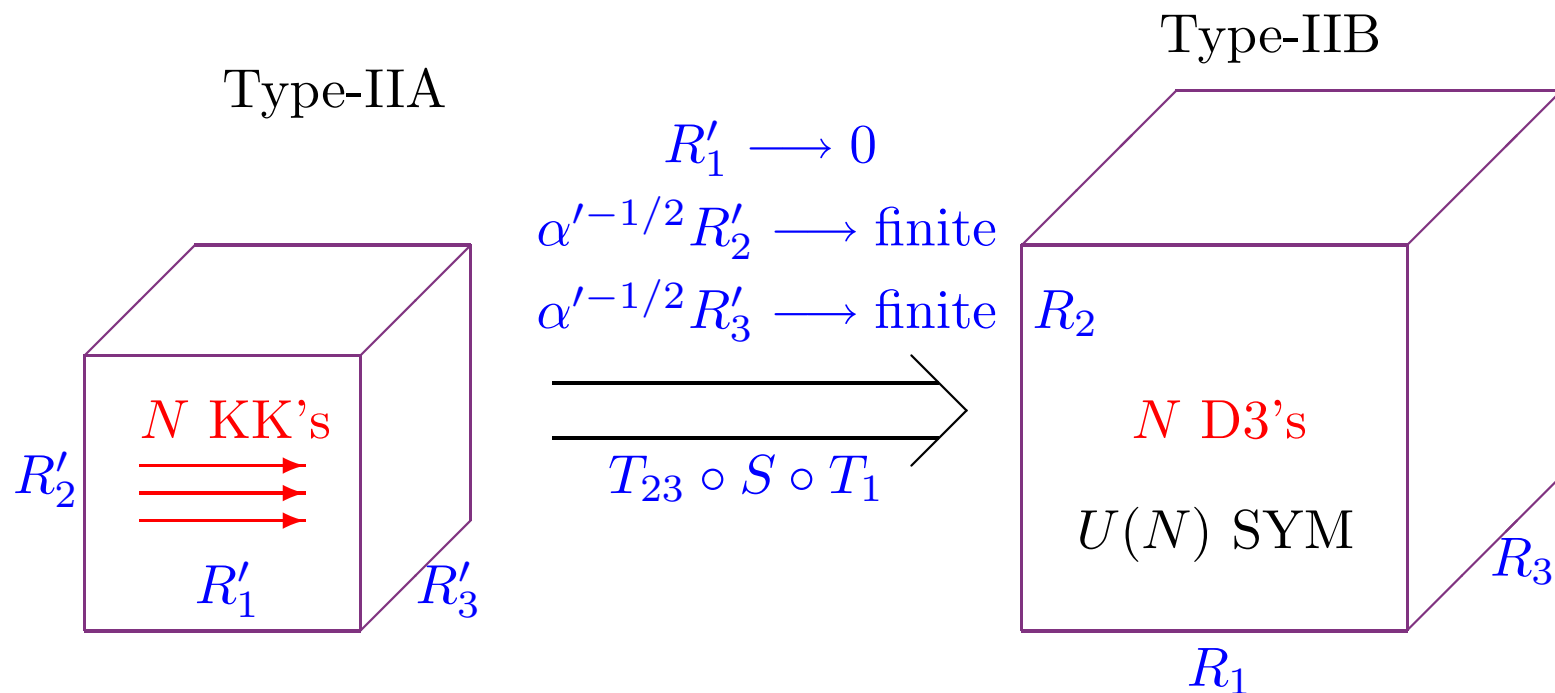
$$[\zeta] = \text{Length}^\Delta$$

| Theory    | Deformation $\zeta \mathcal{O}$                                                                                                         | $\Delta$ |                            |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------|----------|----------------------------|
| NCYM      | $(1/2g_{YM}^2) \theta^{\mu\nu} \text{Tr} \{ F_{\mu\sigma} F_{\nu\tau} F^{\sigma\tau} + F_{\sigma\tau} F^{\mu\nu} F_{\mu\nu} + \dots \}$ | 2        | $\zeta \rightarrow \theta$ |
| Dipole    | $(1/2g_{YM}^2) L_{IJ}^\mu \text{Tr} \{ F_{\mu\nu} (\Phi^I D^\nu \Phi^J + \dots) + \dots \}$                                             | 1        | $\zeta \rightarrow L$      |
| This talk | .....                                                                                                                                   | 3        |                            |

# Douglas-Hull construction of NCYM



# Our Construction (1)



$$g_s = \alpha' R_3'^{-1} R_2'^{-1}$$

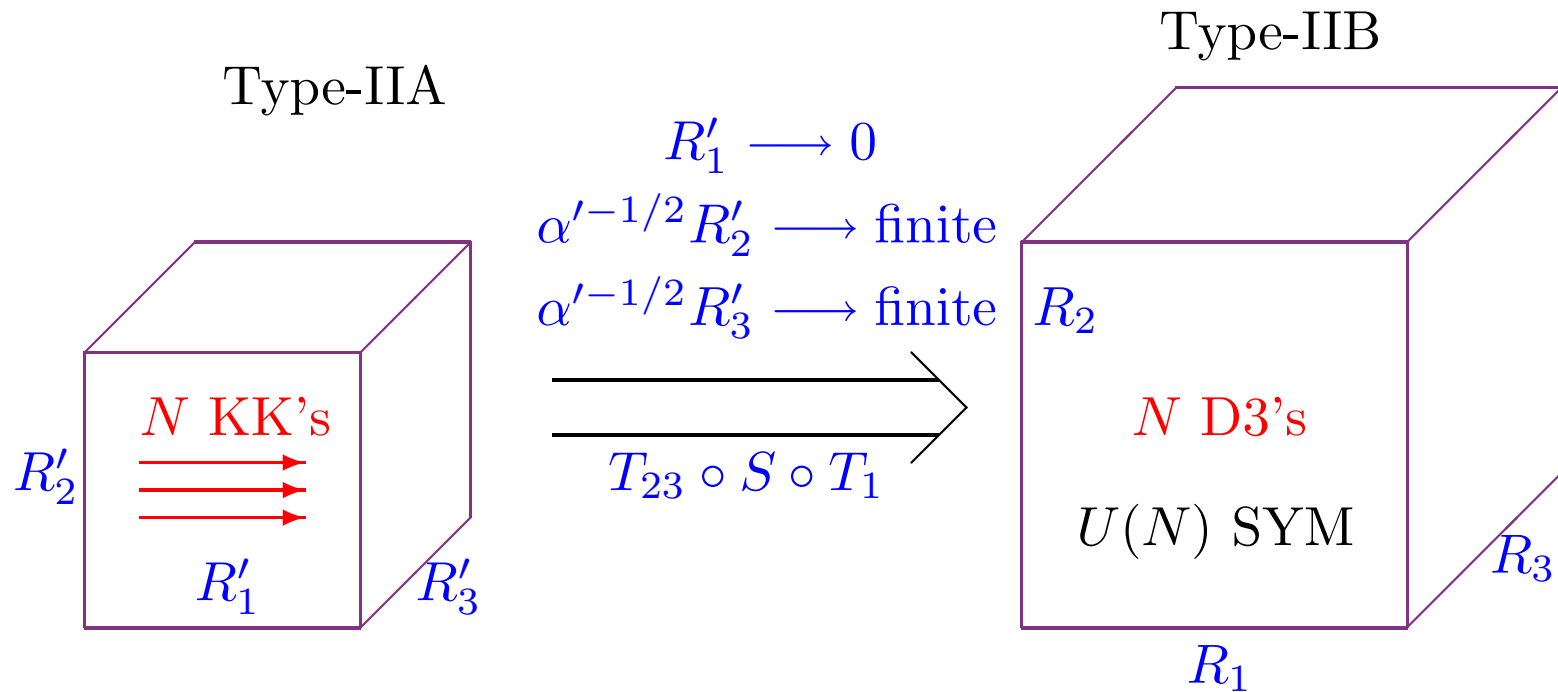
$$R_2 = \alpha'^{\frac{5}{4}} g_s'^{\frac{1}{2}} R_1'^{-\frac{1}{2}} R_2'^{-1}$$

$$R_1 = \alpha'^{\frac{3}{4}} g_s'^{-\frac{1}{2}} R_1'^{-\frac{1}{2}}$$

$$R_3 = \alpha'^{\frac{5}{4}} g_s'^{\frac{1}{2}} R_1'^{-\frac{1}{2}} R_3'^{-1}$$

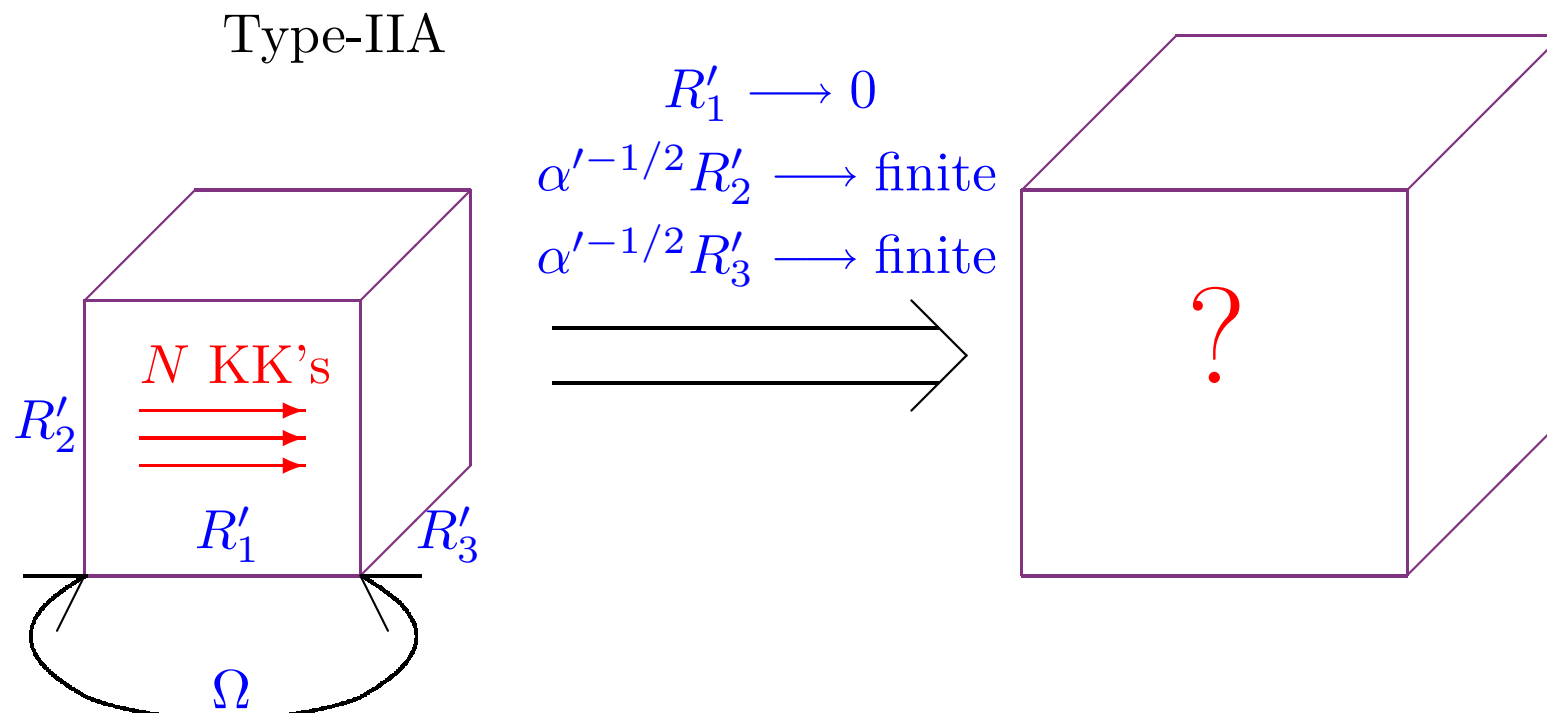


# Our Construction (1)



$$\begin{aligned}
 g_s &\longrightarrow \text{finite} & \alpha'^{-1/2} R_k &\longrightarrow \infty, \quad (k = 1, 2, 3) \\
 R_2/R_1, R_3/R_1 &\longrightarrow \text{finite}
 \end{aligned}$$

# Our Construction (2)



$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1, x_2, x_3, \Omega \vec{y})$$

$$\Omega \in \text{Spin}(6)$$

$$\vec{y} \in \mathbb{R}^6$$

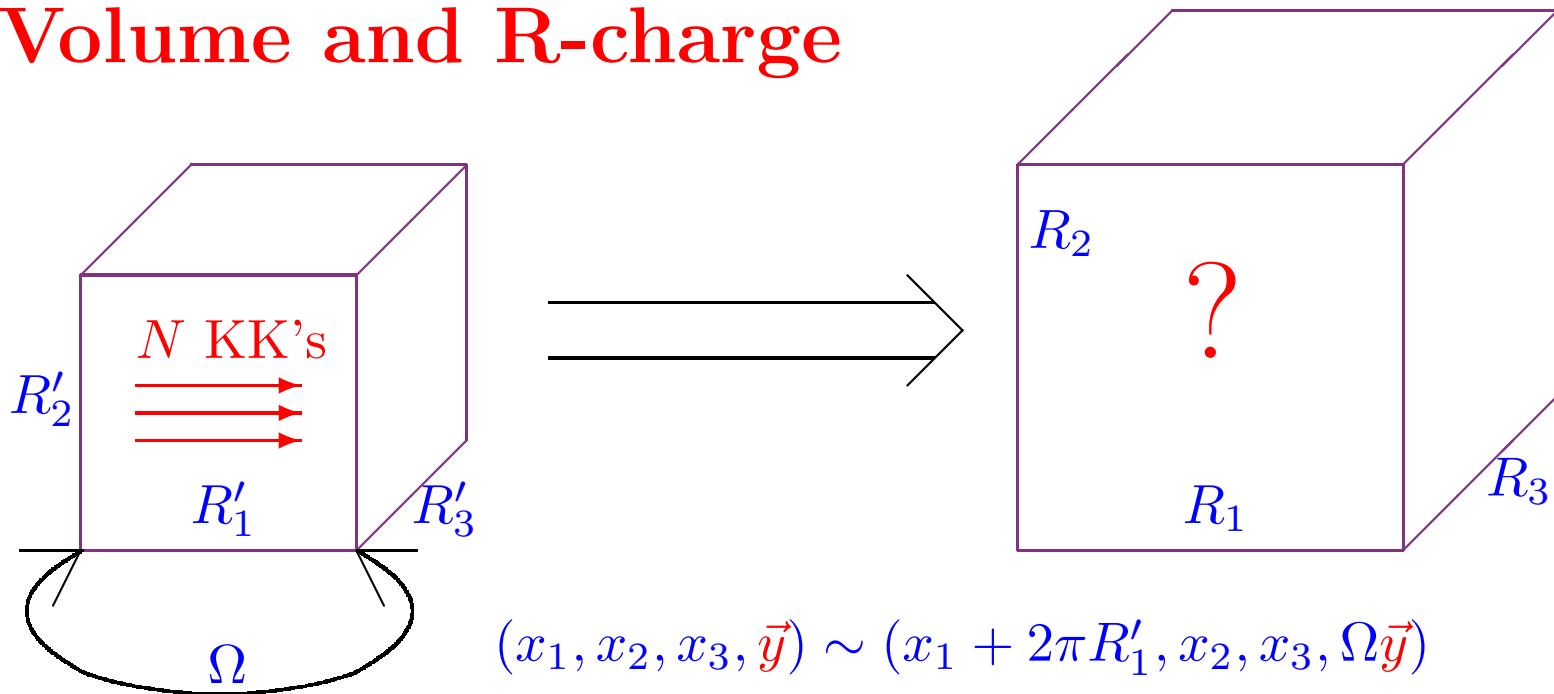
# Scaling $\Omega$

$$\Omega = \exp \left\{ \frac{2\pi}{R_1 R_2 R_3} \zeta \right\} \longrightarrow I, \quad \zeta \rightarrow \text{finite}$$

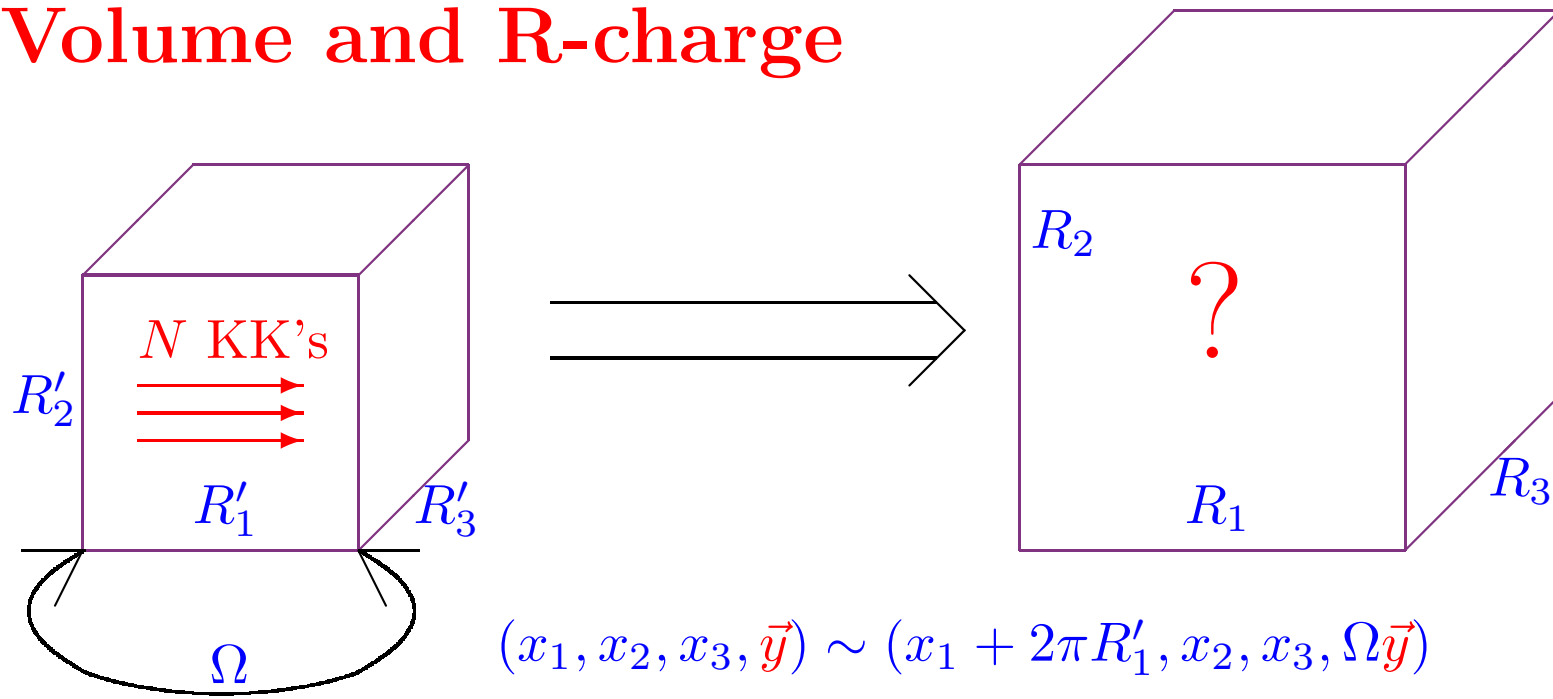
$$\zeta \equiv \begin{pmatrix} & & & & & \\ & \beta_1 & & & & \\ & -\beta_1 & & & & \\ & & \beta_2 & & & \\ & & -\beta_2 & & & \\ & & & \beta_3 & & \\ & & & -\beta_3 & & \end{pmatrix} \in so(6)$$

$$\begin{array}{ccccccc} \text{SUSY:} & \{0\} & \subset & su(2) & \subset & su(3) & \subset & so(6) \\ & \mathcal{N} = 4 & & \mathcal{N} = 2 & & \mathcal{N} = 1 & & \mathcal{N} = 0 \end{array}$$

# Volume and R-charge



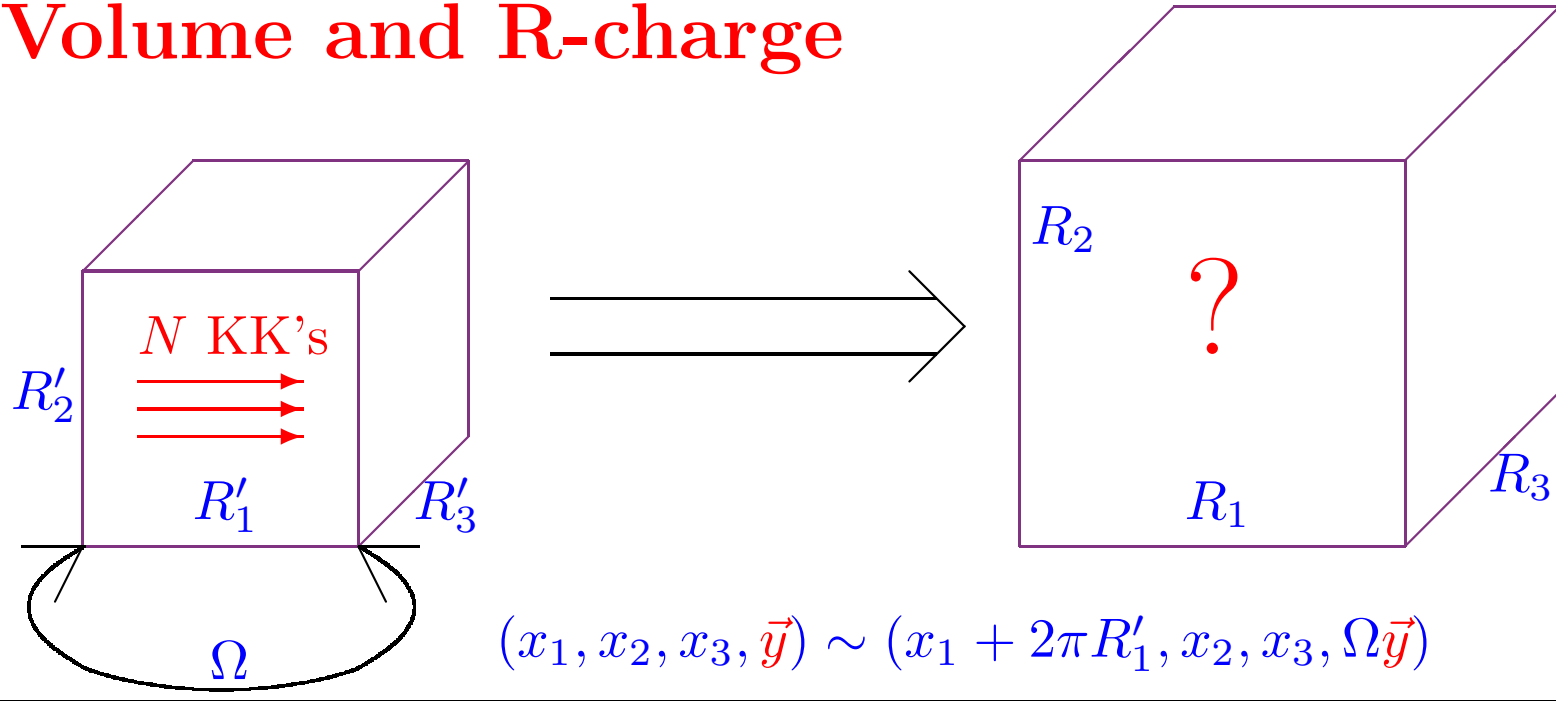
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$$z \equiv y_1 + iy_2, \quad \Omega = \exp \frac{4\pi i \beta}{R_1 R_2 R_3}, \quad f(x_1 + 2\pi R'_1, \Omega z) = f(x_1, z)$$

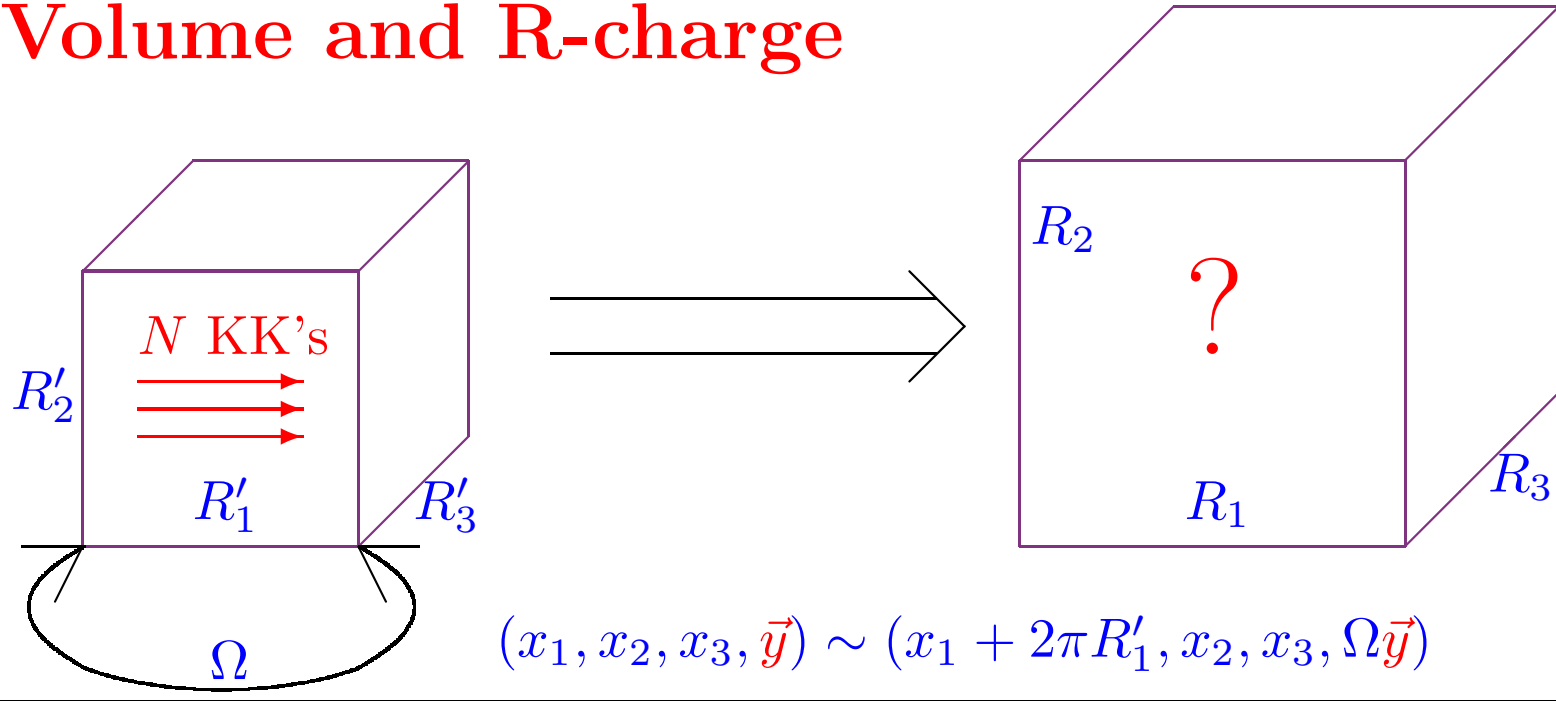
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$$f(x_1, z) = \sum_{p,j} C_{n,j} z^{-j} e^{ipx_1/R'_1} \longrightarrow p \in \mathbb{Z} + \frac{2\beta j}{R_1 R_2 R_3}$$

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$$\Rightarrow \boxed{\text{“Vol(D3)”} = (2\pi)^3 (R_1 R_2 R_3 N + 2\beta j)}$$

# Electric and Magnetic fluxes

$$V \equiv R_1 R_2 R_3,$$

$$\mathbf{P} \equiv \sum_{i=1}^3 \frac{k_i}{R_i} \hat{\mathbf{n}}_i, \quad \mathbf{E} \equiv \sum_{i=1}^3 \frac{e_i R_i}{2\pi V} \hat{\mathbf{n}}_i, \quad \mathbf{B} \equiv \sum_{i=1}^3 \frac{m_i R_i}{2\pi V} \hat{\mathbf{n}}_i.$$

$$E = 2j\beta \frac{M_s^4}{g_s} + \frac{2\pi^2 V^2}{|NV + 2j\beta|} \left( \frac{g_{YM}^2}{2\pi} \mathbf{E}^2 + \frac{2\pi}{g_{YM}^2} \mathbf{B}^2 \right) + \left| \mathbf{P} - \frac{4\pi^2 V^2}{|NV + 2j\beta|} \mathbf{E} \times \mathbf{B} \right|.$$

$$\mathcal{N} = 2 \implies \beta \equiv \beta_1 = \beta_2, \quad \beta_3 = 0.$$

cf. NCSYM [Ho, Morariu & Zumino, Hofman & Verlinde, Konechny & Schwarz, Pioline & Schwarz, ... 1998-9].



# Supergravity Dual

$$ds^2 = \frac{R^2}{r^2} K^{-\frac{1}{2}} \left[ dx^2 + dy^2 + dz^2 - \left( dt - \frac{4\pi N}{r^2} \vec{n}^T \zeta d\vec{n} \right)^2 \right] \\ + \frac{R^2}{r^2} K^{\frac{1}{2}} dr^2 + R^2 K^{\frac{1}{2}} d\Omega_5^2$$

$$C'_4 = \frac{\pi N}{r^4} K^{-1} dt \wedge dx \wedge dy \wedge dz \\ - \frac{\pi N}{g_s \alpha'^2 r^6} K^{-1} \vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz,$$

$$K \equiv 1 + \frac{16\pi^2 N^2}{r^6} \vec{n}^T \zeta^T \zeta \vec{n}$$

$$\vec{n} \in S^5, \quad d\Omega_5^2 = \sum_{I=1}^6 dn_I^2, \quad R^4 \equiv 4\pi g_s N \alpha'^2$$

## IR limit

$$ds^2 = \frac{R^2}{r^2} [dr^2 + dx^2 + dy^2 + dz^2 - dt^2] + R^2 d\Omega_5^2 \\ + \frac{8\pi N R^2}{r^4} \vec{n}^T \zeta d\vec{n} dt + \dots$$

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$$L_{N=4} \longrightarrow L_{N=4} + \zeta \mathcal{O} + \dots$$

# Short multiplets

| SYM Operator                                   | desc            | SUGRA                                               | dim     | spin                         | $SU(4)_R$   |
|------------------------------------------------|-----------------|-----------------------------------------------------|---------|------------------------------|-------------|
| $\text{Tr } X^k$                               | -               | $h_\alpha^\alpha \quad a_{\alpha\beta\gamma\delta}$ | $k$     | $(0, 0)$                     | $(0, k, 0)$ |
| ...                                            | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+ \bar{\lambda} \lambda X^k$     | $Q^3 \bar{Q}$   | $A_{\mu\alpha}$                                     | $k + 5$ | $(\frac{1}{2}, \frac{1}{2})$ | $(1, k, 1)$ |
| ...                                            | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+ F_-^2 X^k$                     | $Q^4 \bar{Q}^2$ | $A_{\mu\nu}$                                        | $k + 6$ | $(1, 0)$                     | $(0, k, 0)$ |
| ...                                            | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+ F_- \bar{\lambda} \lambda X^k$ | $Q^3 \bar{Q}^3$ | $h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$      | $k + 7$ | $(\frac{1}{2}, \frac{1}{2})$ | $(1, k, 1)$ |
| ...                                            | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+^2 F_-^2 X^k$                   | $Q^4 \bar{Q}^4$ | $h_\alpha^\alpha \quad a_{\alpha\beta\gamma\delta}$ | $k + 8$ | $(0, 0)$                     | $(0, k, 0)$ |

taken from [D'Hoker & Freedman, 2003]

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| ...                                        | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+ F_-^2 X^k$                 | $Q^4 \bar{Q}^2$ | $A_{\mu\nu}$                                        | $k + 6$ | $(1, 0)$                     | $(0, k, 0)$ |
| ...                                        | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+ F_- \bar{\lambda} \lambda$ | $Q^3 \bar{Q}^3$ | $h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$      | 7       | $(\frac{1}{2}, \frac{1}{2})$ | $(1, 0, 1)$ |
| ...                                        | ...             | ...                                                 | ...     | ...                          | ...         |
| $\text{Tr } F_+^2 F_-^2 X^k$               | $Q^4 \bar{Q}^4$ | $h_\alpha^\alpha \quad a_{\alpha\beta\gamma\delta}$ | $k + 8$ | $(0, 0)$                     | $(0, k, 0)$ |

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# IR-limit

$$L = L_{N=4} + \zeta \mathcal{O}^{(\Delta+4)} + \dots$$

$$\zeta \mathcal{O} \rightarrow \zeta_{AB}^\mu T_{\mu\nu} J^{\nu AB} + \zeta_{AB}^\mu \epsilon^{ABCDEF} \epsilon_{\mu\nu\sigma\tau} X^C \partial_\nu X^D \partial_\sigma X^E \partial_\tau X^F + \dots$$

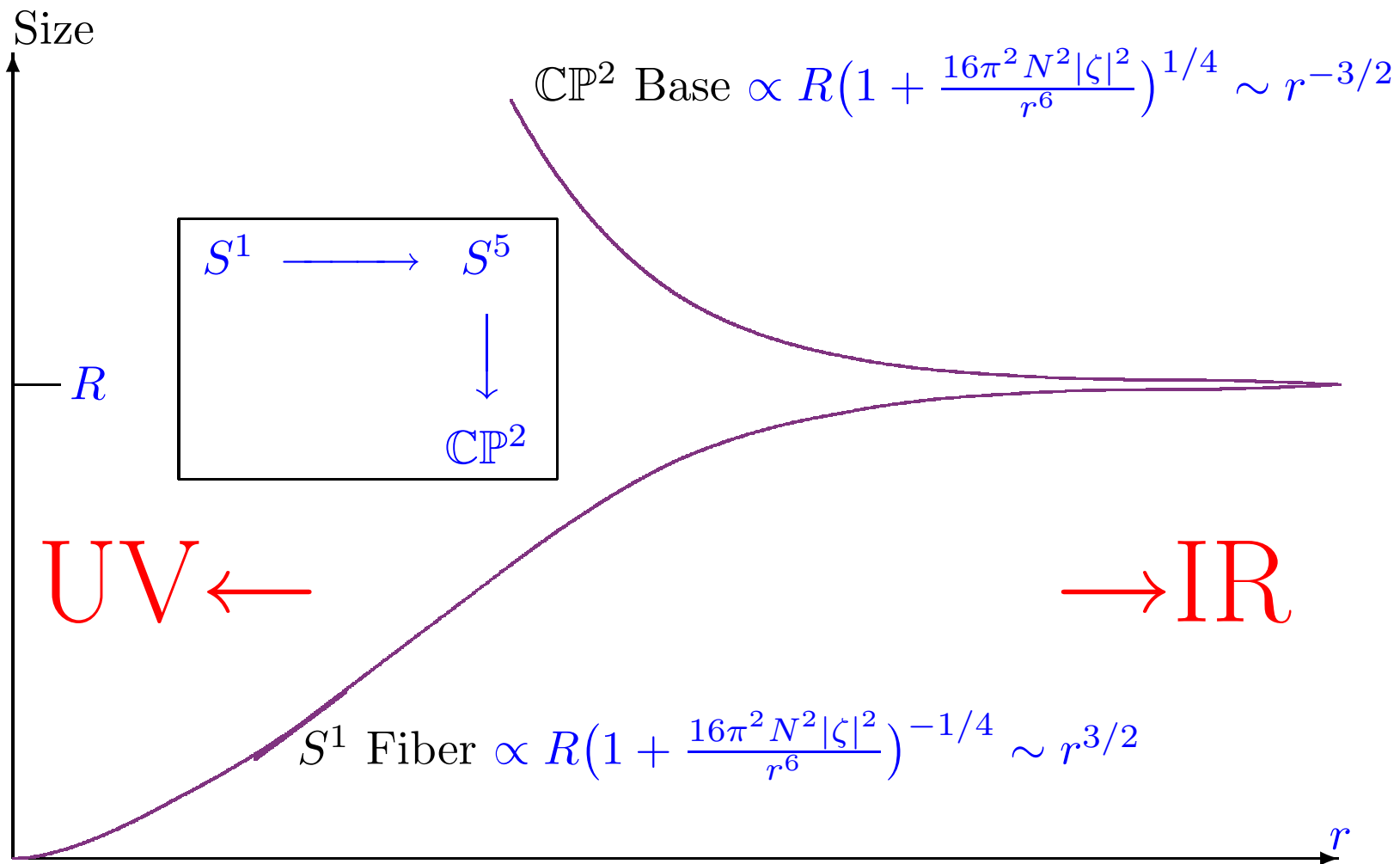
$$J^{\nu AB} = \text{R-current}$$

$$\begin{aligned} \int \zeta \mathcal{O} d^4x \rightarrow & \zeta^k_{i\alpha\dot{\alpha}} \int d^4x d^4\theta \{ -10 \bar{\Phi}_k \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}_n D_\alpha \Phi^n \\ & -2i \bar{\Phi}_k \Phi^i \bar{\Phi}_n \partial_{\alpha\dot{\alpha}} \Phi^n + 2i \bar{\Phi}_k \Phi^i \partial_{\alpha\dot{\alpha}} \bar{\Phi}_n \Phi^n \\ & -32 \bar{\Phi}_k \Phi^i W_\alpha \bar{W}_{\dot{\alpha}} + 4\sqrt{2} i \epsilon^{imn} \bar{\Phi}_m \bar{D}_{\dot{\alpha}} \bar{\Phi}_n W_\alpha \bar{\Phi}_k \\ & +4\sqrt{2} i \epsilon_{kmn} \Phi^i \Phi^m D_\alpha \Phi^n \bar{W}_{\dot{\alpha}} \} \end{aligned}$$

## UV limit (1)

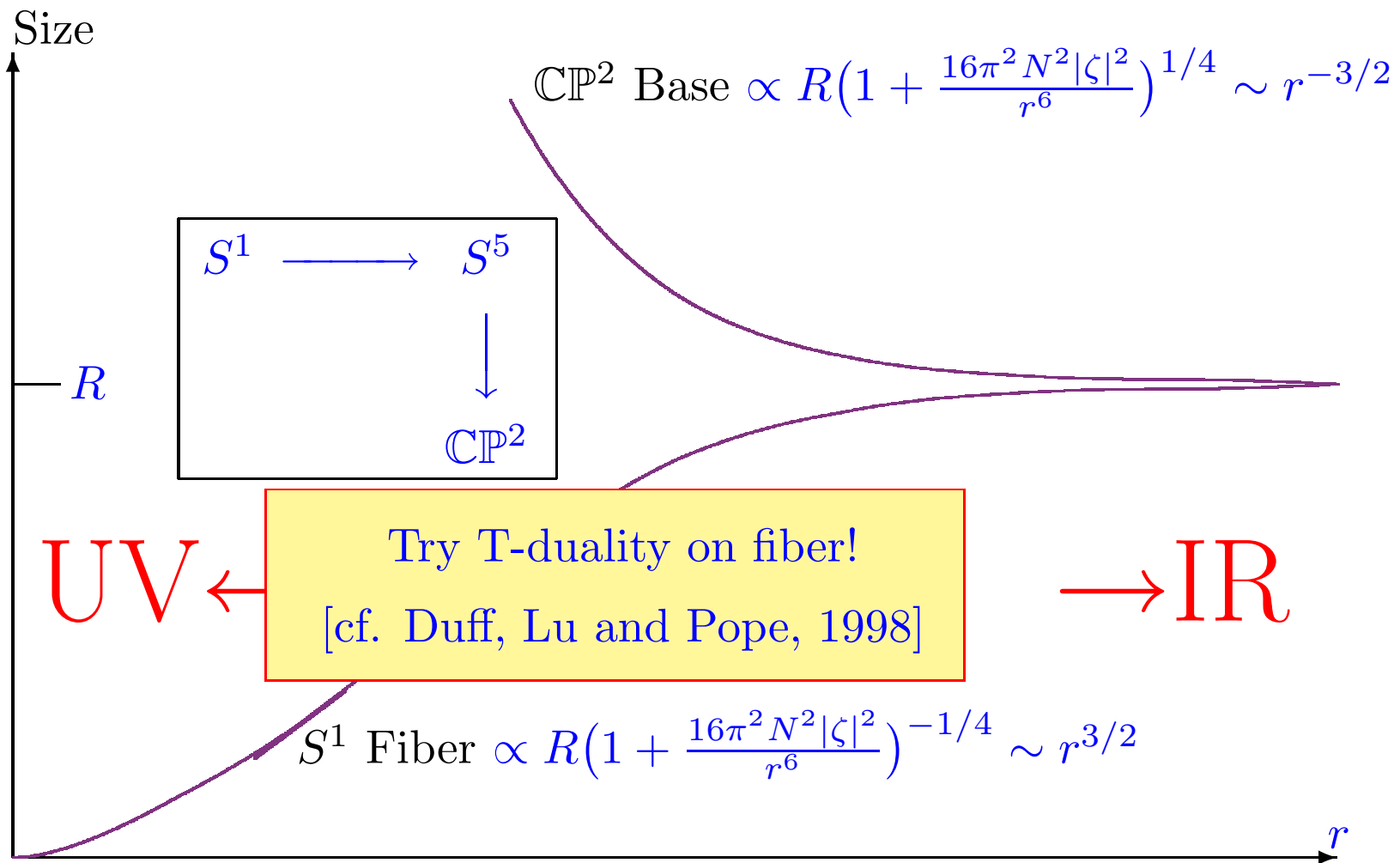
$$\begin{array}{ccc} S^1 & \longrightarrow & S^5 \\ & & \downarrow \\ & & \mathbb{CP}^2 \end{array}$$

# UV limit (1)





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## UV Limit (2)

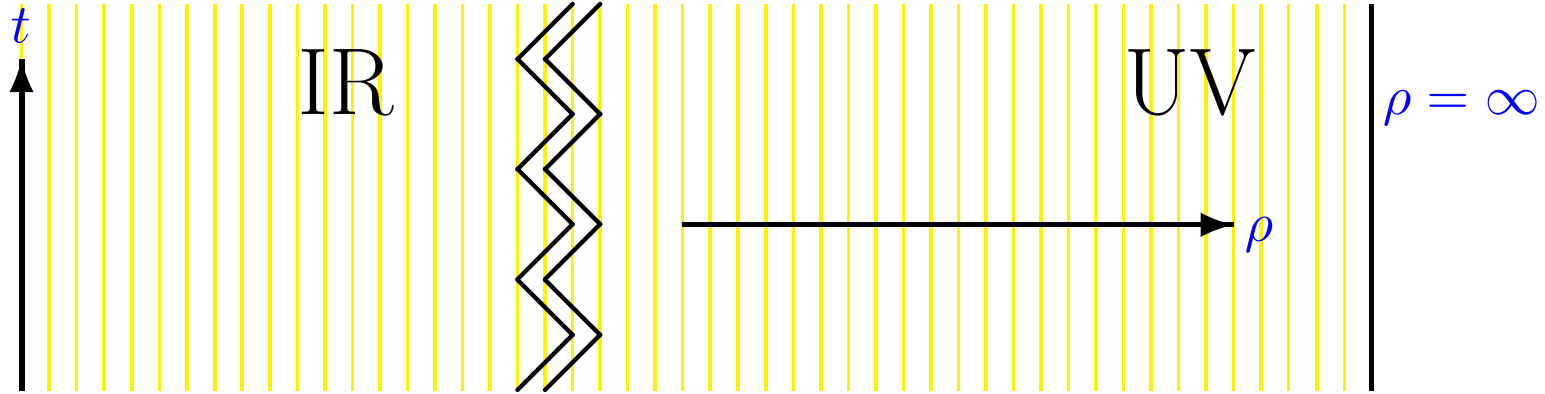
T-duality on fiber: Type-IIB  $\Rightarrow$  Type-0A  $\Rightarrow$  M.

$$\begin{aligned}\ell_P^{-2} ds_M^2 &= (4\pi N)^{-1/3} g_s^{-1} \rho^2 [\Delta^{-2/3} (dx^2 + dy^2 + dz^2) - \Delta^{1/3} dt^2] \\ &\quad + (4\pi N)^{2/3} \Delta^{1/3} \rho^{-2} d\rho^2 + (4\pi N)^{2/3} \Delta^{1/3} ds_B^2 \\ &\quad + (4\pi N)^{-1/3} \Delta^{1/3} (g_s^{-1} d\xi^2 + g_s d\eta^2),\end{aligned}$$

$$\begin{aligned}\ell_P^{-3} G_4 &= 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi \\ &\quad + \frac{2}{3} (4\pi N)^{-2} g_s^{-3} \beta d\left(\frac{\rho^6}{\Delta}\right) \wedge dx \wedge dy \wedge dz,\end{aligned}$$

$$\Delta \equiv 1 + (4\pi N)^{-1} g_s^{-3} \beta^2 \rho^6, \quad (\rho \propto 1/r)$$

# Extreme UV Limit



$$\begin{aligned} \ell_P^{-2} ds_M^2 &\approx -(4\pi N)^{-2/3} g_s^{-2} \beta^{2/3} \rho^4 dt^2 \\ &\quad + (4\pi N)^{1/3} g_s \beta^{-4/3} \rho^{-2} (dx^2 + dy^2 + dz^2) \\ &\quad + (4\pi N)^{1/3} g_s^{-1} \beta^{2/3} (d\rho^2 + \rho^2 ds_B^2) + (4\pi N)^{-2/3} \beta^{2/3} \rho^2 (g_s^{-2} d\xi^2 + d\eta^2), \end{aligned}$$

$$\ell_P^{-3} G_4 \approx 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi - 4g_s^3 \beta^{-3} \rho^{-7} d\rho \wedge dx \wedge dy \wedge dz.$$

## Extreme UV Limit (2)

$$\rho \rightarrow \infty$$

- Curvature  $\rightarrow 0$ .
- Geodesically complete.
- Redshift in frequency ( $g_{00} \rightarrow \infty$ )
- Blueshift in wavelength ( $g_{xx}, g_{yy}, g_{zz} \rightarrow 0$ )

# Degrees of Freedom

UV regime of PFT – holographically dual to weakly-coupled background (?)

Is the spectrum discrete?

# Degrees of Freedom

UV regime of PFT – holographically dual to weakly-coupled background (?)

Is the spectrum discrete?

For particle trajectories with fixed  $(E, \vec{p})$ :

$$\rho \leq \rho_{\max} \equiv (4\pi N)^{1/6} g_s^{1/2} \beta^{-1/3} \left( \frac{E}{|\vec{p}|} \right)^{1/3}$$

Suggests a discrete spectrum!

cf. Little-String-Theory. [Aharony & Berkooz & Kutasov & Seiberg, 1998]

# Effects of $TJ$ coupling

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[illegible]

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A horizontal sequence of green  $R$  nodes connected by a black line. The sequence starts with a  $R$  node, followed by a series of  $R$  nodes, then a central node labeled  $c > 1$ , followed by another series of  $R$  nodes, and ends with a  $R$  node. The entire sequence is enclosed in a purple rectangular box.

- In dense matter:

$$\Delta V_{\text{eff}} \approx \zeta \langle T^{00} \rangle Q^{(R)}, \quad Q^{(R)} \equiv \int J^0 d^3x = (\text{R-charge}).$$

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A horizontal sequence of 20 green  $R$  nodes connected by black lines. The 10th node from the left is labeled with a green  $c > 1$  above it. The sequence ends with a black arrow pointing to the right.

- In dense matter:

$$\Delta V_{\text{eff}} \approx \zeta \langle T^{00} \rangle Q^{(R)}, \quad Q^{(R)} \equiv \int J^0 d^3x = (\text{R-charge}).$$

## Trap R-charged particles?

## More on $c > 1$

Extreme UV:

$$\begin{aligned}\ell_P^{-2} ds_M^2 \approx & -(4\pi N)^{-2/3} g_s^{-2} \beta^{2/3} \rho^4 dt^2 \\ & + (4\pi N)^{1/3} g_s \beta^{-4/3} \rho^{-2} (dx^2 + dy^2 + dz^2) \\ & + (4\pi N)^{1/3} g_s^{-1} \beta^{2/3} d\rho^2 + \dots\end{aligned}$$

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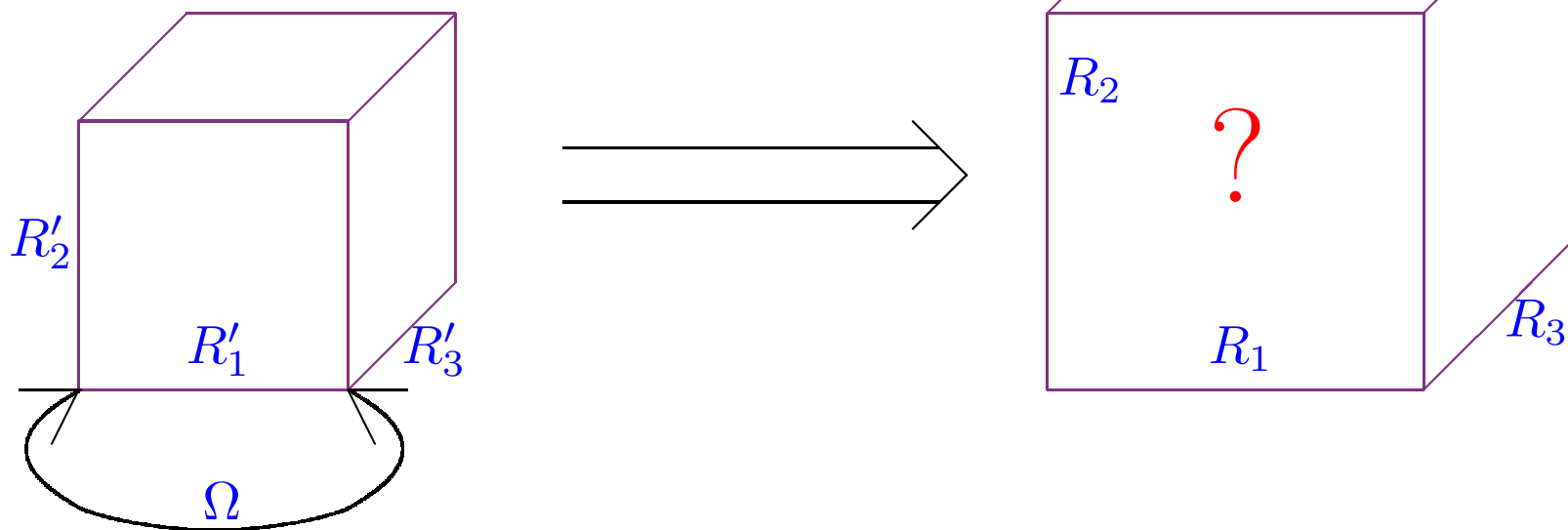
For a particle of mass  $m$ :

$$-\ell_P^2 m^2 \geq -(4\pi N)^{2/3} g_s^2 \beta^{-2/3} \frac{E^2}{\rho^4}.$$

Causality:  $v_{\max} = \sqrt{-\frac{g_{00}}{g_{xx}}} = (4\pi N)^{-1/2} g_s^{-3/2} \beta \rho^3.$

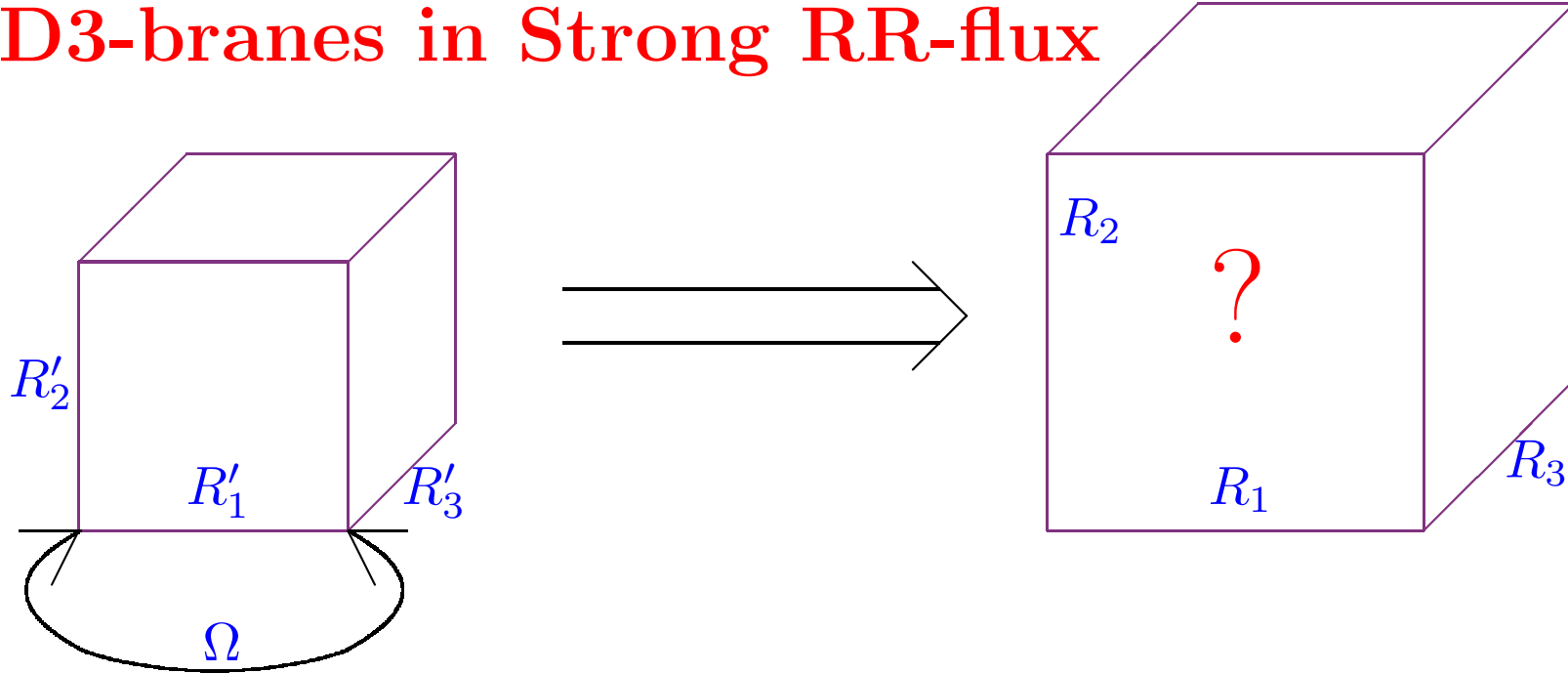
$$\Rightarrow \boxed{E \geq \ell_P m v^{2/3} \beta^{-1/3}}$$

## D3-branes in Strong RR-flux





# D3-branes in Strong RR-flux



$$f \equiv 1 + \frac{1}{g_s^2 \alpha'^4} r^2 \vec{n}^T \zeta^T \zeta \vec{n} \quad [\text{cf. Russo \& Tseytlin, 1995}]$$

$$ds^2 = f^{-\frac{1}{2}} \sum_{i=1}^3 dx_i^2 + f^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2 - dt^2) - \frac{f^{-\frac{1}{2}} r^4}{g_s^2 \alpha'^4} (\vec{n}^T \zeta d\vec{n})^2,$$

$$C'_4 = \frac{1}{4g_s^2 \alpha'^2} f^{-1} r^2 \vec{n}^T \zeta d\vec{n} \wedge dx_1 \wedge dx_2 \wedge dx_3$$

## Summary

- Lorentz violating  $SO(3)$ -invariant QFT.
- Decoupled from Gravity.
- Nonlocal.
- Characteristic  $TJ$  coupling.

## Open issues

- Fundamental formulation?
- Theories other than  $\mathcal{N} = 4$  SYM?
- Tests?
- Relation to Little-String-Theory?
- cf. M-theory duals of  $(p, q)$  5-branes.  
[Witten, 1997]

## More open issues

- Which objects have  $c > 1$ ?  
cf. nonrelativistic dispersion relations of NCYM solitons [Landsteiner & Lopez & Tytgat, Bak & Lee & Park, Hashimoto & Itzhaki, 2000].

## Even more open issues

- Relation to nonassociative structure and nongeometrical twists? [Ramgoolam, Medeiros & Ramgoolam, Bouwknegt & Hannabuss & Mathai, Shelton & Taylor & Wecht, Ellwood & Hashimoto, Nastase, ..., 2004-6]
- Lightlike limit? cf. Brane probes of limits of Melvin twists. [..., Hashimoto & Sethi, Robins & Sethi, Hashimoto & Thomas, Sheikh-Jabbari, Alishahiha & Safarzadeh & Yavartanoo, Gimon & Hashimoto & Hubeny & Lunin & Rangamani, Alishahiha & OG, Varadarajan & OG ]